### Example 11.3

The effect of fertilizer treatment on the density of *Bellis perennis* (daisy) at two locations on the University of Worcester campus, 2003

At the University of Worcester, the lawn and cricket pitch were divided and half of each plot was treated with fertilizer. Students extended their original study (Example 11.2) and, using stratified random sampling with quadrats, recorded the numbers of *Bellis perennis* in these four locations (cricket pitch with fertilizer, cricket pitch without fertilizer, lawn with fertilizer, and lawn without fertilizer) (Table 11.7). The data are organized into blocks and these will be referred to as blocks A–D.

### Table 11.7

The density of *Bellis perennis* (daisies) in two locations, with or without fertilizer treatments at the University of Worcester

<table>
<thead>
<tr>
<th>Fertilizer treatment</th>
<th>The density of <em>B. perennis</em> in each quadrat in two locations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lawn</td>
</tr>
<tr>
<td>Fertilizer</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>9 (A)</td>
</tr>
<tr>
<td></td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>12</td>
</tr>
<tr>
<td>Median</td>
<td>9</td>
</tr>
<tr>
<td>Range</td>
<td>4</td>
</tr>
<tr>
<td>No fertilizer</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>5 (C)</td>
</tr>
<tr>
<td></td>
<td>7</td>
</tr>
<tr>
<td>Median</td>
<td>5</td>
</tr>
<tr>
<td>Range</td>
<td>3</td>
</tr>
</tbody>
</table>
### BOX W11.2 How to calculate the Scheirer–Ray–Hare two-way non-parametric ANOVA

#### 1. General hypotheses to be tested

- **H₀**: There is no difference between the median density of *Bellis perennis* between the lawn and the cricket pitch.
- **H₁**: There is a difference between the median density of *Bellis perennis* between the lawn and the cricket pitch.
- **H₀**: There is no difference in the median density of *Bellis perennis* between the areas treated with fertilizer and those not so treated.
- **H₁**: There is a difference in the median density of *Bellis perennis* between the areas treated with fertilizer and those not so treated.
- **H₀**: There is no difference between the median density of *Bellis perennis* due to an interaction between location and fertilizer treatment.
- **H₁**: There is a difference between the median density of *Bellis perennis* due to an interaction between location and fertilizer treatment.

#### 2. Have the criteria for using this test been met?

Yes (11.7.2).

#### 3. How to work out \( F_{\text{calculated}} \)

**A. Calculate general terms**

1. First add together every observation in the complete data set (grand total).
   \[
   \sum x_T = 5 + 6.5 + 6.5 + \ldots + 9.5 + 12 + 12 = 136
   \]

2. Square each and every observation and add all these squared values together.
   \[
   \sum (x^2) = 5^2 + 6.5^2 + 6.5^2 + \ldots + 9.5^2 + 12^2 + 12^2 = 136
   \]
   \[
   = 25 + 42.25 + 42.25 + \ldots + 90.25 + 144 + 144 = 1492
   \]

3. Add all the observations in a sample (\( \sum x_s \)). This is the sample total. Do this for each sample (Table W11.2).
   \[
   \sum x_s = 27.5 + 10 = 37.5
   \]
   \[
   \sum x_s^2 = 27.5^2 + 10^2 + 41.5^2 = 756.25 + 3249 + 100 + 1722.25 = 5827.5
   \]
   \[
   \text{Divide this by the number of observations in a sample (} n_s \text{).}
   \]
   \[
   5827.5 / 4 = 1456.875.
   \]

4. For each column add all the observations together (\( \sum x_C \)) and square the total (\( \sum x_C^2 \)).
   \[
   \sum x_{CP} = 27.5 + 10 = 37.5, \quad \sum x_{CP}^2 = (37.5)^2 = 1406.25
   \]
   \[
   \sum x_L = 57 + 41.5 = 98.5, \quad \sum x_L^2 = (98.5)^2 = 9702.25
   \]

Add these squared column values together and divide this total by the number of observations in the column (\( n_c \)).

\[
(1406.25 + 9702.25) / 8 = 11108.5 / 8 = 1388.5625
\]

*(Continued)*
5. For each row add all the observations \((\sum x_r)\) and square the total \((\sum x_r)^2\).

\[
(\sum x_r) = 27.5 + 57 = 84.5, \quad (\sum x_r)^2 = 7140.25
\]

\[
(\sum x_{NF}) = 10 + 41.5 = 51.5, \quad (\sum x_{NF})^2 = 2652.25
\]

Add these squared values together and divide this value by the number of observations in the row \((n_r)\).

\[
\frac{(7140.25 + 2652.25)}{8} = 9792.5 / 8 = 1224.0625
\]

6. Square the result from step 1 and divide this by \(N\). \(N\) is the total number of observations in all the samples.

\[
(136)^2 / 16 = 18496 / 16 = 1156
\]

B. Calculate the Sums of Squares (SS)

- 7. \(SS_{\text{Total}} = \text{result from step 2} – \text{result from step 6}\)
  \[= 1492 − 1156 = 336.\]

- 8. \(SS_{\text{samples}} = \text{result from step 3} – \text{result from step 6}\)
  \[= 1456.875 − 1156 = 300.875.\]

- 9. \(SS_{\text{treatment 1 columns}} = \text{result from step 4} – \text{result from step 6}\)
  \[= 1388.5625 − 1156 = 232.5625.\]

- 10. \(SS_{\text{treatment 2 rows}} = \text{result from step 5} – \text{result from step 6}\)
  \[= 1224.0625 − 1156 = 68.0625.\]

- 11. \(SS_{\text{interaction}} = \text{result from step 8} – \text{result from step 9} – \text{result from step 10}\)
  \[= 300.875 − 232.5625 − 68.0625 = 0.25.\]

- 12. \(SS_{\text{within}} = \text{result from 7} – \text{result from 8}\)
  \[= 336 − 300.875 = 35.125.\]

C. Construct and complete an ANOVA calculation table

i. Draw an ANOVA table (Table W11.2).

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>SS</th>
<th>Degrees of freedom</th>
<th>(S^2)</th>
<th>(H_{\text{calculated}})</th>
<th>(H_{\text{critical}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable 1 (location)</td>
<td>(Step 9)</td>
<td>232.5625</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variable 2 (fertilizer treatment)</td>
<td>(Step 10)</td>
<td>68.0625</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interaction</td>
<td>(Step 11)</td>
<td>0.25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Within samples</td>
<td>(Step 12)</td>
<td>35.125</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ii. Calculate the degrees of freedom (\(\nu\)) (Table W11.3).

\[\nu_{\text{treatment 1 columns}} = \text{number of columns} − 1\]

\[\nu_{\text{treatment 2 rows}} = \text{number of rows} − 1\]

\[\nu_{\text{interaction}} = (\text{number of columns} − 1)(\text{number of rows} − 1)\]

\[\nu_{\text{within}} = N − (\text{number of rows} \times \text{number of columns})\]

\[\nu_{\text{total}} = N − 1\]

(Continued)
Table W11.3  ANOVA table for Scheirer–Ray–Hare analysis of data from Example 11.3 The effect of fertilizer treatment on the density of *Bellis perennis* (daisy) at two locations on the University of Worcester campus, 2003

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>SS</th>
<th>Degrees of freedom</th>
<th>$S^2$</th>
<th>$H_{\text{calculated}}$</th>
<th>$H_{\text{critical}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable 1 (location)</td>
<td>232.5625</td>
<td>2 − 1 = 1</td>
<td></td>
<td>232.5625/22.4 = 10.382</td>
<td>$p = 0.01$</td>
</tr>
<tr>
<td>Variable 2 (Fertilizer treatment)</td>
<td>68.0625</td>
<td>2 − 1 = 1</td>
<td></td>
<td>68.0625/22.4 = 3.0385</td>
<td>$p = 0.05$</td>
</tr>
<tr>
<td>Interaction</td>
<td>0.25</td>
<td>1 × 1 = 1</td>
<td></td>
<td>0.25/22.4 = 0.01116</td>
<td>$p = 0.05$</td>
</tr>
<tr>
<td>Within samples</td>
<td>35.125</td>
<td>16 − (2 × 2) = 12</td>
<td></td>
<td>35.125/12 = 2.927</td>
<td>$H_{\text{critical}} = 3.84$</td>
</tr>
<tr>
<td>Total</td>
<td>336.0</td>
<td>15</td>
<td></td>
<td>336/15 = 22.4</td>
<td></td>
</tr>
</tbody>
</table>

iii. At this point the analysis diverges from that described for a two-way parametric ANOVA in Chapter 10. First, a Mean Square (MS) is calculated by adding all the Sum of Squares (SS) values together and dividing the total by the total degrees of freedom (Table 11.15).

\[
\frac{232.5625 + 68.0625 + 0.25 + 35.125}{15} = \frac{336}{15} = 22.4
\]

iv. An $H$-value is calculated instead of an $F$-value as $SS/MS_{\text{total}}$ (Table 11.15).

Table 11.15  ANOVA table for Scheirer–Ray–Hare analysis of data from Example 11.3 The effect of fertilizer treatment on the density of *Bellis perennis* (daisy) at two locations on the University of Worcester campus, 2003

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>SS</th>
<th>$\nu$</th>
<th>MS</th>
<th>$H_{\text{calculated}}$</th>
<th>$H_{\text{critical}}$ for a given $p$ value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable 1 (location)</td>
<td>232.5625</td>
<td>1</td>
<td>22.4</td>
<td>232.5625/22.4 = 10.382</td>
<td>$p = 0.01$</td>
</tr>
<tr>
<td>Variable 2 (fertilizer)</td>
<td>68.0625</td>
<td>1</td>
<td>22.4</td>
<td>68.0625/22.4 = 3.0385</td>
<td>$p = 0.05$</td>
</tr>
<tr>
<td>Interaction</td>
<td>0.25</td>
<td>1</td>
<td>22.4</td>
<td>0.25/22.4 = 0.01116</td>
<td>$p = 0.05$</td>
</tr>
<tr>
<td>Within samples</td>
<td>35.125</td>
<td>12</td>
<td>336/15 = 22.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>336.0</td>
<td>15</td>
<td>336/15 = 22.4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. To find $H_{\text{critical}}$

The critical values for $H$ are found in a chi-squared table (Appendix d, Table D1) for the degrees of freedom associated with the Sum of Squares (SS) (Table 11.15). Since you are testing three pairs of hypotheses there will be three critical $H$-values to find (Table 11.15).

5. The rule

The rule is the same as that given for the parametric two-way ANOVA (Box 10.12) in that if $H_{\text{calculated}}$ is greater than $H_{\text{critical}}$, then you may reject the null hypothesis.
6. What does this mean in real terms?

In our example it can be seen that there is a significant difference ($H = 10.38, p = 0.01$) between the median density of daisies in the two locations but there is no significant difference for the fertilizer treatment or the interaction. The outcome is therefore the same as that for the non-parametric ANOVA described in Boxes 10.5.A–10.5.C.