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1. Deanne drives from her home to her office, which is 40 km away. What is her distance from the office after driving
   (a) 13 km?  
   (b) \(d\) km?

2. A prize of £2400 is shared equally among a team of football players. Find the amount each player gets if there are
   (a) 15 players,  
   (b) \(m\) players.

3. Simplify these expressions.
   (a) \(a \times 5 \times a\)  
   (b) \(3b \times 7c\)
   (c) \(5m \div 8n\)  
   (d) \(p \div 6 - q \times 11\)

4. Write these statements as algebraic expressions.
   (a) Subtract \(6a\) from the product of \(3b\) and \(c^3\).
   (b) Divide the sum of \(2x\) and \(5y\) by \(7z\)
5. A box of chocolates costs £6. A box of biscuits costs £4. Find the total price of
   (a) two boxes of chocolates and five boxes of biscuits,

   (b) $m$ boxes of chocolates and $n$ boxes of biscuits.

6. A mathematics book is 2 cm thick. A dictionary is 5 cm thick. Find the total height of a stack of
   (a) three mathematics books and four dictionaries,

   (b) $p$ mathematics books and $q$ dictionaries.

7. There are 500 grams of butter in a refrigerator. Find the remaining amount of butter if
   (a) 40 grams are used for cooking and 130 grams are used for baking,

   (b) $c$ grams are used for cooking and $b$ grams are used for baking.

8. There are 1500 ml of milk in a jug. Find the remaining amount of milk in the jug after
   (a) 8 cups of coffee are each topped up with 90 ml of milk,

   (b) $x$ cups of coffee are each topped up with $y$ ml of milk.

9. A metal bar is 156 cm long. A length of $L$ cm is cut from it. The remaining length is cut into $m$ equal
    pieces. Express the length of each piece in terms of $L$ and $m$. 
10. Kylie’s age is four years more than three times her daughter’s age. Find Kylie’s age if her daughter is
(a) 11 years old,
(b) \(n\) years old.

11. A bag weighs \(B\) grams. A squash ball weighs 24 grams. A tennis ball weighs 57 grams. Find the total mass of a bag containing
(a) three squash balls and four tennis balls,
(b) \(s\) squash balls and \(t\) tennis balls.

12. In his triathlon training, Paul swims \(s\) km.
(a) His running distance is 10 km more than twice his swimming distance. Express his running distance in terms of \(s\).
(b) His cycling distance is 5 km less than three times his running distance. Express his cycling distance in terms of \(s\).

13. In a compound of carbon, hydrogen and oxygen, there is one oxygen atom. The number of hydrogen atoms is two more than twice the number of carbon atoms. Find the total number of atoms in the compound if there are
(a) three carbon atoms,
(b) \(n\) carbon atoms.
5.2 Evaluation of Algebraic Expressions and Formulae

LEVEL 1

1. Work out the value of $3n + 7$ when
   (a) $n = 5$,  
   (b) $n = -8$.

2. Find the value of $\frac{24}{2p-3}$ when
   (a) $p = 0$,  
   (b) $p = 3$.

3. Given that $T = 10 + 3u - 4v$, find the value of $T$ if
   (a) $u = 2$ and $v = -1$,  
   (b) $u = -5$ and $v = -6$.

4. Given the formula $V = x^2h$, calculate the value of $V$ when $x = 3$ and $h = 5$.

5. Given the formula $S = 6p - 3q^2$, work out the value of $S$ if $p = 8$ and $q = -2$. 
6. Find the value of $3a(b - c)$ when
   
   (a) $a = 4$, $b = 7$ and $c = -1$,  
   
   (b) $a = -5$, $b = -3$ and $c = 6$.

7. Work out the value of $p^2 - q^2 + 3r^2$ when
   
   (a) $p = 2$, $q = 3$ and $r = 1$,  
   
   (b) $p = -4$, $q = -2$ and $r = 5$.

8. Given that $E = \frac{1}{2}m(v^2 - u^2)$, calculate the value of $E$ when
   
   (a) $m = 3$, $u = 5$ and $v = 1$,  
   
   (b) $m = 4$, $u = 3$, $v = 11$.

9. If $T = \frac{mn}{p^3 + 1}$, calculate the value of $T$ when
   
   (a) $m = 8$, $n = 35$ and $p = 3$,  
   
   (b) $m = -21$, $n = 10$ and $p = -2$.

10. If $y = kx^{n+1}$, work out the value of $y$ when
    
    (a) $k = 5$, $x = 4$ and $n = 2$,  
    
    (b) $k = -7$, $x = 6$ and $n = 0$. 

11. The price for \( n \) tickets for a concert is given by the expression £25\( n \).
   (a) Find the price for four tickets.

   (b) What does the number 25 in the expression 25\( n \) represent?

12. The power \( P \) watts of an electrical appliance is given by the formula \( P = VI \), where \( V \) volts is the voltage of the appliance and \( I \) amperes is the current through the appliance. Find the power of a heater if the voltage is 200 volts and the current is 7 amperes.

13. The parking space for a car is 12 m\(^2\) and the parking space for a motorcycle is 4 m\(^2\). A car park has \( m \) parking spaces for cars and \( n \) parking spaces for motorcycles.
   (a) Express the total parking space for cars and motorcycles in terms of \( m \) and \( n \).

   (b) The total area of the car park is 400 m\(^2\) and the area not used for parking spaces is \( A \) m\(^2\). Write a formula connecting \( m \), \( n \) and \( A \).

   (c) If \( m = 24 \) and \( n = 10 \), find the value of \( A \).

   (d) Explain why you would not want \( A \) to be 0.
5.3 Algebraic Expressions in the Real World

**LEVEL 1**

1. The bar model shows the prices of two books, where the price of book A is £2\(x\).

   ![Book A and Book B bar model]

   (a) Express \(y\) in terms of \(x\).

   (b) If \(x = 5\), find the price of book B.

2. The bar model shows two pieces of ribbon cut from a long ribbon.

   ![Long ribbon with pieces]

   (a) Express \(r\) in terms of \(L\), \(p\) and \(q\).

   (b) The ribbon is 127 cm long. Piece 1 is 48 cm long and piece 2 is 37 cm long. Find the length of the remaining piece.

3. The bar model shows the capacities of a pot and a bucket.

   ![Pot and bucket bar model]

   (a) Express the capacity of the bucket in terms of \(N\).

   (b) If \(N = 8\), find the capacity of the bucket.
4. The total mass of an apple and an orange is 450 grams. Let \( x \) grams be the mass of the apple.
   (a) Express the mass of the orange in terms of \( x \).

   (b) The mass of the apple is 230 grams. Find the mass of the orange.

5. Amir takes three seconds more to run 200 m than twice the time he takes to run 100 m. Let \( T \) seconds be his time taken to run 100 m.
   (a) Express his time taken to run 200 m in terms of \( T \).

   (b) If he takes 12 seconds to run 100 m, how many seconds does he take to run 200 m?

6. In a company, the number of female staff is six more than half of the number of male staff. Let \( m \) be the number of male staff and \( f \) be the number of female staff.
   (a) Express \( f \) in terms of \( m \).

   (b) If there are 18 male staff, find the number of female staff.

7. A tank contains 25 litres of water initially. A tap pours 12 litres of water into the tank every minute. Let \( V \) litres be the volume of water in the tank after \( t \) minutes.
   (a) Express \( V \) in terms of \( t \).

   (b) Find the volume of water in the tank after six minutes.
8. The process of making a frame involves cutting, welding and finishing. The time taken for welding is 10 minutes more than the time for cutting. The time for finishing is twice the time for welding. Let $x$ minutes be the time for cutting.

(a) Express the time for welding and the time for finishing in terms of $x$.

(b) Let $T$ minutes be the time required to make the frame. Find a formula connecting $x$ and $T$.

(c) If the time for cutting is 15 minutes, find the time required to make the frame.

9. Let $n$ be a date number in the first two weeks of a month.

(a) Express, in terms of $n$, the next two date numbers that are on the same day of the week as $n$.

(b) Let $S$ be the sum of $n$ and the two numbers in (a). Write a formula connecting $n$ and $S$.

(c) Hence find the value of $S$ when $n = 8$. 

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5.4 Simplification of Linear Expressions

**LEVEL 1**

1. Expand these expressions.
   (a) 2(4 + a)                        (b) 5(3b − 2)
   (c) (5c − 6)(4)                     (d) (1 + 7d)(6)

2. Expand these expressions.
   (a) −3(2m + 5)                      (b) −6(1 − 3n)
   (c) (4p − 7q)(−5)                   (d) (−3r − 8s)(−9)

3. Simplify these expressions.
   (a) 2(a + 3) + 3(a + 8)             (b) 5(2b + 1) + 4(b + 7)
   (c) (2c − d)(6) + 5(−4c + 3d)       (d) (3m − 2n)(7) + 3(2m − n)

4. Simplify these expressions.
   (a) 3(p + 5) − 2(4p + 6)            (b) 5(2q − 1) − 4(q − 3)
   (c) (3r − s)(−7) − 2(5r − 9s)       (d) (−2x + y)(6) − 8(−3x + 4y)
5. Expand these expressions.
   (a) \(2(3a + 4b + 1)\)
   (b) \(5c - 3d + 6\)
   (c) \(4(3g - 4h - 7k)\)
   (d) \((-2m - 5n + 3p)(7)\)

6. Expand these expressions.
   (a) \(-3(2q + 3r + 4)\)
   (b) \(-4(2s - 5t - 1)\)
   (c) \(-5(u - 2v + 3w)\)
   (d) \((-2x - 8y - 9z)(-2)\)

7. Expand these expressions.
   (a) \(a(3x + 2y)\)
   (b) \(-2b(5p - q)\)
   (c) \(c(4m - 5n)\)
   (d) \((4r - 5s)(-d)\)

8. Simplify these expressions.
   (a) \(3(2a + 3b + 1) + 4(3a - 4b - 5)\)
   (b) \((x - 5y)(-3) - (3x - 5y + z)(7)\)
9. The usual price of a shirt is £p. There is a discount of £10 on each shirt.
(a) Express the total sale price of \(n\) shirts in terms of \(n\) and \(p\) in expanded form, i.e. without using brackets.

(b) If the usual price of a shirt is £23, find the total sale price of five shirts.

10. The width of a chair is \((2x + 7)\) cm. Seven chairs can be placed along a wall, leaving a gap of 30 cm at one end. Let \(L\) cm be the length of the wall.
(a) Express \(L\) in terms of \(x\).

(b) If \(x = 25\), find the length of the wall.

11. The frame for a window consists of six rectangles of width \((3x + y)\) cm and height \((x + 2y)\) cm as shown.
(a) Express the total length of the material used to make the frame in terms of \(x\) and \(y\) in expanded form.

(b) If \(x = 8\) and \(y = 6\), find the total length of the material used to make the frame.
5.5 Proof

LEVEL 1

1. Determine whether or not each of these is a statement. If it is a statement, state whether it is a true statement or a false statement.
   (a) \(1 \times 2 \times 3 \times 4 = 24\)

   (b) \(1 \times 2 \times 3 \times 4\)

   (c) Volume of a cube = edge length \(\times\) edge length

   (d) Odd number + odd number = even number

2. Prove the statement ‘If \(n\) is an even number, then \(n - 1\) is an odd number.’

3. Prove the statement ‘If \(a\) is an odd number and \(b\) is an even number, then \(a - b\) is an odd number.’

4. Prove the statement ‘If \(m\) is a multiple of 6, then \(m\) is a multiple of 3.’
5. If \( n \) is a multiple of 8, show that \( \frac{n}{4} \) is an integer.

6. If \( m \) is an even number, show that \( m^2 \) is a multiple of 4.

7. Given that \( p \) is a multiple of 7 and \( q \) is a multiple of 3, show that the statement
   \[(a) \quad \text{‘} pq \text{ is a multiple of 21’ is true,}
   \]
   \[(b) \quad \text{‘} p - q \text{ is a multiple of 4’ is false.}
   \]

8. Jenny found that \( 1^2 = 1 \), \( 9^2 = 81 \) and \( 11^2 = 121 \). The last digit of all these square numbers is 1. She claims that the last digit of the square of an odd number is 1.
   \[(a) \quad \text{Has Jenny correctly proved her claim? Explain your answer.}
   \]
   \[(b) \quad \text{Show that Jenny’s claim is false.} \]
9. Harry worked out that $11 + 12 + 13 = 36$ and $17 + 18 + 19 = 54$. He claimed that ‘the sum of three consecutive integers must be even.’ Show that his claim is false.

10. If $n$ is an integer, show that
   (a) $6n + 3$ is a multiple of 3,
   (b) $6n + 4$ is an even number,
   (c) $6n + 5$ is an odd number.

11. If $m$ and $n$ are multiples of 5, show that the statement
   (a) ‘$m + n$ is a multiple of 10’ is false,
   (b) ‘$mn$ is a multiple of 25’ is true.

12. Lisa proposes that ‘if $p$ is a prime number, then $p + 1$ must not be a prime number.’
    Show that her proposal is not true.