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$E = mc^2$ is a well-known formula proposed by Albert Einstein, the physicist who developed the theory of relativity in the 1900s. It indicates that a huge amount of energy would be released if matter were converted into energy. Do you know what physical quantity each letter in the formula represents?

**LET’S LEARN TO**

- use letters to represent numbers or variables
- interpret algebraic notations
- evaluate algebraic expressions and formulae
- express real-world situations in algebraic terms
- simplify linear expressions
- prove some simple algebraic properties
1. **Index Notation**
   For example,
   \[8^2 = 8 \times 8 = 64\]
   \[8^3 = 8 \times 8 \times 8 = 512\]
   \[8^4 = 8 \times 8 \times 8 \times 8 = 4096\]

2. **Four Operations on Numbers**
   (a) **Addition**
   \[4 + 5 = 9\]
   \[7 + (−4) = 7 − 4 = 3\]
   \[(−6) + 2 = −6 + 2 = −4\]
   \[(−5) + (−4) = −5 − 4 = −9\]
   (b) **Subtraction**
   \[6 − 3 = 3\]
   \[8 − (−4) = 8 + 4 = 12\]
   \[(−7) − 8 = −7 − 8 = −15\]
   \[(−9) − (−2) = −9 + 2 = −7\]
   (c) **Multiplication**
   \[3 \times 4 = 12\]
   \[5 \times (−4) = −20\]
   \[(−5) \times 7 = 7 \times (−5) = −35\]
   \[(−2) \times (−9) = −[2 \times (−9)] = −[−18] = 18\]
   (d) **Division**
   \[7 \div 2 = \frac{7}{2}\]
   \[12 \div (−3) = \frac{12}{−3} = −4\]
   \[(−15) \div 6 = −\frac{15}{6} = −\frac{5}{2}\]
   \[(−8) \div (−24) = \frac{−8}{−24} = \frac{8}{24} = \frac{1}{3}\]

3. **Algebraic Expressions**
   (a) An algebraic expression involves numbers and letters that are connected with operation symbols. The letters can represent unknown quantities.
   For example, \(3ab − 2, 4x + 5y + 6, \frac{a}{x} − \frac{b}{7}\) are algebraic expressions.
   (b) When the values of the letters in an algebraic expression are known, you can use substitution to find the value of the expression.
   For example, when \(x = 3\) and \(y = −7\),
   \[4x + 5y + 6 = 4(3) + 5(−7) + 6\]
   \[= 12 − 35 + 6\]
   \[= −17\]

4. **Collecting Like Terms**
   You can simplify an algebraic expression by adding or subtracting like terms.
   For example, \(2x + 3x = 5x\)
   and \(5y − 2y = 3y\).
   This process is called **collecting like terms**.

5. **Addition and Subtraction of Linear Expressions**
   You can add and subtract linear expressions by removing brackets and collecting like terms.
   For example, \((2a + 3b) − (5a − 4b) = 2a + 3b − 5a + 4b\)
   \[= 2a − 5a + 3b + 4b\]
   \[= −3a + 7b\]
5.1 Use of Letters in Algebra

A Use of Letters

Mrs Hall bought 3 tins of cookies.

If each tin contains 10 cookies, the total number of cookies = \(3 \times 10 = 30\).

If each tin contains 25 cookies, the total number of cookies = \(3 \times 25 = 75\).

If each tin contains \(n\) cookies, the total number of cookies = \(3 \times n\).

In algebra, you can use letters (like \(a, b, c, \ldots, P, Q, R, \ldots\)) to represent numbers.

In the example above, \(3 \times n\) is an algebraic expression, where \(n\) is called a variable and it may represent 10, 25 or any positive integer.

B Basic Notation in Algebra

In algebra, there are rules for writing algebraic expressions. The operation symbols ‘+’, ‘−’, ‘×’, ‘÷’ and ‘=’ have the same meanings in algebra as in arithmetic.

Example 1

Tom is 12 years old now. Find his age after
(a) 4 years,       (b) 7 years,       (c) \(x\) years.

Solution

Present age = 12 years
(a) Age after 4 years = 12 + 4 = 16 years
(b) Age after 7 years = 12 + 7 = 19 years
(c) Age after \(x\) years = (12 + \(x\)) years

Note: The algebraic expression 12 + \(x\) is the sum of 12 and \(x\).

Try it! 1

Molly is 9 years old now. Find her age after
(a) 3 years,       (b) 10 years,       (c) \(t\) years.
Example 2

The width of a rectangle is half of its length. Find the width of the rectangle when its length is
(a) 20 cm, (b) 34 cm, (c) \( y \) cm.

**Solution**

From the information given, the width of the rectangle = \( \frac{1}{2} \times \) the length of the rectangle.

(a) When the length of the rectangle is 20 cm, the width of the rectangle = \( \frac{1}{2} \times 20 = 10 \) cm.

(b) When the length of the rectangle is 34 cm, the width of the rectangle = \( \frac{1}{2} \times 34 = 17 \) cm.

(c) When the length of the rectangle is \( y \) cm, the width of the rectangle = \( \frac{1}{2} \times y = \left( \frac{y}{2} \right) \) cm.

**Note:**
- \( \frac{y}{2} \) is an algebraic expression in variable \( y \).
- \( \frac{y}{2} = y + 2 \)

Try It! 2

A worker’s salary is one-third of a manager’s salary. Find the worker’s salary when the manager’s salary is
(a) £45 000, (b) £60 000, (c) £\( m \).

Example 3

In a bowling tournament, Devi’s score is 5 points less than 3 times Jeremy’s score. Find Devi’s score if Jeremy’s score is
(a) 48 points, (b) 90 points, (c) \( z \) points.

**Solution**

(a) If Jeremy scores 48 points, 3 times Jeremy’s score = \( 3 \times 48 = 144 \) points
5 points less than 3 times Jeremy’s score = \( (3 \times 48 - 5) \) points = 139 points

\[ \therefore \text{Devi’s score is } 139 \text{ points.} \]

(b) If Jeremy scores 90 points, 3 times Jeremy’s score = \( 3 \times 90 = 270 \) points
5 points less than 3 times Jeremy’s score = \( (3 \times 90 - 5) \) points = 265 points

\[ \therefore \text{Devi’s score is } 265 \text{ points.} \]
(c) If Jeremy scores $z$ points,
3 times Jeremy's score $= 3 \times z$
5 points less than 3 times Jeremy's score $= (3 \times z - 5)$ points
$\therefore$ Devi's score is $(3z - 5)$ points.

The travel time that Harry takes to reach school is 8 minutes more than twice the travel time that Rohanna takes. Find the travel time taken by Harry if the time taken by Rohanna is
(a) 15 minutes,  (b) 21 minutes,  (c) $t$ minutes.

Example 4

The price of a papaya is £2 and the price of a mango is £1. Find the total price of
(a) four papayas and five mangoes,
(b) $x$ papayas and $y$ mangoes.

Solution

(a) Price of 4 papayas $= £2 \times 4$
Price of 5 mangoes $= £1 \times 5$
$\therefore$ total price $= £(2 \times 4 + 1 \times 5)$
$= £13$

(b) Price of $x$ papayas $= £2 \times x$
Price of $y$ mangoes $= £1 \times y$
$\therefore$ total price $= £(2 \times x + 1 \times y)$
$= £(2x + y)$

Note: 
- $2x + y$ is an algebraic expression in two variables $x$ and $y$.
- $2 \times x$ or $x \times 2$ is written as $2x$, where the multiplication sign is omitted and the number is placed in front of the variable.
- $1 \times y$ or $y \times 1$ is written as $y$ but NOT $1y$.

Try It! 4

The price of a cup is £5 and the price of a plate is £12. Find the total price of
(a) three cups and two plates,
(b) $n$ cups and $m$ plates.
This table summarises and compares the arithmetic expressions and algebraic expressions for the basic operations.

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<th>Arithmetic expression</th>
<th>Statement</th>
<th>Algebraic expression</th>
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| **Addition** | • Add 7 to 5  
• Sum of 5 and 7  
• 5 plus 7 | $5 + 7 = 7 + 5$ | • Add $b$ to $a$  
• Sum of $a$ and $b$  
• $a$ plus $b$ | $a + b = b + a$ |
| **Subtraction** | • Subtract 11 from 18  
• Take away 11 from 18  
• 18 minus 11 | $18 - 11 \neq 11 - 18$ | • Subtract $c$ from $d$  
• Take away $c$ from $d$  
• $d$ minus $c$ | $d - c \neq c - d$ (if $c \neq d$) |
| **Multiplication** | • Multiply 2 by 6  
• 2 times 6  
• Double 6  
• Product of 2 and 6 | $2 \times 6 = 6 \times 2$ | • Multiply $g$ by $h$  
• $g$ times $h$  
• Product of $g$ and $h$ | $g \times h = h \times g = gh$ |
| **Division** | • Divide 24 by 3  
• Quotient of 24 divided by 3 | $\frac{24}{3} \neq 3 + 24$ | • Divide $x$ by $y$, where $y \neq 0$  
• Quotient of $x$ divided by $y$, where $y \neq 0$ | $\frac{x}{y} \neq \frac{x}{y}$ (if $y \neq x$ and $x$ and $y$ are non-zero) |

**Example 5**

Express these statements as algebraic expressions.
(a) Add $4p$ to $q$.
(b) Half of the sum of $x$ and $y$.
(c) Subtract $24k$ from the product of $e$ and $f$.
(d) Divide $(x - y)$ by 6.

**Solution**
(a) Add $4p$ to $q = q + 4p$
(b) Half of the sum of $x$ and $y = \frac{1}{2} \times (x + y) = \frac{1}{2}(x + y) = \frac{x + y}{2}$
(c) Subtract $24k$ from the product of $e$ and $f$
\[= e \times f - 24k \]
\[= ef - 24k \]
(d) Divide $(x - y)$ by 6 \[= (x - y) + 6 \]
\[= \frac{x - y}{6} \]

**Try it! 5**
Express these statements as algebraic expressions.
(a) The sum of $3p$ and $2q$.
(b) Half of the sum of $2p$ and $q$.
(c) Subtract $5k$ from the product of $a$ and $b$.
(d) Divide $(3 + z)$ by 5.

**Remark**
In this chapter, assume that a variable is non-zero when it appears as the denominator.

**Remark**
As the whole bracket $(x + y)$ is multiplied by $\frac{1}{2}$, the whole term should appear as the numerator when the answer is written as a fraction.

**Discuss**
When $x = 6$ and $y = 8$, what are the values of $\frac{x + y}{2}$, $x + \frac{y}{2}$ and $\frac{x + y}{2}$?
Index Notation

In arithmetic, \( 7 \times 7 \times 7 = 7^3 \) and \( 5 \times 5 \times 5 \times 5 = 5^4 \). In algebra, the notation is similar.

- \( a \times a = a^2 \), read as \( a \) squared;
- \( b \times b \times b = b^3 \), read as \( b \) cubed;
- \( c \times c \times c \times c = c^4 \), read as \( c \) to the power of four;
- \( d \times d \times d \times d \times d = d^5 \), read as \( d \) to the power of five.

Note: Since \( a^2 = a \times a \) and \( 2a = a + a \),
- \( a^2 \neq 2a \).

Since \( b^3 = b \times b \times b \) and \( 3b = b + b + b \),
- \( b^3 \neq 3b \).

Similarly, \( c^4 \neq 4c \) and \( d^5 \neq 5d \).

Simplify these expressions.
(a) \( b \times 1 \times c \times b \)  
(b) \( 7x \times 2y \times x \times x \)  
(c) \( p \div q \times 3 \)

**Solution**
(a) \( b \times 1 \times c \times b = 1 \times b \times b \times c = b^2 c \)  
(b) \( 7x \times 2y \times x \times x = 7 \times x \times 2 \times y \times x \times x = 14x^3y \)  
(c) \( p \div q \times 3 = \frac{p}{q} \times 3 = \frac{3p}{q} \)

Try It!
(a) \( d \times e \times 1 \times e \)  
(b) \( 4m \times 5n \times m \times m \)  
(c) \( 7t + y \times 2 \)

Distinguish between \( 7x^2 \) and \( (7x)^2 \).

**Solution**
\[
\begin{align*}
7x^2 &= 7 \times x \times x \\
(7x)^2 &= 7x \times 7x \\
&= 7 \times x \times 7 \times x \\
&= 49x^2 \\
\therefore (7x)^2 &= 7 \times x \times x \\
&= 49x^2 \\
\therefore (7x)^2 &= 7 \times x \times x \\
&= 49x^2 \\
\end{align*}
\]

Try It!
Distinguish between \( 3z^2 \) and \( (3z)^2 \).
Express these statements as algebraic expressions.

(a) Subtract \( n \) from the quotient of \( m \) divided by \( t \).

(b) Add the product of \( a \) and \( b \) to the square of \( c \).

Solution

(a) Quotient of \( m \) divided by \( t = \frac{m}{t} \)

The required expression is \( \frac{m}{t} - n \).

(b) Product of \( a \) and \( b = ab \)

Square of \( c = c^2 \)

Add \( ab \) to \( c^2 = c^2 + ab \)

The required expression is \( c^2 + ab \).

Note: The answer in (a) cannot be written as \( \frac{n - m}{t} \).

However, the answer in (b) can be written as \( ab + c^2 \).

Express these statements as algebraic expressions.

(a) Multiply the sum of \( c \) and \( d \) by \( u \).

(b) Subtract the quotient of \( a \) divided by \( b \) from the cube of \( v \).

**EXERCISE 5.1**

**LEVEL 1**

1. A man’s monthly income is £3600. Find his savings when his expenditure is
   (a) £2500,         (b) £3400,
   (c) £x.

2. Mr Lin is 5 cm taller than his wife. Find Mr Lin’s height if the height of his wife is
   (a) 160 cm,         (b) 168 cm,
   (c) \( h \) cm.

3. Find the number of days in
   (a) 5 weeks,         (b) 12 weeks,
   (c) \( n \) weeks.

4. John’s score is \( \frac{3}{4} \) of Lucy’s score. Find John’s score if Lucy’s score is
   (a) 76,         (b) 92,
   (c) \( s \).

5. A scholarship of £6000 is shared equally among some students. Find the share for each student if there are
   (a) three students,         (b) five students,
   (c) \( n \) students.

6. There are 63 pills in a container. A patient takes two pills every day. Find the remaining number of pills after
   (a) one day,         (b) three days,
   (c) \( m \) days.

7. The number of Singapore stamps in a stamp album is 23 more than twice the number of British stamps. Find the number of Singapore stamps in the album when there are
   (a) 10 British stamps,         (b) \( p \) British stamps.
8. The time taken by an aeroplane to travel from City A to City B is 20 minutes more than one-eighth of the time taken by a train. Find the time taken in hours by the aeroplane if the time taken by the train is
   (a) 12 hours,  (b) \( t \) hours.

9. Simplify these expressions.
   (a) \( a \times 5 \)  (b) \( b \times b \times 4 \)
   (c) \( 2c + d \)  (d) \( e + f \times g \)
   (e) \( 6h \times 3k \)  (f) \( 9m + 9n \)
   (g) \( 3p \times p \times 5p \)  (h) \( 4q \times 5r \times q \)
   (i) \( s \div 6 + 1 \times t \)  (j) \( u + 6v \div 9w \)

10. Express these statements as algebraic expressions.
    (a) Add 5 to the product of \( h \) and \( k \).
    (b) Subtract 3\( m \) from the quotient of \( n \) divided by \( p \).
    (c) Divide the sum of 2\( t \) and 3\( u \) by \( v \).
    (d) Multiply the product of \( y \) and \( z \) by 7\( y \).

LEVEL 2

11. The price of a pineapple is £2 and the price of a watermelon is £5. Find the total price of
    (a) five pineapples and one watermelon,  (b) \( x \) pineapples and \( y \) watermelons.

12. The mass of Book A is 759 g and the mass of Book B is 400 g. Find the total mass of
    (a) four copies of Book A and three copies of Book B,  (b) \( p \) copies of Book A and \( q \) copies of Book B.

13. Miss Brooks has £500 in her pocket. Find the amount left if she spends
    (a) £120 on a skirt and £30 on her dinner,  (b) \( £ k \) on a skirt and \( £ d \) on her dinner.

14. Mr Taylor goes to a movie with his family. He has two free tickets. Find the amount he has to pay for the tickets if
    (a) there are five family members and each ticket costs £11,  (b) there are \( n \) family members and each ticket costs £\( p \).

15. There are four rotten eggs in a carton. The good eggs in the carton are shared among some households. Find the number of eggs that each household gets if there are
    (a) 100 eggs in the carton and 12 households,  (b) \( n \) eggs in the carton and \( m \) households.

LEVEL 3

16. A car salesman has a basic salary of £2000 a month. For every car sold, he gets a commission of £800. Let \( n \) be the number of cars that he sells in a month.
    (a) Find his monthly salary when \( n = 18 \).
    (b) Express his monthly salary in terms of \( n \).

17. A grocer has \( p \) cartons of oranges. Each carton contains \( q \) oranges and \( r \) of them are rotten. Express, in terms of \( p \), \( q \) and \( r \),
    (a) the total number of oranges in all the cartons,
    (b) the total number of good oranges.

18. A metal bar is \( L \) cm long. It is melted and recast into a wire of length \( k \) cm longer than \( n \) times the length of the bar. The wire is then cut into \( m \) equal pieces. Express, in terms of \( k \), \( L \), \( m \) and \( n \),
    (a) the wire,  (b) each piece.

19. Mia is \( x \) years old. Dario, her brother, is 10 years older than her. Their mother is three times as old as Mia. Their father is twice as old as Dario. Write down the expression, in terms of \( x \), for
    (a) Dario’s age,  (b) their mother’s age,
    (c) their father’s age.

20. A test paper consists of Section A and Section B with a number of questions in each section. Copy and complete the tables to find the total score of the test paper in each case.

<table>
<thead>
<tr>
<th></th>
<th>Score</th>
</tr>
</thead>
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<tr>
<td>Section A</td>
<td>8 questions with each question carrying 5 marks</td>
</tr>
<tr>
<td>Section B</td>
<td>4 questions with each question carrying 15 marks</td>
</tr>
<tr>
<td><strong>Total score</strong></td>
<td></td>
</tr>
</tbody>
</table>
### Evaluation of Algebraic Expressions

**A** Evaluation of Algebraic Expressions

As mentioned in the Section 5.1 introduction, if each tin contains $n$ cookies, then, in three tins, the total number of cookies = $3n$.

If there are 17 cookies in a tin, the total number of cookies can be evaluated by putting $n = 17$ into the above expression.

Thus, the total number of cookies = $3(17) = 51$.

Similarly, if there are 30 cookies in a tin, then $n = 30$ and the total number of cookies = $3(30) = 90$.

This process of replacing each variable with its value to find the actual value of an algebraic expression is called **substitution**.
The monthly salary of a salesperson is \( £(2000 + 0.05m) \), where \( m \) is the number of goods sold in that month. If she sold 10,000 goods in January, find her salary in that month.

**Solution**

Given that the monthly salary = \( £(2000 + 0.05m) \), when \( m = 10,000 \), her salary in January = \( £[2000 + 0.05(10,000)] \)
\[
= £[2000 + 500] \\
= £2500
\]

When \( n \) people share a reward of £36,000, each person gets \( £\frac{36000}{n} \). Find the amount each person gets if it is shared among four people.

### Example 9

When \( x = 5 \) and \( y = 2 \), find the values of these expressions.

(a) \( 3x + 4y - 7 \)  
(b) \( x(y - x) \)  
(c) \( \frac{2x - 3y}{6} \)

**Solution**

When \( x = 5 \) and \( y = 2 \),

(a) \( 3x + 4y - 7 = 3(5) + 4(2) - 7 \)
\[
= 15 + 8 - 7 \\
= 16
\]

(b) \( x(y - x) = 5(2 - 5) \)
\[
= 5(-3) \\
= -15
\]

(c) \( \frac{2x - 3y}{6} = \frac{2(5) - 3(2)}{6} \)
\[
= \frac{10 - 6}{6} \\
= \frac{4}{6} \\
= \frac{2}{3}
\]

### Try It! 9

When \( x = 4 \) and \( y = 3 \), find the values of these expressions.

(a) \( 2x + 7y - 10 \)  
(b) \( y(x - 2y) \)  
(c) \( \frac{5y - 3x}{12} \)

### Example 10

When \( p = \frac{1}{2} \), \( q = 5 \) and \( r = -2 \), evaluate these expressions.

(a) \( 6pq + 7r \)  
(b) \( 4p(q^2 - 3r) \)  
(c) \( \frac{q - pr}{qr} \)

**Solution**

When \( p = \frac{1}{2} \), \( q = 5 \) and \( r = -2 \),

(a) \( 6pq + 7r = 6\left(\frac{1}{2}\right)(5) + 7(-2) \)
\[
= 15 - 14 \\
= 1
\]
B  Formulae

You have learnt that the area of a rectangle is given by

\[
\text{area} = \text{length} \times \text{width}
\]

Let \( l \) and \( w \) represent the length and the width respectively in cm, and let \( A \) represent the area in cm\(^2\), then the relationship above can be expressed as

\[
A = lw
\]

This equality connecting two or more variables is called a formula. When the values of \( l \) and \( w \) are known, you can find the value of \( A \) in the formula by substitution.

Example 12

The area, \( A \) cm\(^2\), of a rectangle of length \( l \) cm and width \( w \) cm is given by the formula \( A = lw \). Find the area of a rectangle which has a length of 12 cm and a width of 7 cm.

\[
A = lw
\]

When \( l = 12 \) and \( w = 7 \),

\[
A = 12 \times 7 = 84
\]

\( \therefore \) the area of the rectangle is 84 cm\(^2\).

Try It! 12

The perimeter, \( P \) metres, of a rectangular field of \( l \) metres by \( w \) metres is given by the formula \( P = 2l + 2w \). Find the perimeter of a rectangular field with dimensions 60 m by 45 m.

\[
P = 2l + 2w
\]

\[
P = 2 \times 60 + 2 \times 45 = 120 + 90 = 210
\]

\( \therefore \) the perimeter of the field is 210 m.
Given the formula $S = \frac{1}{2} n(n + 1)$, find the value of $S$ when

(a) $n = 10, \quad S = \frac{1}{2} \times 10 \times (10 + 1) = 55$

(b) $n = 17, \quad S = \frac{1}{2} \times 17 \times (17 + 1) = 153$

Note: The formula $S = \frac{1}{2} n(n + 1)$ relates the variables $n$ and $S$. When the value of $n$ changes, the value of $S$ also changes.

Given the formula $D = \frac{1}{2} n(n - 3)$, find the value of $D$ when

(a) $n = 5, \quad D = \frac{1}{2} \times 5 \times (5 - 3) = 5$

(b) $n = 12, \quad D = \frac{1}{2} \times 12 \times (12 - 3) = 45$

If $s = ut + \frac{1}{2} at^2$, find the value of $s$ when $u = 20, \ t = 3$ and $a = 10$.

$s = ut + \frac{1}{2} at^2$

When $u = 20, \ t = 3$ and $a = 10, \quad s = 20 \times 3 + \frac{1}{2} \times 10 \times 3^2 = 60 + 45 = 105$

Note: The formula $s = ut + \frac{1}{2} at^2$ relates the variables $a, u, s$ and $t$.

If $s = \frac{v^2 - u^2}{2a}$, find the value of $s$ when $u = 3, \ v = 12$ and $a = 6$. 

$s = \frac{12^2 - 3^2}{2 \times 6} = \frac{144 - 9}{12} = \frac{135}{12} = 11.25$
**LEVEL 1**

1. Find the value of $3x - 1$ when
   (a) $x = 1$,   (b) $x = \frac{4}{5}$,
   (c) $x = -1$,   (d) $x = 0$.
2. Find the value of $25 - 4y$ when
   (a) $y = 0$,   (b) $y = \frac{5}{2}$,
   (c) $y = -3$,   (d) $y = -2$.
3. Find the value of $3a + 4b$ when
   (a) $a = 7$ and $b = 5$,   (b) $a = \frac{1}{2}$ and $b = \frac{1}{2}$,
   (c) $a = -1$ and $b = 2$,   (d) $a = -3$ and $b = -1$.
4. Find the value of $5 - x - y$ when
   (a) $x = -3$ and $y = 4$,   (b) $x = 2$ and $y = -5$.
5. Find the value of $3p - 2q + 7$ when
   (a) $p = -1$ and $q = 4$,   (b) $p = \frac{1}{2}$ and $q = -\frac{1}{2}$.
6. Find the value of $\frac{2a + b}{3}$ when
   (a) $a = -1$ and $b = -1$,   (b) $a = \frac{1}{2}$ and $b = -4$.
7. Find the value of $x(3x - y) + 2y$ when
   (a) $x = 4$ and $y = -1$,   (b) $x = 0$ and $y = 3$.
8. Find the value of $(2m + n)(m - n + 1)$ when
   (a) $m = 5$ and $n = 3$,   (b) $m = 11$ and $n = 6$,
   (c) $m = 0$ and $n = -3$,   (d) $m = -1$ and $n = -2$.
9. Find the value of $x$ in each of these formulae.
   (a) $x = 2a + 1$; given that $a = 3$
   (b) $x = b^2 - 2b$; given that $b = 5$
   (c) $x = 4x(c + 5)$; given that $c = 8$
   (d) $x = \frac{3d + 4}{5d - 2}$; given that $d = 1$

10. Find the value of $y$ in each of these formulae.
    (a) $y = m(3 + 2n)$; given that
        (i) $m = 4$ and $n = 6$,
        (ii) $m = 2$ and $n = 1$
    (b) $y = \frac{p^2 - q^2}{2}$; given that
        (i) $p = 7$ and $q = 3$,
        (ii) $p = 1$ and $q = 2$.  
    (c) $y = \frac{r + x}{s}$; given that
        (i) $r = 6$ and $s = 3$,
        (ii) $r = 1$ and $s = -2$.
    (d) $y = \frac{(4a)^2}{3x - 5x}$; given that
        (i) $t = 4$ and $x = 2$,
        (ii) $t = 2$ and $x = -1$.

**LEVEL 2**

11. Find the value of $b^2 - 4ac$ when
    (a) $a = 1$, $b = 5$ and $c = 3$,
    (b) $a = 2$, $b = 7$ and $c = \frac{3}{4}$,
    (c) $a = -1$, $b = -2$ and $c = 5$.

12. Find the value of $p^2q - (2q)^2$ when
    (a) $p = 8$ and $q = 3$,   (b) $p = \frac{9}{2}$ and $q = 4$,
    (c) $p = -3$ and $q = -2$.

13. Find the value of $a^2 + 3b^2 - c^2$ when
    (a) $a = 1$, $b = 2$ and $c = 3$,
    (b) $a = 5$, $b = 4$ and $c = 6$,
    (c) $a = -1$, $b = -2$ and $c = -3$.

14. Find the value of \(\frac{x^2 - 2xy}{z^2 - 4y^2}\) when
    (a) $x = 9$, $y = 3$ and $z = 6$,
    (b) $x = 8$, $y = 1$ and $z = 2$,
    (c) $x = 6$, $y = -2$ and $z = 3$.

15. If $T = \frac{v^2 - u^2}{2w}$, find the value of $T$ when $u = 0$,
    $v = 5$ and $w = 4$.

16. If $z = kt^n - 1$, find the value of $z$ when $k = -2$, $t = 5$ and $n = 4$.  


17. The daily wage of a worker is given by the expression £15\(t\), where \(t\) is the number of working hours.
   (a) Find the daily wage of the worker when \(t = 8\).
   (b) What do you think the number 15 in the expression stands for?

18. The price of a square frame of side length \(x\) cm is £\(\left(\frac{1}{4}x^2 + 5x\right)\). Find the price of a square frame of side length
   (a) 10 cm, (b) 20 cm.

19. The formula to convert a temperature of \(H^\circ\text{F}\) (Fahrenheit) to \(S^\circ\text{C}\) (Celsius) is given by
   \[S = \frac{5}{9}(H - 32)\].
   (a) The boiling point of water is 212\(^\circ\text{F}\). Express this in \(^\circ\text{C}\).
   (b) The temperature in Glasgow on a certain day was 77\(^\circ\text{F}\). Express this temperature in \(^\circ\text{C}\).

20. The admission fee, £\(F\), to a theme park for a family of \(x\) adults and \(y\) children is given by
   \[F = 25x + 20y\].
   (a) Find the total admission fee to the theme park for a family of two adults and three children.
   (b) What do you think the numbers 25 and 20 stand for in the formula?

21. The body mass index (BMI), \(I\), of a person is given by the formula \(I = \frac{m}{h^2}\), where \(m\) is the mass in kg and \(h\) is the height in metres of the person.
   Judy is 1.63 m tall and her mass is 51 kg. Find her BMI, giving your answer to 3 significant figures.

22. The three sides of a triangle are \(a\) cm, \(b\) cm and \(c\) cm respectively. The perimeter of the triangle is \(P\) cm.

23. The price, £\(P\), for a birthday cake of radius \(r\) cm and height \(h\) cm is given by the formula
   \[P = \frac{1}{25}r^2h\].
   (a) Find the price for a cake of radius 10 cm and height 8 cm.
   (b) If the height of the cake in (a) is increased to 10 cm, what would the increase in price be?
5.3 Algebraic Expressions in the Real World

You can use algebraic expressions and formulae to express the relationship between two or more quantities in your daily life. Here are some examples.

Example 15

The monthly salary of a manager is four times as much as an administrator.

(a) Let $c$ be the monthly salary of the administrator. Express the monthly salary of the manager in terms of $c$.

(b) Let $m$ be the monthly salary of the manager. Write a formula connecting $c$ and $m$.

Solution

(a) Monthly salary of the manager

\[ \text{}= \mathbf{E}(4 \times c) \]

\[ = \mathbf{4}c \]

(b) From (a), you have $m = 4c$.

Example 16

Crest Secondary School takes in 200 students a year. There are $n$ students in each class.

(a) Express, in terms of $n$, the number of classes in the school in the first year.

(b) Given that $n = 25$, find the number of classes in the first year.

(c) Let $m$ be the total number of students enrolled in $t$ years. Write a formula connecting $m$ and $t$.

(d) Find the value of $m$ when $t = 4$.

Solution

(a) Since the number of classes $= \frac{\text{the total number of students in the year}}{\text{the number of students in each class}}$,

\[ \text{the number of classes in the first year} = \frac{200}{n} \] .... (1)

(b) Substituting $n = 25$ into (1),

\[ \text{the number of classes} = \frac{200}{25} \]

\[ = 8 \]

(c) The total number of students enrolled

\[ = \text{the number of students in one year} \times \text{the number of years} \]

\[ \therefore m = 200 \times t \]

\[ m = 200t \] .... (2)
(d) Substituting \( t = 4 \) into (2),
\[
m = 200 \times 4
= 800
\]

In a department store, the price of a cast iron casserole dish is £126 while the price of a stainless steel pot is £s.

(a) Express the number of stainless steel pots that can be bought for £126 in terms of \( s \).

(b) If \( s = 18 \), find the number of stainless steel pots that can be bought for £126.

(c) Mrs Arnot bought \( n \) cast iron casserole dishes as rewards for her staff. Let £\( m \) be the amount she paid for the dishes. Write a formula connecting \( m \) and \( n \).

(d) Find the value of \( m \) when \( n = 7 \).

Hannah has twice as much money in her savings account as Marta. Neela has £24 less in her account than Hannah. Let £\( h \), £\( m \) and £\( n \) be the balances in Hannah’s, Marta’s and Neela’s accounts respectively.

(a) Express \( h \) and \( n \) in terms of \( m \).

(b) If Marta’s account balance is £65, find the account balances of Hannah and Neela.

Solution

(a) Twice Marta’s balance = £(2 \times m)
\[
= £2m
\]
∴ \( h = 2m \) ...... (1)

Neela’s balance = £24 less than Hannah’s balance
\[
= £h - £24
= £(h - 24)
= £(2m - 24) \text{ From (1), } h = 2m.
∴ \( n = 2m - 24 \) ...... (2)

(b) Substituting \( m = 65 \) into (1),
\[
h = 2 \times 65
= 130
\]

Substituting \( m = 65 \) into (2),
\[
n = 2(65) - 24
= 130 - 24
= 106
\]
∴ Hannah’s account balance is £130 and Neela’s account balance is £106.

In an examination, Ava’s score is four marks more than Ben’s score, and Chetan’s score is half of Ava’s score. Let \( a \), \( b \) and \( c \) be the scores of Ava, Ben and Chetan respectively.

(a) Express \( a \) and \( c \) in terms of \( b \).

(b) If Ben’s score is 80, find the scores of Ava and Chetan.
EXERCISE 5.3

LEVEL 1

1. The price of a ballpoint pen is £5 more than the price of a ruler. If the price of the ruler is £x, express the price of the ballpoint pen in terms of x.

2. The area of a kitchen is 12 m² less than the area of a sitting room. If the area of the sitting room is s m², express the area of the kitchen in terms of s.

3. A father’s age is 9 times the age of his son. If the son is y years old, express the age of the father in terms of y.

4. The length of car A is 45 cm less than the length of car B. Let x cm and y cm be the lengths of car A and car B respectively. Express x in terms of y.

5. The speed of a car is five times the speed of a bicycle. Let c km/h and b km/h be the speeds of the car and the bicycle respectively. Express c in terms of b.

6. The mass of a cherry is \( \frac{1}{10} \) of the mass of an apple.

Let c grams and a grams be the masses of the cherry and the apple respectively. Write a formula connecting a and c.

LEVEL 2

7. The total price for a burger and a drink is £5. Let £x be the price of the burger.

(a) Express the price of the drink in terms of x.
(b) If the price of the burger is £3.50, find the price of the drink.

8. The time taken by Jayden to finish a mathematics assignment is 10 minutes more than twice the time taken by Ella. Let t minutes be the time taken by Ella.

(a) Express the time taken by Jayden in terms of t.
(b) If the time taken by Ella to finish the assignment is 25 minutes, find the time taken by Jayden.

9. In a dance club, the number of boys is three less than half the number of girls. Let x and y be the number of boys and girls in the club respectively.

(a) Express x in terms of y.
(b) If there are 24 girls, find the number of boys.

10. Ronan pays £50 for three movie tickets. Let £x be the price of each ticket and £y be the change he gets.

(a) Write a formula connecting x and y.
(b) If the price of each ticket is £9, find the amount of change Ronan gets.

11. The present age of Laura is x years old.

(a) Find her age five years ago in terms of x.
(b) Laura’s mother is four times as old as Laura was five years ago. Express the present age of Laura’s mother in terms of x.

12. Zoey has y books.

(a) A year ago, Zoey had half as many books as she has now. Express the number of books that Zoey had a year ago in terms of y.
(b) A year ago, Eva had four more books than Zoey. Express the number of books that Eva had a year ago in terms of y.

LEVEL 3

13. Leah, Violet and Lucy buy a gift for their father, and they share the cost. Leah’s share is three times as much as Violet’s share. Lucy’s share is half of Leah’s share. Let £v be the amount of Violet’s share.

(a) Express Leah’s share in terms of v.
(b) Express Lucy’s share in terms of v.
(c) If Violet pays £60, find the price of the gift.
14. Max has half as much money in his savings account as Peter. Asmat has £12 more in his account than Max. Let £m, £p and £a be the balances in Max’s, Peter’s and Asmat’s accounts respectively.
   (a) Express m and a in terms of p.
   (b) If Peter’s account balance is £40, find the account balances of Max and Asmat.

15. In a school test, Ana’s mark is three less than Delphine’s mark, and Chris’s mark is twice Ana’s mark.
Let a, d and c be the marks of Ana, Delphine and Chris respectively.
   (a) Express a and c in terms of d.
   (b) If Delphine’s mark is 40, find Ana’s and Chris’s marks.

16. There are some £5 notes, £10 notes and £20 notes in Mr Patel’s wallet. The number of £20 notes is one more than twice the number of £5 notes. The number of £10 notes is one less than the number of £5 notes. Let x, y and z be the number of £5, £10 and £20 notes respectively.
   (a) Express y and z in terms of x.
   (b) Express the total amount of money in terms of x.
   (c) If there are four £5 notes, find the total amount of money in the wallet.

5.4 Simplification of Linear Expressions

How can you simplify a linear expression like 3(2x + 1) – 2(x – 1)? You need to expand the expression. This means that you write the expression as a sum or product of terms by removing brackets. You can explore the process using algebra discs.

### CLASS ACTIVITY 1

**Objective:** To perform multiplication of algebraic expressions and simplify them using algebra discs.

**Tasks**

1. Given an algebraic expression, you can group its terms and then multiply it by an integer.

   For example, expand –3(4x).

### RECALL

–3(4) is the negative of 3 groups of 4s. The ‘−’ sign in –3(4) means flipping over (or changing the signs of) all 1 discs in the groups to –1 discs.

### DISCUSS

Does 3(−4x) = −3(4x)?
For example, expand $-2(-5x)$.

![Diagram showing expansion of $-2(-5x)$]

This means 2 groups of $-5x$ which give a total of $-10x$.

The ‘$-$’ sign means flipping over the algebra discs or changing the sign of the term $-10x$.

$\therefore -2(-5x) = 10x$

Expand these expressions using algebra discs.

(a) $3(4x)$
(b) $3(-4x)$
(c) $-3(-4x)$
(d) $2(5x)$
(e) $2(-5x)$
(f) $-2(-5x)$

2. You can expand a linear expression in a similar way.

For example, expand $2(3x - 1)$.

![Diagram showing expansion of $2(3x - 1)$]

$\therefore 2(3x - 1) = 6x - 2$

Expand these expressions using algebra discs.

(a) $-2(3x - 1)$
(b) $-2(-3x - 1)$
(c) $3(x + 2y)$
(d) $-3(x + 2y)$

3. In general, you can simplify an expression by first removing the brackets and then collecting like terms.

For example, simplify $2(3x - 2) - 3(-x - 1)$.

![Diagram showing simplification of $2(3x - 2) - 3(-x - 1)$]

$\therefore 2(3x - 2) - 3(-x - 1) = 6x - 4$

This is 2 groups of $3x - 2$, i.e. $(3x - 2) + (3x - 2)$.

$-3(-x - 1) = 3x + 3$

This is the negative of 3 groups of $-x - 1$, i.e. $-[(-x - 1) + (-x - 1) + (-x - 1)]$. 
Collect like terms. $9x - 1$

$2(3x - 2) - 3(-x - 1)$  
$= 6x - 4 + 3x + 3$  
$= 6x + 3x - 4 + 3$  
$= 9x - 1$

For example, simplify $(5y + 2) + 3(-2y + 1)$.

$2(3x - 2) - 3(-x - 1)$  
$= 6x - 4 + 3x + 3$  
$= 6x + 3x - 4 + 3$  
$= 9x - 1$

For example, simplify $(5y + 2) + 3(-2y + 1)$.

$(-3)(-x)$ means that $-x$ is multiplied by 3, followed by the changing of the sign.

REMARK

$3(-2y + 1) = -6y + 3$

The signs of all terms in the bracket remain unchanged after the bracket is removed.
For example, simplify \(-2(2x + y) - 4(-x + y)\).

\[
-2(2x + y) - 4(-x + y) = -4x - 2y + 4x - 4y
\]

\[
= -4x + 4x - 2y - 4y
\]

\[
= -6y
\]

\textbf{Remark}

\(-2(2x + y)\) can be seen as the negative of 2 groups of \(2x + y\), i.e., \(-2(2x + y) = -[2(2x + y)]\).

\textbf{REMARK} \quad \text{Since there is a ‘-’ sign in front of the bracket, all terms inside the bracket will change signs after the bracket is removed.}

\textbf{REMARK} \quad \text{Since there is a ‘-’ sign in front of the bracket, all terms inside the bracket will change signs after the bracket is removed.}

Simplify these expressions using algebra discs.

\textbf{(a)} \quad 3(-2x + 1) + 2(4x - 1) \quad \textbf{(b)} \quad 2(3x - 2) - 3(-x - 2)
\textbf{(c)} \quad -3(x - 2y) - 2(-2x + y) \quad \textbf{(d)} \quad -4(x + 1) - 3(2x - 2)

4. Two algebraic expressions are said to be equivalent if they can be simplified to the same expression. For example, since \(-3(-4x + 2) = 12x - 6\) and \(3(4x - 2) = 12x - 6\), \(-3(-4x + 2)\) and \(3(4x - 2)\) are equivalent expressions.

Work in groups to evaluate and find pairs of equivalent expressions from this list using algebra discs.

\textbf{(a)} \quad 4(-5x) \quad \textbf{(b)} \quad -3(2x - 1)
\textbf{(c)} \quad 6(x - 1) \quad \textbf{(d)} \quad 2(5x + 1) - 5(-2x + 1)
\textbf{(e)} \quad -4(-5x) + 9 \quad \textbf{(f)} \quad 3(2x + 1)
\textbf{(g)} \quad -6(-x + 1) \quad \textbf{(h)} \quad 3(-2x + 1)
\textbf{(i)} \quad -4(5x) \quad \textbf{(j)} \quad 3(4x - 5) - 4(-2x - 6)
\textbf{(k)} \quad 2(3x + 5) - 8 \quad \textbf{(l)} \quad -10(-2x + 1) + 7

5. Expand \(a(bx)\), where \(a\) and \(b\) are numbers. You may substitute \(a\) and \(b\) with some numbers to help you explain.

6. Expand \(a(x + y)\), where \(a\) is a number. You may substitute \(a\) with some numbers to help you explain.
In Class Activity 2, you used algebra discs to help you understand the expansion of expressions. In fact, the process of expansion is actually the **distributive law** of multiplication over addition:

\[ a(x + y) = ax + ay \]

\( a(x + y) \) is expanded to \( ax + ay \), or \( ax + ay \) is the expanded form of \( a(x + y) \).

This law can be generalised and applied in these ways:

1. \((x + y)a = xa + ya\)
   \[
   = ax + ay
   \]
   Multiplication can be distributed over addition from the right.

2. \(a(x - y) = ax - ay\)
   \[
   = 2x - 2y
   \]
   Multiplication can be distributed over subtraction.

3. \(a(x + y + z) = ax + ay + az\)
   \[
   = 3x + 3y + 3z
   \]
   Multiplication can be distributed over the sum of several terms.

You use the distributive law when removing brackets in a linear expression.

**Example 18**

Expand these expressions.
(a) \(2(3x + 4y)\)
(b) \(-4(5a - 3b) + 7(a - 2b)\)

**Solution**
(a) \(2(3x + 4y)\) = \(2(3x) + 2(4y)\) = \(6x + 8y\)
(b) \(-4(5a - 3b) + 7(a - 2b)\) = \(-4(5a) - (-4)(3b) + 7(a) - 7(2b)\) = \(-20a + 12b + 7a - 14b\) = \(-13a - 2b\)

**Try it!! 18**

Expand these expressions.
(a) \(5(2x + 7y)\)
(b) \(-3(-4a + 8b) + 10(2a + b)\)
Expand these expressions.

(a) \( a(2x + 7y + 5z) \)

(b) \( (2p + 3q - 4r)(-6b) \)

**Solution**

(a) \( a(2x + 7y + 5z) = a(2x) + a(7y) + a(5z) = 2ax + 7ay + 5az \)

(b) \( (2p + 3q - 4r)(-6b) = (2p)(-6b) + (3q)(-6b) - (4r)(-6b) = -12bp - 18bq + 24br \)

Expand these expressions.

(a) \( c(4x + 6y + 9z) \)

(b) \( (3m - 6n + p)(-4d) \)

Simplify \( 4(9x - 2y + 1) - 6(6x - 2y) \).

**Solution**

\[
4(9x - 2y + 1) - 6(6x - 2y) = 4(9x) - 4(2y) + 4(1) - 6(6x) - 6(-2y)
= 36x - 8y + 4 - 36x + 12y
= 36x - 36x - 8y + 12y + 4
= 4y + 4
\]

Simplify \( 3(6p + 2q - 1) - 4(3p - q) \).

Simplify \( (2x + 8y - 1)(-3) + 5(3x - y - 4) \).

**Solution**

\[
(2x + 8y - 1)(-3) + 5(3x - y - 4) = (2x)(-3) + (8y)(-3) - (1)(-3) + 5(3x) + 5(-y) + 5(-4)
= -6x - 24y + 3 + 15x - 5y - 20
= 9x - 29y - 17
\]

Simplify \( -4(2p - 5q + 6) + (-3p - 2q + 1)(10) \).
LEVEL 1

1. Expand these expressions.
   (a) 3(2 + a)  
   (b) 4(7b + 5c)  
   (c) (2d - 6e)(5)  
   (d) (-3g - 4h)(2)  
   (e) -4(7 - 5n)  
   (f) -5(-3p + 9q)  
   (g) 6(-x - 5y)  
   (h) -1(5u - 16v)

2. Simplify these expressions.
   (a) 4(x + 7) + 3(x + 5)  
   (b) 3(2a + 4b) + 9(3a - 2b)  
   (c) -4(2x + 1) + 3(-x - 2)  
   (d) 2(3u - 5) - 3(2u + 1)  
   (e) -5(3x - y) - 7(-2x + 3y)  
   (f) -2(-3u + 4v) + 5(-u + 3v)  
   (g) (4x - 2y)(2) - (-3x + 4y)  
   (h) b(-5v - 4) - 2b(v + 4)

LEVEL 2

3. Expand these expressions.
   (a) 6(2r - 3s + 4t)  
   (b) a(-5x + 3y - 8z)  
   (c) -2(6a - 18b - 24c)  
   (d) (4a - 8b + 12c)(-5)

4. Simplify these expressions.
   (a) 3(2a - 3b - 5c) - 5(a - 3c)  
   (b) (x - 3y)(-2) + (x - y + 3)(6)  
   (c) 4(u - 3v + w) - 2(4u - v - w)  
   (d) -1(m - 2n) + 4(-2m - 2n + 2p)

LEVEL 3

5. There are (2a + b) books in a pile. The thickness of each book is 2 cm. Find the height of the pile of books in terms of a and b, expressing your answer in expanded form.

6. Oranges in a box are arranged in six rows and each row consists of (2m - 3) oranges. There are five oranges left over.
   (a) Express, in expanded form, the number of oranges in the box in terms of m.
   (b) Find the number of oranges in the box when m = 7.

7. In making the frame of a rectangular box, a carpenter needs four pieces of wood of length (2x + 3y) cm and eight pieces of length (2x + y) cm.

   (a) Express the total length of wood required in terms of x and y.
   (b) If x = 30 and y = 10, find the total length of wood required.

8. Some shortcuts in arithmetic make use of the distributive law. For example, 37 x 99 = 37 x (100 - 1) = 3700 - 37 = 3663

   Devise another two such shortcuts.

9. Some matchsticks are used to form a pattern of n squares as shown below.

   (a) Copy and complete the following table.

<table>
<thead>
<tr>
<th>n</th>
<th>Total number of matchsticks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

   (b) Find the total number of matchsticks used to form n squares in terms of n.
5.5 Proof

A mathematical statement (or simply a statement) is a sentence which is either true or false. For example, these are statements.

1. \(1 + 2 + 3 + 4 = 10\).
2. 17 is a prime number.
3. The cube root of 8 is 4.
4. \(\pi\) is a fraction.

Here, the first two statements are true while the last two statements are false.

These are not statements.

5. \(1 + 2 + 3 + 4\)
6. \(\pi\) is an interesting number.
7. \(17 + x = 20\).

Here, \(1 + 2 + 3 + 4\) is an expression. It cannot be true or false. Whether \(\pi\) is an 'interesting number' is arguable and is not a statement because it does not have a definite answer. Whether \(17 + x = 20\) is true or false depends on the value of the variable \(x\). If \(x = 3\), it is true. Otherwise, it is false. Hence, \(17 + x = 20\) is not a statement.

To prove that a statement is true or false, you have to apply your knowledge and use the assumptions given in the statement.

Consider even and odd integers.

You know that even integers are multiples of 2. Hence, an even integer can be expressed as \(2n\), for some integer \(n\). For instance, 6 = \(2 \times 3\), 14 = \(2 \times 7\) and 26 = \(2 \times 13\).

Each odd integer is 1 more than an even number. Hence, an odd integer can be expressed as \(2n + 1\), for some integer \(n\), for instance, 7 = \(2 \times 3 + 1\), 15 = \(2 \times 7 + 1\) and 27 = \(2 \times 13 + 1\).

### Example 22

Prove the statement ‘If \(a\) is an even number and \(b\) is an odd number, then \(a + b\) is an odd number.’

**Solution**

Let \(a = 2m\),
and \(b = 2n + 1\) for some integers \(m\) and \(n\).

\[
a + b = 2m + (2n + 1) = 2m + 2n + 1 = 2(m + n) + 1
\]

Since \(m\) and \(n\) are integers, \(p = m + n\) is an integer.

\(2p + 1\) is in the form of an odd number.

Hence, \(a + b\) is an odd number.

**Try It! 22**

Prove the statement ‘If \(c\) and \(d\) are even numbers, then \(c + d\) is an even number.’
If \( p \) is a multiple of 3 and \( q \) is a multiple of 4, determine whether each of these statements is true or false.

(a) \( p + q \) is a multiple of 7.
(b) \( pq \) is a multiple of 12.

**Solution**

(a) Take \( p = 6 \), which is a multiple of 3 and \( q = 20 \), which is a multiple of 4.
Then \( p + q = 6 + 20 \)
\[ = 26 \]
\[ = 2 \times 13 \]
which is not a multiple of 7.
Hence, the statement ‘\( p + q \) is a multiple of 7.’ is false.

(b) Let \( p = 3m \) \hspace{1em} (since \( p \) is a multiple of 3)
and \( q = 4n \) \hspace{1em} (since \( q \) is a multiple of 4)
for some integers \( m \) and \( n \).
Then \( pq = (3m)(4n) \)
\[ = 12mn \]
\[ = 12r \] \hspace{1em} Since \( m \) and \( n \) are integers, \( r = mn \) is an integer.
Hence, \( pq \) is a multiple of 12.
\[ \text{i.e. the statement ‘} pq \text{ is a multiple of 12’ is true.} \]

**Note:** In (a), an example is used to disprove the statement. This example is called a **counterexample**.
However, numerical examples cannot be used to prove that a statement is always true. For instance, in (b), if you just write
\( p = 6 \) is a multiple of 3 and \( q = 20 \) is a multiple of 4,
\[ pq = 6 \times 20 \]
\[ = 120 \]
\[ = 12 \times 10 \]
which is a multiple of 12, and conclude that \( pq \) is always a multiple of 12, then your proof is wrong.
This is because you cannot use a particular example to cover all the possible values of \( p \) and \( q \).

**Remark**

If \( p \) is a multiple of another number \( y \), then \( p = y \times m \), for some integer \( m \).

If \( x \) is a multiple of 2 and \( y \) is a multiple of 5, determine whether each of these statements is true or false.

(a) \( x + y \) is a multiple of 7.
(b) \( xy \) is a multiple of 10.
1. Determine whether or not each of these is a statement. If it is a statement, state whether it is a true statement or a false statement.
   (a) \( 3 \times 8 = 27 \)
   (b) The positive square root of 36 is 6.
   (c) \( 12 - y = 5 \)
   (d) All prime numbers are odd numbers.

2. Prove the statement 'If \( n \) is an odd number, then \( n + 1 \) is an even number.'

3. Prove the statement 'If \( a \) and \( b \) are odd numbers, then \( a + b \) is an even number.'

4. Prove the statement 'If \( c \) is an odd number and \( d \) is an even number, then \( cd \) is an even number.'

5. Prove the statement 'If \( m \) is a multiple of 10, then \( m \) is a multiple of 5.'

6. Is the statement 'If \( n \) is a multiple of 6, then \( \frac{n}{3} \) is an integer' true or false. Explain your answer.

7. If \( p \) and \( q \) are even numbers, show that \( pq \) is a multiple of 4.

8. If \( r \) is a multiple of 3 and \( s \) is a multiple of 5, show that \( rs \) is a multiple of 15.

9. Given that \( x \) is a multiple of 3 and \( y \) is a multiple of 6, determine whether each of these statements are true.
   (a) \( xy \) is a multiple of 18.
   (b) \( x + y \) is a multiple of 6.

10. If \( m \) is an even number, determine whether \( m^2 \) is an even number or an odd number.

11. Cathy worked out \( 19 + 20 = 39 \) and \( 27 + 28 = 45 \). She claimed that 'the sum of two consecutive integers is odd.' Is her claim valid? Justify your answer.

12. Let \( n \) be the smallest integer of three consecutive integers.
   (a) Express the other two integers in terms of \( n \).
   (b) Is it true that 'the sum of three consecutive integers is a multiple of 3'? Justify your answer.
In a Nutshell

Basic Notation
In algebra, symbols are used to represent numbers. The signs ‘+’, ‘−’, ‘×’, ‘÷’ and ‘=’ have the same meanings in both algebra and arithmetic.

- add = \( a + b \)
- subtract = \( a - b \)
- product = \( ab \) or \( a \times b \)
- quotient = \( \frac{a}{b} \) or \( a ÷ b \)

(For division, \( b \neq 0 \).

- \( a \times 3 = 3a = a + a + a \)
- \( a \times 1 = 1 \times a = a \)
- \( a \times a = a^2 \)
- \( a \times a \times a = a^3 \)

Algebraic Expressions
An algebraic expression involves numbers and letters that are connected with operations such as ‘+’, ‘−’, ‘×’ and ‘÷’.

For example,
- \( 2n + 1 \)
- \( 3a + 4b - 5c \)
- \( 7 - \frac{x}{y} \)

Substitution
You substitute each variable in an algebraic expression with its value to evaluate the expression.

For example,
- when \( n = 3, \)
  \( 2n + 1 = 2(3) + 1 = 7 \)
- when \( a = 2, b = -1 \) and \( c = 5, \)
  \( 3a + 4b - 5c = 3(2) + 4(-1) - 5(5) = 6 - 4 - 25 = -23 \)

Algebraic Expressions in the Real World
Let \( x \) be a variable for a quantity in the situation. Then, some other quantities in the situation can be expressed in terms of \( x \).

Formulae
A formula is an equality relating two or more quantities.

For example,
- \( A = \pi r^2 \)
- \( P = 2(a + b) \)

Substitution
The value of a variable in a formula can be found by substitution if the values of other variables are known.

For example, when \( r = 3, \)
\( A = \pi (3)^2 = 9\pi \)
when \( a = 3 \) and \( b = 4, \)
\( P = 2(3 + 4) = 14 \)

Distributive Law
- \( a(x + y) = (x + y)a = ax + ay \)
- \( a(x - y) = (x - y)a = ax - ay \)

For example,
\( 4(−2x + 3y) = 4(−2x) + 4(3y) = −8x + 12y \)
\( (7x - 2y)(−5) = (7x)(−5) - (2y)(−5) = −35x + 10y \)

Proof
A proof is a sequence of logical reasoning to show a mathematical statement is correct.
A counterexample is an example to show that a mathematical statement is incorrect.
1. Simplify these expressions.
   (a) \(5s \times 3t + 1 \times u\)
   (b) \((a \times 4 - b \times b) + 2c\)
   (c) \(3x - b + c - 5 \times y\)

2. Write these statements as algebraic expressions.
   (a) Subtract \(c \times c\) from \(d \times 5\).
   (b) Divide the product of \(6a\) and \(4\) by \(8b\).

3. Find the values of these expressions when \(n = 10\).
   (a) \(3n + 8\)
   (b) \(9(1 - 2n)\)

4. Given the formula \(p = \frac{2c + d}{3}\), find the value of \(p\)
   when \(c = -5\) and \(d = -8\).

5. Given the formula \(E = \frac{1}{2} m(v^2 - u^2)\), find the value of \(E\)
   when \(m = 5\), \(v = 11\) and \(u = 7\).

6. The capacity of a bowl is 30 cm\(^3\) more than the capacity of a cup. Let \(x\) cm\(^3\) be the capacity of the cup. Express the capacity of the bowl in terms of \(x\).

7. Simplify these expressions.
   (a) \(4(2m - 1) + 3(4m + 1)\)
   (b) \(-3(4x + y) + 2(5x - 8y)\)
   (c) \(-7(2x - y + 9) - 4(-3x + y - 5)\)

8. Prove that the sum of two consecutive odd numbers is an even number.

9. If \(a\) is a multiple of 4 and \(b\) is a multiple of 8, which of these statements are correct? Explain your answer.
   (a) \(a + b\) is a multiple of 4.
   (b) \(ab\) is divisible by 12.

10. Ethan is \(p\) years old now.
    (a) Find his age in \(t\) years’ time.
    (b) Ethan’s father is three years older than four times Ethan’s age now. Express his father’s age in terms of \(p\).
    (c) If \(p = 8\), find the age of Ethan’s father.

11. A piece of wire is 100 cm long. One piece of length \(a\) cm and two pieces of length \(b\) cm each are cut from it.
    (a) Express the length of the remaining part in terms of \(a\) and \(b\).
    (b) (i) If the remaining part is bent into a square, express the length of a side of the square in terms of \(a\) and \(b\).
        (ii) When \(a = 24\) and \(b = 16\), find the length of a side of the square.

12. There are \(x\) boys and \(y\) girls in a class. Half of the boys and a third of the girls join a learning camp.
    (a) Express, in terms of \(x\) and \(y\), the total number of students in the class.
    (b) When \(x = 18\) and \(y = 24\), find the number of students joining the camp.

13. Nadia’s savings after \(n\) months are £\((2500 + 300n)\).
    (a) Find the amount of savings she has after
        (i) 5 months, (ii) 1 year.
    (b) What might the numbers 2500 and 300 in the expression represent?
    (c) After \(n\) months, Nadia uses all her savings to buy gold coins costing £\(g\) each.
        (i) Express the number of gold coins she buys in terms of \(g\) and \(n\).
        (ii) When \(g = 100\) and \(n = 6\), find the number of gold coins Nadia buys.

14. Luke has \(x\) £5 notes and \(2y\) £10 notes in his wallet. Jacob has three times as many of each type of banknote as Luke.
    (a) What is the total value of Luke’s banknotes?
    (b) How many banknotes does Jacob have?
    (c) What is the total value of Jacob’s banknotes?
    (d) What is the total value of the banknotes of both boys?
    Express your answers in terms of \(x\) and \(y\).

Write in Your Journal

Julie says that \(3a^2 + 2a + 5a^2 = 10a^2\) and \(7b \times 5b^2 \times 6 = 210b\). Do you agree? How do you check?