EDEXCEL A LEVEL MATHS
YEAR 1 + YEAR 2
MECHANICS TEACHER BOOK
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Background

This chapter introduces the notion of a ‘force’ as the effect that one body can have on another body, for example, when a book falls to the ground because of the gravitational pull of the Earth. There are many different types of force (such as magnetic, electric, nuclear) but mechanics is limited to contact forces, such as tensions in strings, thrusts in rods, reactions between surfaces in contact and to gravitational forces, otherwise known as weights.

The study of forces on objects which are stationary is called ‘statics’. However, this chapter deals more with ‘dynamics’, the study of moving objects. It looks at Newton’s three laws of motion and particularly how the second law links the magnitude of a force with the acceleration of the body on which it acts.

Chapter 8 has four sections.
Chapter content

8.1 Forces 1

Students are reminded that a force is a vector which can be defined either by its magnitude and direction or by its components. Following on from Chapter 6, students should be comfortable with converting between the two forms, and with adding vectors and resolving in perpendicular directions, etc.

The key idea in this section is how to model situations as a set of non-negligible forces acting on a body. Typical forces used in mechanics include: contact forces such as normal reactions between touching bodies, thrusts in rods and tensions in strings and also weights as gravitational forces.

The initial focus is Newton’s first law and the conditions for equilibrium. Students are likely to find little difficulty with the idea of static equilibrium but they may have less confidence in the idea of dynamic equilibrium — when a body moves with constant velocity. A discussion of their experience on a train may help convince them that this is an equilibrium situation.

8.2 Dynamics 1

When an object is not in equilibrium, the non-zero, resultant force causes it to accelerate. Newton’s second law gives the relationship between force, mass and acceleration as a vector equation \( F = ma \). Here the importance of using consistent units should be emphasised: students should always use SI units in this equation. Applications include situations where several forces, given in either magnitude–direction form or component form, act on a body.

8.3 Motion under gravity

The key idea here is the difference between weight (a force due to gravity) and mass (a measure of the amount of material in an object). The link between the two is the equation \( W = mg \) where \( g \) is the acceleration due to gravity. The value of \( g \) depends on the mass and radius of the Earth and Newton’s gravitational constant; its value can show small changes due to local conditions near the Earth’s surface. In practice, \( g \) always points vertically downwards and its value will be specified in the question: typical values are 10, 9.8 or 9.81 \( \text{m s}^{-2} \). Students may have previously met these ideas in physics.

8.4 Systems of forces

The key idea here is Newton’s third law and the idea of equal and opposite actions and reactions. That is, pairs of forces that cancel one another out. This is important for coupled systems of objects which can be treated as a whole or as isolated components. To help identify the forces acting on a system of objects, diagrams may be drawn in ‘exploded’ form, where small gaps are added for clarity.

This section draws together ideas from throughout the chapter and makes use of all three of Newton’s laws. Typical problems involve lifts and pulley systems.
Investigation

The two investigations here are into the two aspects of mechanics: statics and dynamics.

Note at AS-level students may be asked to add forces in different directions but they will not be asked to resolve them into components.

Investigation 1 – Statics

**Question** How would you confirm experimentally that the triangle of forces is a valid way of finding the resultant for two (or more) forces?

Students are likely to offer various methods and the issue is likely to be which of them is the most viable and practicable to undertake. The method suggested involves three and, later, four forces, so the triangle law has to be extended to a polygon. Refer to Section 6.1 to refresh their understanding and then discuss the extension to more than two forces.

**Equipment**

Three spring balances  
A large wooden or plastic board about one metre square  
A sheet of paper to fit the board; string; pencil

**Method**

Secure the board horizontally with the paper fixed to it.  
Attach the three spring balances to an object with lengths of string. Stretch the spring balances so that the forces in them hold the object in equilibrium. Mark the lines of the string on the paper and record the readings on the balances.

**Analysis**

The three forces, with their magnitudes and directions known, have a resultant of zero as the object is held at rest in equilibrium. Confirm this result by drawing a triangle of forces to scale.

**An extension**

By fixing the board vertically and knowing the weight of the object, four forces now hold the object in equilibrium. Take measurements and draw a scale diagram to confirm the result.

Investigation 2 – Dynamics

**Question** How would you confirm experimentally that the kinematic (SUVAT) equations and Newton’s second law provide a good model to predict the motion of moving objects?

Refer back to Investigation 2 in Chapter 7 involving weights $W_1$ and $W_2$ and a smooth pulley. If the values of $W_1$ and $W_2$ are recorded, then the investigation can be extended to compare the experimental results with results calculated from the methods in Sections 7.2 and 7.3. Accelerations and times can be calculated and the times taken to fall can be compared with the experimental data.

Discuss whether the comparison sufficiently confirms that the model adopted is accurate. Refer to the flow diagram in the Introduction to this book. Are there any adjustments that could be made to the experiment, the model or both to make the comparison more favourable?
8.1 Forces 1

Simplification

Example 1

a A car is towing a caravan using a tow bar of negligible weight.
   i Draw a force diagram for the car and for the caravan.
   ii Describe all the forces in your diagram.
   iii Draw a force diagram for the tow bar and explain whether the tow bar is in thrust or tension.

b Repeat a iii when the car is reversing and pushing the caravan.

For clarity, the diagram is shown slightly 'exploded'. In practice the weights, \( W_1 \) and \( W_2 \), act at the (unknown) centres of gravity and normal reactions, \( R_x \) and \( R_y \), act at the tyres.

The force \( T \) in the tow bar holds the car back and pulls the caravan forward; hence the directions shown in the diagram.

Remember that normal means perpendicular.

For the tow bar alone, imagine being the tow bar. You will 'feel' you are being stretched at both ends and in a state of tension.

When reversing, the tow bar 'feels' squeezed at both ends, pushed in by both the car and the caravan.

Example 2

a Find the resultant \( \mathbf{R} \) of the three forces \( \mathbf{F}_1 = 2\mathbf{i} + 3\mathbf{j} \), \( \mathbf{F}_2 = 4\mathbf{i} - 6\mathbf{j} \) and \( \mathbf{F}_3 = -3\mathbf{i} + \mathbf{j} \)

b These three forces and a force \( \mathbf{F}_4 \) hold an object in equilibrium. Find \( \mathbf{F}_4 \)

\[
\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = (2\mathbf{i} + 3\mathbf{j}) + (4\mathbf{i} - 6\mathbf{j}) + (-3\mathbf{i} + \mathbf{j}) = 3\mathbf{i} - 2\mathbf{j}
\]

Either

\[
\begin{align*}
\mathbf{F}_4 &= \mathbf{0} - (3\mathbf{i} - 2\mathbf{j}) \\
&= -3\mathbf{i} + 2\mathbf{j}
\end{align*}
\]

Simply add the \( \mathbf{i} \) and \( \mathbf{j} \) components.

For equilibrium, you can think either of the total of all the forces being zero or of the final force \( \mathbf{F}_4 \), balancing \( \mathbf{R} \) by being equal in magnitude but opposite in direction.
Simplification questions 8.1

1. A block of weight 20 N is placed on a rough, level surface. It is pulled at constant velocity by a horizontal force, \( P \). Draw a diagram to show the forces acting on the block.
   Make sure that each label you use is defined.

2. A block of weight \( W \) is being pulled up a rough slope at a constant velocity by a rope of negligible mass.
   a. Draw a diagram to show the forces acting on the block.
   Make sure that each label you use is defined.
   b. Redraw your diagram to show the forces acting on the block when it is being lowered down the slope at constant velocity.
   c. In which case, a or b, is the tension in the rope larger?
      Give your reasons.
   d. Does your answer to part c change if resistance is negligible?
      Explain your answer.

3. A train consists of an engine attached to a wagon by a fixed bar of negligible weight. The train travels at a constant speed of 10 km h\(^{-1}\) along a straight stretch of track.
   a. Draw a force diagram for
      i. The engine,
      ii. The wagon,
      iii. The fixed bar,
      iv. The train as a whole.
      Define each force shown on your diagram.
   b. The train goes into reverse. Which forces, if any, change and how do they change?

4. Two climbers, A and B, have had an accident. Both climbers hang in mid-air with A above B. Climber A (with weight \( W_A \)) is held to the mountain by one rope. Climber B (with weight \( W_B \)) is tied to A by a second rope and hangs below A.
   What are the tensions in the two ropes?

5. For each set of forces acting on an object,
   i. Find the resultant force as a vector.
   ii. Find the single additional force that will bring the object into equilibrium.
   \( \mathbf{F}_1 = 5 \mathbf{i} + 6 \mathbf{j} \) and \( \mathbf{F}_2 = 1 - 4 \mathbf{j} \)
   \( \mathbf{F}_1 = -3 \mathbf{i} + 3 \mathbf{j} \) and \( \mathbf{F}_2 = -8 \mathbf{i} - 9 \mathbf{j} \), and \( \mathbf{F}_3 = 11 \mathbf{i} + 5 \mathbf{j} \)
   \( \mathbf{F}_1 = 4 \mathbf{i} + 5 \mathbf{j} \), \( \mathbf{F}_2 = -2 \mathbf{i} - 8 \mathbf{j} \) and \( \mathbf{F}_3 = 13 \mathbf{i} + 12 \mathbf{j} \)
   \( \mathbf{F}_1 = 42 \mathbf{i} + 53 \mathbf{j} \), \( \mathbf{F}_2 = -22 \mathbf{i} - 26 \mathbf{j} \) and \( \mathbf{F}_3 = 33 \mathbf{i} - 39 \mathbf{j} \)

6. Find the exact magnitude of the force \( \mathbf{F} \) when
   a. \( \mathbf{F} = 5 \mathbf{i} + 12 \mathbf{j} \)
   b. \( \mathbf{F} = 6 \mathbf{i} - 8 \mathbf{j} \)
   c. \( \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 \) where \( \mathbf{F}_1 = 2 \mathbf{i} + 3 \mathbf{j} \) and \( \mathbf{F}_2 = 4 \mathbf{i} - 1 \mathbf{j} \)
   d. \( \mathbf{F} = \mathbf{F}_1 - \mathbf{F}_2 \) where \( \mathbf{F}_1 = 3 \mathbf{i} - 4 \mathbf{j} \) and \( \mathbf{F}_2 = 1 + 2 \mathbf{j} \)

7. This object is in equilibrium.
   Find the magnitudes of the forces \( \mathbf{T} \) and \( \mathbf{U} \) acting on the object.

8. a. Explain why these two objects are not in equilibrium.
   b. Find the resultant force acting on each object in vector form.
   c. Calculate the magnitude and direction of each resultant force.
Example 1

Three children pull on a sledge S sitting on ice. Child A pulls due north with a force of 10 N. Child B pulls due south with 7 N. Child C pulls due east with 8 N.

a) On what bearing does the sledge move and what is the resultant force pulling it?

b) How might your result change if the sledge were not on ice?

\[ R = \sqrt{10^2 + (-7)^2} = 10.7 N \]

\[ \tan \theta = \frac{7}{8} \]

\[ \theta = 36.9 ^\circ \]

The sledge moves on a bearing {eq}\theta \text{ N}\text{E} \text{S}\text{W} \text{ of } 8.5 \text{ N}.

Example 2

The top of a vertical pole is held in equilibrium by four horizontal wires. The wires pull on the pole with forces of 2 N due north, 8 N due west, 6 N due south and a fourth force, \( F \). Find the magnitude and direction of the force \( F \).

\[ \sqrt{F_x^2 + F_y^2} = \sqrt{2^2 + 6^2} = 6.3 \text{ N} \]

\[ \theta = 53.1 ^\circ \]

The force \( F \) has magnitude 6.3 N and acts on a bearing \( 53.1 ^\circ \)S\text{E}W.
Extension questions 8.1

1 A man is taking three dogs for a walk.
   Dog A pulls with a force of 7 N due east.
   Dog B pulls with a force of 5 N due south.
   Dog C pulls with a force of 8 N due west.
   In what direction and with what force is the man pulled?

2 A car is at rest on a frozen pond. Cables are attached and winches used to slide the car off the ice.
   Winch A applies a force of 1000 N due west.
   Winch B applies a force of 1500 N due north.
   Winch C applies a force of 600 N due east.
   a On what bearing does the car move?
   b What is the resultant force on the car?
   c How would your results for parts a and b change if the car was on a rough surface?

3 Four springs are attached to a point particle. Three of the forces are: 17 N acting due east, 8 N acting due south and 15 N acting due north. If the particle is moving at a constant speed of 2 m s⁻¹ on a bearing 135°, what is the fourth force?

4 A metal object of negligible weight is held in equilibrium by three magnets. The forces exerted on the object by two of the magnets are 30 N due north and 25 N due west.
   a Find the magnitude and direction of the force exerted by the third magnet.
   b Check your answer by drawing a vector diagram of the three forces using a scale where 1 cm represents 5 N.

5 A boat moves under horizontal forces exerted by the wind, the tide and its engine. The forces are 160 N due south, 120 N due east and 200 N due west.
   a Calculate the magnitude and direction of the resultant horizontal force on the boat.
   b Check your answers by drawing a scale diagram where 1 cm represents 20 N.

6 A sheet of metal is acted on by the forces shown in the diagram.
   ![Diagram of forces](image)
   In practice, would the sheet of metal be in equilibrium? Give your reasons.

7 A pony is pushed by two children and it refuses to move. They push with forces of 50 N and 100 N which are perpendicular to each other. The direction of the resultant of these two forces is due east. Find
   a The magnitude of the resultant,
   b The directions in which the children are pushing, giving your answers as bearings,
   c The force which the pony is exerting in its refusal to move.

8 A cable-car of weight 10 000 N carries skiers. It comes to rest in mid-air where it is held by two cables inclined at angles of 30° and 60° to the vertical. Find the tensions in the two cables.
   Draw a vector diagram of the forces involved and calculate the two tensions.

9 Three identical cubes are stacked in a column and placed on a level table top. The cubes have rough sides and each one weighs 2.5 N. A horizontal force of 1 N to the right is applied to the middle cube. None of the cubes move.
   a Draw a diagram showing all the forces acting on this system.
   b What is the frictional force exerted by the top cube on the middle cube?
   c What is the force on the table?

10 Three forces of 10 N, 3 N and 5 N act on a massless body sitting on a smooth surface. The directions of the forces are not given. Is it possible for the body to be in equilibrium? Explain your answer.
Common misconceptions / Exam tips

Some students might find it strange that an object can be in equilibrium when it is moving (with a constant velocity). Remind them that there is static equilibrium, when the object does not move, and dynamic equilibrium, when the object has a constant velocity and no acceleration because no force is acting on it.

- **Contact forces between surfaces**
  Students are often less knowledgeable about contact forces, that is, forces that occur when two bodies touch and their surfaces press on one another. This contact force always has a component at right angles to the surfaces, called the *normal reaction*. If there is a tendency for the two surfaces to move relative to each other, or if they actually do move, there is also a component of the contact force along the direction of the surfaces, usually called the *resistance* or *resistive force*. If the surface contact is smooth, then the resistive force is zero.

  Explain that, if both these components (normal reaction and resistance) exist, they can be combined using Pythagoras’ Theorem and their resultant is called the ‘total reaction’ between the two objects, but this is not often done – exam questions usually prefer to keep the components separate.

- **Tensions and thrusts**
  Students can confuse these two types of force. Have them imagine an object at rest on an icy surface. If a string is fixed to it and the string pulled, the object would move across the ice because of the force in the string. If the string could feel this happening, it would feel that it was being stretched, so the force in it is a tension.

  However, the object on ice could be moved instead by having a rod pushing it from behind. If the rod could feel what was happening, it would feel it was being squashed by the object. The rod is thrusting the object forward – the force is a thrust.

- **Resolving a force**
  Students may not fully understand what it means to ‘resolve’ a force. If you resolve a force, you find its component in a particular direction. You often find two components of a force at right-angles to each other, in which case you have resolved in two directions.

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**Exercise 8.1B Student Book commentary**

**Questions 1 – 3** These are straightforward questions. Refer students who are struggling to Example 3 in this section of the Student Book.

**Question 4** Ensure that parallel forces balance vertically and horizontally by writing two equations. Solve the equations, which are sometimes simultaneous equations.

**Questions 5 & 6** Find the magnitude of the components in the two directions. Refer students who are struggling to Examples 3 and 4.

**Question 7** The forces cannot balance in both directions simultaneously, so there will be a resultant force and thus no equilibrium.

**Question 8** Ask students to discuss possible explanations with a partner and then insist that they write their explanation as a full sentence.

**Question 9** The question is straightforward and relies on drawing a correct force diagram.

**Question 10** Ask students to discuss possible explanations. As necessary, remind them that they can treat the engine and carriage in isolation.

**Question 11** Tension means the rod is being stretched, thrust means it is being compressed.

**Question 12** Draw a diagram of the resultant force and use trigonometry in a right-angled triangle to find its two components in the two principal directions. Equate the forces which act due east/west on the two diagrams. Repeat for forces which act north/south. Solve these two equations for X and Y.
8.2 Dynamics 1

Simplification

Example 1

a. Find the acceleration created by a force of 10 N acting on a mass of 5 kg.

\[ a = \frac{F}{m} = \frac{10}{5} = 2 \text{ m/s}^2 \]

b. The forces \( \left( \begin{array}{c} 3 \\ 1 \end{array} \right) \) N and \( \left( \begin{array}{c} 2 \\ 5 \end{array} \right) \) N act on a mass of 2 kg.

Find the acceleration of the mass.

\[ a = \frac{\sum F}{m} = \frac{3 + 2}{2} = \frac{5}{2} \text{ m/s}^2 \]

This very simple example in one-dimension should be compared with the example of part b below in two-dimensions.

Check that the units in \( F = ma \) are all SI units. The equation is not valid if they are not.

The equation \( F = ma \) requires the resultant force which is found by adding the two given forces.

Example 2

A van with a mass of 1200 kg is travelling at speed when the driver brakes and the van decelerates at \(-1.5\) m s\(^{-2}\). Find the braking force provided by the van’s brakes.

Newton’s second law, applied horizontally

\[ -F = m \cdot \text{acceleration} = 1200 \times (-1.5) = -1800 \text{ N} \]

Taking the forward direction as positive, both the braking force \( F \) and the deceleration are negative.

Always check for SI units when using \( F = ma \)

Example 3

A mass of 5 kg is on a horizontal table subject to the forces shown.

a. Find the normal reaction, \( R \)

b. Find the acceleration, \( a \)

a. Resolving vertically

\[ N = 5g = 5 \times 9.8 = 49 \text{ N} \]

b. Newton’s second law, applied horizontally

\[ 90 - 150 = 5 \cdot a \]

\[ a = \frac{W - R}{5} = \frac{-60}{5} = -12 \text{ m/s}^2 \]

The object is in vertical equilibrium, so the vertical forces balance.

The horizontal forces do not balance, so there is a horizontal acceleration: \( \text{use } F = ma \)

The acceleration is negative. It is slowing down (or decelerating).
1. Calculate the acceleration of an object with a mass of $m$ kg if a force of $F$ N acts on it, where
   
   a. $m = 10$ kg and $F = (2i + 5j)$ N
   b. $m = 2$ kg and $F = -3j$ N
   c. $m = 4$ kg and $F = \left(\frac{4}{2j} - 2\right)$ N
   d. $m = 0.5$ kg and $F = \left(\frac{-4}{6}\right)$ N

2. A particle of mass 4 kg is acted upon by a single force, $F$, of magnitude 20 N.
   
   a. Find the magnitude of the particle’s acceleration if
      
      i. $F$ is pulling the particle forward,
      
      ii. $F$ is resisting the particle’s motion.
   
   b. In each case, explain how your answers indicate whether the particle is speeding up or slowing down.

3. Two toy cars, A and B, are pulled along a rough horizontal surface by horizontal forces of 60 N and 10 N. These diagrams show all the forces acting on the cars.

4. Calculate the acceleration, $a$, and the forces, $F$ and $R$, in these diagrams of blocks moving under two systems of forces.

5. A sledge of mass 15 kg is pushed on a horizontal icy surface where resistance is negligible by a force of 30 N. If there is no one on the sledge, what is its acceleration?
   
   a. If a child of mass 25 kg is sitting on the sledge, what is the acceleration now?

6. The ice in Question 5 begins to melt into slush and friction is no longer negligible. The resistive force on the sledge has a constant value of 5 N.
   
   a. If the 15 kg sledge with no one on it is pushed by a force of 30 N, what is the acceleration?
   
   b. Find the acceleration if the 25 kg child is sitting on the sledge.

7. A car of mass 1000 kg is travelling along a horizontal road with its engine providing a driving force of 1500 N. If it experiences a resisting force of 300 N, find
   
   a. The resultant horizontal force on the car,
   
   b. The car’s acceleration.

8. A car is at rest on a horizontal road when the driver sets off along the road with an acceleration of 2.5 m s$^{-2}$. If the car has a mass of 800 kg and there is a constant resistance to its motion of 200 N, calculate the driving force provided by the car’s engine.
Extension

Example 1

A lift is travelling vertically upwards in a lift shaft at a speed of 4 m s
when it begins to decelerate. It comes to rest over a distance of 10 m

a Find its deceleration.

\[ v^2 = u^2 + 2as \]
\[ 0 = 4^2 + 20 \times a \]
\[ a = \frac{-4}{5} = -0.8 \text{ ms}^{-2} \]

b If the lift and its passengers have a total mass of 8000 kg, find the resultant force necessary to produce this deceleration and state its direction.

[Equation and calculations]

A kinematic equation is needed to find \( a \). \( u, v, s \) are known, \( a \) is needed, so use the equation without \( t \) which is \( v^2 = u^2 + 2as \)

The negative sign of \( a \) indicates a deceleration.

Having found \( a \), use \( F = ma \) vertically. Take the upward direction as positive, so the negative sign of \( F \) means that \( F \) is a downward force. This resultant will comprise several forces such as the lift’s weight, tension in the cables, friction, etc. This problem does not explore these.

Example 2

A train of mass 180 tonnes starts from rest and takes 2 minutes to reach a speed of 72 km h
Its engine produces a constant tractive force \( T \) of 35 kN

Calculate the force \( U \) of the resistances to motion.

\[ v = \frac{72 \times 1000}{60 \times 60} = 20 \text{ m s}^{-1} \]
\[ t = \frac{2 \times 60}{120} = 1 \text{ s} \]
\[ u = 0 \]
\[ v = u + at \]
\[ 20 = 0 + 120a \]
\[ a = \frac{20}{120} = \frac{1}{6} \text{ m s}^{-2} \]

Newton’s second law, applied horizontally
\[ T - U = ma \]
\[ 35000 - U = 180 \times 1000 \times \frac{1}{6} \]
\[ U = 35000 - 30000 = 5000 \text{ N} \]

Draw a diagram showing all the forces, including the vertical reaction \( R \) and weight \( W \) which do not appear in the working. Note that the word tractive means ‘pulling’.

Change units into SI units.

Choose \( v = u + at \) because there is no \( s \).

Change units to SI units.

1 tonne = 1000 kg

The resistances to motion might include: air resistance, headwind or resistance between wheels and track.
Extension questions 8.2

1 A particle $P$ with a mass of 3 kg is pulled by two strings with tensions $\mathbf{T} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$ N and $\mathbf{U} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$ N. Its motion is resisted by a frictional force $\mathbf{F} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ N.

a Find, in vector form, the resultant force on the particle and its acceleration.

b Find the magnitude of the resultant force and the acceleration.

c Show that the resultant of $\mathbf{T}$ and $\mathbf{U}$ acts in the same line as $\mathbf{F}$

Explain why this this is always the case regardless of the values of $\mathbf{T}$, $\mathbf{U}$ and $\mathbf{F}$

2 A lorry of mass 3000 kg is travelling on a straight horizontal road at a speed of 800 N when the driver disengages the engine and applies a horizontal braking force of 1200 N. Calculate the time taken for the lorry to come to rest.

3 A crate stands stationary on a level floor. When a horizontal force of 60 N is applied to it, it begins to move and reaches a speed of 12 m s$^{-1}$ in the first 24 metres. If the resistance to its motion is a constant 18 N, find the mass of the crate.

4 A small aeroplane of mass 2000 kg accelerates from 270 km h$^{-1}$ to 360 km h$^{-1}$ over a horizontal distance of 5 km. If there is a constant resistance to its motion of 12 kN, find the thrust produced by its engine.

5 The brakes fail on a train of mass $4 \times 10^5$ kg. It is travelling at 2 m s$^{-1}$ when it hits the buffers and it is brought to rest in 0.8 metres. Find the resisting force exerted on the train by the buffers.

What assumptions have you made?

6 A car of mass 1200 kg is driven on a straight horizontal road at a speed of 60 km h$^{-1}$. The total resistance to its motion caused by friction and air resistance is constant at 360 N.

a What must be its driving force for the car to travel

i At constant speed,

ii With an acceleration of 0.5 m s$^{-2}$?

b If the driver free-wheels to rest, what is the car’s deceleration?

c If it free-wheels with the brakes applied and it comes to rest with a deceleration of 0.75 m s$^{-2}$, what must be the force exerted by its brakes?

d Describe one improvement you would make to this model of the car’s motion.

7 A van with a mass of 1200 kg is being driven on a level road at 20 m s$^{-1}$ against a resistance of 200 N. The driver brakes suddenly and the brakes provide a retarding force of 3000 N.

Calculate

a The van’s deceleration,

b The time for the van to come to rest,

c The distance travelled in coming to rest.

8 A particle $P$ of weight $\mathbf{W}$ N is held in equilibrium by two strings with tensions $\mathbf{T} = -10 \mathbf{i} + 20 \mathbf{j}$ N and $\mathbf{U} = 10 \mathbf{i} + 25 \mathbf{j}$ N. Given that the $x$-axis is horizontal and the $y$-axis is vertical, find

a The weight $\mathbf{W}$,

b The initial acceleration of $P$ if the first string breaks, giving your answer

i In vector form,

ii As a magnitude and direction.
Common misconceptions / Exam tips

- **Newton's second law**
  Newton initially expressed his second law in terms of the rate of change of momentum. Only when mass is constant can $F = ma$ be used. (The law can be adapted for systems where the mass changes, such as space rockets burning fuel, but its application is subtle and is beyond the scope of A-level maths.)

- **Acceleration and deceleration**
  By now, students will know that deceleration or retardation is negative acceleration. When using $F = ma$, students can become confused with directions when an object is slowing down. They should decide which way is positive and then write their vector equation accordingly for forces and accelerations (or decelerations).

- **Resolving forces**
  When forces hold an object in equilibrium, the resultant force is zero because the forces balance each other. When writing a force equation for equilibrium, students can choose either to add all the forces together (taking account of direction) and equate the total to zero, or to balance the forces in one direction with those in the other direction. For example, they could write either $F_1 + F_2 + F_3 = 0$ or $F_1 + F_2 = F_3$.

- **Tension and thrust**
  Students find tension easier to comprehend than thrust, presumably because they have more often experienced tension in strings, wires and chains. Thrust occurs in rods and struts. Students should imagine what it 'feels' like to be a string or rod; do they feel that they are being pulled apart (tension) or pushing outwards (thrust)?

**Exercise 8.2B Student Book commentary**

There is a common strategy for all the problems in this exercise. That is, the acceleration, $a$, occurs in both the five kinematic equations (SUVAT) and the equation of motion $F = ma$. Each problem is solved by finding $a$ in one equation and then using it in the other.

**Question 1 & 2** Given $u$, $v$ and $s$, use $v^2 = u^2 + 2as$ to find $a$, then use it in $F = ma$.

**Question 3** Find $a$ from $F = ma$ and then use it in the equation without $v$, namely, $s = ut + \frac{1}{2}at^2$.

**Question 4** No resultant force is given in this question, as it was in questions 1 – 3. First, find $a$ from $v^2 = u^2 + 2as$, as $t$ is not known or wanted. Then write an equation of motion using $F - R = ma$, where $F$ is the driving or tractive force and $R$ is the resistance.

**Question 5** Given that it is constant velocity, part a is an equilibrium situation implying zero resultant force. In part b, you are given $u$, $v$ and $s$ and so can use $v^2 = u^2 + 2as$ to find $a$, then use it in $F = ma$. Part c would make a good topic for discussion: students must write a complete sentence to summarise their argument for what happens.

**Question 6** This is straightforward when written as separate horizontal and vertical equations.

**Question 7** The weight and mass indicate that the acceleration due to gravity, $g$, should be calculated. Since it involves a parachute, students should see that air resistance is important. Typically air resistance is proportional to $v^2$ and can lead to the idea of 'terminal' velocity.

**Question 8** The lorry's engine creates the driving force that is applied by the tyres to the road.

**Question 9** This is a multistage problem. Students should be clear that different forces and accelerations will apply in each stage and should be careful not to mix them up. The results from one stage of calculation will typically be needed to act as the inputs to the next stage – here it is the upward velocity of the box just before the tension is reduced from 60 N to 40 N.

**Question 10** Students need to realise that adding two forces of fixed magnitude gives the greatest resultant force when they are parallel and the least when they are anti-parallel. Pythagoras relates $a$, $b$ and 10.

**Question 11** Use Pythagoras to find the magnitude of the resultant force to use in $F = ma$.

**Question 12** Newton's third law applied vertically and his second law applied horizontally give two simultaneous equations for $R$ and $m$. 
8.3 Motion under gravity

### Simplification

**Example 1**

- **a**  A mass of 5 kg is attached to a vertical string and is lifted so that it accelerates at 3 m s\(^{-2}\). Calculate the tension in the string.

- **b**  The same mass is now at rest on a floor and is lifted vertically by the string so that it is 12 m above the floor after 4 seconds. Calculate the tension in the string.

\[
\begin{align*}
\text{Let } T &= \text{tension in the string} \\
&= \text{Newton's second law, applied vertically} \\
&= T - mg = ma \\
&= T - 5 \times 9.8 = 5 \times 3 \\
&= 15 + 49 = 64 \text{ N} \\
\end{align*}
\]

\[
\begin{align*}
\text{Part } b \text{ is essentially the same as part } a, \text{ but the first step calculates the acceleration } a \\
\text{using a kinematic equation.}
\end{align*}
\]

**Example 2**

A crane is lifting a load of mass 120 kg with an acceleration of 0.8 m s\(^{-2}\). If half the load falls off the crane accidentally and the tension in the crane's cable stays the same, find the new acceleration of the remaining part of the load.

\[
\begin{align*}
\text{Before the accident:} \\
&= \text{tension in the string} \\
&= \text{Newton's second law, applied vertically} \\
&= T - 120g = 120 \times 0.8 \\
&= 1176 + 96 = 1272 \text{ N} \\
\end{align*}
\]

\[
\begin{align*}
\text{After the accident:} \\
&= \text{acceleration} \\
&= \text{Newton's second law, applied vertically} \\
&= 1272 - 60g = 60a \\
&= 1272 - 588 = 684 \\
&= 684 = 11.4 \text{ m s}^{-2} \\
\end{align*}
\]

The tension stays the same so a solution strategy is to find \( T \) before the accident and use it to find the acceleration after the accident.
Simplification questions 8.3

You may assume that $g = 9.8\, \text{m s}^{-2}$ unless stated otherwise

1. The label on a bag of flour gives its mass as 2 kg. Find its weight in N, taking $g$ as
   \[ \begin{align*}
   &\text{a} \quad 9.81 \, \text{m s}^{-2} \\
   &\text{b} \quad 9.8 \, \text{m s}^{-2} \\
   &\text{c} \quad 10 \, \text{m s}^{-2}
   \end{align*} \]
   How many significant figures are you justified to use in your answers?

2. A toy battery-powered car runs with constant velocity in a straight line across a rough table. If the mass of the car is 250 grams, find
   \[ \begin{align*}
   &\text{a} \quad \text{The reaction between the car and the table,} \\
   &\text{b} \quad \text{The horizontal force on the car if its tractive force is 0.2 N}
   \end{align*} \]

3. A bucket and contents on a building site have a total mass of 8.5 kg. They are being pulled vertically upwards by a light rope. Find the tension in the rope if the bucket and contents
   \[ \begin{align*}
   &\text{a} \quad \text{Have a constant velocity,} \\
   &\text{b} \quad \text{Are accelerating at a rate of 0.2 m s}^{-2}, \\
   &\text{c} \quad \text{Are decelerating at a rate of 0.2 m s}^{-2}
   \end{align*} \]

4. A crate is pulled horizontally across a rough floor by a horizontal rope. Take $g$ as 10 m s$^{-2}$
   \[ \begin{align*}
   &\text{a} \quad \text{Name all the forces that act on the crate.} \\
   &\text{b} \quad \text{If the mass of the crate is 80 kg, find the normal reaction between the crate and the floor.} \\
   &\text{c} \quad \text{If the tension in the rope is 70 N and the force resisting motion is 20 N, calculate the acceleration of the crate.}
   \end{align*} \]

5. A crane on a building site uses a thin steel cable to lift a 500 kg load with an upward acceleration of 0.3 m s$^{-2}$. Calculate the tension in the cable. State an assumption you have made.

6. The maximum tension allowed in the lifting cable of a crane is 8000 N. Find the maximum load, in kg, that it can lift
   \[ \begin{align*}
   &\text{a} \quad \text{At constant velocity,} \\
   &\text{b} \quad \text{With an acceleration of 0.2 m s}^{-2}
   \end{align*} \]

7. A boy on a bridge drops a $\frac{1}{2}$ kg stone into a deep lake. On striking the surface of the lake, it sinks with an acceleration of 1.5 m s$^{-2}$. Find the resistance of the water to the motion of the stone.

8. A stone of 250 grams is moving across the icy surface of a frozen lake. Find
   \[ \begin{align*}
   &\text{a} \quad \text{The normal reaction between the stone and the ice,} \\
   &\text{b} \quad \text{Its deceleration if the resistance to its motion} \\
   &\quad \text{i} \quad \text{Can be neglected,} \\
   &\quad \text{ii} \quad \text{Is 0.5 N}
   \end{align*} \]

9. An object P is held in static equilibrium by a spring balance AB. The reading on the spring balance is 2 kg
   \[ \begin{align*}
   &\text{a} \quad \text{The end B of the spring balance} \\
   &\quad \text{is lifted so that the balance and the object P move vertically} \\
   &\quad \text{upwards with an acceleration of 1.2 m s}^{-2} \text{ and negligible} \\
   &\quad \text{resistance to the motion. What will be the reading on the} \\
   &\quad \text{balance in newtons?} \\
   &\text{b} \quad \text{What will the reading be if the balance and object have a downward} \\
   &\quad \text{acceleration of 1.2 m s}^{-2}\text{?}
   \end{align*} \]

10. A box of mass 3 kg is at rest on a rough, horizontal surface.
    \[ \begin{align*}
    &\text{a} \quad \text{A horizontal string attached to the box is} \\
    &\quad \text{pulled with a tension of 20 N but the box} \\
    &\quad \text{does not move.} \\
    &\quad \text{What is the magnitude of these forces?} \\
    &\quad \text{i} \quad \text{The resistance force,} \\
    &\quad \text{ii} \quad \text{The normal reaction between the box} \\
    &\quad \text{and the surface.}
    \end{align*} \]

    \[ \begin{align*}
    &\text{b} \quad \text{If the string is removed and the surface} \\
    &\quad \text{moves upwards with an acceleration of} \\
    &\quad \text{0.2 m s}^{-2}, \text{find the normal reaction between} \\
    &\quad \text{the box and surface.}
    \end{align*} \]

    \[ \begin{align*}
    &\text{c} \quad \text{If the surface without the string moves} \\
    &\quad \text{downwards with an acceleration of 12 m s}^{-2}, \text{describe the motion and explain what} \\
    &\quad \text{happens to the normal reaction between the} \\
    &\quad \text{box and surface.}
    \end{align*} \]
An object with a weight of 5 N on Earth is lifted 8 m vertically from rest by a force $F$ in a time of 2.0 s. The same object on the Moon’s surface is lifted the same distance by the same force in 1.14 s. Find the acceleration $g'$ due to gravity on the Moon’s surface. Take $g = 10$ m s$^{-2}$ on Earth.

**On Earth**

Weight = $mg$

$W = \frac{5}{10} = 0.5 \text{ kg}$

$s = ut + \frac{1}{2}at^2$

$t = 0 + \frac{1}{2} \times a \times 2^2$

$a = 4 \text{ m s}^{-2}$

Newton’s second law, applied vertically

$F - mg = ma$  or  $F - W = ma$

$F = 0.5 \times 4 + 5 = 7 \text{ N}$

**On the Moon**

$s = ut + \frac{1}{2}at^2$

$t = 0 + \frac{1}{2} \times a \times 1.14^2$

$a = 12.3 \text{ m s}^{-2}$

Newton’s second law, applied vertically

$F - mg' = ma$

$7 - 0.5g' = 0.5 \times 12.3$

$g' = \frac{7 - 0.5 \times 12.3}{0.5} = 1.7 \text{ m s}^{-2}$

The acceleration due to the Moon’s gravity is 1.7 m s$^{-2}$.
Extension questions 8.3

Take $g$ as $9.8 \text{ m s}^{-2}$ unless told otherwise.

1. a. An object weighs 20 N on Earth and 3.3 N on the Moon. Find the acceleration due to gravity on the Moon’s surface.
   b. Jupiter is the most massive planet in the solar system where $g$ has a value of $24.8 \text{ m s}^{-2}$. Find the weight of the object weighing 20 N on Jupiter.

2. a. A rock weighing 30 N falls 10 m vertically from rest onto the Earth’s surface. How long does it take to hit the ground?
   b. The same rock falls the same distance from rest on the Moon’s surface. How long does it take to hit the ground on the Moon?

3. a. What vertical forces act on a space-rocket during lift-off?
   b. If a space-rocket has a mass of $2 \times 10^6$ kg and its engines produce a thrust of $30 \times 10^4$ N on lift-off, calculate its acceleration.
   c. An astronaut in the rocket has a mass of 80 kg. Calculate
      i. The vertical contact force between him and his seat during lift-off,
      ii. How many times greater this force is than it was before lift-off.

4. a. A lift of 500 kg in a lift-shaft is raised and lowered by a vertical cable attached to its roof. Find the tension in the cable if
   i. It moves with constant velocity,
   iii. Its upward acceleration is $1.5 \text{ m s}^{-2}$,
   iv. Its upward deceleration is $1.5 \text{ m s}^{-2}$,
   v. Its downward acceleration is $1.5 \text{ m s}^{-2}$
   b. What assumptions are you making about the lift and its cable?

5. An empty bottle of mass 0.25 kg is released from rest at a point 12 m below the surface of a deep well. The water in the well resists its motion with a vertical force of 1.2 N. It reaches the surface after 4 seconds. Calculate the upward force on the bottle due to the buoyancy of the water.

6. A parachutist with a mass of 60 kg jumps from rest in a stationary helicopter and free-falls for 5 seconds. The parachute then opens and she is subject to a constant vertical resisting force of 800 N until she reaches the ground with no speed. How far did she fall?

7. A bullet of mass 50 grams is fired into a fixed piece of wood at a speed of $120 \text{ m s}^{-1}$
   a. i. If it comes to rest after penetrating 9 cm, what is the resisting force of the wood on the bullet?
      ii. What assumption have you made about the wood?
   b. If the wood is only 2 cm thick, with what speed does the bullet emerge from the wood?

8. A train of mass $9 \times 10^5$ kg is slowing down on a straight track. It takes 20 s and 30 s to travel two successive 500 metres. Find the resisting force which is acting on the train if
   a. The engine is disengaged,
   b. The tractive force of the engine is 80 kN

9. Point P is 5 m above the surface of a deep lake. A stone of mass 8 kg falls from rest at P into the lake. On entering the lake, it experiences a vertical resistance to its motion equal to half its weight. Find the time taken from P for the stone to sink 5 m below the surface of the lake.
Common misconceptions / Exam tips

There are few aspects of this topic where additional misconceptions arise. This section is a specific example of the previous Section 8.2, Dynamics 1, and has fewer aspects that students might misunderstand.

- **Negative vectors**
  As in previous sections, students must ensure that correct signs are used when they represent forces and accelerations using vector notation. It helps to avoid errors if students decide, from the start, which direction is positive and hold to it, particularly when forces or accelerations are negative.

- **The gravitational constant**
  In those problems that involve motion on the Moon, or elsewhere beyond Earth, students have to take care with the use of the gravitational constant, \( g \); that is, the acceleration due to gravity. Students may forget that, although the mass of an object does not change, its weight depends on the value of \( g \) and so will vary from planet to planet.

**Exercise 8.3B Student Book commentary**

Several questions in this exercise use the same strategy as in Section 8.2; namely, using acceleration \( a \) as the link between the kinematics equations and \( F = ma \)

- **Question 1** Use a kinematic equation to find acceleration \( a \). The known quantities are \( u, t, s \), so choose \( s = ut + \frac{1}{2}at^2 \) (no \( v \)). Then write an equation of motion \( F = ma \) to find the tension.

- **Question 2** This is a 2-dimensional problem, so there are two equations. Vertical equilibrium gives one equation by resolving vertically. The other equation is a horizontal equation of motion, \( F = ma \)

- **Question 3** Remind students that mass \( m \) is constant across space, but that weight depends on the local value of \( g \). Use the equation \( W = mg \) on the Earth and on the Moon, but with different values of \( g \)

- **Question 4** The strategy here is the reverse of that in Question 1. Firstly, use \( F = ma \) to find the acceleration \( a \). Then, use the appropriate kinematic equation, which is \( v^2 = u^2 + 2as \)

- **Question 5** In the exam, a value for the acceleration due to gravity, \( g \), will be given. Discuss with students if there is any symmetry between the upward and downward motions: they are time reversals of one another. That is, if you filmed the ball being thrown up and then falling you could play the film backwards and see the ball move in a perfectly sensible way.

- **Question 6** Exam questions may ask students to identify any assumptions made in a question’s scenario and explain the effects on the answer of changing these assumptions.

- **Question 7** The effect of the steadyinig rope is the same as increasing \( g \) by 20%.

- **Question 8** Here it may be useful to reinforce the difference between mass and weight.

- **Question 9** The questions says the two tensions are the same, what would happen if they weren’t? The tray would start to rotate.

- **Question 10** Part a is the same strategy as for Question 2

- **Question 11** This question could usefully involve a whole-class discussion. Ask students for their strategy before they begin any calculations. The first step is to consider the time taken to hit the ground under free-fall. This time is the link with the controlled lowering using the string. In both situations, use a kinematic equation and so find the acceleration when being lowered to the ground. This acceleration then links with an equation of motion, \( F = ma \), for the lowering and so gives the tension in the string.
8.4 Systems of forces

Simplification

Example 1

An object of mass \( m \) is placed on a horizontal tray.

a Find an expression for the reaction \( R \) between the tray and the object if the tray travels

i Vertically upwards with an acceleration \( a \)

ii Vertically downwards with an acceleration \( a \)

b Under what conditions is the reaction \( R \) zero?

\[
\begin{align*}
\text{i Newton's second law,} \\
\text{applied upwards} \\
R - mg &= ma \\
R &= m(a + g) \\

\text{ii Newton's second law,} \\
\text{applied downwards} \\
mg - R &= ma \\
R &= m(g - a) \\

b \text{ R = 0 if the downward acceleration} a = g
\end{align*}
\]

When moving up, \( R > 0 \) for all positive values of \( a \). Note that, if \( a = 0 \), \( R = mg \), as expected.

Draw a diagram of the forces on the object: the reaction \( R \) and its weight \( mg \). Assume SI units are used throughout.

When moving down, \( R = 0 \) when \( a = g \). The tray and the mass are both falling freely under gravity and are not pressing on each other.

A man whose true weight is 75 N stands on bathroom scales in a moving lift and appears to weigh 70 N. What is the acceleration of the lift?

Let \( W = mg = \text{man's true weight} \)

\[
75 = m \times 9.8 \\
m = \frac{75}{9.8} = 7.65 \text{ kg}
\]

Use Newton's second law, assuming lift is accelerating upwards

\[
R - mg = ma \\
R = mg + ma \\
\Rightarrow R > mg, \text{ which is not the case.} \\
The assumption is wrong.
\]

Assuming lift is accelerating downwards

\[
mg - R = ma \\
75 - 70 = 7.65 \times a \\
a = \frac{5}{7.65} = 0.65 \text{ m/s}^2 \text{ downwards}
\]

First find the man's mass.

Start by assuming the lift is accelerating upward and draw a diagram.

The scales measure the reaction \( R \) between the man's feet and the scales on the lift's floor. The reaction \( R = 70 \text{ N} \).
Simplification questions 8.4

Take \( g \) as 9.8 m s\(^{-2} \) unless told otherwise.

1. A 200 kg metal box is being lifted onto a container ship by a crane. Find the tension in the cable of the crane when the box is moving vertically with
   a. A constant upward speed of 0.5 m s\(^{-1} \),
   b. An upward acceleration of 1.2 m s\(^{-2} \),
   c. Downward acceleration of 0.8 m s\(^{-2} \)

2. An object of mass 4 kg is placed on a horizontal tray. Find the reaction between the tray and the object if they move vertically with an acceleration of 2.0 m s\(^{-2} \)
   a. Upwards,
   b. Downwards.

3. A caravan of mass 750 kg is towed on a level road by a car of mass 1000 kg. Both vehicles experience forces opposing their motion due to air resistance of 200 N on the caravan and 100 N on the car. The tractive force supplied by the car’s engine is a constant 3600 N.
   Take \( g \) = 10 m s\(^{-2} \)
   a. Draw a diagram indicating all the external forces acting on the car and caravan.
   b. Use Newton’s second law to write a horizontal equation of motion for
      i. The car alone,
      ii. The caravan alone,
      iii. The car and caravan as one combined unit.
   c. Calculate
      i. The normal reaction between the car and the road,
      ii. The acceleration of the car and caravan,
      iii. The force in the tow bar.
   d. What other resistances to motion have you assumed to be negligible?

4. A car of mass 750 kg tows a trailer of mass 100 kg on a horizontal road. There are resistances to motion due to air friction which total 100 N for the car and 80 N for the trailer.
   If they move from rest with an acceleration of 0.6 m s\(^{-2} \),
   a. Calculate the tractive force supplied by the car’s engine,
   b. Is the force in the tow bar a tension or a thrust and what is its magnitude?

5. A passenger of mass 70 kg travels vertically upwards in a lift of mass 300 kg
   a. What is the tension in the lift’s cable if
      i. The acceleration is 1.5 m s\(^{-2} \),
      ii. They travel at a steady speed?
   b. What is the reaction between the passenger and the floor of the lift in both the situations in i and ii?

6. A man of mass 80 kg, travelling in a lift of mass 200 kg, is standing on a set of bathroom scales. What is the reading on the scales when they travel
   a. Upward at a constant speed of 3 m s\(^{-1} \),
   b. Downward at a constant speed of 3 m s\(^{-1} \),
   c. Upward with an acceleration of 0.2 m s\(^{-2} \),
   d. Downward with an acceleration of 0.2 m s\(^{-2} \)?

7. A 60 kg box rests on the floor of a 400 kg lift which is held by a vertical cable. Find the tension in the cable and the normal reaction between the box and the floor when the lift is
   a. At rest,
   b. Moving up with an acceleration of 1.5 m s\(^{-2} \),
   c. Moving up at constant velocity,
   d. Moving down with constant velocity,
   e. Accelerating downwards at 2.0 m s\(^{-2} \)

8. Two particles P and Q, both of mass 4 kg, are attached to the two ends of a taut string which passes over a smooth pulley at the edge of a smooth table. P rests on the table and Q hangs freely below the pulley.
   a. After P and Q are released from rest, find
      i. The tension in the string,
      ii. The acceleration of P and Q.
   b. Name three assumptions you have made in your model of the situation.
   c. If the table and pulley were rough, how would your answers to part a change?
A crane uses a chain to lift two boxes, one below the other, with an acceleration of 2 m s\(^{-2}\). The chain is fixed to the upper box of mass 20 kg. The lower box of mass 10 kg is connected by a rope to the upper box. Find the tensions in the chain and rope.

Since there are two unknowns you will need two equations. Using \(F = ma\), there are three equations available: for the lower box alone; for the upper box alone; and for both boxes together.

**Lower box**

Newton's second law
\[ S - 10g = 10 \times 2 \]
\[ S = 20 + 9.8 = 118 \text{ N} \]

**Upper box**

Newton's second law
\[ T - 5 - 20g = 20 \times 2 \]
\[ T = 40 + 196 + 118 = 354 \text{ N} \]
Tension in the rope = 118 N
Tension in the chain = 354 N

**Check - both boxes**

Newton's second law
\[ T - 20g - 10g = (20 + 10) \times 2 \]
\[ T = 60 + 196 + 98 = 354 \text{ N} \]

The tension \( S \) in the rope pulls down on the upper box and pulls upwards on the lower box. The tension \( T \) in the chain pulls upwards on the upper box.

Substitute for \( S \) from above.

When considering both boxes together, the two forces \( S \) on the diagram become an internal force. They cancel each other out. So the equation of motion for both boxes together does not involve \( S \).

A 10 kg mass on a smooth horizontal table is attached by two strings to masses of 8 kg and 4 kg hanging from smooth pulleys over opposite edges of the table. Find their acceleration.

Since it is heavier the 8 kg mass will fall, the 4 kg mass will rise and the 10 kg mass move to the right; all with the same acceleration. Since there are three unknowns, the two tensions and an acceleration, you need three equations.

Let \( a \) = the acceleration of the 10 kg mass to the right.

Newton's second law

- 8 kg mass downwards: \( gS - T = 8a \)  \( \text{(1)} \)
- 4 kg mass upwards: \( S - 4g = 4a \)  \( \text{(2)} \)
- 10 kg mass rightwards: \( T - S = 10a \)  \( \text{(3)} \)

\[ (gS - T) + (S - 4g) + (T - S) = 8a + 4a + 10a \]
\[ 4g = 22a \]
\[ a = \frac{4g}{22} = 1.8 \text{ m s}^{-2} \]

Smooth pulleys ensure \( T \) and \( S \) are constant in the strings.

Add the three equations to eliminate \( S \) and \( T \)

Note that the 10 kg mass is in vertical equilibrium, so \( R = 10g \), but this does not affect the horizontal motion.
Extension questions 8.4

1 Two particles of mass 2 kg and 3 kg are attached to the two ends of an inextensible string passing over a smooth fixed pulley so that both particles hang freely. They are released from rest with the 3 kg particle 1.2 m above a horizontal plane. Find
   a Their common acceleration,
   b The time taken for the heavier particle to hit the plane.

2 Two point masses $m$ and $M$ (with $m < M$) are connected by a light string which passes over a smooth pulley fixed to a ceiling. The masses are released from rest.
   a Show that the acceleration $a$ of the masses is given by $a = \frac{M - m}{M + m}g$
   b Find expressions for the tension in the string and the force holding the pulley to the ceiling.
   c What assumptions have you made in your model of this situation?

3 A mass $m$ is placed on a smooth table and attached to a taut string passing over a smooth pulley at the edge of the table. The other end of the string is attached to a mass $M$ which hangs freely. Find expressions in terms of $m$ and $M$ for the tension in the string and the force exerted on the pulley by the table.

4 A 10 kg mass is held by a spring balance fixed to the ceiling of a lift. The lift moves upwards and then downwards with acceleration $a$ m s$^{-2}$. The reading on the spring balance when moving up is double that when moving down.

Find the value of $a$

5 A train comprises an engine of 5000 kg and one carriage of mass 1000 kg. The forces resisting their motion are 2 kN and 1 kN respectively and the engine’s tractive force is 8 kN.
   a Calculate the tension in the coupling between the engine and the carriage.
   b When the train is travelling at a steady speed of 100 km h$^{-1}$, what is the magnitude to which the engine’s tractive force is reduced?

6 A train has an engine of mass 8 tonnes and two wagons, each of mass 1 tonne. The resistances to their motion are one-twentieth of their weight. The train accelerates at 0.4 m s$^{-2}$

Take $g$ as 10 m s$^{-2}$ and calculate
   a The tension in the coupling between
      i The two wagons,
      ii The engine and first wagon,
   b The driving force of the engine.

7 Two similar boxes X and Y of mass 8 kg are connected by a string passing over a fixed pulley. Inside box Y is a 2 kg mass Z.

a Find, in terms of $g$,
   i The acceleration of the boxes,
   ii The reaction between Y and Z.

b State three assumptions you have made about this model.

8 A 4 kg mass P on a smooth table is connected to a light inextensible string which passes over a smooth light pulley Q at the edge of the table and under a smooth moveable pulley R of mass 2 kg. The other end of the string is tied to a fixed point S. The lengths of the string from which R hangs are both vertical.

Calculate, in terms of $g$,
   a The accelerations of P and R,
   b The tension in the string.
### Common misconceptions / Exam tips

- **Resolving**
  Students may not have met the word 'resolve' as a specialised word used in mechanics. To resolve a force is to break it into two (usually) perpendicular components. The directions are often horizontal and vertical and, sometimes, a component is zero. When an object is in equilibrium, students can write, for example, 'resolve horizontally' which means they will balance the horizontal components of all the forces acting on the object.

- **True weight and apparent weight**
  Students readily understand that, when an object rests on a table or floor, resolving vertically gives the simple equation: reaction \( R = \text{weight}, \ W \). But they may not realise that, when the table or floor is accelerating vertically, as in a lift, Newton's second law, \( F = ma \) is needed instead of Newton's third law. Discuss the scenario of a person of weight \( W \) standing on bathroom scales in a lift which is accelerating up or down. The scales measure the reaction \( R \) between the feet and the floor, which is not the true weight \( W \) of the person. \( R \) is the apparent weight of the person. See the Example 2 in the simplification part of this section. (The quickest way to lose weight is not to diet, but to live in a lift which is accelerating downwards!)

- **Internal forces**
  When objects are joined together by tow bars or couplings, as when a car tows a trailer, the force in the tow bar or coupling affects both parts of the system: both the car and the trailer. Some students do not readily understand that this force becomes an internal force of the system when the equation of motion is written for the whole scenario as one unit — so this internal force does not appear in the equation. See Example 1 in the Extension part of this section.

### Exercise 8.4B Student Book commentary

**Question 1** Refer to Example 3, part a in this section of the Student Book.

**Question 2** Refer to Example 4 in this section of the Student Book. In part d, students should appreciate that the two tensions are pulling the pulley in the directions of the two strings, but they are being resisted by a reaction which holds the pulley in equilibrium. Being in equilibrium, the resultant force on the pulley is zero. The magnitude of the reaction is found using the triangle law of addition.

**Question 3** For parts a and b, refer to Example 4 in this section of the Student Book. For parts c and d, the appropriate kinematic equations are \( v = u + at \) and \( s = ut + \frac{1}{2}at^2 \) respectively.

**Question 4** For parts a and b, refer to Example 3 in this section of the Student Book. Part c needs \( v = u + at \) and part d needs both \( s = ut + \frac{1}{2}at^2 \) for the first part of the motion and \( v^2 = u^2 + 2as \) for the second — free fall — part of the motion.

**Question 5** Students may benefit from a whole-class discussion of a solution strategy for this two-part problem. Ask 'What links the two parts of the problem; that is, what variables have the same values before and after the departure of the 3 kg mass?' Answer, their distances above the ground (given) and their speed, which can be found in the first part of the problem.

So in the first part, find the tension and acceleration as in Example 3 part a in this section of the Student Book and then use a kinematic equation (\( u, s, a \) are known; \( v \) is needed) to find the speed. In the second part after the 3 kg has gone, use an equation of motion to find the new acceleration and a kinematic equation to find the time (\( u, s, a \) are known, \( t \) is needed).

**Question 6** This is a straightforward, algebraic version of Example 3 part a in this section of the Student Book. The distance \( s \) referred to in the problem is not needed in the solution.