Exam-style assessment

Motion in a straight line

1. The speed-time graph shown relates to a car travelling between two sets of traffic lights. The car accelerates from rest and reaches a speed of 20 m s\(^{-1}\) after 8 seconds. It maintains this constant speed until, at time \(T\) seconds, the driver applies the brakes to bring the car to rest at the next set of lights. The total journey takes 24 seconds.

![Speed-time graph](image)

(a) What assumption regarding the car’s acceleration has been made when drawing the graph? (1)

(b) Calculate the acceleration of the car during the first stage of the journey. (2)

(c) Given that the distance between the two sets of lights is 340 m, calculate
   (i) the value of \(T\) (5)
   (ii) the car’s acceleration during the final stage of the journey.

2. A train P, travelling at a constant 25 m s\(^{-1}\), is overtaken by a second train, Q, travelling at a constant 40 m s\(^{-1}\). At the instant that the trains are level, the driver of Q disengages the engine, as a result of which the train undergoes a constant acceleration of \(-0.5\) m s\(^{-2}\).

Calculate
   (a) the furthest distance ahead of P reached by Q (6)
   (b) the length of time before the trains are again level. (3)

3. A runner starts the last 400 m lap of a distance race running at a constant speed of 5 m s\(^{-1}\). After 30 s she realises that a rival is catching up, so she accelerates for 10 s to a speed of \(u\) m s\(^{-1}\), which she then maintains to the end of the race. The whole lap takes her 60 s.

![Speed-time graph](image)
Calculate
(a) the distance she covers in the first 30 s (1)
(b) the value of \( u \) (3)
(c) her acceleration when she realises she is being overhauled. (2)

4. A racing car leaves the pit lane travelling at 90 km\( h^{-1} \). It moves with constant acceleration and 8 seconds later is travelling at 270 km\( h^{-1} \).

Calculate
(a) its acceleration in ms\(^{-2} \) (4)
(b) the distance it travels during this period. (3)

5. A ball is projected vertically upwards from ground level at a speed of 35 ms\(^{-1} \).

(a) Find the speed of the ball after 2 seconds. (2)
(b) Calculate the maximum height reached by the ball. (2)
(c) Find the length of time for which the ball is more than 50 m above the ground. (4)
(d) What modelling assumption have you made in calculating your results? If this assumption were not justified, what effect would you expect this to have on your results for parts a and b? (2)

6. A train starts from rest at station A. It accelerates uniformly to a speed of 20 ms\(^{-1} \), then maintains this speed for a time before braking uniformly to rest at station B. The distance AB is 2 kilometres and the whole journey takes 130 seconds. The magnitude of the train’s acceleration is twice that of its deceleration.

(a) Sketch a speed-time graph to show this journey. (2)
(b) Calculate how long the train spends travelling at 20 ms\(^{-1} \). (2)
(c) Find the acceleration of the train. (3)

7. A car is travelling along a motorway at a speed of \( U \) ms\(^{-1} \) when the driver observes that the matrix signs are warning of a hold-up ahead. Four seconds after noticing this, she decides to reduce speed, and over the next six seconds brakes uniformly so that her speed reduces to 18 ms\(^{-1} \).

(a) Sketch a speed-time graph to illustrate this 10-second period. (2)
(b) During the given period the car covers 320 m. Calculate
(i) the value of \( U \) (5)
(ii) the deceleration of the car.
### Question Scheme Marks

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1.</strong> (a)</td>
<td>Acceleration is constant in each stage.</td>
<td>B1</td>
</tr>
<tr>
<td>(b)</td>
<td>$\frac{20}{8} = 2.5 \text{ ms}^{-2}$</td>
<td>M1 A1</td>
</tr>
<tr>
<td>(c)</td>
<td>(i) Area of trapezium = 340 m</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$\frac{24 + (T - 8)}{2} \times 20 = 340$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$16 + T = 34$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$T = 18 \text{ s}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>(ii) $-\frac{20}{6} = -3\frac{1}{3} \text{ ms}^{-2}$</td>
<td>M1 A1ft</td>
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<tr>
<td><strong>2.</strong> (a)</td>
<td>Furthest distance when Q speed is $25 \text{ ms}^{-1}$</td>
<td>M1</td>
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<tr>
<td></td>
<td>Use $v = u + at$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>$25 = 40 - 0.5t$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$t = 30$</td>
<td>M1</td>
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<tr>
<td></td>
<td>For P: $s = ut \Rightarrow s = 25 \times 30 = 750 \text{ m}$</td>
<td>M1</td>
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<tr>
<td></td>
<td>For Q: $s = \frac{(u + v)t}{2} \Rightarrow s = \frac{(40 + 25) \times 30}{2} = 975 \text{ m}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$975 - 750 = 225 \text{ m}$</td>
<td>M1 A1</td>
</tr>
<tr>
<td>(b)</td>
<td>P: $s = 25t$ and Q: $s = 40t - 0.25t^2$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Solve: $25t = 40t - 0.25t^2$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$0 = 60t - t^2 = t(60 - t)$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$t = 60 \text{ s}$</td>
<td>M1</td>
</tr>
<tr>
<td><strong>3.</strong> (a)</td>
<td>$30 \times 5 = 150 \text{ m}$</td>
<td>B1</td>
</tr>
<tr>
<td>(b)</td>
<td>Area of trapezium + Area of rectangle = 250 m</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$\frac{105 + u}{2} + 20u = 250$</td>
<td>M1</td>
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<tr>
<td></td>
<td>$25 + 5u + 20u = 250$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$25u = 225$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$u = 9 \text{ ms}^{-1}$</td>
<td>M1</td>
</tr>
<tr>
<td>(c)</td>
<td>Using $v = u + at$:</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$9 = 5 + 10a$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$10a = 4$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$a = 0.4 \text{ ms}^{-2}$</td>
<td>A1</td>
</tr>
</tbody>
</table>
4. (a) Using $v = u + at$:

$270 = 90 + 8a$

$8a = 180$

$a = 22.5$

Converting: $\times 1000 \div 3600 \Rightarrow a = 6.25 \text{ ms}^{-2}$

(b) Converting speeds: $u = 25 \text{ ms}^{-1}$, $v = 75 \text{ ms}^{-1}$

Using $s = \frac{(u + v)t}{2}$:

$s = \frac{(25 + 75) \times 8}{2} = 400 \text{ m}$

[If formula using acceleration is used, M1 A1ft]

5. (a) Using $v = u + at$:

$v = 35 - 9.8 \times 2 = 15.4 \text{ ms}^{-1}$

(b) Using $v^2 = u^2 + 2as$:

$0 = 35^2 - 2 \times 9.8 \times s \Rightarrow s = 62.5 \text{ m}$

(c) Using $s = ut + \frac{1}{2}at^2$:

$50 = 35t - 4.9t^2$

$49t^2 - 350t + 500 = 0$

$t = 5.169 \text{ or } 1.974$

Time is $5.169 - 1.974 = 3.20 \text{ s}$

(d) Air resistance is negligible. The speed would be less in (a) and the height would be less in (b).

6. (a) Area of trapezium = 2000 m

$\frac{(130 + t) \times 20}{2} = 2000 \Rightarrow t = 70 \text{ s}$

(b) Calculate $T$: $(130 - 2T) - T = 70 \Rightarrow T = 20 \text{ s}$

So acceleration is 1 ms$^{-2}$
7. (a)

(b) (i) Area under graph = 320 m

\[ 18 \times 10 + \frac{(10 + 4)(u - 18)}{2} = 180 + 7u - 126 = 54 + 7u \]

\[ = 320 \Rightarrow u = 38 \text{ ms}^{-1} \]

(ii) Using \( v = u + at \): \( 18 = 38 + 6a \Rightarrow a = 3.33 \text{ ms}^{-2} \)
### Vectors

1. A swimmer, who can swim at 2 ms\(^{-1}\) in still water, leaves a point A on the bank of a river 50 m wide and flowing at 1 ms\(^{-1}\). The point B is directly opposite A on the other bank.

   (a) The swimmer sets a course directly towards B.
      (i) Draw a vector diagram to show the velocities of the swimmer and the river.
      (ii) Calculate her resultant speed.
      (iii) Find how far downstream of B she will reach the opposite bank. (5)

   (b) If the swimmer wished to reach the opposite bank at B
      (i) in what direction should she set her course?
      (ii) how long will she take to cross? (5)

2. A ship has an initial position vector (-5i + 3j) km relative to a lighthouse. The ship is moving with constant velocity (2i - j) kmh\(^{-1}\). (Take East and North to be the directions of the i and j unit vectors respectively).

   (a) Find an expression for the position vector of the ship at time \(t\) hours. (2)

   (b) Find the time at which the ship is due east of the lighthouse. (2)

   (c) Find the distance and bearing of the ship from the lighthouse after 5 hours. (5)

3. Two people, A and B, are running across a field. At a certain instant A has position vector (10i + 20j) m relative to an origin O, and B is 30 m due east of A. A is running with constant velocity (i + 3j) ms\(^{-1}\) and B is running due north at a speed of 6 ms\(^{-1}\). At time \(t\) seconds their position vectors are \(r_A\) and \(r_B\). (Take East and North to be the directions of the i and j unit vectors respectively).

   (a) Find \(r_A\) and \(r_B\) in terms of \(t\). (4)

   (b) Find the vector \(\overrightarrow{AB}\) at time \(t\). (2)

   (c) Find the bearing of B from A when \(t = 5\). (4)

   (d) Find when the two people are again 30 m apart. (3)

4. A particle P, of mass 2 kg, is moving with velocity (-i + 4j) ms\(^{-1}\) when a force of (2i - 4j) N is applied. After \(t\) seconds its velocity is \(v\) ms\(^{-1}\).

   (a) Find the acceleration vector of P. (2)

   (b) Express \(v\) in terms of \(t\). (2)

   (c) Find the value of \(t\) when P is moving parallel to the vector i. (1)

   (d) Find the speed and direction of motion of P when \(t = 4\). (4)
5. Two particles, A and B, are moving with constant velocities on a horizontal plane. B has velocity $2\mathbf{i}$ m s$^{-1}$. Initially A has position vector $(3\mathbf{i} + \mathbf{j})$ m relative to an origin O, and B has position vector $(-2\mathbf{i} + 4\mathbf{j})$ m. Three seconds later, A has position vector $(6\mathbf{i} + 10\mathbf{j})$ m.

At time $t$ seconds, the position vectors of the particles are $\mathbf{r}_A$ and $\mathbf{r}_B$ respectively and the particles are a distance $d$ m apart.

(a) Find the velocity vector of A. (2)
(b) Find $\mathbf{r}_A$ and $\mathbf{r}_B$ in terms of $t$. (4)
(c) Find the vector $\mathbf{AB}$ in terms of $t$. (2)
(d) Find the value of $t$ for which the line joining the particles makes an angle of 45° with the $i$-direction. (2)
(e) Show that $d^2 = 10t^2 - 28t + 34$, and hence find the value of $t$ when the particles are 15 m apart. (5)

6. Two ships, A and B, are moving relative to an origin O. East and North correspond to the $i$ and $j$ directions. The position vectors of the ships are $\mathbf{r}_A$ and $\mathbf{r}_B$ respectively.

Initially, $\mathbf{r}_A = (-2\mathbf{i} + 4\mathbf{j})$ km and $\mathbf{r}_B = (4\mathbf{i} - 3\mathbf{j})$ km. Ship A is moving with constant velocity $(2\mathbf{i} + \mathbf{j})$ km h$^{-1}$, and ship B with constant velocity $v\mathbf{j}$ km h$^{-1}$.

(a) Find expressions for $\mathbf{r}_A$ and $\mathbf{r}_B$ at time $t$ hours. (4)
(b) Five hours after the start, an observer at O sees the ships “one behind the other” along the same line of sight. Find
   (i) the value of $v$ (ii) the bearing of both ships from O at that instant. (6)
(c) Find the vector $\mathbf{AB}$ at time $t$ hours. Hence find
   (i) the time at which A is due east of B (ii) the distance between the ships when $t = 6$. (6)
### Exam-style mark scheme

#### Vectors

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<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
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<tbody>
<tr>
<td><strong>1. (a)</strong></td>
<td><img src="image" alt="Diagram" /></td>
<td></td>
</tr>
<tr>
<td>(i)</td>
<td></td>
<td></td>
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<tr>
<td>(ii)</td>
<td>$v^2 = 1^2 + 2^2 \Rightarrow v = 2.24 \text{ ms}^{-1}$</td>
<td>M1 A1</td>
</tr>
<tr>
<td>(iii)</td>
<td>2:50 is the same ratio as 1: $x \text{ so } x = 25 \text{ m}$</td>
<td>M1 A1</td>
</tr>
<tr>
<td>(b)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(i)</td>
<td>$\sin^{-1} \frac{1}{2} = 30^\circ$ upstream to AB</td>
<td>M1 A1</td>
</tr>
<tr>
<td>(ii)</td>
<td>Resultant speed: $2^2 - 1^2 = 1.73 \text{ ms}^{-1}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Time: $t = \frac{d}{s} \Rightarrow t = \frac{50}{1.73} = 28.9 \text{ s}$</td>
<td>M1 A1</td>
</tr>
<tr>
<td><strong>2. (a)</strong></td>
<td>$(-5\mathbf{i} + 3\mathbf{j}) + (2\mathbf{i} - \mathbf{j}) t = ((2t - 5)\mathbf{i} + (3 - t)\mathbf{j}) \text{ km}$</td>
<td>M1 A1</td>
</tr>
<tr>
<td>(b)</td>
<td>$\mathbf{j}$ vector = 0 at $t = 3\text{s}$, where $\mathbf{i}$ vector = 1</td>
<td>M1 A1</td>
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<tr>
<td>(c)</td>
<td>Substitute $t = 5$: Vector $(5\mathbf{i} - 2\mathbf{j}) \text{ km}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Distance: $\sqrt{5^2 + 2^2} = 5.39 \text{ km}$</td>
<td>M1 A1 ft</td>
</tr>
<tr>
<td></td>
<td>Bearing: $90^\circ + \tan^{-1} \frac{2}{5} = 111.8^\circ$</td>
<td>M1 A1 ft</td>
</tr>
<tr>
<td><strong>3. (a)</strong></td>
<td>$\mathbf{r}_A = 10\mathbf{i} + 20\mathbf{j} + (\mathbf{i} + 3\mathbf{j})t = (t + 10)\mathbf{i} + (3t + 20)\mathbf{j}$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>$\mathbf{r}_B = 40\mathbf{i} + 20\mathbf{j} + (6\mathbf{j})t = 40\mathbf{i} + (6t + 20)\mathbf{j}$</td>
<td>M1 A1</td>
</tr>
<tr>
<td>(b)</td>
<td>$\mathbf{r}_B - \mathbf{r}_A = 40\mathbf{i} + (6t + 20)\mathbf{j} - [(t + 10)\mathbf{i} + (3t + 20)\mathbf{j}] = (30 - t)\mathbf{i} + 3t\mathbf{j}$</td>
<td>M1 A1 ft</td>
</tr>
<tr>
<td>(c)</td>
<td>$\mathbf{A}\mathbf{B} = 25\mathbf{i} + 15\mathbf{j}$ at $t = 5$</td>
<td>M1</td>
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<tr>
<td></td>
<td>Angle: $\tan^{-1} \frac{25}{15} = 59^\circ$</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>Bearing 059°</td>
<td>A1</td>
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<tr>
<td>(d)</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>$(30 - t)^2 + (3t)^2 = 30^2$</td>
<td>M1</td>
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<tr>
<td></td>
<td>$900 - 60t + t^2 + 9t^2 = 900$</td>
<td>M1</td>
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<tr>
<td></td>
<td>$10t^2 - 60t = 0$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$10t(t - 6) = 0$</td>
<td>M1 simplification</td>
</tr>
<tr>
<td></td>
<td>So time = 6 seconds</td>
<td>A1</td>
</tr>
</tbody>
</table>

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Mechanics M1
4. (a) Using $F = ma$: \(2i - 4j = 2a \Rightarrow a = (i - 2j) \text{ ms}^{-2}\)

(b) Using $v = u + at$:
\[
v = (\mathbf{i} - 4\mathbf{j}) + (\mathbf{i} - 2\mathbf{j})t = (1 - 1)\mathbf{i} + (4 - 2t)\mathbf{j}
\]

(c) $\mathbf{j}$ component $= 0$ so $t = 2$ s.

(d) At $t = 4$: Speed: $\sqrt{3^2 + (-4)^2} = 5 \text{ ms}^{-1}$
Direction: $\tan^{-1}\left(\frac{-4}{3}\right) = -53.1^\circ$ (53.1° clockwise from +ve $\mathbf{i}$)

5. (a) Velocity $= \text{change in position} \div \text{time} = \frac{3\mathbf{i} + 9\mathbf{j}}{3} = (\mathbf{i} + 3\mathbf{j}) \text{ ms}^{-1}$

(b) $\mathbf{r}_A = (3\mathbf{i} + \mathbf{j}) + (\mathbf{i} + 3\mathbf{j})t = (t + 3)\mathbf{i} + (3t + 1)\mathbf{j}$
$\mathbf{r}_B = (-2\mathbf{i} + 4\mathbf{j}) + (2t)\mathbf{i} = (2t - 2)\mathbf{i} + 4\mathbf{j}$

(c) $\mathbf{r}_B - \mathbf{r}_A = (2t - 2)\mathbf{i} + 4\mathbf{j} - [(t + 3)\mathbf{i} + (3t + 1)\mathbf{j}] = (t - 5)\mathbf{i} + (3 - 3t)\mathbf{j}$

(d) $t - 5 = 3 - 3t \Rightarrow t = 2$ s

(e) $d^2 = (t - 5)^2 + (3 - 3t)^2 = t^2 - 10t + 25 + 9 - 18t + 9t^2 = 10t^2 - 28t + 34$
Solve quadratic: $10t^2 - 28t + 34 = 10t^2 - 28t - 191 = 0$
$t = 3.19$ or $5.99$
and taking positive value gives $t = 5.99$ s

6. (a) $\mathbf{r}_A = (-2\mathbf{i} + 4\mathbf{j}) + (2\mathbf{i} + \mathbf{j})t = (2t - 2)\mathbf{i} + (t + 4)\mathbf{j}$
$\mathbf{r}_B = (4\mathbf{i} - 3\mathbf{j}) + (v\mathbf{j})t = 4\mathbf{i} + (vt - 3)\mathbf{j}$

(b) (i) At $t = 5$, A is at $8\mathbf{i} + 9\mathbf{j}$
For B to be in line, $(5v - 3) = 4.5 \Rightarrow v = 1.5$

(ii) Direction: $\tan^{-1}\frac{9}{8} = 48.4^\circ$ from positive $\mathbf{i}$, hence bearing 041.6°

(c) $\mathbf{r}_B - \mathbf{r}_A = 4\mathbf{i} + (1.5t - 3)\mathbf{j} - [(2t - 2)\mathbf{i} + (t + 4)\mathbf{j}] = (6 - 2t)\mathbf{i} + (0.5t - 7)\mathbf{j}$
(i) $\mathbf{j}$ component must be zero so $t = 14$ hours
(ii) At $t = 6$, vector is $-6\mathbf{i} - 4\mathbf{j}$
Magnitude: $\sqrt{(-6)^2 + (-4)^2} = 7.21 \text{ km}$
1. An object A, of mass 50 kg, is supported by two ropes. The first is fixed at point B, as shown, and the second passes over a pulley at C, on the same level as B, and is then fixed to the ground at D.

\[ \text{B} \quad \text{C} \quad \text{D} \]

\[ \text{A} \]

The rope sections AB and AC make angles of 60° and 40° respectively with the vertical.

(a) State one assumption you might reasonably make about

(i) the rope  
(ii) the pulley  
(iii) the object A.  

(b) Draw a diagram showing the forces acting at A.

(c) Calculate the tension in the rope section

(i) AB  
(ii) AC  
(iii) CD.

2. The diagram shows an object of mass 4 kg resting on a rough plane inclined at 30° to the horizontal. The coefficient of friction between the object and the plane is \( \mu \).

A horizontal force, \( P \), is applied to the object, as shown. When \( P = 40 \), the object is on the point of moving up the plane.

(a) Find the value of \( \mu \).

(b) Find the new value of \( P \).
3. The diagram shows a light string AC. The end A is fixed to a wall and a particle of weight 30 N is attached to C. A second particle, of weight $W$ N, is attached part way along the string at B.

![Diagram](image)

A horizontal force, $P$ N, is applied to C so that the system rests in equilibrium with the sections AB and BC of the string making angles of 30° and 60° respectively with the vertical, as shown.

(a) By drawing a force diagram, or otherwise, show that the tension in the section BC of the string is 60 N, and find the value of $P$. (4)

(b) Explain why the tension in the section AB of the string is $2P$ N. (2)

(c) Calculate the value of $W$. (2)

4. An object of weight $W$ N rests on a rough plane inclined at 30° to the horizontal. A force of $P$ N, applied parallel to the plane as shown, is just sufficient to prevent the object from sliding down the plane. The coefficient of friction between the object and the plane is $\mu$.

(a) Show that $P = W(\sin 30° - \mu \cos 30°)$. (3)

The force acting on the object is doubled and it is then on the point of sliding up the plane.

(b) Write down a second equation connecting $P$ and $\mu$. (1)

(c) Hence show that $P = \frac{1}{3}W$ and find the value of $\mu$. (4)
5. The diagram shows a particle, A, of weight 60 N, which is held in equilibrium by two strings, AB and AC. AB is attached to the wall and is horizontal. AC is attached to the ceiling and makes an angle of \( \alpha \) with the horizontal.

![Diagram of particle A with strings AB and AC](image)

The tension in AC is twice that in AB.

(a) Find the value of \( \alpha \). 
(b) Calculate the tensions in the strings.

6. The diagram shows a smooth ring, A, of weight \( W \) N, threaded onto a string of length 1.8 m. The ends of the string are attached to points B and C on the same level. A force of \( P \) N is applied to the ring so that it rests in equilibrium with the section AC of the string vertical. In this position AC = 0.8 m.

![Diagram of smooth ring A with force P](image)

(a) What can you deduce about the tensions in AB and AC from the modelling assumption that the ring is smooth?
(b) Find the tensions in AB and AC in terms of \( W \).
(c) Show that \( P = \frac{1}{3}W \).

7. The diagram shows three cables attached to the top of a telegraph pole. They exert horizontal forces of 150 N, 120 N and 200 N as shown.

![Diagram of telegraph pole with cables](image)

(a) Find, in terms of \( \mathbf{i} \) and \( \mathbf{j} \), the resultant force acting on the pole.
A support cable is to be attached to the pole. It will connect the top of the pole to the ground and will ensure that the horizontal forces acting on the top of the pole are in equilibrium.

(b) Calculate the magnitude and direction of the horizontal force which the support cable must exert on the top of the pole. (5)

(c) Calculate the tension in the support cable if it makes an angle of 70° with the ground. (2)
<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
</table>
| 1. (a)          | (i) Light, inextensible  
(ii) Smooth  
(iii) A particle – zero size | B1 B1 B1 |
| (b)             | ![Diagram](image) | D2 |
| (c)             | (i) (ii) $T_1 \sin 60^\circ = T_2 \sin 40^\circ$  
$T_1 \cos 60^\circ + T_2 \cos 40^\circ = 50g$  
Since $\cos 60^\circ = 0.5$,
$T_1 = 2(50g - T_2 \cos 40^\circ)$  
$\Rightarrow 2 \sin 60^\circ(50g - T_2 \cos 40^\circ) = T_2 \sin 40^\circ$  
Solving for tensions gives: $T_2 = 431$ N, $T_1 = 320$ N | M1 M1 M1 A1 A1 B1ft |
| (iii) 431 N     |        |       |
| 2. (a)          | Resolve parallel to plane: $P \cos 30^\circ = F + 4g \sin 30^\circ$  
Resolve perpendicular to plane: $R = P \sin 30^\circ + 4g \cos 30^\circ$  
Friction: $F = \mu R$  
Substitute for $R$ into parallel equation using friction: $P \cos 30^\circ = \mu(P \sin 30^\circ + 4g \cos 30^\circ) + 4g \sin 30^\circ$  
Substitute for $P = 40$ and $g = 9.8$  
Solve for $\mu = \frac{40 \cos 30^\circ - 39.2 \sin 30^\circ}{40 \sin 30^\circ + 39.2 \cos 30^\circ} = 0.279$ | M1 M1 M1 A1 |
| (b)             | Parallel equation becomes: $P \cos 30^\circ + F = 4g \sin 30^\circ$  
Solve for $P$ in $P \cos 30^\circ + 0.279(P \sin 30^\circ + 4g \cos 30^\circ) = 4g \sin 30^\circ$  
$P = \frac{4g \sin 30^\circ - 0.279 \times 4g \cos 30^\circ}{\cos 30^\circ + 0.279 \sin 30^\circ} = 10.1$ N (using exact value for $\mu$) | M1 M1ft |

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3. (a) Resolving vertically at C:
\[ 30 = T_{BC} \cos 60^\circ \Rightarrow T_{BC} = 60 \text{ N} \]
Resolving horizontally at C:
\[ P = 60 \sin 60^\circ = 52.0 \text{ N} \]
(b) If the tension in AB is \( T \), then resolving horizontally for the whole system gives \( T \cos 60^\circ = P \), and hence \( T = 2P \).
(c) Resolving vertically at B:
\[ 2P \cos 30^\circ = W + 60 \cos 60^\circ \Rightarrow W = 60.1 \text{ N} \]
4. (a) Friction \( F \) and normal reaction \( R \) give \( F = \mu R = \mu W \cos 30^\circ \)
Resolving up the slope gives \( P + F - W \sin 30^\circ = 0 \)
Substituting for \( F \) gives \( P = W(\sin 30^\circ - \mu \cos 30^\circ) \), as required.
(b) \( 2P = W(\sin 30^\circ + \mu \cos 30^\circ) \)
(c) Adding the two equations gives \( P = \frac{1}{3}W \), as required.
Substituting for \( P \) in (b):
\[ \frac{2}{3}W = W(\sin 30^\circ + \mu \cos 30^\circ) \Rightarrow \mu = \frac{\frac{2}{3} - \sin 30^\circ}{\cos 30^\circ} = 0.192 \]
5. (a) Resolving horizontally at A:
\[ T_{AB} = T_{AC} \cos \alpha \Rightarrow \cos \alpha = T_{AB} / 2T_{AB} = 0.5 \Rightarrow \alpha = 60^\circ \]
(b) Resolving vertically at A:
\[ T_{AC} \sin 60^\circ = 60 \Rightarrow T_{AC} = 69.28 \text{ N} \]
\[ T_{AC} = 2T_{AB} \Rightarrow T_{AB} = 34.64 \text{ N} \]
6. (a) The tensions are equal.
(b) If \( \theta = BAC \), \( \cos \theta = \frac{4}{5} \)
\[ T_{AC} + T_{AB} \times \frac{4}{5} = W \Rightarrow \frac{9}{5}T = W \Rightarrow T = \frac{5W}{9} \text{ N} \]
(c) \( \sin \theta = \frac{3}{5} \Rightarrow P = T_{AB} \times \frac{3}{5} = \frac{5W}{9} \times \frac{3}{5} = \frac{1}{3}W \)
7. (a) \( i : 200 \cos 25^\circ + 120 \cos 60^\circ - 150 \cos 75^\circ = 202.4 \text{ N} \)
\( j : 150 \sin 75^\circ + 120 \sin 60^\circ - 200 \sin 25^\circ = 164.3 \text{ N} \)
Resultant Force: \( (202.4i + 164.3j) \text{ N} \)
(b) Magnitude: \( \sqrt{202.4^2 + 164.3^2} = 260.7 \text{ N} \)
Direction: \( \tan^{-1} \frac{164.3}{202.4} = 39.1^\circ \)
But acts down to the left so \( 39.1 - 180 = -140.9^\circ \) to positive i.
(c) Magnitude is horizontal component so \( \frac{260.7}{\cos 70^\circ} = 762 \text{ N} \)
1. A body of mass \( m \) kg is projected from a point A up the line of greatest slope of a plane inclined at 20° to the horizontal, as shown. The initial speed of the body is 10 m s\(^{-1}\).

\[ \begin{align*}
\text{A} & \quad \text{20°} \\
10 \text{ms}^{-1} & \\
\end{align*} \]

(a) Assuming that the plane is smooth, find
(i) the acceleration of the body
(ii) the distance it travels up the plane before coming instantaneously to rest.  

In fact the plane is rough. The coefficient of friction between the body and the plane is \( \mu \). The body comes to rest after 9 m.

(b) Calculate the value of \( \mu \).  

(c) Show that the body slides back down the plane, arriving at A with speed 4.55 m s\(^{-1}\).

2. Bodies of mass 2 kg and 3 kg are attached to either end of a light inextensible string passing over a smooth fixed pulley. Initially the system is held at rest with the 2 kg body on the ground and the 3 kg body suspended 4 m above the ground, as shown. The system is then released.

\[ \begin{align*}
\text{3kg} & \quad \text{4m} \\
\text{2kg} & \\
\end{align*} \]

(a) Calculate
(i) the acceleration of the bodies
(ii) the tension in the string.  

(b) In what way have you used the assumption that
(i) the string is inextensible
(ii) the pulley is smooth?  

The 3 kg body strikes the ground and does not rebound.

(c) Find the greatest height above the ground reached by the 2 kg body.
3. A body of mass 5 kg rests on a rough horizontal plane. The coefficient of friction between the body and the plane is \( \mu \). A force of \( P \) N acts at 30° to the horizontal, as shown.

![Diagram of 5 kg body with force P acting at 30°](image)

(a) Initially \( P = 25 \) and the body is in limiting equilibrium. Calculate the value of \( \mu \). (4)

(b) The value of \( P \) is now increased to 40. Find the acceleration of the body. (4)

4. A car of mass 900 kg is towing a trailer of mass 100 kg along a horizontal road. There are resistance forces of 60 N and 20 N acting on the car and the trailer respectively.

![Diagram of car towing trailer](image)

(a) The engine applies a driving force of 480 N. Calculate
(i) the acceleration of the car and trailer
(ii) the tension in the towbar. (4)

(b) The brakes are now applied, bringing the car to a halt from a speed of 10 ms\(^{-1}\) in a distance of 50 m. Find
(i) the magnitude of the braking force
(ii) the force in the towbar. (6)

5. A particle of mass \( m \) kg is moving with constant speed 10 ms\(^{-1}\) across a smooth horizontal surface when it encounters a rough area, 20 m wide, where the coefficient of friction is 0.2.

Calculate
(a) the acceleration of the particle (3)
(b) the speed with which the particle leaves the rough area (2)
(c) the length of time it takes for the particle to cross the rough area. (2)

6. Particles A and B, of mass 4 kg and 3 kg respectively, are connected together by means of a light inextensible string. Particle A rests on a smooth plane inclined at 40° to the horizontal. The string passes over a smooth pulley at the top of the slope and particle B hangs freely, as shown.

![Diagram of particles A and B](image)
The system is released from rest.

(a) Write down an equation of motion for each of the two particles. (3)

(b) Hence calculate
   (i) the acceleration of the system
   (ii) the tension in the string. (4)

(c) Explain how you used the fact that the string is inextensible. (1)

7. A man of mass 80 kg stands in a lift of mass 300 kg which is raised and lowered on a cable. The lift accelerates upwards at 0.5 ms\(^{-2}\), travels for a while at a constant speed, then decelerates to rest at 1 ms\(^{-2}\).

   Calculate
   (a) the tension in the cable during the first stage of the motion (2)
   (b) the reaction force between the man and the floor of the lift during the second and third stages of the motion. (3)

   The cable must not be subjected to a tension of more than 10000 N. The average person has a mass of 80 kg and the lift always accelerates upwards at 0.5 ms\(^{-2}\).

   (c) Calculate the maximum number of people who should use the lift at the same time. (4)

8. A car, of mass 700 kg, is towing a trailer, of mass 100 kg. The (constant) resistance forces on the car and trailer are 200 N and 140 N respectively.

   (a) Initially it accelerates at 0.7 ms\(^{-2}\). Calculate
      (i) the driving force, \(P\), of the engine
      (ii) the tension, \(T\), in the towbar. (4)

   (b) When the car and trailer reach a speed of 28 ms\(^{-1}\), the trailer breaks free. Assuming that the driving force and the resistances remain the same, calculate
      (i) the time taken for the trailer to come to rest
      (ii) the distance between the car and the trailer at that moment. (9)

9. Annabel and Mihir are standing 21 m apart on an ice rink. Each holds one end of a light inextensible rope. Annabel has mass 50 kg and Mihir 100 kg.

   At a certain instant, Annabel starts to “haul in” the rope, so that it is maintained at a constant tension of 70 N as the two people get closer together.
(a) Making the modelling assumption that the ice rink is smooth, find when and where the two people would collide with each other. (7)

In fact the coefficient of friction between each of the two people and the ice is 0.1.

(b) Explain what effect this will have on the motion of
   (i) Annabel (3)
   (ii) Mihir. (3)

(c) Find when and where they will collide with each other.
## Newton's laws of motion

### Question Scheme Marks

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
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</table>
| 1. (a)          | (i) Using \( F = ma \) : \( -mg \sin 20^\circ = ma \) \( \Rightarrow \) \( a = -g \sin 20^\circ = -3.35 \) ms\(^{-2} \)  
(ii) Using \( v^2 = u^2 + 2as \) :  
\[ 0 = 100 - 2 \times 3.35s \Rightarrow s = 14.9 \text{ m} \] | M1  
| | Calculate \( a \) using \( v^2 = u^2 + 2as \) :  
\[ 0 = 100 + 2a \times 9 \Rightarrow a = -5.56 \text{ ms}^{-2} \] | M1  
| | Now \( -mg \sin 20^\circ - F = ma \) where \( a = -5.56 \) and \( F = \mu R \)  
Solve:  
\[ -mg \sin 20^\circ - \mu mg \cos 20^\circ = -5.56m \]  
\[ \mu = \frac{5.56 - g \sin 20^\circ}{g \cos 20^\circ} = 0.239 \]  
(c)  
\[ mg \sin 20^\circ - 0.239 mg \cos 20^\circ = ma \Rightarrow a = 1.15 \text{ ms}^{-2} \]  
\[ v^2 = 2as = 2 \times 1.15 \times 9 = 20.7 \Rightarrow v = 4.55 \text{ ms}^{-1} \] | M1 A1ft  
| | | 15 |
| 2. (a) | (i)(ii) \( 3g - T = 3a \) and \( T - 2g = 2a \)  
Adding gives \( g = 5a \) \( \Rightarrow \) \( a = 1.96 \) ms\(^{-2} \)  
\[ T = 2a + 2g = 23.5 \text{ N} \] | M1  
| | (b) | M1 A1ft  
| | (i) Both bodies have the same acceleration  
(ii) The tensions are the same | B1  
| | (c) Using \( v^2 = u^2 + 2as \), \[ v^2 = 2 \times 1.96 \times 4 \Rightarrow v = 3.96 \text{ ms}^{-1} \]  
Extra distance travelled by 2 kg mass:  
\[ 0 = 15.68 - 2gs \Rightarrow s = 0.8 \text{ m} \]  
Greatest height: \( 4 + 0.8 = 4.8 \text{ m} \) | M1 A1ft  
| | | 12 |
| 3. (a) | Resolving:  
\[ P \sin 30^\circ + R = 5g \Rightarrow R = 36.5 \]  
\[ P \cos 30^\circ = F = \mu R \]  
\[ \mu = \frac{25 \cos 30^\circ}{36.5} = 0.593 \] | M1 A1  
| | (b) Resolving vertically and using N2L:  
\[ 40 \sin 30^\circ + R = 5g \Rightarrow R = 29 \]  
\[ 40 \cos 30^\circ - 0.593 \times 29 = 5a \Rightarrow a = 3.49 \text{ ms}^{-2} \] | M1 A1  
| | | 8 |
4. (a) (i) Considering as a single particle:
Using \( F = ma \Rightarrow 480 - 80 = 1000a \Rightarrow a = 0.4 \text{ ms}^{-2} \)

(ii) Considering car only:
\[ 480 - 60 - T = 900 \times 0.4 \Rightarrow T = 60 \text{ N} \]

(b) (i) Deceleration using \( v^2 = u^2 + 2as \)
\[ 0 = 100 + 100a \Rightarrow a = -1 \text{ ms}^{-2} \]
Using N2L on whole system: \( F + 80 = 1000 \times 1 \Rightarrow F = 920 \text{ N} \)

(ii) Considering car only:
\[ 920 + 60 + T = 900 \times 1 \Rightarrow T = -80 \text{ N i.e. 80 N Thrust} \]

5. (a) Using N2L:
\[ -\mu R = ma \]
\[ -0.2mg = ma \Rightarrow a = -0.2g = -1.96 \text{ ms}^{-2} \]

(b) Using \( v^2 = u^2 + 2as : \)
\[ v = \sqrt{100 - 2 \times 1.96 \times 20} = 4.65 \text{ ms}^{-1} \]

(c) Using \( v = u + at \) (or equivalent):
\[ 4.65 = 10 - 1.96t \Rightarrow t = 2.73 \text{ s} \]

6. (a) For A: \( T - 4g \sin 40^\circ = 4a \)
For B: \( 3g - T = 3a \)

(b) (i) Adding equations: \( 7a = 3g - 4g \sin 40^\circ = 4.20 \Rightarrow a = 0.60 \text{ ms}^{-2} \)
(ii) Substituting \( a \) into either equation: \( T = 27.6 \text{ N} \)

(c) The accelerations of the two particles are equal.

7. (a) Using N2L: \( T - 380g = 380 \times 0.5 \Rightarrow T = 3914 \text{ N} \)

(b) Constant speed: \( R = 80g = 784 \text{ N} \)
During deceleration: \( 80g - R = 80 \times 1 \Rightarrow R = 704 \text{ N} \)

(c) Using \( T = 10000 \text{N}: 10000 - (300 + 80n)g = (300 + 80n) \times 0.5 \)
Solving for \( n: \)
\[ 10000 - 300g - 150 = 40n + 80ng \]
\[ 6910 = n(40 + 80g) \]
\[ n = 6910 \div 824 = 8.39 \]
No more than 8 average-sized people should use the lift.

8. (a) (i) Using N2L on whole system: \( P - 340 = 800 \times 0.7 \Rightarrow P = 900 \text{ N} \)
(ii) Considering car only: \( 900 - 200 - T = 700 \times 0.7 \Rightarrow T = 210 \text{ N} \)

(b) (i) Using N2L: \(-140 = 100a \Rightarrow a = -1.4 \text{ ms}^{-2} \)
Then using \( v = u + at: 0 = 28 - 1.4t \Rightarrow t = 20 \text{ s} \)
(ii) \( s \) for trailer: \( s = 28 \times 20 - 0.5 \times 1.4 \times 20^2 = 280 \text{ m} \)
For the car: \( a = \frac{\left(900 - 200\right)}{700} = 1 \text{ ms}^{-2} \)
This means \( s \) for car: \( s = 28 \times 20 + 0.5 \times 1 \times 20^2 = 760 \text{ m} \)
Distance between them is therefore 480 m
9. (a) Annabel’s acceleration: \(70 = 50a\) \(\Rightarrow a = 1.4 \text{ ms}^{-2}\)
Mihir’s acceleration: \(70 = 100a\) \(\Rightarrow a = 0.7 \text{ ms}^{-2}\)
Since accelerations in the ratio 2:1, distance travelled by skaters will be in same ratio, hence Annabel travels 14 m, Mihir 7 m.
\[14 = \frac{1}{2} \times 1.4 \times t^2.\] Taking the positive root, \(t = 4.47 \text{ s}\)

(b) (i) Annabel has resistance force \(F = \mu R\) where \(R = 50 \text{ g}\)
New acceleration: \(70 - 0.1 \times 50g = 50a\) \(\Rightarrow a = 0.42 \text{ ms}^{-2}\)
(ii) Mihir’s max resistance force is \(0.1 \times 100g\) which is greater than 70 N, He will remain stationary.

(c) They will collide where Mihir is standing.
For Annabel: \(s = \frac{1}{2}at^2\) \(\Rightarrow 21 = 0.2t^2\) \(\Rightarrow t^2 = 100\)
Taking the positive root, \(t = 10 \text{ seconds}\).
1. Three particles, A, B and C, of mass 0.2 kg, 1.0 kg and 0.5 kg respectively, are at rest in a straight line on a smooth horizontal plane, as shown.

An impulse of magnitude 4 Ns is applied to the particle A in the direction AB.

(a) Calculate the speed with which A starts to move. (2)

Particle A collides with particle B. After the collision, particle A is travelling in the opposite direction at 10 ms\(^{-1}\).

(b) Find the velocity of particle B after the collision. (2)

Particle B now collides with particle C and the two become stuck together.

(c) Show that the common velocity of B and C is 4 ms\(^{-1}\). (2)

2. A particle A, of mass 3 kg and travelling at a speed of 8 ms\(^{-1}\), collides with a second particle, B, of mass 5 kg and travelling at a speed of 6 ms\(^{-1}\).

(a) The two particles become stuck together. Calculate their common velocity if, before the collision they were travelling

(i) in the same direction

(ii) in opposite directions. (4)

(b) In the case where they were travelling in opposite directions, calculate the magnitude of the impulse particle A receives as a result of the collision. (2)

3. Three particles, A, B and C, have masses of 3 kg, 2 kg and \(m\) kg. They lie in a straight line on a smooth horizontal plane. B is stationary, and A and C are moving towards B with speeds of 3 ms\(^{-1}\) and 1 ms\(^{-1}\) respectively, as shown.

A strikes B first, and after the collision A is travelling in the same direction but with its speed reduced to 1 ms\(^{-1}\).

(a) Calculate the velocity of B after the collision. (2)

B and C now collide and become stuck together. Their common velocity after the collision is \(v\) ms\(^{-1}\).

(b) Find \(v\) in terms of \(m\). (2)

(c) Given that there is a third collision between the particles, show that \(m > 2\). (3)
4. A particle of mass 0.2 kg slides over a horizontal plane towards a vertical wall, along a line perpendicular to the wall. Its initial speed is 20 ms\(^{-1}\). There is a frictional resistance force of 1 N.

(a) The particle takes 2 seconds to reach the wall. Calculate
   (i) the distance it travels to reach the wall
   (ii) the speed with which it hits the wall. (6)

As a result of colliding with the wall the particle receives an impulse of 3 Ns.

(b) Calculate the speed with which the particle rebounds from the wall. (2)

(c) Assuming that the resistance force continues to act, find how far from the wall the particle comes to rest. (2)

5. A particle of mass 2 kg is moving with constant velocity \(\mathbf{u}\). Initially its position vector relative to an origin O is \((\mathbf{i} + 6\mathbf{j})\) m. Three seconds later its position vector is \((7\mathbf{i} + 3\mathbf{j})\) m.

(a) Find the velocity of the particle. (2)

It then receives an impulse of \((-10\mathbf{i} + 6\mathbf{j})\) Ns.

(b) Find
   (i) the speed of the particle after a further 2 seconds
   (ii) the position of the particle at that time. (6)

6. Particles A and B, of mass 0.2 kg and 0.5 kg respectively, are moving directly towards each other on a smooth horizontal surface. Particle A is moving with speed 4 ms\(^{-1}\) and particle B at 2 ms\(^{-1}\). As a result of the collision with B, particle A has its speed reduced to 2 ms\(^{-1}\) and its direction of travel reversed.

(a) Find the speed and direction of motion of particle B after the collision. (3)

(b) Calculate the magnitude of the impulse exerted by particle A on particle B during the collision. (2)

(c) Given that the particles were in contact for 0.04 seconds, find the magnitude of the average force exerted by particle A on particle B. (2)
## Impulse and momentum

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
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<tbody>
<tr>
<td>1. (a)</td>
<td>Impulse = change in momentum</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4 = mv \Rightarrow v = \frac{4}{0.2} = 20 \text{ ms}^{-1}</td>
<td>M1 A1</td>
</tr>
<tr>
<td>(b)</td>
<td>Conservation of momentum:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.2 \times 20 = 0.2 \times (-10) + 1 \times v_B \Rightarrow v_B = 6 \text{ ms}^{-1}</td>
<td>M1 A1</td>
</tr>
<tr>
<td>(c)</td>
<td>6 \times 1 = 1.5v \Rightarrow v = 4 \text{ ms}^{-1}</td>
<td>M1 A1</td>
</tr>
<tr>
<td>2. (a)</td>
<td>(i) Conservation of momentum:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 \times 8 + 5 \times 6 = 8v \Rightarrow v = 6.75 \text{ ms}^{-1} in the same direction</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>(ii) 3 \times 8 + 5 \times (-6) = 8v \Rightarrow v = -0.75 \text{ ms}^{-1} i.e. 0.75 \text{ ms}^{-1} in the direction of B.</td>
<td>M1 A1</td>
</tr>
<tr>
<td>(b)</td>
<td>Impulse = change in momentum:</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>3 \times 8 - 3 \times (-0.75) = 26.25 Ns</td>
<td></td>
</tr>
<tr>
<td>3. (a)</td>
<td>Conservation of momentum:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 \times 3 = 3 \times 1 + 2v \Rightarrow v = 3 \text{ ms}^{-1}</td>
<td>M1 A1</td>
</tr>
<tr>
<td>(b)</td>
<td>2 \times 3 - 1 \times m = (2 + m)v \Rightarrow v = \frac{6 - m}{2 + m}</td>
<td>M1 A1</td>
</tr>
<tr>
<td>(c)</td>
<td>v must be less than 1</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>This gives: \frac{6 - m}{2 + m} &lt; 1 \Rightarrow 6 - m &lt; 2 + m \Rightarrow m &gt; 2</td>
<td>M1 A1</td>
</tr>
<tr>
<td>4. (a)</td>
<td>(i) Using N2L: -1 = 0.2a \Rightarrow a = -5 \text{ ms}^{-2}</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>Using s = ut + \frac{1}{2}at^2 \Rightarrow s = 20 \times 2 - 2.5 \times 4 = 30 \text{ m}</td>
<td>M1 A1</td>
</tr>
<tr>
<td>(ii)</td>
<td>Using v = u + at \Rightarrow v = 20 - 5 \times 2 = 10 \text{ ms}^{-1}</td>
<td>M1 A1</td>
</tr>
<tr>
<td>(b)</td>
<td>Impulse = change in momentum:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 = 0.2 \times 10 - 0.2v \Rightarrow v = -5 \text{ ms}^{-1} (i.e. 5 \text{ ms}^{-1} away from the wall)</td>
<td>M1 A1</td>
</tr>
<tr>
<td>(c)</td>
<td>Using v^2 = u^2 + 2as :</td>
<td></td>
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<tr>
<td></td>
<td>0 = 25 - 10s \Rightarrow s = 2.5 \text{ m}</td>
<td>M1 A1</td>
</tr>
</tbody>
</table>

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5. (a) Velocity = change in position/time:
\[
[(7\mathbf{i} + 3\mathbf{j}) - (\mathbf{i} + 6\mathbf{j})] \div 3 = (2\mathbf{i} - \mathbf{j}) \text{ ms}^{-1}
\]
(b) Impulse = change in momentum:
\[
(-10\mathbf{i} + 6\mathbf{j}) = 2\mathbf{v} - 2(2\mathbf{i} - \mathbf{j}) \Rightarrow 2\mathbf{v} = -6\mathbf{i} + 4\mathbf{j} \Rightarrow \mathbf{v} = (-3\mathbf{i} + 2\mathbf{j}) \text{ ms}^{-1}
\]
Speed = \sqrt{(-3)^2 + 2^2} = \sqrt{13} = 3.61 \text{ ms}^{-1}
(ii) \quad 7\mathbf{i} + 3\mathbf{j} + 2(-3\mathbf{i} + 2\mathbf{j}) = (\mathbf{i} + 7\mathbf{j}) \text{ m}

6. (a) Using conservation of momentum:
\[
0.2 \times 4 + 0.5 \times (-2) = 0.2 \times (-2) + 0.5v \Rightarrow v = 0.4 \text{ ms}^{-1}
\]
Speed is 0.4 ms\(^{-1}\). It is opposite to its original direction.
(b) Impulse = change in momentum:
\[
0.5 \times 0.4 - 0.5 \times (-2) = 1.2 \text{ Ns}
\]
(c) \[
\frac{1.2}{0.04} = 30 \text{ N}
\]
1. A uniform beam AB of mass 40 kg and length 3 m, rests on two supports, one at A and the other at C, where BC = 1 m, as shown.

A load of mass 10 kg is attached to the beam at B.

(a) Draw a diagram showing the forces acting on the beam. (2)
(b) Calculate the magnitude of the reaction forces at A and C. (4)

The load at B is now increased to \( m \) kg, as a result of which the beam is on the point of tipping about C.
(c) Calculate \( m \). (3)

2. A uniform beam AB of mass 20 kg and length 4 m rests on a support at A and is held in a horizontal position by a vertical string attached to the beam at B. A load of mass 8 kg is placed on the beam at C, where AC = 1 m.

(a) (i) Calculate the reaction at the support at A. (4)
(ii) Find the tension in the string.

The string has a breaking strain of 170 N.
(b) How close to B could the 8 kg load be moved without breaking the string? (4)

3. The diagram shows a wheelbarrow standing on horizontal ground. The total weight of the barrow and its load is 500 N, which acts through a point 0.4 m from the wheel and 0.6 m from the leg of the barrow. The handles protrude 0.5 m from the leg.

(a) Calculate the reaction force between
(i) the ground and the wheel (ii) the ground and the leg. (4)

(b) A vertical force of \( P \) N is applied to the end of the handles. Find the value of \( P \) if as a result the leg is on the point of lifting from the ground. (3)
4. A non-uniform beam AB, of length 4 m and weight 240 N, is placed on two supports a distance of 3 m apart.

Initially the end A is placed on one of the supports. In this position the reaction forces at the two supports are equal.

(a) Find the distance of the centre of gravity of the beam from A. (1)

The beam is then moved so that the end B is on one of the supports.

(b) Find the reaction forces at the supports in this position. (4)

(c) With the beam in the second position, what is the weight of the heaviest particle which could be placed at A without tipping the beam? (2)

5. A uniform rod AB, of mass 3 kg and length 1 m, rests in a horizontal position. It is held in equilibrium by two supports, one above the rod at C and one below the rod at D, where AC = 10 cm and AD = 30 cm. A load of mass 5 kg hangs from the end B of the rod.

(a) Calculate the forces at the supports C and D. (4)

(b) If the coefficient of friction between the rod and the supports is 0.6, calculate the least force, applied at B in the direction BA, which would cause the rod to move. (2)

(c) Would the force needed to maintain the motion increase, decrease or remain the same as the rod moved in the direction BA? Explain your answer. (1)
### 1. (a)

![Diagram showing forces and distances](image)

Taking moments about A:
\[40g \times 1.5 + 10g \times 3 = R_c \times 2\]
\[882 = 2R_c\]
\[R_c = 441 \text{ N}\]

Resolving vertically:
\[R_A + R_C = 40g + 10g\]
\[R_A = 50g - 441\]
\[R_A = 49 \text{ N}\]

(b) Taking moments about A:
\[40g \times 1.5 + 10g \times 3 = R_c \times 2\]
\[882 = 2R_c\]
\[R_c = 441 \text{ N}\]

(c) \(R_A = 0\) on point of tipping about C.
Taking moments about C:
\[40g \times 0.5 = (mg) \times 1\]
\[20g = mg\]
\[m = 20 \text{ kg}\]

### 2. (a)

(i) Taking moments about B:
\[(20g \times 2) + (8g \times 3) = 4R \Rightarrow R = 156.8 \text{ N}\]

(ii) Resolving vertically: \(R + T = 28g \Rightarrow T = 117.6 \text{ N}\)

(b) If \(T = 170 \text{ N}, R = 28g - 170 \Rightarrow 104.4 \text{ N}\)
Taking moments about B:
\[104.4 \times 4 - 20g \times 2 = 8gx\]
\[x = 25.6 \div 8g = 0.327 \text{ m from B}\]

### 3. (a)

(i) Taking moments about the leg:
\[500 \times 0.6 = R_w \times 1 \Rightarrow R_w = 300 \text{ N}\]

(ii) Taking moments about the wheel:
\[500 \times 0.4 = R_L \times 1 \Rightarrow R_L = 200 \text{ N}\]

(b) Reaction at leg = 0:
Taking moments about wheel:
\[500 \times 0.4 = P \times 1.5 \Rightarrow P = 133 \frac{1}{3} \text{ N}\]
4. (a) By consideration of symmetry, 1.5 m from end A.

(b) Taking moments about B:
\[ 240 \times 2.5 = 3R_{Support} \Rightarrow R_{Support} = 200 \text{ N} \]
By resolving vertically, \( R_B = 240 - 200 = 40 \text{ N} \)

(c) Reaction force at B must = 0. Taking moments about other support:
\[ 240 \times 0.5 = 1 \times P \Rightarrow P = 120 \text{ N} \]

5. (a) Taking moments about D:
\[ 5g \times 0.7 + 3g \times 0.2 = R_C \times 0.2 \Rightarrow R_C = 201 \text{ N} \]
Resolving vertically:
\[ R_C + 8g = R_D \Rightarrow R_D = 279 \text{ N} \]

(b) Using \( F = \mu R \):
\[ F = 0.6(201 + 279) = 288 \text{ N} \]

(c) Reaction forces decrease so friction forces decrease and the applied force needed decreases.