1. The flowchart shows an algorithm.

(a) Trace the operation of the algorithm shown in the flowchart, recording your results in a copy of the table below.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Print out</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1, 1</td>
</tr>
</tbody>
</table>

(b) Explain what the flowchart achieves.

2. Copper piping is supplied in 240 cm lengths. A plumber needs to cut the following pieces (all lengths in cm):
   40, 60, 40, 110, 50, 40, 140, 40, 150, 60, 30, 50, 180, 50, 100

(a) Use the first-fit algorithm to devise a cutting plan. State the number of 240 cm lengths required and the amount of piping wasted.

(b) Show that the first-fit decreasing algorithm gives a more efficient cutting plan.

(c) Explain how you know that it is not possible to improve on the cutting plan found in part b.
3. A list of names is recorded as follows:
   Sam, Pat, Tim, Lee, Ben, Ali, Mat, Val, Fay
(a) Explain why it is not possible to perform a binary search of the list in its present form. (1)
(b) Use a suitable algorithm to reconfigure the list so that a binary search can be conducted. (4)
(c) Carry out a binary search of your revised list for (i) Sam (ii) Nan. (5)

4. Carry out a quick sort to arrange the following list in ascending order. Indicate the pivots you use at each stage.
   5, 20, 14, 15, 8, 13, 11, 17, 8, 9, 12 (5)

5. The flowchart shows how a loan of £P, borrowed at R% interest, is paid off in annual instalments of £A.

   Using a copy of the table below, trace the operation of the flowchart with starting values P = 1000, R = 5 and A = 350.

<table>
<thead>
<tr>
<th>A</th>
<th>R</th>
<th>P</th>
<th>T</th>
<th>Print out</th>
</tr>
</thead>
<tbody>
<tr>
<td>350</td>
<td>5</td>
<td>1000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   (5)
6. This list of numbers is to be sorted into descending order.
   42, 60, 65, 30, 27, 38, 22, 21, 24, 24, 20, 11, 16
   (a) (i) Perform three passes of the bubble sort algorithm.
   (ii) Explain how you know after the third pass that the list is in order. (5)
   The numbers correspond to the lengths, in cm, of pieces which Sara wants to cut from cables of length 1 m.
   (b) (i) Use the first-fit decreasing algorithm to find a possible cutting plan.
   How many 1 m cables would Sara need using this plan?
   Sara notices that the total length of the pieces is exactly 4 m. She uses up two cables completely by cutting lengths of 65, 24 and 11 from the first and 60, 24 and 16 from the second.
   (ii) Show that she can complete the task with just two more cables. (4)

7. The table shows a class list of 13 children, in alphabetical order, together with their ages.

<table>
<thead>
<tr>
<th></th>
<th>Name</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Adams (A)</td>
<td>13y 5m</td>
</tr>
<tr>
<td>2</td>
<td>Cayley (C)</td>
<td>12y 10m</td>
</tr>
<tr>
<td>3</td>
<td>Dare (D)</td>
<td>12y 6m</td>
</tr>
<tr>
<td>4</td>
<td>Fernandez (F)</td>
<td>13y 0m</td>
</tr>
<tr>
<td>5</td>
<td>Gilliam (G)</td>
<td>13y 3m</td>
</tr>
<tr>
<td>6</td>
<td>Horowitz (H)</td>
<td>12y 8m</td>
</tr>
<tr>
<td>7</td>
<td>Klee (K)</td>
<td>12y 10m</td>
</tr>
<tr>
<td>8</td>
<td>Moss (M)</td>
<td>13y 9m</td>
</tr>
<tr>
<td>9</td>
<td>Nugent (N)</td>
<td>13y 4m</td>
</tr>
<tr>
<td>10</td>
<td>Page (P)</td>
<td>12y 11m</td>
</tr>
<tr>
<td>11</td>
<td>Ridout (R)</td>
<td>13y 11m</td>
</tr>
<tr>
<td>12</td>
<td>Sebag (S)</td>
<td>13y 1m</td>
</tr>
<tr>
<td>13</td>
<td>Wilson (W)</td>
<td>13y 2m</td>
</tr>
</tbody>
</table>

   (a) (i) List the names in the order in which they would appear after one pass of a bubble sort to put them in ascending age order (initial letters only need be written).
   (ii) What is the maximum number of comparisons that would be needed to perform a complete bubble sort of a list of 13 items? (4)
   (b) Perform a binary search of the original list to find the age of (i) Page (ii) Joshi. (6)

8. Here is a list of numbers:
   27, 45, 16, 32, 38, 49, 28, 52, 17
   (a) Sort the list of numbers into ascending order using the quick sort algorithm.
       Indicate clearly the pivots you use and show the state of the list after each pass. (4)
   On a school theatre trip, the serial numbers of the programmes the students bought were as given in the list. In the interval it was announced that programme numbers 17 and 46 had won first and second prize respectively in the lucky programme competition.
   (b) Perform binary searches of your sorted list to decide if anyone from the school had won a prize. (5)
9. Draught-proofing tape is supplied in 2 metres lengths. George wants to cut pieces with lengths, in cm, as follows:

80, 40, 70, 50, 130, 40, 60, 100, 110, 50, 120, 120

(a) Use the first-fit algorithm to devise a possible recording plan. (2)

(b) Use the quick sort algorithm to reconfigure the list so that the first-fit decreasing algorithm may be employed. (5)

(c) Use the first-fit decreasing algorithm to produce an improved recording plan. (2)
### Question 1

(a) | A | B | C | Print out |
---|---|---|---|----------|
1  | 1 | 1 | 0 |          |
1  | 1 | 1 | 1 | 1, 1     |
8  | 7 | 2 | 2 | 8        |
27 | 19| 3 | 3 | 27       |
64 | 37| 4 | 4 | 64       |
125| 61| 5 | 5 | 125      |
216| 91| 6 | 6 | 216      |

(b) The algorithm prints the integers and their cubes.

**Marks:**
- M1 next line
- M1
- A1 print out column correct

### Question 2

(a) Length 1: 40, 60, 40, 50, 40
Length 2: 110, 40, 60, 30
Length 3: 140, 50, 50
Length 4: 150
Length 5: 180
Length 6: 100

6 lengths needed, 300 cm waste.

(b) Length 1: 180, 60
Length 2: 150, 60, 30 (re-sort)
Length 3: 140, 100
Length 4: 110, 50, 50
Length 5: 50, 40, 40, 40, 40

5 lengths needed, 60 cm waste.

(c) Total of required lengths is 1140 cm.
   This is more than $4 \times 240$, so 5 lengths required.

**Marks:**
- M1
- A1, A1
- M1 implied re-sort
- M1
- E1
- E1

### Question 3

(a) Binary search requires a sorted list.

(b) Use of Bubble Sort or Quick Sort.

Bubble sort:
- SPTLBAMVF
- PSLBAMTFV
- PLBAMSFTV
- LBAMPFSTV
- BALMFPSTV
- ABLMPFTSV
- ABFLMPSTV
- ABFLMPSTV

No swaps in this pass, so list is in order.

**Marks:**
- E1
- M1 must be seen.
- M1 A1 A1 for either
Quick sort:

**SPTABAMVF**

The pivot is B, giving **AB** **SPTLVMVF**
The pivot is L, giving **ABFL** **SPTMV**
The pivot is T, giving **ABFLSP** **MTV**
The pivot is P, giving **ABFLM** **PSTV**

(c) (i) Compare with Mat, search in **PSTV**
    Compare with Tim, search in **PS**
    Compare with Sam – found.
(ii) Compare with Mat, search in **PSTV**
    Compare with Tim, search in **PS**
    Compare with Sam, search in **P**
    Compare with Pat.
    Search list now empty, so Nan not in list.

4.  
5 20 14 15 8 13 11 17 8 9 12  
The pivot is 13, giving 5 8 11 8 9 12 13 20 14 15 17  
The pivots are 8 and 15, giving 5 8 8 11 9 12 13 14 15 20 17  
The pivots are 8, 9 and 17, giving 5 8 8 9 11 12 13 14 15 17 20  
The pivot is 12, giving 5 8 8 9 11 12 13 14 15 17 20

5.  

<table>
<thead>
<tr>
<th>A</th>
<th>R</th>
<th>P</th>
<th>T</th>
<th>Print out</th>
</tr>
</thead>
<tbody>
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<td>1000</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>1050</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>700</td>
<td>1</td>
<td>1, 700</td>
</tr>
<tr>
<td></td>
<td></td>
<td>735</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>385</td>
<td>2</td>
<td>2, 385</td>
</tr>
<tr>
<td></td>
<td></td>
<td>404.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>54.25</td>
<td>3</td>
<td>3, 54.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>56.9625</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td>4</td>
<td>4, 0</td>
</tr>
</tbody>
</table>

(a) (i)  

| Pass 1 | 60 | 65 | 42 | 30 | 27 | 38 | 22 | 21 | 24 | 24 | 20 | 11 | 16 | 5  |
| Pass 2 | 65 | 60 | 42 | 38 | 30 | 27 | 24 | 24 | 22 | 21 | 20 | 16 | 11 | 5  |
| Pass 3 | 65 | 60 | 42 | 38 | 30 | 27 | 24 | 24 | 22 | 21 | 20 | 16 | 11 | 5  |

(ii) There were no swaps in the third pass so the list is in the right order.
| (b) | (i) 5 cables:  
|     | Cable 1: 65, 30  
|     | Cable 2: 60, 38  
|     | Cable 3: 42, 27, 24  
|     | Cable 4: 24, 22, 21, 20, 11  
|     | Cable 5: 16  
| (ii) | Cable 3: 42, 38, 20  
|     | Cable 4: 30, 27, 22, 21  
|     | (ii) $12 + 11 + 10 + \ldots + 2 + 1 = 78$  
| (b) | (i) 7 Klee, search 8 – 13  
|     | 11 Ridout, search 8 – 10  
|     | 9 Nugent, search 10  
|     | 10 Page – found.  
| (ii) | 7 Klee, search 1 – 6  
|     | 4 Fernandez, search 5 – 6  
|     | 6 Horowitz, search now empty, name not found.  
| 8. (a) | 27 45 16 32 38 49 28 52 17  
|     | Pivot is 38.  
|     | 27 16 32 28 17 38 45 49 52  
|     | Pivots 32 and 49.  
|     | 27 16 28 17 32 38 45 49 52  
|     | Pivots 28.  
|     | 27 16 17 28 32 38 45 49 52  
|     | Pivot 16  
|     | 16 27 17 28 32 38 45 49 52  
|     | Pivot 17  
|     | 16 17 27 28 32 38 45 49 52  
| (b) | Compare 32, search 16 – 28  
|     | Compare 27, search 16 – 17  
|     | Compare 17, found  
|     | Compare 32, search 38 – 52  
|     | Compare 49, search 38 – 45  
|     | Compare 45, sublist now empty, number not found.  
| 9. (a) | Tape 1: 80, 40, 70  
|     | Tape 2: 50, 130  
|     | Tape 3: 40, 60, 100  
|     | Tape 4: 110, 50  
|     | Tape 5: 120  
|     | Tape 6: 120  

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(b) 80 40 70 50 130 40 60 100 110 50 120 120
   Pivot 60
   80 70 130 100 110 120 120 60 40 50 40 50
   Pivots 100 and 40
   130 110 120 120 80 70 60 50 50 40 40 40
   Pivots 120, 70 and 50
   130 120 120 110 100 80 70 60 50 50 40 40
   Pivot 120
   130 120 120 110 100 80 70 60 50 50 40 40

(c) Tape 1: 130, 70
    Tape 2: 120, 80
    Tape 3: 120, 60
    Tape 4: 110, 50, 40
    Tape 5: 100, 50, 40

   M1  A1
   A1
   A1
   A1
   M1  A1ft

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1. The network shown models the paths between features in an ornamental garden. The numbers on the arcs represent the walking distances, in metres, between the features. The owner wishes to install lighting along some of the paths so that at night visitors can access all the features along lighted paths.

(a) Use Kruskal’s algorithm to identify which paths should be lit to minimise the amount of lighting needed. List the order in which you choose the arcs and indicate any which you reject. (3)

(b) Before installing the lights, the owner decides that she wants the path from AD to be lit. Which of the paths from your solution to part (a) would you now leave out of the lighting scheme? Give reasons for your answer. (2)

2. The table gives the adjacency matrix for a network. The weights correspond to distances in metres.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>–</td>
<td>8</td>
<td>4</td>
<td>11</td>
<td>15</td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td>B</td>
<td>8</td>
<td>–</td>
<td>12</td>
<td>7</td>
<td>5</td>
<td>14</td>
<td>–</td>
</tr>
<tr>
<td>C</td>
<td>4</td>
<td>12</td>
<td>–</td>
<td>9</td>
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<td>–</td>
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<tr>
<td>D</td>
<td>11</td>
<td>7</td>
<td>9</td>
<td>–</td>
<td>10</td>
<td>–</td>
<td>8</td>
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<tr>
<td>E</td>
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<td>–</td>
<td>8</td>
<td>10</td>
<td>–</td>
<td>15</td>
<td>11</td>
</tr>
<tr>
<td>F</td>
<td>9</td>
<td>5</td>
<td>12</td>
<td>–</td>
<td>15</td>
<td>–</td>
<td>6</td>
</tr>
<tr>
<td>G</td>
<td>6</td>
<td>14</td>
<td>–</td>
<td>8</td>
<td>11</td>
<td>6</td>
<td>–</td>
</tr>
</tbody>
</table>

(a) Use Prim’s algorithm on this table to find the minimum spanning tree for the network. Start from vertex A and record in the top row the order in which you choose the vertices. State the total length of the minimum spanning tree. (4)

(b) Draw your minimum spanning on a copy of this diagram.
3. The diagram shows a network representing nine residence blocks on a university campus. The arcs correspond to footpaths between the blocks and the numbers on the arcs represent distances in metres.

The university wishes to install lighting so that it is possible to walk between any two locations along lit paths. They want to minimise the amount of lighting which needs to be installed.

(a) Use Prim’s algorithm, starting from location A, to find the paths which should be lit. State the order in which you choose the arcs and the total length of the minimum spanning tree. (3)

(b) After lights are installed, it is proposed to open a late evening shop at one of the residence blocks. Suggest, with reasons, the best location for the shop. (2)
### Minimum spanning trees

#### Question Scheme Marks

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (a)</td>
<td>Choose $EG$ 40 $BF$ 45 $AB$ 55 $BC$ 60 $ED$ 65</td>
<td>M1M1 A1</td>
</tr>
<tr>
<td></td>
<td>Reject $AC$ $EG$ 90 Total length 355 m</td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>Leave out $FG$. $AD$ forms a cycle with previous spanning tree so leave out the longest arc in that cycle.</td>
<td>A1 E1</td>
</tr>
<tr>
<td>2. (a)</td>
<td>Total length = 36 m</td>
<td>M1 M1 A1</td>
</tr>
<tr>
<td>(b)</td>
<td>Total length = 36 m</td>
<td></td>
</tr>
<tr>
<td>3. (a)</td>
<td>Order: $AC$, $CD$, $DH$, $DG$, $HI$, $IF$, $IE$, $FB$. Total length = 1350 m</td>
<td>M2 A1</td>
</tr>
<tr>
<td>(b)</td>
<td>Locate at $H$. The furthest anyone has to walk is 540 m, all other locations would increase this.</td>
<td>A1 E1</td>
</tr>
</tbody>
</table>

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1. The diagram shows the times, in minutes, to travel by car between nine villages.

(a) Use Dijkstra's algorithm to find the least time needed to drive from $A$ to $I$. Show your working on a copy of the following diagram.

(b) List the vertices corresponding to the quickest route. Explain how you identified the route.

2. The diagram shows the bus routes between nine villages. The numbers on the arcs correspond to the bus fares, in pence.
(a) Use Dijkstra’s algorithm to find the cheapest route for a person wishing to travel by bus from A to I. Show your working on a copy of the diagram below. State the route and the total cost of travel.

(b) Use Kruskal’s algorithm to decide which routes should be subsidised and calculate the total cost of the subsidy.
### Question Scheme Marks

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
</table>
| 1. (a)          | ![Diagram](image1.png)  

Least time is 23 minutes. 
Route is $ABCEFHI$. 
Found by working back from $I$ and identifying edges where weight is the same as the difference between the labels. | M1 A1 A1 ft | |
| 2. (a)          | ![Diagram](image2.png)  

Route $ACFHI$ 
Total cost $= 190p = £1.90$ | M1 A1 A1 A1 | |

(b) $HI, CD, DE$ or $EH, EH$ or $DE, AB$ or $DF, DF$ or $AB$, reject $CF, AC$, Reject $BD$ and $HF$ and $EF, GI$ 
Total cost $= 330 \times 1000 = 330000p = £3300$ | A1 A1 | |
1. The diagram shows the main routes between eight towns. The numbers on the arcs correspond to the distances, in kilometres, between the towns. The total weight for the whole network is 271 km.

![Diagram of eight towns and their routes with distances labeled.]

The local council has decided that all the roads should be inspected for potholes. The inspection team will start and finish at the depot at town A.

(a) Explain why it is not possible to inspect all the roads without travelling more than once along some of the roads. (1)

(b) Showing your working carefully, identify which arcs should be repeated to allow the inspection in the minimum distance. Calculate the total length of the inspection route. (4)

(c) What effect would it have on your solution if the inspection team were to start at town A and finish at town H? (2)

2. (a) (i) Explain what is meant by an Eulerian graph.

(ii) State how you can tell if a given graph is Eulerian. (2)

(b) The diagram shows a network of paths in a park. The numbers on the arcs represent the lengths, in metres, of the paths. The total length of all the paths is 1970 m.

![Diagram of a park network with path lengths labeled.]

A worker needs to travel along every path at least once to collect litter. He starts and finishes at A.

(i) Use the route inspection algorithm to find the minimum distance the worker needs to travel. State the paths which will be traversed more than once.
It is decided to place a skip for the litter at some point other than A. The worker will start from A and travel each path at least once before putting the litter in the skip.

(ii) Where should the skip be placed so that the distance covered before depositing the litter is a minimum? Give a possible route that the worker could follow. (7)

3. The diagram shows the lengths, in kilometres, of the main roads between six towns. The total of all the distances is 32 km.

![Diagram of road network](image)

(a) Use Dijkstra’s algorithm to find the shortest route from A to E. State the route and its total length. (4)

(b) A political party worker wishes to drive along each road at least once in order to erect posters for an election.

(i) The party headquarters is at D. The worker wants to start and finish her task at headquarters. Find the distance she will need to travel, and state a possible route she could take.

(ii) Someone suggests she could do the job on her way home from work, and would then only need to travel 32 km. If she works at E, where does she live? Explain your answer. (5)

4. The table shows the lengths, in metres, of the paths connecting six hides at a nature reserve.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>–</td>
<td>12</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>15</td>
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<tr>
<td>B</td>
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<tr>
<td>C</td>
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<td>23</td>
</tr>
<tr>
<td>F</td>
<td>15</td>
<td>18</td>
<td>19</td>
<td>17</td>
<td>23</td>
<td>–</td>
</tr>
</tbody>
</table>

The nature trust wants to upgrade some of the paths to enable wheelchair access to all the hides.

(a) Use Prim’s algorithm on the table, starting from hide A, to decide on the most economical strategy. Show the order in which the paths are chosen, and state the total length of path which needs upgrading. (4)
5. The diagram shows the network of rail routes between seven locations. There are two sets of rails along each route. The numbers on the arcs correspond to the lengths, in kilometres, of the routes. The total length of all the routes is 90 km.

An inspection wagon needs to travel along all the routes to inspect the rails for defects.

(a) Each route must be travelled twice to inspect both sets of rails, starting and finishing at the depot at D. Explain how you know that no extra travel will be necessary, so that the total distance the inspection wagon travels is 180 km.

(b) Explain why some routes will still need to be travelled twice.

(c) Use the route inspection algorithm to decide which routes must be repeated. Calculate the total distance the wagon travels, and give a possible inspection route.
### Route inspection

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1.</strong> (a)</td>
<td>Odd vertices at A, F, G and H. Graph not Eulerian.</td>
<td>E1</td>
</tr>
<tr>
<td></td>
<td>(b) Pair off arcs.</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Working shown: ( AF + GH = 51, AG + FH = 50, AH + FG = 64 )</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>Best pairing ( AG + FH ) so repeat ( AC, CE, EG ) and ( FH ).</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Total distance = ( 271 + 50 = 321 ) km</td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td>Just repeat ( FE ) and ( EG ). Total distance = ( 271 + 29 = 300 ) km.</td>
<td>A1 A1</td>
</tr>
<tr>
<td><strong>2.</strong> (a)</td>
<td>(i) A route exists which travels every arc exactly once and returns to the start.</td>
<td>E1</td>
</tr>
<tr>
<td></td>
<td>(ii) All nodes have an even degree.</td>
<td>B1</td>
</tr>
<tr>
<td>(b)</td>
<td>(i) Odd nodes A, B, D, H</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Pairings ( AB + DH = 290, AD + BH = 390, AH + BD = 240 )</td>
<td>M1 A1</td>
</tr>
<tr>
<td></td>
<td>Best pairing ( (AH + BD) ) gives repeated arcs ( AE, EH ) and ( BD ).</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Minimum distance = ( 1970 + 240 = 2210 ) m</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>(ii) Place skip at ( H ) since its further from ( A ) than ( B ) or ( D ).</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>Only ( BD ) needs repeating: Total distance = ( 1970 + 50 = 2020 ).</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Possible route: ( ACGHEABDBCDGFEDFH )</td>
<td>B1</td>
</tr>
<tr>
<td><strong>3.</strong> (a)</td>
<td><img src="image" alt="Diagram" /></td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Total length = 8 km</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Route ( ACDE )</td>
<td>A1</td>
</tr>
<tr>
<td>(b)</td>
<td>(i) Repeat ( AC, CD, DE ) between odd vertices ( A ) and ( E ).</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Distance = ( 32 + 8 = 40 ) km</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Possible route ( DBACDACDECFED ) (others exist)</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>(ii) She lives at ( A ) (the other odd node)</td>
<td>E1</td>
</tr>
</tbody>
</table>
4. (a) Each arc is doubled, so every node is of even degree. The graph is then Eulerian, so the total journey = sum of arc weights = $2 \times 90 = 180$ km.

(b) The graph is not Eulerian, because there are odd nodes at A, C, D and G. Need to travel twice between pairs of odd nodes.
   \[ AC + DG = 23, \quad AD + CG = 23, \quad AG + CD = 22 \]

5. (a) Each arc is doubled, so every node is of even degree. The graph is then Eulerian, so the total journey = sum of arc weights = $2 \times 90 = 180$ km.

(b) The graph is not Eulerian, because there are odd nodes at A, C, D and G. Need to travel twice between pairs of odd nodes.
   \[ AC + DG = 23, \quad AD + CG = 23, \quad AG + CD = 22 \]

(c) Best pairing is AG + CD, repeat AF, FG and CD

Total distance = $90 + 22 = 112$ km

Possible route: DEABCDGFAFGBFGCD
1. This adjacency table shows which workers A, B, C, D, E and F are qualified to do which tasks 1, 2, 3, 4, 5 and 6.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

(a) Draw a bipartite graph to show this information.  

(b) Worker D wants to do task 2. Explain why a complete matching of workers to tasks is not possible if this is allowed.  

(c) Initially worker A is allocated task 1, worker C task 4, worker D task 2, worker E task 6 and worker F task 5.

   (i) Show this allocation on your diagram in a distinctive way.  
   (ii) Apply the maximum matching algorithm to obtain a complete matching. State the alternating path you use and the complete matching obtained.  

2. Six children – Alice, Belle, Chulchit, Dwayne, Edwina and Fe – are to be entered in six athletics events – 1 (high jump), 2 (long jump), 3 (javelin), 4 (shot), 5 (discus) and 6 (pole vault). The table shows the events they are good at.

<table>
<thead>
<tr>
<th>Child</th>
<th>Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alice</td>
<td>1, 2</td>
</tr>
<tr>
<td>Belle</td>
<td>2, 3, 4</td>
</tr>
<tr>
<td>Chulchit</td>
<td>3, 6</td>
</tr>
<tr>
<td>Dwayne</td>
<td>4</td>
</tr>
<tr>
<td>Edwina</td>
<td>4, 5</td>
</tr>
<tr>
<td>Fe</td>
<td>1, 2</td>
</tr>
</tbody>
</table>

(a) Illustrate this information on a bipartite graph.  

Initially the teacher allocates Alice to event 1, Belle to 2, Chulchit to 3, Dwayne to 4 and Edwina to 5.

(b) (i) Show this in a distinctive way on your bipartite graph.  
   (ii) Show that there are two possible alternating paths, and find the complete matching that results from each.
3. Six tutors – A, B, C, D, E and F - at a college are asked to staff an information desk during an open day. There are six time slots which need covering. The adjacency matrix shown indicates which staff members are available for which slots.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>F</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

(a) Draw a bipartite graph to show this information. (2)

(b) Initially A signs up for slot 6, B for slot 1, E for slot 3 and F for slot 5. Starting from this partial matching, use the maximal matching algorithm to obtain a complete matching. Show all your working clearly. (3)
**Question Scheme Marks**

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Scheme</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (a)</td>
<td>Bipartite graph.</td>
<td>D2</td>
</tr>
<tr>
<td><img src="image" alt="Diagram" /></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b)</td>
<td>B is only qualified for task 2 so if it is allocated to D, be cannot be matched.</td>
<td>E1</td>
</tr>
<tr>
<td>(c) (i)</td>
<td></td>
<td>B1 bold lines or equivalent</td>
</tr>
<tr>
<td><img src="image" alt="Diagram" /></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td>Alternating path $B-2=D-5=F-1=A-3$ Changing status: $B=2-D=5-F=1-A=3$ Complete matching is ${A3, B2, C4, D5, E6, F1}$</td>
<td>M1 A1 B1</td>
</tr>
</tbody>
</table>
2. (a)(bi)

(bii) Path 1: \(F-1=A-2=B-3=C-6\)
Matching \{A, B, C, D, E, F\}

Path 2: \(F-2=B-3=C-6\)
Matching \{A, B, C, D, E, F\}

3. (a)

(b) Possible paths: \(C-3=E-4\) then \(D-5=F-6=A-1=B-2\)
\(D-5=F-6=A-1=B-3=E-4\) then \(C-3=B-2\)
Matching \{A, B, C, D, E, F\}

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1. Pepita’s Pizzas buy in ready-made pizza bases, with which they make two styles of pizza – “Trad” and “Extra Pep”. Trad contains 40 g of cheese, 20 g of tomato and 20 g of peperoni. Extra Pep contains 20 g of cheese, 20 g of tomato and 60 g of peperoni. They have all the bases they need, together with 120 kg of cheese, 80 kg of tomatoes and 180 kg of peperoni. They make 20 p profit on a Trad pizza and 30 p on an Extra Pep. They want to maximise their profit.

(a) Formulate this situation as a linear programming problem. (4)

(b) Draw a graph to illustrate the constraints, clearly labelling the feasible region. (5)

(c) Draw on your graph the objective line corresponding to a profit of £600. Hence or otherwise find the optimal solution to the problem, stating the maximum profit and the corresponding number of each type of pizza. (3)

2. A company makes two types of decorative braiding for furniture – “Regal” and “Antique”.

To make 1 metre of Regal they use 3 m of crimson thread, 1 m of purple thread and 6 m of gold thread. To make 1 metre of Antique they use 1 m of crimson, 3 m of purple and 4 m of gold thread.

They have 600 m of crimson, 600 m of purple and 1400 m of gold thread in stock. They make a profit of 60 p per metre of Regal braid and 50 p per metre of Antique braid.

They make \( x \) metres of Regal and \( y \) metres of Antique, and they wish to maximise their profit.

(a) Express this situation as a linear programming problem. (4)

(b) (i) Draw a graph to show the constraints. Label the feasible region and draw a possible position for the objective line.

(ii) Find the optimal values of \( x \) and \( y \), and state the maximum profit the company can make. (7)

3. The constraints in a linear programming problem are

\[
\begin{align*}
2x + y & \leq 30 \\
x + 4y & \leq 50 \\
x + y & \geq 12 \\
x & \geq 4, \ y & \geq 4
\end{align*}
\]

(a) Draw a graph to show these constraints. Indicate the feasible region clearly. (5)

The objective function is \( P = 3x + 2y \).

(b) Draw on your graph the objective line corresponding to \( P = 36 \). (1)

(c) Find the maximum value of \( P \) and state the corresponding values of \( x \) and \( y \). (2)

(d) Find the minimum value of \( P \) and state the corresponding values of \( x \) and \( y \). (2)
1. (a) Let \( x \) be the number of Trad pizzas, \( y \) the number of Extra Peps.

Maximise \( P = 0.2x + 0.3y \)

Subject to \( 2x + y \leq 6000 \) (or equivalent)
\( x + y \leq 4000 \) (or equivalent)
\( x + 3y \leq 9000 \) (or equivalent)
\( x \geq 0, y \geq 0 \)

(b) 

(c) Objective line.
Maximum Profit: £1050 at \( x = 1500, y = 2500 \)

2. (a) Maximise \( P = 0.6x + 0.5y \)
Subject to \( 3x + y \leq 600 \)
\( x + 3y \leq 600 \)
\( 6x + 4y \leq 1400 \) \( \Rightarrow \) \( 3x + 2y \leq 700 \)
\( x \geq 0, y \geq 0 \)

(b) 

(i) 

(ii) \( x = 157 \frac{1}{7}, y = 128 \frac{4}{7}, P = £155 \frac{5}{7} \)
3. (a)(b)

(c) Max $P = 50$, $x = 10$, $y = 10$

(d) Min $P = 28$, $x = 4$, $y = 8$
1. A project consists of tasks $A, B, C, \ldots, I$. The tasks are related by the following precedence table.

<table>
<thead>
<tr>
<th>Task</th>
<th>Must be preceded by</th>
<th>Duration (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>–</td>
<td>4</td>
</tr>
<tr>
<td>$B$</td>
<td>–</td>
<td>7</td>
</tr>
<tr>
<td>$C$</td>
<td>$A$</td>
<td>8</td>
</tr>
<tr>
<td>$D$</td>
<td>$B$</td>
<td>6</td>
</tr>
<tr>
<td>$E$</td>
<td>$B, C$</td>
<td>5</td>
</tr>
<tr>
<td>$F$</td>
<td>$A$</td>
<td>3</td>
</tr>
<tr>
<td>$G$</td>
<td>$B, C, E$</td>
<td>7</td>
</tr>
<tr>
<td>$H$</td>
<td>$B, C, E$</td>
<td>5</td>
</tr>
<tr>
<td>$I$</td>
<td>$D, E, H$</td>
<td>4</td>
</tr>
</tbody>
</table>

(a) Draw an activity network, representing the activities by the arcs of the network, to illustrate this project. (2)

(b) Perform a forward and a backward pass for your network, and hence state

(i) the optimal project duration

(ii) the critical activities. (6)

(c) An unexpected problem causes activity $E$ to take 3 days longer than planned. Assuming this activity starts at the earliest possible time, what effect, if any, does the delay have on the overall project duration? (1)

2. A project is illustrated by means of the activity network shown. The durations are in days.

(a) On a copy of the network given overleaf, analyse the project to find

(i) the optimal project duration

(ii) the critical activities. (6)
3. A project involves ten tasks A, B, ..., J. The relation between the tasks is shown in the precedence table.

<table>
<thead>
<tr>
<th>Task</th>
<th>Must be preceded by</th>
<th>Duration (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>–</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>–</td>
<td>6</td>
</tr>
<tr>
<td>C</td>
<td>A, B</td>
<td>5</td>
</tr>
<tr>
<td>D</td>
<td>A, B</td>
<td>8</td>
</tr>
<tr>
<td>E</td>
<td>C, D</td>
<td>10</td>
</tr>
<tr>
<td>F</td>
<td>D</td>
<td>12</td>
</tr>
<tr>
<td>G</td>
<td>D</td>
<td>12</td>
</tr>
<tr>
<td>H</td>
<td>E, F</td>
<td>6</td>
</tr>
<tr>
<td>I</td>
<td>E, F</td>
<td>3</td>
</tr>
<tr>
<td>J</td>
<td>G, H</td>
<td>4</td>
</tr>
</tbody>
</table>

The diagram shows the first part of an activity network for this project.

(a) Explain why the dummy activity (2, 3) is needed. (1)
(b) Copy and complete the network, using one more dummy activity. (2)
(c) Analyse the network by performing a forward and a backward pass to find the earliest and latest event times. Hence state
   (i) the minimum project duration
   (ii) the critical activities. (6)
(d) State the amount of float in the non-critical activities. (1)
4. The diagram shows an activity network on which a forward and a backward pass have been performed.

(a) State the critical activities. (1)

(b) On a grid, construct a cascade (Gantt) chart to illustrate the project. (4)

(c) Each task in the project requires one worker. Show, by constructing a schedule, that the project can be completed on time by three workers. (3)

5. The precedence table shows a project comprising eleven activities.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Must be preceded by</th>
<th>Duration (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>–</td>
<td>5</td>
</tr>
<tr>
<td>B</td>
<td>–</td>
<td>5</td>
</tr>
<tr>
<td>C</td>
<td>A</td>
<td>12</td>
</tr>
<tr>
<td>D</td>
<td>A</td>
<td>7</td>
</tr>
<tr>
<td>E</td>
<td>–</td>
<td>4</td>
</tr>
<tr>
<td>F</td>
<td>–</td>
<td>3</td>
</tr>
<tr>
<td>G</td>
<td>B, C</td>
<td>3</td>
</tr>
<tr>
<td>H</td>
<td>D</td>
<td>6</td>
</tr>
<tr>
<td>I</td>
<td>E, F</td>
<td>8</td>
</tr>
<tr>
<td>J</td>
<td>H, I</td>
<td>10</td>
</tr>
<tr>
<td>K</td>
<td>G, J</td>
<td>6</td>
</tr>
</tbody>
</table>

(a) Draw an activity network to show this project. (2)

(b) Perform a forward and a backward pass on your network to find the earliest and latest event times. State the minimum project duration. (4)

(c) List the critical activities and calculate the float on the non-critical activities. (2)

(d) If each activity required one worker, state how you know that the project cannot be completed in the minimum time by just two workers. (1)
1. (a)  

![Diagram of Project Network](image)

- Duration = 23 days  
- Critical activities are A, C, G, I

(b)  

![Diagram of Project Network](image)

(i) Duration = 23 days  
(ii) Critical activities are A, C, G, I  
(iii) Float on E is 2 days. 3 days delay will extend the project by 1 day.

2. (a)  

![Diagram of Project Network](image)

(i) Duration = 23 days.  
(ii) Critical activities are A, E, F, K
3. (a) \(A\) and \(B\) are parallel activities but each must be defined by a unique pair of events.

(b)(c)

(i) Project duration: 36 days
(ii) Critical activities: \(B, D, F, H, J\)

(d) \(A = 2, C = 5, E = 2, G = 6, I = 7\)
4. (a) $A, E, J$ and $N$

(b) Gantt Chart

(c) Critical activities: $A, D, H, J, K$

5. (a)(b)

(c) Critical activities: $A, D, H, J, K$

(d) The total of all the weights = 69 hours. This is more than twice the project duration, so at least three workers are needed.