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### About this book

This book is designed to complement the Student Books in the series *AQA A Level Maths* and provides you with extra support for mechanics. An introductory chapter offers advice on modelling real-world situations using mechanics; indicates how the various topics within mechanics are related to one another; and provides a summary of the relevant notation and language required. There is then one section in this book for every mechanics section in the Student Book. Each of these sections provides a recap of the main ideas in the Student Book and illustrates their application with examples of exam-style questions and model student answers followed by three pages of exam-style questions for you to attempt yourself.

Full details of the mechanics content of the AS level (7356) and A level (7357) exam specifications can be found on the AQA web site: [http://www.aqa.org.uk/subjects/mathematics/as-and-a-level](http://www.aqa.org.uk/subjects/mathematics/as-and-a-level)

At AS Level, mechanics is tested in section B of paper 1; there are 27 marks and approximately 30 minutes available. At A Level, mechanics is tested in section B of paper 2; there are 50 marks and approximately 60 minutes available.

### Answers

The back of this book contains short answers to all the questions. Full mark schemes are password protected for teachers, online: [https://global.oup.com/education/content/secondary/series/aqaalevelmaths-answers](https://global.oup.com/education/content/secondary/series/aqaalevelmaths-answers)

### Formulae

In the exam, you will be provided with a ‘Formulae for A-level Mathematics’ booklet for use in AS Level and A Level Maths qualifications. The relevant mechanics formulae are provided at the end of this book.

### Calculators

All papers are calculator papers. You must make sure that you have a calculator and that you know how to use it. The rules on which calculators are allowed can be found in the Joint Council for General Qualifications document ‘Instructions for conducting examinations’ (ICEL).
Recap

A force is a push or pull. Its effect is to either start something moving or alter the motion of an already moving body.

- In Mechanics, you meet several different kinds of forces.
  - A rod under tension can exert a pull or under thrust can exert a push.
  - A string under tension can exert a pull.
    (A string cannot exert a push.)
  - Gravity acts on any massive body to give a vertically downwards force – weight; see section 8.3
  - Two bodies in contact that press against one another give rise to a normal reaction. The direction of this force is at right angles, that is, normal to the surface.
  - Friction is the force exerted by a ‘rough surface’ that acts to stop a body moving.
    If a surface exerts no or negligible friction you say that it is ‘smooth’.
  - An engine, or motor, can exert a driving force.

Newton’s first law
A body will continue in a state of equilibrium unless acted upon by a force.

Equilibrium means either at rest or moving at a constant velocity, that is, at a constant speed in a fixed direction.

If you are in a train or lift moving at a steady speed you cannot tell that you are not at rest – you are in equilibrium. When you speed up, slow down or go round a corner you do feel a force acting and you are no longer in equilibrium.

- Force is a vector quantity: it has both magnitude and direction.
- The unit of force is the newton, N.
- You specify a force by either giving its components, relative to a pair of basis vectors \( \mathbf{i} \) and \( \mathbf{j} \), or giving its magnitude and direction, for example, up-down, left-right, east-west or a bearing, say 090° (measured clockwise from north).

\[
\begin{align*}
R &= \sqrt{P^2 + Q^2} \\
\theta &= \tan^{-1}\left(\frac{Q}{P}\right) \\
P &= R \cos \theta \\
Q &= R \sin \theta
\end{align*}
\]

Finding a magnitude usually involves Pythagoras’ theorem.
Finding the direction usually involves trigonometry.

- You can add forces by either adding the \( \mathbf{i} \) and \( \mathbf{j} \) components separately or by using a vector triangle; see chapter 6
- When a body is in equilibrium you can resolve the forces: either set the sum of all the forces to 0 or set the sum of the forces in one direction equal to the sum of the forces in the opposite direction.
Example 1

A block of weight 24 N rests on two supports A and B

The reaction from B is twice as big as the force from A

a Sketch a force diagram.

b Work out the size of the reactions from A and B

![Diagram with forces](image)

Do not worry about why the two reactions are different.

Explain that you are resolving vertically.

Example 2

A stone has four forces acting on it: 10 N from the north, 5 N from the east, 7 N from the south and 12 N from the west.

Calculate the resultant force.

![Diagram with forces](image)

Draw a diagram showing all the forces.

The net force to the East is 12 – 5 = 7

Use Pythagoras and trigonometry to find the magnitude and direction.

You may be asked to give direction as a bearing, if not, you could say 23.2° south of east.

Exam tips

- Look for key words that imply equilibrium; for example, at rest or constant velocity.
- Draw a large, clear diagram that shows all the forces acting on the body.
- If you want to show direction of motion ↑ or acceleration → draw the symbols separate from the body.
- Explain what you are doing; for example, say ‘resolving forces horizontally.’
  You may do this using abbreviations, such as, R →.
- Remember to give units.
- Be prepared to identify any assumptions and say how changing the assumption might affect your answer.
  - a smooth surface ignore friction
  - a rough surface include friction
  - a body is a particle ignore any effects of its size
  - a light object ignore gravity for the object
  - a light string tension is the same along its length
  - an inextensible string ignore any stretching
  - a smooth pulley tension is the same on each side of the pulley


Exam practice questions

1 Three forces act on a point particle: 10 N east, 15 N west and 25 N south.

What is the bearing of the resultant force? [1]

Circle your answer.

\[180^\circ \quad 191^\circ \quad 011^\circ \quad 259^\circ\]

2 a Steve places his briefcase on a horizontal table.

Complete the diagram to show the forces acting on the briefcase. [1]

<table>
<thead>
<tr>
<th>Briefcase</th>
</tr>
</thead>
</table>

b A cat pushes the briefcase horizontally but the briefcase does not move.

Complete the diagram to show the forces acting on the briefcase. [2]

<table>
<thead>
<tr>
<th>Briefcase</th>
</tr>
</thead>
</table>

3 Calculate the resultant force in these situations.

Identify any that are in equilibrium.

a [1]

<table>
<thead>
<tr>
<th>30 N</th>
<th>50 N</th>
<th>10 N</th>
<th>80 N</th>
</tr>
</thead>
</table>

---

Forces and Newton's laws Forces 1

---
4 Three forces \((5i - 2j), (3i - j)\) and \((6i + 7j)\) act on a body. Calculate the magnitude and direction of the resultant force.

5 Three forces act on a body \((6i + 5j), (-10i - 13j)\) and \((\alpha i + bj)\)

Find the integer values of \(\alpha\) and \(b\) that give the smallest possible resultant force that acts in a direction 45° below the x-axis.

6

![Diagram](image)

You are given that this body is in equilibrium and that forces \(X\) and \(Y\) are positive.

Find the value of \(X\) and the value of \(Y\).
Recap

- **Dynamics** is the study of forces and how they affect motion.
- **Newton's second law** is the most important rule used in dynamics. It connects the resultant force acting on a body to the acceleration of the body.

  Newton's second law

  A total force of $F$ N acting on a body of mass $m$ kg gives it an acceleration $a$ m s$^{-2}$ where
  
  $F = ma$

- The force that appears in Newton's second law is the total force acting on the body.
  - Force must be measured in newtons, N
  - Mass must be measured in kilograms, kg
  - Acceleration must be measured in metres per second squared, m s$^{-2}$

  Forces can be given in component form.
  A force $\mathbf{F} = (10\mathbf{i} + 25\mathbf{j})$ N acting on a particle of mass 10 kg gives it an acceleration $\mathbf{a} = (11 + 2.5\mathbf{j})$ m s$^{-2}$

- The force that appears in Newton's second law is the total force acting on the body.
- Given a constant acceleration you can apply the *svnat* equations, see section 7.3, to calculate velocities, distances and times.

  The *svnat* equations

  \[
  \begin{align*}
  v &= u + at \\
  s &= ut + \frac{1}{2}at^2 \\
  v^2 &= u^2 + 2as \\
  s &= \frac{1}{2}(u + v)t 
  \end{align*}
  \]

- The most important part of an answer in a dynamics problem is likely to be a diagram showing all the forces acting on a body.
  - Make space for a large diagram (start a new page if necessary).
  - Do not rush this part of the answer.
  - Show all forces, including:
    - a body's weight – the force due to gravity
    - friction on rough surfaces
    - normal reactions where bodies touch
    - driving forces – due to engines
  - Show any acceleration using a double arrow $\rightarrow$

For a box being dragged along a rough surface

![Diagram of forces acting on a box]

- Friction
- Normal reaction
- Tension
- Weight
- Acceleration

---

[Image link to MyMaths 2186, 2293 search]
Example 1

The driving force acting on a car of mass 900 kg is 5000 N and the total resistive forces are 2000 N.

Calculate the acceleration of the car.

\[
\begin{align*}
5000 - 2000 &= 9000 \\
\text{Newton's second law, applied to the right} \\
5000 - 2000 &= 900a \\
3000 &= 900a \\
a &= \frac{3000}{900} \\
&= 3.33 \text{ m s}^{-2}
\end{align*}
\]

First draw a diagram.

Explain to the examiner what you are doing. You could use an abbreviation, such as N2L or NII.

5000 - 2000 gives the ‘total force’ in the direction of the acceleration.

Remember to include units.

Example 2

A car, of mass 750 kg, is 60 m from some traffic lights, and it’s travelling at 50 km h\(^{-1}\) when the lights change from green to red. Show that if the driver applies an average braking force of 1200 N, the car will not stop before the lights. Assume the braking force is the only force acting on the car.

As there is only one force, a diagram is not necessary.

\[
\begin{align*}
50 \text{ km h}^{-1} &= \frac{50}{3.6} = 13.9 \text{ m s}^{-1} \\
\text{Let } a &= \text{ deceleration required to stop in time} \\
\gamma^2 &= \gamma^2 + 2 \times a \\
0 &= 13.9^2 + 2 \times a \times 60 \\
a &= -1.61 \\
\text{Newton's second law} \\
F &= ma \\
= 750 \times 1.61 \\
= 1208 \text{ N} \\
1200 \text{ N } < 1208 \text{ N}
\end{align*}
\]

Convert the speed to m s\(^{-1}\).

Chose the appropriate \(suvat\) equation.

A negative sign \(\Rightarrow\) deceleration.

Use Newton’s second law. If you use the negative \(a\), you will get a negative, braking, force.

Remember to give a conclusion.

Exam tips

- Start by drawing a large, clear diagram.
- Always tell the examiner what you are doing, for example, by writing ‘using Newton’s second law’ or an abbreviation, such as ‘N2L’.
- Remember that the \(F\) in \(F = ma\) means total force.
- A negative acceleration, or deceleration, simply means the velocity is decreasing in your chosen direction.
- Remember to use SI units, including in your answer.
Exam practice questions

1. A particle of mass 2 kg is acted on by a force of 12 N
   What is the acceleration of the particle?     [1]

   Circle your answer.
   6 m s\(^{-2}\)  24 m s\(^{-2}\)  10 m s\(^{-2}\)  14 m s\(^{-2}\)

2. A crate of mass 100 kg is pushed across a smooth surface with a force of 20 N
   a. Calculate the velocity of the crate after 15 seconds if it initially starts from rest.  [3]

   b. After another 50 seconds the crate slides onto a rough surface and there is now a frictional force of 16 N.
      After another 15 seconds the pushing force of 20 N is removed and the crate eventually stops moving.
      What is the total length of time that the crate will have been in motion?  [8]

(continued on next page)
A metal sphere of mass 2 kg and weight 20 newtons is suspended from the roof of a train by a light inextensible string.

The train is accelerating at 1 m s$^{-2}$

What angle does the string make with the vertical?
4 a  A lift is moving upwards and decelerating at 2 m s\(^{-2}\).
A woman of weight 600 N is standing in the lift.

What is the contact force from the floor on the woman? Use \( g = 9.8 \) m s\(^{-2}\)  

4 b  Repeat part a if the lift is accelerating upwards at 2 m s\(^{-2}\)
8.3 Motion under gravity

Recap

- The **mass** of a body is a property of the body.
  It is the same everywhere.
- Mass is measured in kilograms, kg, or related units such as milligrams, grams and tonnes.
  
  - 1000 milligrams = 1 gram
  - 1000 grams = 1 kilogram
  - 1000 kilograms = 1 tonne
- **Weight** is the force that acts on a massive body due to gravity.
  - Using Newton’s second law, \( F = ma \), the mass and weight of a body are related by the acceleration due to gravity.
  - To calculate the weight of a body multiply its mass by the acceleration due to gravity, \( g \). \( W = mg \)
- Acceleration due to gravity varies depending on position.

<table>
<thead>
<tr>
<th>Location</th>
<th>( g ) in ( \text{m s}^{-2} )</th>
<th>1 kg weighs</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>In the Arctic</td>
<td>9.83</td>
<td>9.83 N</td>
<td></td>
</tr>
<tr>
<td>In Peru</td>
<td>9.76</td>
<td>9.76 N</td>
<td></td>
</tr>
<tr>
<td>On the Moon</td>
<td>1.62</td>
<td>1.62 N</td>
<td></td>
</tr>
<tr>
<td>(Lunar gravity = ( \frac{1}{6} ) Earth gravity)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>In free fall</td>
<td>0</td>
<td>nothing</td>
<td></td>
</tr>
</tbody>
</table>

- In questions (because distances are small) you should take \( g \) to be a constant.
  - The average value of \( g \) on the Earth is 9.81 m s\(^{-2}\).
  - Exam papers may suggest you use 9.81, 9.6 or even 10

In questions involving weight you should expect to have to use other knowledge.

Newton’s second law, \( F = ma \), will allow you to calculate any acceleration.

The \textit{suvat} equations will allow you to calculate subsequent distances, times and velocities.

\[
W = mg = 2 \times 9.81 = 19.6 \text{ N}
\]

In Newton’s theory of universal gravity the force, \( F \), between two bodies of masses, \( m \) and \( M \), a distance, \( r \), apart is given by

\[
F = G \frac{mM}{r^2}
\]

\[
W = m \times \left( G \frac{M}{r^2} \right) = m \times g
\]

where \( G \) is the gravitational constant.

Clearly the acceleration due to gravity, \( g \), depends on the mass of the body doing the attracting, \( M \) - gravity is weaker on the Moon than on Earth - and on the distance between the bodies, \( r \) - gravity is slightly weaker at the top of a mountain than at sea level.
Example 1
A farmer is ploughing a horizontal field.
The plough has a mass of 550 kg and is being pulled with a force of 800 N.
Given that the plough accelerates at 1.2 m s\(^{-2}\) and that \(g = 9.8\) m s\(^{-2}\) calculate

a. The normal reaction from the field on the plough,
b. The total resistance to motion of the plough.

\[
\text{Weight} = 550 \times 9.8 = 5390\text{ N} \\
\text{N}, \uparrow \\
N - 5390 = 550 \times 0 \\
\text{Normal reaction, } N = 5390\text{ N} \\
b. \text{N}, \rightarrow \\
800 - E = 550 \times 1.2 \\
E = 800 - 660 \\
= 140\text{ N}
\]

You should always draw a diagram.

Example 2
A man of mass 70 kg stands on bathroom scales in an elevator that is moving upwards.
The scales show a reading of 89.5 kg.

a. Taking \(g\) as 9.81 m s\(^{-2}\), calculate the acceleration of the lift.
b. Is the lift accelerating or decelerating?

\[
a. \text{Normal reaction} = 89.5 \times 9.81 = 878\text{ N} \\
\text{Man's weight} = 70 \times 9.81 = 686.7\text{ N} \\
\text{N}, \uparrow \\
878 - 686.7 = 70a \\
a = 191.3 + 70 \\
= 2.73\text{ m s}^{-2} \\
b. \text{The answer is positive, the lift is accelerating upwards.}
\]

Assume the lift is accelerating upwards. If this assumption is wrong, the answer for a will be negative.

Bathroom scales measure the normal reaction they apply to a body. They represent this force as equivalent mass \(-\text{normal reaction} + 9.81\)

Exam tips
- Read the question carefully: are you given a body's mass or weight?
  In everyday life people confuse the two ideas – you must be careful to distinguish between them!
- Remember that Newton's second law uses mass not weight.
- Remember to draw a large, clear diagram.
- If a body is falling freely under gravity its acceleration is 9.81 m s\(^{-2}\) and you should use this value in \(suva\)t calculations, unless the question tells you to use a different value.
  In practice, as a body accelerates downwards, air resistance increases and the acceleration reduces to 0 m s\(^{-2}\) when the air resistance exactly balances the body's weight. The body is said to have reached terminal velocity.
Exam practice questions

Use \(g = 9.8 \text{ m s}^{-2}\) in this exercise.

1. The acceleration due to gravity on the planet Mars is \(3.7 \text{ m s}^{-2}\). A spacecraft has mass 15 000 kg. What is its mass and weight on Mars? [1]

Circle your answer.

A. 55 500 kg, 55 500 N
B. 15 000 kg, 15 000 N
C. 15 000 kg, 55 500 N
D. 15 000 kg, 0 N

2. A woman has a weight of 430 N. Calculate her mass. [1]

3. A palette of 1000 bricks each of mass 2.8 kg is being raised off the ground by a rope at a constant velocity of 4 m s\(^{-1}\).

a. Calculate the tension in the rope. [2]

b. How long will it take the palette of bricks to reach a platform that is 48 m above the ground? [1]

c. Give two modelling assumptions that you have made about the rope. [2]
3  d  A more accurate model of the bricks’ movement assumes that they start from rest, accelerate uniformly at 1 m s\(^{-2}\) for the first 24 m and then decelerate at 1 m s\(^{-2}\) for the second 24 m.

i  Calculate the tension in the rope for the first 24 m.  \[2\]

ii Calculate the tension in the rope for the final 24 m. \[1\]

e  Calculate the time taken to raise the bricks to 48 m under this model. \[4\]
Jo has a mass of 81 kg. She stands on her bathroom scales in a lift.

What is the reading on the bathroom scales when the lift is

a moving at constant velocity, [1]

b accelerating upwards at 2 m s$^{-2}$, [4]

c moving upwards and decelerating at 1 m s$^{-2}$, [3]

d moving downwards and decelerating at 1 m s$^{-2}$? [3]
Recap

- When bodies interact they exert forces on one another.
  - A massive body resting on a surface presses down with its weight.
    The surface pushes back with a **normal reaction**.
  - The Earth pulls a massive body downwards.
    The massive body pulls the Earth upwards.
  - A body being dragged across a rough surface feels friction, resisting its motion.
    The surface feels a force trying to drag it along with the body.
- Newton’s third law says that these pairs of forces are **equal and opposite**.

**Newton’s third law**

If one body exerts a force $F$ on a second body then the second body exerts an equal and opposite force $-F$ on the first body.

- Bodies necessarily interact when they are connected. For example,
  - a car towing a trailer
  - masses connected by a light string over a frictionless pulley

The rod joining the car and trailer can be in thrust or tension.

When the car is braking, the thrust pushing the trailer back is equal to the thrust pushing the car forward.

The string joining the masses can only be in tension.

The tension in the string is constant.
The 6 kg weight is being pulled up by the same force as the 8 kg mass.

- The forces acting on connected bodies can either be
  - **external forces** such as weight or friction
    - You regard the Earth or a rough surface as an immovable object and ignore any forces acting on them.
  - or
  - **internal forces** such as tension in a string or tension or thrust in a tow bar.
- You can either
  - consider the connected bodies as one single body
    - In this case you only consider the external forces.
  - consider the individual bodies separately
    - In this case you must consider both the internal and external forces.

However, usually you will need to do both.
A family car has mass 800 kg and is pulling a trailer of mass 200 kg. It is accelerating at 4 m s\(^{-2}\).

The total resistance to motion for the car is 150 N and for the trailer is 250 N.

**a** Find the driving force of the car.

**b** Find the force in the tow bar and determine whether it is a thrust or a tension.

\[ F = 150 \text{ N} \]
\[ R = 250 \text{ N} \]
\[ a = 4 \text{ m s}^{-2} \]

**Example 2**

A mass of 4 kg is held at rest 50 cm above the ground. It is attached, by a light, inextensible string that passes over a smooth, light pulley, to a mass of 2 kg that lies at rest on a smooth, horizontal surface. The 4 kg mass is then released from rest.

**a** Calculate the tension in the string.

**b** After what time will the 4 kg mass hit the ground.

\[ F = 4 \times 9.8 = (2 + 4)a \]
\[ a = 6.53 \text{ m s}^{-2} \]
\[ v = \frac{1}{2} \times 2 \times 6.53 = 13.1 \text{ N} \]

**Exam tips**

- Draw a clear diagram showing all the forces.
- Be aware of which forces are external and which internal.
- Usually consider the whole system first (ignoring internal forces).
- Remember that a string can only be in tension, whereas a bar can be in tension or thrust.
1 A lift is moving downwards at constant speed.

Calculate the normal reaction from the floor of the lift on a man of mass 80 kg. [1]

Circle your answer.

0 N     8.16 N     80 N     784 N

2 A 9 kg mass is held at rest and is connected to a 4 kg mass by a string that passes over a pulley. A 3 kg mass hangs below the 4 kg mass also connected by a shorter length of string.

a Calculate the acceleration of the system after the 9 kg mass is released. [4]

b Show that the tension in the longer string is 77.2 N [3]
2. c Calculate the tension in the shorter string.

3. A train consists of a miniature locomotive pulling two carriages.
   The mass of the locomotive is 600 kg and the mass of each carriage is 800 kg.
   The resistance to motion of the locomotive is 100 N and each carriage has resistance 300 N.

   a. The train accelerates from rest at 1.5 m s\(^{-2}\)
      Calculate the driving force of the train.
3 b  The couplings between the carriages and between the locomotive and the carriage are all rigid.
Calculate the force in the coupling between the carriages.
Is it a tension or a thrust?  

[3]

c  The maximum speed of the train is 5 m s\(^{-1}\)
Calculate the braking force needed to bring the train to rest in 10 metres when travelling at full speed.  

[5]