Advance sample materials

11 pages of sample content written for the 2017 specification
# Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Large data set explained</td>
<td>1</td>
</tr>
<tr>
<td>How the topics interrelate</td>
<td>6</td>
</tr>
<tr>
<td>Notation and language</td>
<td>7</td>
</tr>
<tr>
<td>Chapter 9: Collecting, representing and interpreting data</td>
<td></td>
</tr>
<tr>
<td>9.1           Sampling</td>
<td>9</td>
</tr>
<tr>
<td>9.2           Central tendency and spread</td>
<td>14</td>
</tr>
<tr>
<td>9.3           Single variable data</td>
<td>19</td>
</tr>
<tr>
<td>9.4           Bivariate data</td>
<td>24</td>
</tr>
<tr>
<td>Chapter 10: Probability and discrete random variables</td>
<td></td>
</tr>
<tr>
<td>10.1          Probability</td>
<td>29</td>
</tr>
<tr>
<td>10.2          Binomial distribution</td>
<td>34</td>
</tr>
<tr>
<td>Chapter 20: Probability and continuous random variables</td>
<td></td>
</tr>
<tr>
<td>20.1          Conditional probability</td>
<td>49</td>
</tr>
<tr>
<td>20.2          Modelling with probability</td>
<td>54</td>
</tr>
<tr>
<td>20.3          The Normal distribution</td>
<td>59</td>
</tr>
<tr>
<td>20.4          Using the Normal distribution as an approximation to the binomial</td>
<td>64</td>
</tr>
<tr>
<td>Chapter 21: Hypothesis testing 2</td>
<td></td>
</tr>
<tr>
<td>21.1          Testing correlation</td>
<td>69</td>
</tr>
<tr>
<td>21.2          Testing a Normal distribution</td>
<td>74</td>
</tr>
<tr>
<td><strong>Answers</strong></td>
<td>79</td>
</tr>
</tbody>
</table>

*An advance sample version of this chapter is provided in this book.*

| Chapter 21: Hypothesis testing 1                                        |      |
| 11.1          Formulating a test                                         | 39   |
| 11.2          The critical region                                        | 44   |
Formulating a test

Recap

- A **hypothesis test** assumes that the **null hypothesis**, $H_0$, is true and examines if there is sufficient evidence to reject it in favour of the **alternative hypothesis**, $H_1$.
- You can test the probability of success in a binomial distribution, the null hypothesis will be $H_0: p = p_1$.
- The alternative hypothesis can be either
  - $H_1: p > p_1$, or $H_1: p < p_1$ for a **one-tailed test**
  - $H_1: p \neq p_1$ for a **two-tailed test**.

To test if a coin is fair you can use a two-tailed or a one-tailed test. Let $p$ = the probability of 'heads'

A two-tailed test would have hypotheses

$H_0: p = \frac{1}{2}$ and $H_1: p \neq \frac{1}{2}$

If you suspected that the coin was biased towards 'tails' a one-tailed test would have hypotheses

$H_0: p = \frac{1}{2}$ and $H_1: p < \frac{1}{2}$

- A **test statistic** is a value of the random variable found from a sample.
- The **critical region** is the set of values of the random variable that are sufficiently unlikely to occur under the null hypothesis.
  
  Precisely how unlikely is determined by the given **significance level**.
- The **critical value** is the boundary of the critical region.
  
  The critical value lies just inside the critical region.
- If the test statistic is inside the critical region then you reject the null hypothesis.
- The region outside the critical region is called the **acceptance region**.
- If the test statistic is inside the acceptance region then you accept the null hypothesis.

Suppose you wish to perform a one-tailed test at the 5% significance level to see if a coin is biased towards tails.

- Flip the coin 30 times and count the number of heads, $H$ – this is your test statistic.
- Assuming the coin is fair (the null hypothesis) the probability of getting 10 or fewer 'heads' is $0.0494 = 4.94\%$
  
  but the probability of getting heads 11 or fewer times is $0.1002 > 5\%$
- Therefore the critical region for the test statistic is $H \leq 10$.
- That is the critical value is 10.
- If your test statistic is in the region $H \leq 10$ then you can reject the null hypothesis at the 5% significance level as there would be evidence to suggest the coin is biased towards 'tails'.

Assuming the null hypothesis, $H_0$ is true then the significance level tells you how likely you are to **incorrectly** reject $H_0$ when applying the test. As you decrease the significance level, the critical region gets smaller, and this error is less likely to occur. That is, if you do reject $H_0$ it is more likely to be because $H_0$ is not true than you have been unlucky.
Based on past data, the probability of a day in June at Leeming weather station having a maximum humidity level of over 95% is assumed to be 0.58. In June one year, 23 days had a humidity of over 95%. A scientist tests to see if that June had been unusually humid. The critical value at the 1% significance level is 24.

a. For this test state
   i. the hypotheses
   ii. the test statistic
   iii. the critical region
   iv. the acceptance region

b. What conclusions can you draw from the test statistic?
   a. i. \( H_0 : p = 0.58 \)  \( H_1 : p > 0.58 \)
      ii. The test statistic is 23
      iii. \( X \geq 24 \) is the critical region
      iv. \( X \leq 23 \) is the acceptance region
   b. 23 lies in the acceptance region, therefore we do not reject the null hypothesis. There is insufficient evidence at the 1% significance level to suggest that this particular June was unusually humid.

Example 2

A hypothesis test is set up with \( H_0 : p = \frac{1}{4} \) and \( H_1 : p \neq \frac{1}{4} \). The critical values at the 10% significance level are \( X = 2 \) and \( X = 9 \).

a. State the critical region for this test.

b. What effect will lowering the significance level have on the probability of incorrectly rejecting the null hypothesis?
   a. \( X \leq 2 \) and \( X \geq 9 \)
   b. The probability of incorrectly rejecting the null hypothesis will decrease in line with the significance level.

Exam tips

One-tailed and two-tailed tests

- In a two-tailed test, the significance value is split in half so that half is at the top of the distribution and half at the bottom. Thus, to do a 10% two-tailed test, you need 5% at the top and bottom.
- Setting up the alternative hypothesis is important in deciding what sort of test to use.
  - If it is though that the claim is simply wrong, then the alternative hypothesis will be \( x \neq a \) a value and a two-tailed test should be used.
  - If it is thought to be too high or too low the alternative hypothesis will involve an inequality and a one-tailed test should be used.
Exam practice questions

1 A random variable $X$ ~ $B(40, p)$
   A one-tailed test has hypotheses $H_0: p = p_1$ and $H_1: p > p_1$
   At the 5% significance level the critical region for the test is $X > 24$
   How do you expect the critical region to change? Explain your answer. [2]

2 A study into peoples’ attitude to crime in 2006 showed that 64% of people thought that national crime rates were increasing. When questioned about *local* crime, an independent random sample of 45 adults included 20 people who agreed that it was increasing.
   a State the null and alternative hypotheses for a test that the view on local crime was different from that on national crime. [2]
   b Let $X$ be a random variable for the number of people in the sample of 45 adults who agree with the proposition that local crime is on the increase.
      Give the distribution of $X$ under the null hypothesis defined in part a. [2]

3 In Camborne in the late 1980s it was known that 24% of all spring/summer days had a mean visibility of over 3500 dm. A sample 40 days was considered during 2015 and 14 were found to have a mean visibility of over 3000 dm.
   a State a condition on the method of choosing the sample so that a binomial probability model can be used to test the suggestion that mean visibility had changed over this period. [1]
   b Assuming that the condition in part a is met, write down the null and alternative hypotheses for this test. [2]
3 c Let the random variable $X =$ the number of days in a sample of 40 with a mean visibility of over 3500 dm.
Give the distribution of $X$ assuming the null hypothesis to be true. [2]

4 a Explain the term ‘critical region’ in the context of hypothesis testing and state why an observation in the critical region suggests that the null hypothesis should be rejected. [2]

b In Leuchars between 1987 and 2015 it is believed that the summer total daily sunshine hours fell significantly. In 1987 the median number of total sunshine hours was 5.1 per day. To test this belief, in 2015 researchers took a sample of 16 summer days and found that 4 had more than the median number of hours of sunshine.
Let $X$ be the number of days in the sample with more than the median number of hours of sunshine.

i State the null and alternative hypotheses for the required test and give the distribution of $X$ assuming that the null hypothesis is true. [2]

ii Given that under the null hypothesis, at a significance level of 5%, the critical value is 4. Write down the result of the test. [1]
4 b iii What is the probability that the conclusion is incorrect given that the null hypothesis is true? [1]

5 a State the conditions under which a random experiment can be modelled by a binomial probability distribution.
You should clearly define the relevant random variable. [4]

b State the probability distribution function, \( P(X = x) \), for a random variable \( X \) which has a binomial distribution, parameters \( n \) and \( p \) [3]

6 In the spring 1987 in Jacksonville the proportion of days on which it rained was found to be 24%. In a sample of 28 days taken 20 years later, rain occurred on 3 days. A statistical test is to be carried out to use this statistic to determine whether the proportion of rainfree days had increased.

a State a condition for the sample to be suitable for use in the test and state why the condition is necessary. [2]

Let \( X \) be a random variable for the number of days in the sample when some rain fell.

b State the hypotheses \( H_0 \) and \( H_1 \) and give the distribution of \( X \). [3]

c At a significance level of 10%, the critical value is 3. State the conclusion of the hypothesis test. [1]
To find the critical region, you assume the null hypothesis in order to calculate probabilities.

The critical value lies inside the critical region.

For the binomial distribution, \( X \sim B(n, p) \), it is very unlikely that there will be an integer, \( m \), such that \( P(X \geq m) \) exactly equals the required significance level.

In practice, for a high-values, one-tailed test, you find the smallest \( m \) such that
\[
P(X \geq m) \leq \text{significance level} \quad [P(X \geq m - 1) > \text{significance level}]
\]

The critical region is \( X \geq m \).

For low-values you find the largest value of \( m' \) such that
\[
P(X \leq m') \leq \text{significance level} \quad [P(X \geq m' + 1) > \text{significance level}]
\]

If you have \( H_0: p = 0.1 \) and \( H_1: p < 0.1 \) and wish to find the critical region at the 5% significance level when there are 50 trials, you must find the largest value of \( x \) such that
\[
P(X \leq x) \leq 0.05
\]
You need to assume \( X \sim B(50, 0.1) \)

By adding up the probabilities you can see that
\[
P(X \leq 1) \leq 0.0138 \quad \text{which is less than 5%}
\]
\[
P(X \leq 2) \leq 0.0530 \quad \text{which is greater than 5%}
\]
The critical value is 1 and the critical region is \( X \leq 1 \)

Alternatively, you can carry out a hypothesis test by finding the \( p \)-value of a test statistic; this is the probability of obtaining the test statistic or worse under the null hypothesis.

Suppose \( H_0: p = 0.3 \) and \( H_1: p \neq 0.3 \) and that out of 20 trials, 10 were successful.

With 20 trials the expected number of successes is \( 20 \times 0.3 = 6 \) so 10 is more than expected. Hence you need to calculate the probability of getting 10 or more successes out of 20 trials.

\[
P(X \geq 10) = 1 - P(X \leq 9)
\]
\[
= 1 - 0.9520
\]
\[
= 0.0480
\]

Remember, since the binomial distribution is discrete,
\[
P(X \geq x) = 1 - P(X \leq x - 1)
\]

In this case the \( p \)-value is 0.0480 (4.8%).

\( H_1: p \neq 0.3 \Rightarrow \) use a two-tailed test: you split the 5% between the two tails.

Therefore you need to compare the \( p \)-value to 2.5%.

4.8% > 2.5% \( \Rightarrow \) do not reject the null hypothesis.

There is insufficient evidence to assume that the probability of success is not 0.3
In 2015, the probability of rain in Jacksonville, Florida was 45% each day. A tourist visits the city for a 10 day holiday and it rains on 7 of the days. She claims that the probability of rain in Jacksonville is greater than 45%.

a  Assuming the number of rainy days can be modelled by a binomial distribution, perform a hypothesis test with a 10% significance level to test the tourist’s claim.

b  Comment on the use of the binomial distribution in this scenario.

\[ H_0: p = 0.45 \quad H_1: p > 0.45 \]

Assume \( X \sim B(10, 0.45) \)

\[ P(X \geq 7) = 1 - P(X \leq 6) = 1 - 0.8980 = 0.1020 \]

\[ 0.1020 > 10\% \Rightarrow \text{Do not reject the null hypothesis} \]

There is insufficient evidence at the 10% significance level to support the tourist’s claim.

b  Possibly unsuitable as the probability of rain may not be independent of whether there was rain the day before.

The probability of a train in the UK being on time has been estimated from past data to be 85%. Individual train companies are to be analysed to see if their punctuality results are significantly different to this figure.

a  Write down the null and the alternative hypotheses.

A binomial model is to be used and a random sample of 50 trains.

b  Find the critical region at the 10% significance level.

c  What is the probability of incorrectly rejecting the null hypothesis in this case?

\[ H_0: p = 0.85 \quad H_1: p \neq 0.85 \]

It is a two-tailed test, so you need 5% in each tail. Use tables or your calculator to find the critical values.

<table>
<thead>
<tr>
<th>( n )</th>
<th>( P(X \leq n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000798</td>
</tr>
<tr>
<td>1</td>
<td>0.007637</td>
</tr>
<tr>
<td>2</td>
<td>0.035483</td>
</tr>
<tr>
<td>3</td>
<td>0.107087</td>
</tr>
<tr>
<td>4</td>
<td>0.237508</td>
</tr>
<tr>
<td>5</td>
<td>0.416371</td>
</tr>
</tbody>
</table>

Example 1

Example 2

Exam tips

Calculating critical values

Your calculator may be able to find binomial probabilities. If your calculator can also find inverse binomial distributions, you should check if it gives the value of \( x \) such that \( P(X \leq x) \) is closest to the required significance level. This may mean that \( P(X \leq x) \) is slightly larger than the required significance level and that the critical value is \( x - 1 \). The correct value can be found by inspecting the values of \( P(X \leq x) \) and \( P(X \leq x - 1) \).

For example, suppose \( X \sim B(20, 0.3) \) and a 10% significance level is required for a low-values, one-tailed test. A calculator may give \( x = 3 \) as \( P(X \leq 3) = 10.7\ldots\% \), whereas the correct critical value is \( x = 2 \) for which \( P(X \leq 2) = 3.54\ldots\% \).
Exam practice questions

1  a  A random variable $X$ has a binomial distribution, $X \sim B(6, p)$. The value of $p$ was known to be 0.72 but is now believed to have decreased.

What are the null and alternative hypotheses in a test of this belief?  

b  The table below gives the probabilities, to 2 significant figures, of all values of $X$ assuming that the null hypothesis is true.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = x)$</td>
<td>0.00048</td>
<td>0.0074</td>
<td>0.048</td>
<td>0.16</td>
<td>0.32</td>
<td>0.33</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Using a significance level of 5%, what values of $X$ would suggest that the belief is incorrect?

Explain your answer.  

2  In 1987, 70% of September days in Leeming had an average daily mean temperature above 13 °C. It is now claimed that this proportion had increased. To test this, a random selection of 16 September days were sampled and 14 were found to have a higher daily mean temperature.

a  Perform a hypothesis test of this claim using a significance level of 10%

You should state clearly the null and alternative hypotheses.

Hypothesis testing  The critical region
2  b  Comment on the reliability of the test.  [1]

3  In 1987, a large investigation among residents at Heathrow found that 6 out of every 10 people questioned believed that maximum wind gust speeds had increased over the last ten years. This year, in an independent, random sample of 25 people, 20 people agreed with this proposition. A test was carried out to determine whether the proportion of people agreeing with the proposition had changed within the whole population.

a  Write down the null and alternative hypotheses for the test.  [2]

b  Perform the test at a significance level of 5%, stating clearly your conclusion.  [3]

4  In 1987, during May to September (inclusive), the median daily maximum relative humidity in Hurn was 97%. In order to test whether this had changed by 2015, 35 days were chosen at random and twelve of them had a humidity above this level.

Use the binomial distribution to test at a significance level of 5% whether this is evidence of a change in the proportion of days with this level of humidity.  [5]
5 A chicken farm has a hatch rate (the percentage of fertilized eggs which hatch after incubation) of 52%. Following the introduction of new incubation equipment, a random sample of 64 eggs produced 39 chicks.

a Explain why the binomial probability distribution could provide a good model for the number of chicks produced from these eggs. [3]

b Complete the following table for $X$, a random variable for the number of chicks from a total of 64 eggs using the *old* equipment. [3]

<table>
<thead>
<tr>
<th>$x$</th>
<th>39</th>
<th>40</th>
<th>41</th>
<th>$\geq$ 42</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = x)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c It is claimed that the new equipment has increased the hatch rate for fertilized eggs. Test, using a significance level of 5%, whether the claim is justified. [2]

6 In the first half of the 18th century, the mean height of an adult English male was 165 cm. A comparative study on the heights of adult Irish males, found 27 with heights over 165 cm in a sample of 40 randomly chosen Irish males.

a Explain why it is reasonable to assume that the probability that a randomly chosen English male exceeds 165 cm is 0.5. [2]

b Let $X$ be the number in the sample of 40 Irish males with heights over 165 cm.

Using a significance level of 2%, test the belief that Irish males are on average taller than English males. [2]
A chicken farm has a hatch rate (the percentage of fertilized eggs which hatch after incubation) of 52%. Following the introduction of new incubation equipment, a random sample of 64 eggs produced 39 chicks.

a Explain why the binomial probability distribution could provide a good model for the number of chicks produced from these eggs. [3]

b Complete the following table for $X$, a random variable for the number of chicks from a total of 64 eggs using the old equipment. [3]

<table>
<thead>
<tr>
<th>$x$</th>
<th>39</th>
<th>40</th>
<th>41</th>
<th>$\geq 42$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = x)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c It is claimed that the new equipment has increased the hatch rate for fertilized eggs. Test, using a significance level of 5%, whether the claim is justified. [2]

In the first half of the 18th century, the mean height of an adult English male was 165 cm. A comparative study on the heights of adult Irish males, found 27 with heights over 165 cm in a sample of 40 randomly chosen Irish males.

a Explain why it is reasonable to assume that the probability that a randomly chosen English male exceeds 165 cm is 0.5. [2]

b Let $X$ be the number in the sample of 40 Irish males with heights over 165 cm. Using a significance level of 2%, test the belief that Irish males are on average taller than English males. [2]
A Level Maths is changing. From September 2017 you will be teaching linear specifications and 100% prescribed content. Plus, increased emphasis is being given to problem-solving and modelling. We have developed brand new resources for the Edexcel specifications to ensure you and your students have everything needed for the changes ahead.

- **Complete support** for the new linear A Level Maths and Further Maths specifications from Edexcel
- Written by a team of experienced authors and practising teachers
- **Bespoke support** for teaching Mechanics, Statistics and the Large Data Set - all now compulsory in the new specification
- Dedicated practice for problem-solving and modelling in every chapter
- **Direct links to MyMaths.co.uk** after every exercise provide additional support and practice

**How to order**

Email schools-orders.uk@oup.com quoting K44238 to order your inspection copies today.

Visit [www.oxfordsecondary.co.uk/edexcelalevelmaths](http://www.oxfordsecondary.co.uk/edexcelalevelmaths) for current prices and to find out more about additional resources in this series.

Prices and publication dates are subject to change.