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An advance sample version of this chapter is provided in this book.
Recap

- A **hypothesis test** assumes that the **null hypothesis**, \( H_0 \), is true and examines if there is sufficient evidence to reject it in favour of the **alternative hypothesis**, \( H_1 \).
- You can test the probability of success in a binomial distribution, the null hypothesis will be
  - \( H_0: p = p_1 \)
- The alternative hypothesis can be either
  - \( H_1: p > p_1 \) or \( H_1: p < p_1 \) for a **one-tailed test**
  - \( H_1: p ≠ p_1 \) for a **two-tailed test**.

To test if a coin is fair you can use a two-tailed or a one-tailed test.

Let \( p \) = the probability of ‘heads’

A two-tailed test would have hypotheses

\[ H_0: p = \frac{1}{2} \quad \text{and} \quad H_1: p ≠ \frac{1}{2} \]

If you suspected that the coin was biased towards ‘tails’ a one-tailed test would have hypotheses

\[ H_0: p = \frac{1}{2} \quad \text{and} \quad H_1: p < \frac{1}{2} \]

- A **test statistic** is a value of the random variable found from a sample.
- The **critical region** is the set of values of the random variable that are sufficiently unlikely to occur under the null hypothesis.
  
  Precisely how unlikely is determined by the given **significance level**.
- The **critical value** is the boundary of the critical region.
  
  The critical value lies just inside the critical region.
- If the test statistic is inside the critical region then you reject the null hypothesis.
- The region outside the critical region is called the **acceptance region**.
- If the test statistic is inside the acceptance region then you accept the null hypothesis.

Suppose you wish to perform a one-tailed test at the 5% significance level to see if a coin is biased towards tails.

- Flip the coin 30 times and count the number of heads, \( H \) – this is your test statistic.
- Assuming the coin is fair (the null hypothesis)
  - the probability of getting 10 or fewer ‘heads’ is 0.0494 = 4.94%  
  - but the probability of getting heads 11 or fewer times is 0.1002 > 5% 
  - Therefore the critical region for the test statistic is \( H ≤ 10 \).
  - That is the critical value is 10.
  - If your test statistic is in the region \( H ≤ 10 \) then you can reject the null hypothesis at the 5% significance level as there would be evidence to suggest the coin is biased towards ‘tails’.

Assuming the null hypothesis, \( H_0 \) is true then the significance level tells you how likely you are to **incorrectly** reject \( H_0 \) when applying the test. As you decrease the significance level, the critical region gets smaller, and this error is less likely to occur. That is, if you do reject \( H_0 \) it is more likely to be because \( H_0 \) is not true than you have been unlucky.
The probability that a randomly chosen household in the UK has bought butter in any given week is assumed to be 70%. A shop keeper in a town in the South-West claims that the probability is higher in his area. He samples 50 households and 40 of them have bought butter in the last week. The critical value at the 5% significance level is 41.

a For this test state

- i the hypotheses
- ii the test statistic
- iii the critical region
- iv the acceptance region

b What conclusions can you draw from the test statistic?

- a $H_0: p = 0.7$, $H_1: p > 0.7$
- b The test statistic is 40
- c $X \geq 41$ is the critical region
- d $X \leq 40$ is the acceptance region

b 40 lies in the acceptance region therefore we do not reject the null hypothesis. There is insufficient evidence at the 5% significance level to support the shop keeper’s claim.

Example 2

A hypothesis test is set up with $H_0: p = \frac{1}{4}$ and $H_1: p \neq \frac{1}{4}$.

The critical values at the 10% significance level are $X = 2$ and $X = 9$.

a State the critical region for this test.

b What effect will lowering the significance level have on the probability of incorrectly rejecting the null hypothesis?

- a $X \leq 2$ and $X \geq 9$
- b The probability of incorrectly rejecting the null hypothesis will decrease in line with the significance level.

Exam tips

One-tailed and two-tailed tests

- In a two-tailed test, the significance value is split in half so that half is at the top of the distribution and half at the bottom. Thus, to do a 10% two-tailed test, you need 5% at the top and bottom.
- Setting up the alternative hypothesis is important in deciding what sort of test to use.
  - If it is thought that the claim is simply wrong, then the alternative hypothesis will be $x \neq a$ value and a two-tailed test should be used.
  - If it is thought to be too high or too low the alternative hypothesis will involve an inequality and a one-tailed test should be used.
1 A random variable $X \sim B(40, p)$
A one-tailed test has hypotheses $H_0: p = p_1$ and $H_1: p > p_1$
At the 5% significance level the critical region for the test is $X > 24$
The same test is repeated with the significance level changed to 10%
What is the new critical region? [1]

2 A study into peoples’ attitude to crime in 2006 showed that 64% of people thought that national crime rates were increasing. When questioned about local crime, an independent random sample of 45 adults included 20 people who agreed that it was increasing.

a State the null and alternative hypotheses for a test that the view on local crime was different from that on national crime. [2]

b Let $X$ be a random variable for the number of people in the sample of 45 adults who agree with the proposition that local crime is on the increase.
Give the distribution of $X$ under the null hypothesis defined in part a. [2]

3 It has been estimated that in response to increased food prices between 2007 and 2013, 22% of households reacted by ‘trading down,’ that is, buying an alternative cheaper product. A sample of 40 adults were asked about their response to food price increases and 14 accepted trading down as the main response.

a State a condition on the method of choosing the sample so that a binomial probability model can be used to test the estimate. [1]
3 b Assuming that the condition in part a is met, write down the null and alternative hypotheses in a test of this estimate.

Let the random variable $X$ = the number of households out of 40 who trade down.

c Give the distribution of $X$ assuming the null hypothesis to be true.

d Given that under the null hypothesis, at a significance level of 5%, the critical value is 14. State the conclusion of the test.

4 a Explain the term 'critical region' in the context of hypothesis testing and state why an observation in the critical region suggests that the null hypothesis should be rejected.

b Between 2010 and 2015 it is believed that the sales of white bread in England fell significantly. In 2010 the median amount of bread bought was 280 g per person per week. To test the belief that sales were falling, in 2015 researchers took a sample of 16 adults from England and found that 4 bought more than 280 g of white bread per week.

Let $X$ be the number in the sample who bought more than 280 g of white bread per week.

i State the null and alternative hypotheses for the required test and give the distribution of $X$ assuming that the null hypothesis is true.

ii Given that under the null hypothesis, at a significance level of 5%, the critical value is 4. Write down the result of the test.
4 b iii What is the probability that the conclusion is incorrect given that the null hypothesis is true? [1]

5 a State the conditions under which a random experiment can be modelled by a binomial probability distribution.
You should clearly define the relevant random variable. [4]

b State the probability distribution function, \( P(X = x) \), for a random variable \( X \) which has a binomial distribution, parameters \( n \) and \( p \) [3]

6 In 2010 the proportion of individuals in England purchasing more than 380 ml of full price liquid milk was found to be 24%. In a sample of 28 individuals taken in 2014, 3 purchased more than this amount. A statistical test is to be carried out to use this statistic to determine whether the amount spent on this type of milk has decreased over this period.

a State a condition for the sample to be suitable for use in the test and state why the condition is necessary. [2]

Let \( X \) be a random variable for the number of individuals in the sample who purchase more than 380 ml of full price liquid milk.

b State the hypotheses \( H_0 \) and \( H_1 \) and give the distribution of \( X \). [3]

c At a significance level of 10%, the critical value is 3. State the conclusion of the hypothesis test. [1]
Hypothesis testing

11.2 The critical region

Recap

- To find the critical region, you assume the null hypothesis in order to calculate probabilities.
- The critical value lies inside the critical region.
- For the binomial distribution, $X \sim B(n, p)$, it is very unlikely that there will be an integer, $m$, such that $P(X \geq m)$ exactly equals the required significance level.

In practice, for a high-values, one-tailed test, you find the smallest $m$ such that

$$P(X \geq m) \leq \text{significance level}$$

$$[P(X \geq m - 1) > \text{significance level}]$$

The critical region is $X \geq m$.

- For low-values you find the largest value of $m'$ such that

$$P(X \leq m') \leq \text{significance level}$$

$$[P(X \geq m' + 1) > \text{significance level}]$$

If you have $H_0: p = 0.1$ and $H_1: p < 0.1$ and wish to find the critical region at the 5% significance level when there are 60 trials, you must find the largest value of $x$ such that $P(X \leq x) \leq 0.05$

You need to assume $X \sim B(60, 0.1)$

By adding up the probabilities you can see that

$P(X \leq 1) \leq 0.0138$ which is less than 5%

$P(X \leq 2) \leq 0.0530$ which is greater than 5%

The critical value is 1 and the critical region is $X \leq 1$

- Alternatively, you can carry out a hypothesis test by finding the $p$-value of a test statistic; this is the probability of obtaining the test statistic or worse under the null hypothesis.

Suppose $H_0: p = 0.6$ and $H_1: p \neq 0.6$ and that out of 10 trials, 9 were successful.

With 10 trials the expected number of successes is $10 \times 0.6 = 6$ so 9 is more than expected.

Hence you need to calculate the probability of getting 9 or more successes out of 10 trials.

$P(X \geq 9) = 0.0403 + 0.0065 = 0.046$

In this case the $p$-value is 0.0468 (4.68%)

$H_1: p \neq 0.6 \Rightarrow$ use a two-tailed test: you split the 5% between the two tails.

Therefore you need to compare the $p$-value to 2.5%

$4.68\% > 2.5\% \Rightarrow$ do not reject the null hypothesis.

There is insufficient evidence to assume that the probability of success is not 0.6
A supermarket sells eggs in boxes of 6. The probability of an egg being broken is 1%. A crate of 20 boxes of eggs contains 3 broken eggs. Assuming that the number of broken eggs can be modelled by a binomial distribution, use a hypothesis test with a 10% significance level to test if this an unusually large number of eggs to be broken.

\[ H_0: p = 0.01 \quad H_1: p > 0.01 \]
Assume \( X \sim B(120, 0.01) \)
\[ P(X \geq 3) = 1 - (0.2994 + 0.3629 + 0.2181) = 0.1186 \]
0.1186 > 10% \( \Rightarrow \) Do not reject the null hypothesis
There is insufficient evidence at the 10% significance level that an unusually large number of eggs were broken.

The probability of a particular bus being on time has been found over time to be 52%
The bus company wants to test if this has changed.

a Write down the null and the alternative hypotheses.
A binomial model is to be used and a random sample of 11 buses.
b Find the critical region at the 5% significance level.
c What is the probability of incorrectly rejecting the null hypothesis in this case?

Let \( p \) = the probability of a bus being on time

\[ H_0: p = 0.52 \quad H_1: p \neq 0.52 \]
Assume \( X \sim B(11, 0.52) \)
\[ P(X \leq 2) = 0.0201 + 0.0037 + 0.0003 = 0.0241 \]
\[ P(X \geq 10) = 0.0076 + 0.0008 = 0.0084 \]
Critical region is \( X \geq 10, X \leq 2 \)
\[ 0.0084 + 0.0241 = 0.0325 \]
There is a 3.25% chance of incorrectly rejecting \( H_0 \)

Create a table on your calculator
\[ f(x) = ^{11}C_x \times 0.52^x \times 0.48^{11-x} \]
It is a two-tailed test, so you need 2.5% in each tail.
\[ P(X = 9) = 0.352 \text{ so too high.} \]

**Exam tips**

**Calculating critical values**

In the exam AQA will not provide tables of cumulative binomial distributions so you must learn how to find critical values using your own calculator. If your calculator can find inverse binomial distributions, you should check if it gives the value of \( x \) such that \( P(X \leq x) \) is closest to the required significance level. This may mean that \( P(X \leq x) \) is slightly larger than the required significance level and that the critical value is \( x - 1 \). The correct value can be found by inspecting the values of \( P(X \leq x) \) and \( P(X \leq x - 1) \).

For example, suppose \( X \sim B(20, 0.3) \) and a 10% significance level is required for a low-values, one-tailed test. A calculator may give \( x = 3 \) as \( P(X \leq 3) = 10.7\ldots\% \), whereas the correct critical value is \( x = 2 \) for which \( P(X \leq 2) = 3.54\ldots\% \).
1 a A random variable $X$ has a binomial distribution, $X \sim B(6, p)$. The value of $p$ was known to be 0.72 but is now believed to have decreased.

What are the null and alternative hypotheses in a test of this belief? [1]

Circle your answer.

A $H_0: p = 0.72$  $H_1: p \neq 0.72$
B $H_0: p = 0.72$  $H_1: p < 0.72$
C $H_0: p < 0.72$  $H_1: p = 0.72$
D $H_0: p = 0.72$  $H_1: p < 0.19$

b The table below gives the probabilities, to 2 significant figures, of all values of $X$ assuming that the null hypothesis is true.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>P($X = x$)</td>
<td>0.00048</td>
<td>0.0074</td>
<td>0.048</td>
<td>0.16</td>
<td>0.32</td>
<td>0.33</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Using a significance level of 5%, what values of $X$ would suggest that the belief is incorrect?

Explain your answer. [2]

2 In 2013, 70% of adults had a higher than recommended percentage of food energy derived from saturated fatty acids. Two years later, it was claimed that this proportion had increased. To test this, a random selection of 16 adults were investigated and 14 were found to have a higher than recommended percentage of food energy derived from saturated fatty acids.

a Perform a hypothesis test of this claim using a significance level of 10%

You should state clearly the null and alternative hypotheses. [5]
2  b  Comment on the reliability of the test.  [1]

3  In 2016, a large investigation found that 6 out of every 10 people questioned agreed that the quantity of butter they bought each week had increased over the last year. This year, in an independent, random sample of 25 people, 20 people agreed with this proposition. A test was carried out to determine whether the proportion of people agreeing with the proposition had changed within the whole population.
   a  Write down the null and alternative hypotheses for the test.  [2]
   b  Perform the test at a significance level of 5%, stating clearly your conclusion.  [3]

4  In 2010 the median amount of soft drinks bought was 1206 ml per person per week. In order to test whether this had changed by 2014, 35 people were asked about the amount of soft drink they had bought in one week. Twelve of them had bought less than 1206 ml per week.
   Test at a significance level of 5% whether this is evidence of a change in the amount of soft drink bought during the four years  [5]
5 A chicken farm has a hatch rate (the percentage of fertilized eggs which hatch after incubation) of 52%. Following the introduction of new incubation equipment, a random sample of 64 eggs produced 39 chicks.

a Explain why the binomial probability distribution could provide a good model for the number of chicks produced from these eggs. [3]

b Complete the following table for X, a random variable for the number of chicks from a total of 64 eggs using the old equipment. [3]

<table>
<thead>
<tr>
<th>x</th>
<th>39</th>
<th>40</th>
<th>41</th>
<th>≥42</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X = x)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c It is claimed that the new equipment has increased the hatch rate for fertilized eggs. Test, using a significance level of 5%, whether the claim is justified. [2]

6 In the first half of the 18th century, the mean height of an adult English male was 165 cm. A comparative study on the heights of adult Irish males found 27 with heights over 165 cm in a sample of 40 randomly chosen Irish males.

a Explain why it is reasonable to assume that the probability that a randomly chosen English male exceeds 165 cm is 0.5. [2]

b Let X be the number in the sample of 40 Irish males with heights over 165 cm. Using a significance level of 2%, test the belief that Irish males are on average taller than English males. [2]
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