Advance sample materials

14 pages of sample content written for the 2017 specification

STATISTICS TEACHER BOOK

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**An advance sample version of this chapter is provided in this book.**

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11.0 Background

This chapter covers the basics of hypothesis testing which is developed further in Chapter 21. This work will be entirely new to students as there is no reference to this type of testing at GCSE level. This work has also not been part of the first statistics modules of pre-2017 specifications so many teachers will not have taught this before.

The basis of hypothesis testing is to test a whether a claim or hypothesis should be accepted or rejected based on the available data. A statistical statement is made—this is known as the null hypothesis and given the notation $H_0$. A statistical test is then performed and the null hypothesis is only rejected if there is sufficient evidence to do so. If this is the case the alternative hypothesis—which contradicts the null hypothesis is accepted instead.

A null hypothesis always suggests that a statistic is equal to a particular value and so includes an equal sign whereas the alternative hypothesis can say that the statistic being tested is lower, higher or simply not equal to the value stated in the null hypothesis.

The significance level of a test gives the probability that under the null hypothesis the null hypothesis will be incorrectly rejected; this is formally known as a type-I error. As the significance level is decreased, the critical region gets smaller and it becomes harder to reject the null hypothesis due to a fluctuation, assuming $H_0$. If the test could be repeated many times then the significance level gives the fraction of times that the null hypothesis would be incorrectly rejected.

It is very important that the early basic ideas in the chapter are understood by all students otherwise the rest of the work will be built on very shaky foundations. It is therefore worth taking the early stages slowly to ensure full understanding.

Chapter 11 has two sections.

### Prior knowledge

**GCSE**

Interpret and analyse the distributions of data sets

### Objectives

**11.1 Formulating a test**

Formulating null and alternative hypotheses

Acceptance/rejection based on a given test statistic and a given critical region

The significance level and interpreting a test result

Calculating the critical region

Calculating and interpreting p-values

**11.2 The critical region**

Calculating critical values and critical regions for the binomial distribution

The significance level of a test and the p-value of a test statistic

### Leads to

**21 Hypothesis testing 2**

Testing hypotheses for the correlation coefficient and the normal distribution

### Resources

**MyMaths**

Hypothesis testing 2115

**Kerboodle assessments**

Chapter tests 11A, B

Online skills test 11

**InvisiPen videos**

Testing a hypothesis 11.1A

Using p-values 11.2B

**ICT resource**

Hypothesis testing 11.2

Statistics student workbook Ch 11
11.1 Formulating a test

Example 1, on page 280 of the student book shows the main idea behind a hypothesis test.

The null hypothesis is a statement with an equals sign in it. Here, \( H_0 \) states that \( p = 0.85 \) to represent the 85% of customers that are satisfied.

The alternative hypothesis contradicts the null hypothesis, but changes depending on the belief of which way the null hypothesis is wrong.

- If you feel that 85% is an overestimate, then \( H_1 \) states that \( p < 0.85 \)
- On the other hand, if you think 85% is an underestimate, then \( H_1 \) states that \( p > 0.85 \)

Both of these will be tested with a one-tailed test as you just want to look either above or below the given value.

- If you just want to say that 85% is wrong, then \( H_1 \) states that \( p \neq 0.85 \)

This will be tested using a two-tailed test as you are looking both below and above the given value.

Once the test has been set up, the next stage is to get the critical values, these are the values that lie on the edge of the critical region. You can then look to see if you should accept or reject the null hypothesis.

This is demonstrated in Example 3. Note that in this section the critical values are given and students are not required to calculate them.

This is an example of a one-tailed test because the value is thought to be too high. Thus the alternative hypothesis is that \( p < 0.7 \)

Students are told that the critical value at a 10% significance level is 14.

This means that the critical region is \( X \leq 14 \) and the acceptance region is \( X \geq 15 \). That is, if 14 or fewer candidates in the sample pass their driving test then the null hypothesis should be rejected whereas if 15 or more candidates in the sample pass, the null hypothesis should be accepted.

11.2 The critical region

This section requires students to calculate the critical values themselves and so identify the critical region. The examples are based on the binomial distribution covered in Chapter 10. If these two chapters are not taught consecutively, you may wish to revise the binomial distribution first.

In these problems students will be presented with a binomial distribution with a number of trials given. The null hypothesis will be a particular value of \( p \) to go with the distribution. The alternative hypothesis will then
be that \( p \) is either greater than, less than or not equal to this value. Given a significance level for the test, usually 1%, 5% or 10%, students then calculate a value of \( X \) which gives this percentage using the binomial formula. Since it is very unlikely that this will produce a probability of exactly 5\% = 0.05, students will need to choose a value that makes the value no more than 5\% T is, as close to 5\% as possible without exceeding it. In some questions, students are just asked to work out a particular value and check whether this would be in the critical region or not.

An alternative to finding the critical values is to use the p-value of the result obtained. Example 2 in this section illustrates this method.

The farmer’s claim that it rains on 80\% of the days in February gives the null hypothesis of \( p = 0.8 \) Because the farmworker just says that it is inaccurate but does not specify too low or too high, the alternative hypothesis becomes \( p \neq 0.8 \)

The significance level required is 10\% so the probability from the binomial distribution is required to be as close to 0.1 as possible whilst not exceeding it. Since it is a two-tailed test, this value is halved as you would want 5\% at either side of the distribution. The farmworker finds that it rained on 26 out of the 28 days so find \( P(X \geq 26) \) from the binomial distribution, \( X \sim B(28, 0.8) \). As this comes to 0.061, this is more than 5\%. This means that there is not sufficient evidence to reject the null hypothesis and so it should be accepted.

**Investigation—Biased dice?**

Ask each student or group of students to create a biased dice. This can be done easily using the net of a dice with a piece of blu-tak taped onto an internal face of the dice. The dice will then be biased to the number opposite the face with the blu-tak on it.

Dice should then be swapped between students or groups.

**Question** How do you decide if a dice is biased?

It is likely that students will suggest rolling the dice many times and looking at the relative frequencies of the faces 1 to 6. Given such data, take the opportunity to quiz the students on how they would decide if the dice is biased – press them to be quantitative.

This should provide the opportunity to naturally cover the material in the chapter.

- **Section 11.1** explains how to formulate null and alternative hypotheses to test against. The null hypothesis is that the dice is not biased and so the probability of getting any number is \( H_0: p = \frac{1}{6} \). The alternative hypothesis will be that either \( H_1: p > \frac{1}{6} \), \( p < \frac{1}{6} \) or \( p \neq \frac{1}{6} \) depending on what is being tested.
- Given a critical region, section 11.1 shows students how to use their results to accept or reject the null hypothesis. This can be repeated with different significance levels to bring out the idea that the lower the significance level the more evidence is needed to reject \( H_0 \) and the lower is the probability of making a wrong decision assuming \( H_0 \) is correct.
- Students can then investigate how to find critical values for themselves, using section 11.2 for guidance. Students should use their calculators or the statistical tables provided to find the cumulative probabilities for a given binomial distribution.
- **Section 11.2** also explains how to use the p-value to test a hypothesis in the absence of knowing the critical values.
Example 1
A manufacturer states that 40% of the chocolates in a large tin are dark.
A customer thinks the fraction is different. To test this they take 20 chocolates from a tin and find that 12 are dark.

a State the hypotheses being tested.

b Let \( N \) = the number of dark chocolates.

What is the distribution of \( N \)?

c For a test with significance 10\% the critical values are 3 and 13.

What is the conclusion of the test? Give your reasons.

\[
\begin{align*}
a & \quad \text{Let } f = \text{the fraction of dark chocolates} \\
H_0: f = 0.4 \\
H_1: f \neq 0.4 \\
b & \quad \text{The critical region is} \\
N \leq 3 \quad \text{or} \quad 13 \leq N \\
c & \quad 12 \text{ is not in the critical region.} \\
\text{The null hypothesis is accepted.}
\end{align*}
\]
1 A variety of rose produces red or white flowers. A supermarket buyer wishes to buy roses where the proportion of white flowers is at least 75%. She takes a sample of 30 roses to test if the proportion of plants with white flowers is sufficiently high.
   a If $p$ is the population proportion of plants with white flowers, give the distribution of $X$, a random variable for the number of plants in the sample with white flowers.
   b State the null and alternative hypothesis for this test.
   c Is the critical region in the left or right tail of the distribution in $H_0$ and, if the observed value of $X$ falls in the critical region, state the final conclusion of the test.

2 A dice is thrown 20 times and the number of sixes recorded. A test of whether or not the dice is fair is to be performed. State fully the null and alternative hypotheses for the test for these questions.
   a Is the dice biased?
   b Is the dice biased towards sixes?
      In each case, state whether it is low, high or both low and high values of the total number of sixes which lead to rejection of the null hypothesis.

3 21.2% of Germans and 18.8% of Austrians are over 65 years old. Explain why it is not appropriate to perform a hypothesis test to investigate whether these proportions are significantly different.

4 In a large survey carried out five years ago, 6 out 10 adults said that daily maximum temperatures had decreased over the last 15 years. This year, in an independent random sample of 30 adults, 23 agreed with this statement.
   Let $N$ be the number of adults in the sample who agreed with the statement.
   a State the null and alternative hypotheses for a test that the number agreeing with the statement had changed.
   b State the distribution of $N$.
   c For a test with significance 5% the critical values is 23.
      What is the conclusion of the test?
      Give your reasons.

5 You wish to investigate whether three coins, labelled A, B and C, are biased. You toss each coin 24 times and note the number, $X$, of heads showing.
   a Write down the null and alternative hypotheses to test this possibility.
   b At a significance level of 5% the critical values are 6 and 18.
      State the conclusion of each test giving your reasons.
      i For coin A, $X = 16$
      ii For coin B, $X = 6$
      iii For coin C, $X = 19$

6 It is estimated that 40% of all days in Camborne have a moderate daily mean wind speed. A sample of 30 days were considered and 15 were found to have this level of wind speed.
   a State a condition on the method of choosing the sample so that a binomial probability model can be used to test the estimate.
   b If the condition in part a is met, write down the null and alternative hypotheses.
      Explain your choice of alternative hypothesis.

7 The proportion of individuals with a fat intake more than the recommended daily allowance was found to be 28%. In a sample of 50 individuals taken 5 years later, 20 had a fat intake more than recommended. A statistical test is to be carried out to determine whether the proportion with a fat intake higher than recommended has increased over this period.
   a State with reasons a condition for the sample to be suitable for use in the test.
   b Let $X$ = the number of individuals in the sample with a fat intake higher than the recommended.
      State $H_0$ and $H_1$ for this test and, given that $H_0$ is true, give the distribution of $X$.
   c The critical value is 20. What is the conclusion of the hypothesis test?
A random variable $X$ has a binomial distribution, $X \sim B(n, 0.7)$

The table shows the critical values for $X$ assuming a two-tailed hypothesis test for two values of the number of trials, $n$.

a  What is the result of these tests? Give your reasons.

i  $H_0: p = 0.7$, $H_1: p \neq 0.7$, significance level = 10%, $n = 20$ and $X = 10$

ii  $H_0: p = 0.7$, $H_1: p \neq 0.7$, significance level = 5%, $n = 20$ and $X = 10$

iii  $H_0: p = 0.7$, $H_1: p < 0.7$, significance level = 5%, $n = 30$ and $X = 28$

b  Comment on your answers to parts i and ii above.

c  Explain why the answer to part a ii changes if $n = 30$.

a  i  Critical region $X \leq 10$ or $X \geq 18$

10 is in the critical region

$\Rightarrow$ Reject the null hypothesis that $p = 0.7$

ii  Critical region $X \leq 9$ or $X \geq 19$

10 is not in the critical region

$\Rightarrow$ No reason to reject the null hypothesis that $p = 0.7$

iii  Critical region $X \leq 16$

28 is not in the critical region

$\Rightarrow$ No reason to reject the null hypothesis that $p = 0.7$

b  The conclusions are different because a larger significance level gives a larger rejection region.

c  New critical region $X \leq 15$ or $X \geq 27$ $\Rightarrow$ reject $H_0$

The average number of successes is $0.7 \times 20 = 14$ for $n = 20$ and $21$ for $n = 30$. In the larger sample $X=10$ is a more extreme value and so less likely under $H_0$.

Example 2

In a test of the hypotheses $H_0: p = 0.4$ and $H_1: p \neq 0.4$ at a significance level of 10% the critical values are 5 and 15.

a  Find an inequality for $P(4 < X < 11)$

b  If $a$ and $b$, $a < b$, are the critical values for the same test with a significance level of 20% find inequalities for $a$ and $b$.
1 In hypothesis testing, it is usual to either ‘reject the null hypothesis’ or state that there is ‘no reason to reject the null hypothesis.’

Why is this language used rather than the more obvious ‘reject’ or ‘accept’?

2 a State the conditions under which the binomial distribution provides a good model for a statistical experiment.

The median summer daily rainfall at Heathrow is 2.2 mm. To test the hypothesis that Camborne has a greater proportion of days where the rainfall is above 2.2 mm, researchers sampled 25 summer days at Camborne and found that 18 had a rainfall greater than 2.2 mm.

b Why is it necessary to know the median rainfall rather than either the mean or mode?

c Let \( X \) be the number of days in the sample with rainfall greater than 2.2 mm.

Give the distribution of \( X \) and state the null and alternative hypotheses for the required test.

d Under the null hypothesis, for a significance level of 5% the critical value is \( 18 \) and \( P(X < 18) = 0.978 \)

i What is the result of the test?

Give your reasons.

ii What is the probability that the conclusion is incorrect given that the null hypothesis is true?

3 In Leuchars during 1987, 12.2% of days were classified as low pressure (mean pressure below 1006 hPa). You wish to investigate a possible change in this proportion. During 2015, a sample of 50 days included 9 classified as low pressure.

Let \( X \) be a random variable for the number of low pressure days in a sample of size 50.

a State the null and alternative hypotheses for a suitable hypothesis test with a significance level of 5% if you are testing for

i a change in the proportion

ii an increase in the proportion.

b Explain why it makes no sense to test whether there is a decrease in this proportion.

c In both of the tests in part a, a large value of \( X \) will lead to the null hypothesis being rejected. Explain why the lower limit for rejection is higher for the test in part a i

4 In a test of the Hypotheses \( H_0: p = 0.34 \) and \( H_1: p < 0.34 \) at a significance level of 10% the critical value is 8.

a Find an inequality for \( P(X \leq 8) \)

b Find an inequality for \( a \) if \( a \) is the critical value for a test at a 20% significance level.

5 In a test of the Hypotheses \( H_0: p = 0.43 \) and \( H_1: p \neq 0.43 \) at a significance level of 5% the critical values are 10 and 40.

a Find inequalities for

i \( P(10 \leq X \leq 40) \)

ii \( P(X < 9) \)

iii \( P(9 < X < 41) \)

b If \( a \) and \( b \), \( a < b \), are the critical values for the same test with a significance level of 10% find inequalities for \( a \) and \( b \).

6 a Successive binomial probabilities can be found using the formula

\[
P(X = x + 1) = \binom{n}{x} \frac{p}{q} \cdot \frac{(n-x)}{(x+1)}
\]

where \( X \sim B(n, p) \) and \( q = 1 - p \).

You are given that if \( n = 50 \) and \( p = 0.6 \), then \( P(X \leq 23) = 0.0314 \) and \( P(X = 23) = 0.0154 \).

Using the recurrence formula, find \( P(X \leq 24) \).

b Over several years during the 1980s it was found that the proportion of early summer days in Beijing when the mean air temperature was above 21 °C was 60%.

To investigate possible climate changes, a sample of 50 days was taken 30 years later and 24 of these had a mean air temperature above 21 °C.

i Explain how the sample should be taken so that it could be used to test the hypothesis that the proportion of days above 21 °C had decreased.

ii Use the results in part a to test this hypothesis.
**Common misconceptions / Exam tips**

**One or two tails**

It is important that students understand the difference between a one-tailed and a two-tailed test. Using graphs may help.

In a two-tailed test, the significance value is split in half so that half is at the top of the distribution and half at the bottom. Thus, to do a 10% two-tailed test, you need 5% at the top and bottom.

Setting up the alternative hypothesis is important. If it is thought that the claim is simply wrong, then the alternative hypothesis will be $x \neq a$ value and a two-tailed test should be used. If it is thought to be too high or too low the alternative hypothesis will involve an inequality and a one-tailed test should be used.

Deciding between using a one-tailed test for low values, a one-tailed test for high values or a two-tailed test, often depends on identifying the key word or phrase in the question that tells you how the probability of success may be different under the alternative hypothesis $H_1$.

For example, if $H_0: p = p_0$, then these words

- too low, underestimate, less often, lower, reduced, worse  
  might ⇒ $H_1: p < p_0$
- too high, overestimate, more often, higher, increased, better  
  might ⇒ $H_1: p > p_0$
- inaccurate, incorrect, wrong, has changed  
  might ⇒ $H_1: p \neq p_0$

The actual interpretation will rely on the context of the question.

**Exercise 11.1B student book commentary**

**Question 1** To decide whether $H_1$ includes an $<, \neq, >$ sign look for the key word/phrase in the description of what is being tested.

- a ‘too low’  ⇒ $H_1: p < 0.6$  [One-tailed, high values]
- b ‘too high’  ⇒ $H_1: p > 0.6$  [One-tailed, low values]
- c ‘inaccurate’  ⇒ $H_1: p \neq 0.6$  [Two-tailed, low and high values]

**Question 2** The key to this question is in understanding what is the critical region; if students struggle ask them to discuss it with a partner. Students who struggle with part b should be directed to example 4 on page 284 of the student book.

**Question 3** Parts a and b are straightforward. In part b, ensure that students do not simply write ‘39 < 41 ⇒ accept $H_0$’ but instead write a full sentence explaining their conclusion in the context of the question. Students who struggle with part c should be directed to example 4 in the student book.

**Question 4** Part d is a new question type. The key idea is that the size of the critical region decreases as the significance level decreases and vice versa. In the question the critical region gets smaller so you are less likely to reject $H_0$ by mistake and so the significance level must be less.

Encourage students to discuss the reasoning behind their answers.

**Question 5** This question extends the ideas in question 4d from a one-tailed test to a two-tailed test however the basic reasoning remains the same.

Students who finish early could be asked to write their own ‘inequality’ questions and swap them with a partner.
The critical region

### Simplification

For each hypothesis calculate:

1. the critical region(s) for a significance level of 10%
2. state the conclusion if $X = 4$

**Example 1**

a) $X \sim B(20, p)$, $H_0: p = 0.4$ and $H_1: p < 0.4$

- **Under $H_0$**, $X \sim B(20, 0.4)$
  - $H_1 \Rightarrow$ one-tailed test, low values of $X$
  - $P(X \leq 4) = 5.09\%$ and $P(X \leq 5) = 12.5\%$
  - **Critical region** $X \leq 4$
  - 4 is in the critical region $\Rightarrow$ **reject $H_0$**

b) $X \sim B(12, p)$, $H_0: p = 0.2$ and $H_1: p > 0.2$

- **Under $H_0$**, $X \sim B(12, 0.2)$
  - $H_1 \Rightarrow$ one-tailed test, high values of $X$
  - $P(X \geq 4) = 20.5\%$ and $P(X \geq 5) = 7.25\%$
  - **Critical region** $X \geq 5$
  - 4 is **not in** the critical region $\Rightarrow$ **accept $H_0$**

c) $X \sim B(30, p)$, $H_0: p = 0.45$ and $H_1: p \neq 0.45$

- **Under $H_0$**, $X \sim B(30, 0.45)$
  - $H_1 \Rightarrow$ two-tailed test, low and high values of $X$
  - Probability in each tail $\leq 2.5$%
  - $P(X \leq 8) = 3.12\%$ and $P(X \leq 9) = 6.94$
  - $P(X \geq 18) = 7.13\%$ and $P(X \geq 19) = 3.34\%$
  - **Critical region** $X \leq 8$ and $X \geq 19$
  - 4 is **in** the critical region $\Rightarrow$ **reject $H_0$**

**Example 2**

A survey of the public’s attitude towards GCSE exams asked whether respondents thought that GCSEs had become easier. Five years ago, 76% of people agreed with this statement. This year, in an independent random sample of 20 people, 12 agreed. Is there evidence that the proportion of people holding this view has decreased? You should use a 10% significance level and give the exact size of the critical region.

- $H_0: p = 0.76$, $H_1: p < 0.76$
- $X \sim B(20, 0.76)$
- $P(X \leq 12) = 8.34\%$ and $P(X \leq 13) = 18.3\%$
- **Critical region** $X \leq 12$
- 12 is in the critical region $\Rightarrow$ **reject $H_0$**

OR

- $p$-value $= P(X \leq 12) = 8.34\%$
- $p$-value $< 10\% \Rightarrow$ **reject $H_0$**

There is evidence that the proportion of people holding this view has decreased.

The exact size of the critical region is 8.34%
For each hypothesis
i find the critical region(s) for a significance level of 10%

ii state the conclusion for the given value of \( X \)

\[
\begin{align*}
\text{a} & \quad X \sim B(20, p), H_0: p = 0.3, \quad H_1: p < 0.3 \\
\text{and} & \quad X = 3 \\
\text{b} & \quad X \sim B(10, p), H_0: p = 0.5, \quad H_1: p > 0.5 \\
\text{and} & \quad X = 2 \\
\text{c} & \quad X \sim B(40, p), H_0: p = 0.55, \quad H_1: p \neq 0.45 \\
\text{and} & \quad X = 16
\end{align*}
\]

A random variable \( X \) has a binomial distribution, parameters \( n = 8 \) and \( p \), a constant. The value of \( p \) was known to be 0.7 but is now believed to have changed.

i Write down the null and alternative hypotheses in a test of this belief.

The table gives the probabilities, to 2 sf, of all values of \( X \) assuming that the null hypothesis is true.

\[
\begin{array}{c|cccccccc}
\text{x} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\hline
\text{P}(X = x) & 0.000066 & 0.0012 & 0.010 & 0.047 & 0.14 & 0.25 & 0.30 & 0.20 & 0.058 \\
\end{array}
\]

ii Using a significance level of 5%, what values of \( X \) would suggest that the belief is incorrect?

A fortune teller claims that in more than 70% of cases, he can predict the marital status of his client. He is tested with a random selection of 8 clients and successfully identifies the marital status of all of them.

Using the probability distribution in part a, perform a hypothesis test of his claim using a significance level of 10%.

You should state clearly the null and alternative hypotheses.

In a large investigation carried out 20 years ago, 60% of days at Hurn had a mean windspeed classified as ‘light’. This year, an independent random sample of 15 days included 12 days with this classification. A test was carried out to determine whether the proportion of days with this classification had increased.

i Write down the null and alternative hypotheses for the test.

b Perform the test at a significance level of 10%, stating clearly the conclusion of the test.

Over a long period, the springtime median daily maximum wind speed at Heathrow was known to be 19 knots. In order to test whether the proportion of days with a maximum wind speed over 19 knots had increased by 2015, 20 spring days were chosen at random and 14 of these had a maximum windspeed greater than 19 knots.

Test at a significance level of 5% whether this is evidence of an increased proportion of days with maximum windspeed over 19 knots.

Copy and complete the table for binomial probabilities with \( n = 8 \), \( p = 0.44 \) Give your answers to 4 dp.

\[
\begin{array}{c|cccccccc}
\text{x} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\hline
\text{P}(X = x) & & & & & & & & & \\
\end{array}
\]

It is known that 44% of springtime days in Leeming have at most a trace of rainfall. In Hurn during the same period it is claimed that this proportion is greater. In a survey of 8 days in Hurn, 6 were found to have at most a trace of rainfall.

Perform a test with a significance level of 5% to investigate this claim. You should state clearly your null and alternative hypotheses.
A newspaper claimed that 1 in 4 adult in the UK is obese. A researcher thinks that this is an underestimate. He decides to carry out a hypothesis test with a 5% significance level using a sample of 40 adults from town A.

a State the null and alternative hypotheses.

b In the sample n of the adults were obese and he accepts $H_0$ for town A.

He then repeats the survey in town B and finds $n + 1$ adults are obese and rejects $H_0$ for town B.

What is the value of $n$?

a Let $p =$ the fraction of obese adults

$$H_0: p = 0.25 \quad H_1: p > 0.25$$

b $N \sim B(40, 0.25)$

$$P(N \geq n) > 0.05 \Rightarrow P(N < n) < 0.95$$

$$P(N \geq n + 1) \leq 0.05 \Rightarrow P(N < n + 1) \geq 0.95$$

$$P(N \leq 15) = 0.945 \Rightarrow \text{accept } H_0$$

$$P(N < 16) = 0.973 \Rightarrow \text{reject } H_0$$

$n = 15$

It is estimated that the proportion of adults in Heathrow who believe that the average daily amount of cloud cover has remained constant over the last five years is 0.4. To test this estimate, a sample of $n$ adults from Heathrow is taken and, $X$, the number who support this belief is noted. The set of $x$-values which leads to a rejection of the estimate at a significance level of 5% is $x \leq 6$, $x \geq 18$.

a Show that $n = 30$

b Explain why it may be useful to consider a test with a smaller significance level.

c Find the critical region for the same test with a significance level of 2%.

a Let $p =$ the fraction of adults who support the belief

$$H_0: p = 0.4 \quad H_1: p \neq 0.4$$

Assume $n = 30$ and $H_0 \Rightarrow X \sim B(30, 0.4)$

$$P(X \leq 6) = 0.017 \quad \text{and} \quad P(X \leq 7) = 0.043$$

$$P(X \geq 18) = 0.021 \quad \text{and} \quad P(X \geq 17) = 0.048$$

∴ the critical region is $x \leq 6, x \geq 18, \therefore n = 30$

b Significance level = probability of rejecting $H_0$ when it is true. Reducing this probability may be important if the consequences of making this mistake are serious.

c $P(X \leq 5) = 0.005 \quad \text{and} \quad P(X \leq 6) = 0.017$

$$P(X \geq 19) = 0.008 \quad \text{and} \quad P(X \geq 18) = 0.021$$

Critical region: $x \leq 5, x \geq 19$
1. a. A hypothesis test is carried out for the null hypothesis that a population fraction equals $p_0$ against the alternative that it is less than $p_0$. Explain why a one-tailed test, with the critical region on the left of the distribution defined in the null hypothesis, is used and not a two-tailed test.

b. A long term study found that 25.2% of days in Scotland had a maximum relative humidity classified as 'high' (above 98%). The same humidity measurement was recorded in Leuchars on a sample of days to test whether they were typical of the whole of Scotland.

i. Write down the null and alternative hypotheses for this test.

ii. Find the maximum and minimum sample size if there were 12 days in the sample with high humidity and the null hypothesis was rejected.

2. A late 1980s investigation into global warming found that 73% of summer days had daily maximum temperatures above 20 °C. It is believed that this fraction was higher during the summer of 2015. To test this, a researcher took a sample of 16 days from summer 2015 and found 14 days when the daily maximum temperature was above 20 °C.

a. State the null and alternative hypotheses for this test.

b. Find the probability that, under the null hypothesis, at least 14 of the daily maximum temperatures would be above 20 °C.

c. State the conclusion about the null hypothesis to be drawn from part b. Use a significance level of 5%.

d. What is the minimum number of days with a maximum temperature above 20 °C which would lead to a rejection of the null hypothesis?

3. A firm wishes to sell a large batch of electrical components. A buyer decides to accept the batch if a random sample of 30 components contains no more than 1 defective component.

a. If the probability of the batch being accepted has to be at least 95%, show that the maximum probability that one component is defective is 0.011 (3 dp).

b. To test whether the probability of any item being defective is 0.011, 300 components are tested and 5 are found to be defective. Perform this test against the alternative hypothesis that the fraction of defective components is greater than 0.011.

4. A five sided spinner, labelled 1 – 5, is spun $n$ times and $X$, the number of ones and twos obtained, is recorded. The spinner is tested to see if it is fair at the 20% significance level. The critical region is $X \leq 2$ and $X \geq 11$.

a. What is the value of $n$?

b. What is the critical region for a test at the 5% significance level?

5. A dice is rolled $n$ times and $X$, the number of fives and sixes obtained, is recorded. The dice is tested to see if it is fair at the 10% significance level. The critical region is $X \leq 4$ and $X \geq 13$.

What is the critical region for a test at the 5% significance level?

6. The random variable $X \sim B(n, p)$. A test of the hypotheses $H_0: p = 0.3$ and $H_1: p > 0.3$ is carried out for a significance level of 10%. The result $X = 11$ gives a p-value that only just falls in the rejection region.

If the test had a significance level of 20%, would $X = 10$ lead to $H_0$ being rejected?

7. In 2012 at Heathrow, 73% of summer days had at most half the sky covered by cloud covered ($\leq 4$ oktas). Later surveys suggest that this proportion has decreased. To test this hypothesis, researchers took a sample of summer days at Heathrow.

a. If $n$ is the sample size and $X$ is the number of days in the sample with $\leq 4$ oktas of cloud cover, write down $H_0$ and $H_1$ for this test.

b. If $n = 8$, find the values contained in the critical region.

c. Given that the critical region is $x \leq 6$, find the value of $n$. 

Calculating critical values

Your calculator may be able to find binomial probabilities. If your calculator can also find inverse binomial distributions, you should check if it gives the value of \( x \) such that \( P(X \leq x) \) is closest to the required significance level. This may mean that \( P(X \leq x) \) is slightly larger than the significance level and that the critical value is \( x - 1 \). The correct value can be found by inspecting the value of \( P(X \leq x) \).

For example, suppose \( X \sim B(20, 0.3) \), and a 10% significance level is required for a low-values, one-tailed test. A calculator may give \( x = 3 \) as \( P(X \leq 3) = 0.1070 \). whereas the correct critical value is \( x = 2 \) for which \( P(X \leq 2) = 0.0432 < 5\% \).

There are websites that will help you calculate these values such as www.statrek.com

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**Exercise 11.2B student book commentary**

**Question 1** In part a, the word ‘overestimate’ signals that alternative hypothesis is \( H_1: p < 0.8 \) and a one-tailed test is required. The probability distribution is \( X \sim B(40, 0.8) \).

Students should use their calculators to find the critical region, that is the smallest value of \( x \) such that \( P(X \leq x) \) is less than 5\%: \( P(X \leq 28) = 0.0875 > 5\% \) and \( P(X \leq 27) = 0.0432 < 5\% \).

Part b, it is not sufficient to state ‘31 > 27, accept \( H_0 \)’; a full sentence is required.

**Question 2** In part a, the word ‘inaccurate’ signals that alternative hypothesis is \( H_1: p \neq 0.6 \) and a two-tailed test is required. The probability distribution is \( X \sim B(75, 0.6) \). In a two-tailed test the probability of rejecting \( H_0 \), the significance level, is split equally between the tails. \( P(X \leq 35) = 0.0113 > 1\% \) but \( P(X \leq 34) = 0.0072 < 1\% \) and \( P(X \leq 55) = 0.0056 < 1\% \).

**Question 3** The word ‘changed’ signals that alternative hypothesis is \( H_1: p \neq 0.15 \) and a two-tailed test is required. The probability distribution is \( X \sim B(60, 0.15) \).

\( P(X \leq 4) = 0.0424 < 5\% \Rightarrow \text{reject } H_0 \) and \( P(X \leq 5) = 0.0968 > 5\% \Rightarrow \text{accept } H_0 \)

**Question 4** This is a one tailed test with \( B(4, 0.5) \) and \( B(24, 0.5) \) with \( H_1: p = 0.5 \) and \( H_1: p > 0.5 \). In part a i, you need \( P(4) = 0.109 \ldots > 0.1 \Rightarrow \text{accept } H_0 \). In part a ii, you need \( P(17) = 0.0319 \ldots < 0.1 \Rightarrow \text{reject } H_0 \).

(Note 16 heads would not be more than \( 2 \times \text{tails} \).)

Strictly the result might be 17, 18, ..., 24 heads but once rejected for 17 it will be rejected for any higher number as the p-value decreases.

**Question 5** In part a, the sum of probabilities = \( 1 = 15k^2 + 5k + 0.35 \Rightarrow k = 0.1 \) or –0.5

Using the positive value of \( k \), the lower bound is \( P(X = 0 \text{ or } 6) = 0.05 + 0.05 = 10\% \).

[An (unattained) upper bound would be \( P(X = 0, 1, 5 \text{ or } 6) = 0.05 + 0.2 + 0.2 + 0.05 = 50\% \).

**Question 6** In part b, for the distribution \( X \sim B(40, 0.1) \) you need to find \( x \) such that \( P(X \leq x) \leq 0.1 \) and \( P(X \leq x + 1) \geq 0.1 \). Testing values give \( x = 1 \).

**Question 7** This is a new type of ‘inverse’ problem. You need to find \( n \) in the distribution \( X \sim B(n, 0.5) \) given that \( P(X \leq 12) \leq 0.01 \) but \( P(X \leq 13) > 0.01 \) and \( P(X \geq 28) \leq 0.01 \) but \( P(X \geq 27) > 0.01 \). You should use trial and improvement to find the value of \( n \).
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