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This Teacher’s Guide is designed to support the component parts of Nelson International Mathematics. The guide covers Student Book 4 and Workbook 4.

Support is presented under the following headings:

- **Concept and skill development** – an overview of the topic, which outlines the objectives covered and the skills students will develop in the section.
- **Vocabulary** – highlights the keywords that you should use in your teaching. Using the correct terminology yourself and encouraging students to use the same, plays an important part in the development of sound mathematical thinking.
- **Resources needed** – a list of the items that you and the students can use for suggested activities.
- **Mental warm-ups** – It is useful to spend about ten minutes each day doing an oral and mental activity so that students get a chance to use known facts, sharpen and improve their mental strategies and practise and consolidate previously learned mental calculation strategies (such as partitioning, compensating or bridging through multiples of ten). You cannot expect the students to recall mental facts quickly unless they have practised and repeated these regularly.

  We have provided a bank of sample mental warm-ups on pages 24–41. In most cases, these activities can be done with the whole class. Students can show answers using place value cards or mini-whiteboards, or write answers in their books or on scrap paper. Alternatively, in some cases, it may be more useful to have different students come up and write answers on the board.

  Some teachers may prefer to choose activities which are linked to the concepts that will be covered in lessons that follow. For example, before teaching the section on calculating a mean in data handling, you may do a mental activity adding sets of two-digit numbers mentally or revise division facts. However, the mental activities are designed to focus on mental calculation strategies, so they will not always link to new concepts being taught.

- **Teaching ideas** – these are listed under **Practical activities** (suggestions for activities that introduce the topics in a lively and engaging way before students tackle theoretical or written work) and **Using the Student’s materials** (notes to take you through the pages of the Student Book with suggestions for class and group work). Many sections also have Workbook activities, which are activities the students can complete in the write-in workbooks.

- **Informal assessment questions to ask** – a list of the types of question that might help you assess the topic, including questions that can stretch higher-attaining students or to give lower-attaining ones more practice.

- **Common errors and misconceptions** – tips and advice to draw your attention to areas that students frequently find difficult or confusing, so you can prepare additional material in advance.
Key to icons

In the Student Book, you will see icons identifying the main syllabus strand covered by a specific activity (or, in most cases, by a cluster of activities).

Number and calculation

Shape and space

Measures

Organising and using data

Note that some syllabuses identify ‘Problem solving’ as a separate strand. However, because problem solving arises in each of the four other strands, we have simply identified problem solving as one of the different kinds of activity you will find in the Student Book.

This table gives you examples to show how and where the problem solving objectives from the Cambridge Primary Mathematics Curriculum Framework are integrated and included in the Student’s Book and Workbook.

Problem solving

Problem solving is integrated throughout the materials. The following table lists the problem solving objectives and gives one or two examples to show where this objective is specifically covered in the book. However, remember, the students will use problem solving skills throughout this course and that there are many opportunities to meet each objective built into the materials.

<table>
<thead>
<tr>
<th>Objectives</th>
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<tr>
<td>4Pt1 Choose appropriate mental or written strategies to carry out a calculation involving addition or subtraction.</td>
<td>SB page 42</td>
</tr>
<tr>
<td>4Pt2 Understand everyday systems of measurement in length, weight, capacity and time, and use these to solve problems as appropriate.</td>
<td>SB pp 31, 33, 35</td>
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<td>4Pt3 Check the results of adding numbers by adding them in a different order or by subtracting one number from the total.</td>
<td>Chapter 6, 14</td>
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<tr>
<td>4Pt4 Check subtraction by adding the answer to the smaller number in the original calculation.</td>
<td>Chapter 6, 14</td>
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<tr>
<td>4Pt5</td>
<td>Check multiplication using a different technique, e.g. check $6 \times 8 = 48$ by doing $6 \times 4$ and doubling.</td>
</tr>
<tr>
<td>4Pt6</td>
<td>Check the result of a division using multiplication, e.g. multiply 4 by 12 to check $48 \div 4$</td>
</tr>
<tr>
<td>4Pt7</td>
<td>Recognise the relationship between 2D shapes and identify the differences and similarities between 3D shapes.</td>
</tr>
<tr>
<td>4Pt8</td>
<td>Estimate and approximate when calculating, and check working.</td>
</tr>
<tr>
<td>4Ps1</td>
<td>Make up a number story for a calculation, including in the context of measures.</td>
</tr>
<tr>
<td>4Ps2</td>
<td>Explain reasons for a choice of strategy when multiplying or dividing.</td>
</tr>
<tr>
<td>4Ps3</td>
<td>Choose strategies to find answers to addition or subtraction problems; explain and show working.</td>
</tr>
<tr>
<td>4Ps4</td>
<td>Explore and solve number problems and puzzles, e.g. logic problems.</td>
</tr>
<tr>
<td>4Ps5</td>
<td>Use ordered lists and tables to solve problems systematically.</td>
</tr>
<tr>
<td>4Ps4</td>
<td>Describe and continue number sequences e.g. 7, 4, 1, −2 ... identifying the relationship between each number.</td>
</tr>
<tr>
<td>4Ps7</td>
<td>Identify simple relationships between shapes, e.g. these polygons are all regular because ...</td>
</tr>
<tr>
<td>4Ps8</td>
<td>Investigate a general statement by finding examples which do or do not satisfy it.</td>
</tr>
<tr>
<td>4Ps9</td>
<td>Explain methods and reasoning orally and in writing; make hypotheses and test them out.</td>
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In the Student Book, and in this Teacher’s Guide, you will see small icons next to some of the practical activities suggested for each topic. These icons indicate a specific type of activity.
This icon indicates a practical activity, which develops mathematical understanding through the use of manipulatives. Typical manipulatives used at this level include: blocks, dice, abaci, counters, measuring sticks, items used for non-standard measures, for example, paperclips or books for length; bottles or tins for capacity, and so on. Sometimes practical activities do not involve manipulatives, for example, they might instead require students to explore their own movements or actions.

This icon indicates an activity involving construction, building or craft work. For example, students might cut out 2D shapes, build 3D shapes from modelling clay, cut out symmetrical shapes, and so on.

This icon indicates an activity involving discussion, debate or any other oral work. For example, students might talk about which method they found easiest or fastest for solving a specific problem. You may also see the same icon for singing, clapping or any other activities that develop listening and speaking skills.

This icon indicates an activity involving writing or drawing. For example, students might fill in answers to number sentences or questions.

This indicates a problem solving activity.

**Fundamental principles**

This series makes the following assumptions about the teaching of mathematics:

- Students need concrete experience in order to acquire sound mathematical understanding.
- Like adults, students learn best when they investigate and make discoveries for themselves.
- Students refine their understanding and develop conceptual structures by talking about their own thinking and what they have done.
- Individual students develop at different rates, some will find certain elements of mathematics difficult, others will understand them quickly.
- Students learn in a variety of different ways; mathematics teaching should provide a rich and wide variety of experiences.
- Students will become more mathematically able if allowed to develop reliable personal methods of working; the formal recording used by mathematicians is very difficult for most students to understand.
- The conventions of mathematics should be taught only once students are confident in their own knowledge, concepts and skills.
- Calculators and computers are tools with which students must become familiar in order to function effectively in the future.
- Students need plenty of opportunities to apply what they have learnt, and to relate their mathematics work to other areas of the curriculum and to their lives in general.
- Students learn mathematics most effectively when they enjoy and see relevance in what they are doing.

This course offers a wide range of mathematical experiences that reflect current thinking on the most effective ways of teaching and learning mathematics at the primary level. It recognises the professionalism of the teacher, and acknowledges that teachers are the best judges of experience appropriate for their own students. It does not impose an inflexible structure. Instead it provides a wide variety of practical activities, pencil and paper exercises and games linked to well-defined purposes or objectives. The teacher selects from this menu to meet the needs of classes, groups and individuals.

Frameworks for teaching

Summary of the approach

The learning framework of this course can be summarised as:

do – talk – record.

Doing
Students develop their skills by manipulating apparatus, playing games, investigating patterns and rules, modelling problems and talking about their ideas with peers before they are expected to record their work.

Talking
Through discussion, students can make sense of what they have been doing. They can then begin to generalise from their experiences. The teacher’s central role is to create such situations and to judge when to intervene.

Most of the activities in this Teacher’s Guide will help you to facilitate discussion, and will encourage students to listen to each other and experiment with different ways of thinking about and solving problems.

Recording
At stage 6, students are likely to have refined their skills and knowledge and developed the use of strategies that they find easy and useful for solving problems. They may still need to use informal and very personal methods (jottings) of recording steps in a process, or keeping track of what they have done. Jottings are an important step in moving towards non-standard methods of calculation (such as diagrams and jumps on a number line) that give the students a foundation for more concise standard written methods of recording.

It is very important that you allow, and in fact encourage, students to make use of jottings as they work. Here are some possible ways of doing this in the classroom:
• Do jottings of your own as you work out solutions. For example, if you are demonstrating how to calculate $144 \times 5$ you might jot the following on the board to show how you are thinking:
  
  $1440$
  $720$
  
• Talk through the jottings as you make them. For example, 144 times 10 is 1440, half of that is 720. This modelling process helps students to see that jottings are important and useful.

• Make space for jottings in the students’ exercise books. You can reinforce the importance of jottings as a means of showing your working by encouraging the students to jot as they work. If you only allow jotting on scrap paper, students may think it is not as important or valuable as their ‘real’ work in their book.

• Limit the use of prepared sheets with boxes for answers and no space for jotting down steps.

• Do activities where jotting is the point of the activity, for example, ask students to represent $\frac{3}{4}$ visually in as many ways as possible, or ask them to work out problems where they will need to jot down interim steps to keep track of the process: for example, how many ways can you find of making one dollar using any combination of 50 cent and 10 cent pieces.

• Ask students to share their jottings and compare them to show that there are different methods of working. This can help the students to see that some strategies are more efficient than others and, in turn, refine their own thinking. In the ‘make a dollar’ task above you may find that some children draw coin combinations, others list them and those who are more able and confident may make a table and work more systematically. All of these methods may provide the correct answers, but obviously some will take longer than others.

In the early stages of using apparatus in a new way, recording may take the form of drawings or words and drawings. Some students will gradually find this time-consuming and will simplify their recording independently. Others may need your suggestions and encouragement. As a teacher, you will need to work out carefully when a student is ready to use a standard mathematical symbol or format, so that recording is based on full comprehension.

Although at stage 6, you will teach students some standard written methods for operations on larger numbers, it remains crucial that you do not force children into formal and standard methods of recording calculations before they have fully grasped the process and are confident in the methods.

**Individual differences**
Everyone learns at their own pace, and in different ways, although experiences may be common. Adults in the real world bring a wide variety of approaches to their work, often ones they have devised for themselves despite many years in school learning standard methods. This course recognises...
individual differences and aims to give students the chance to explore the world of mathematics and solve problems in their own way. The course is also designed to provide equal opportunities to all students who may use it, regardless of their gender or ethnic, cultural or linguistic background.

Developing mental strategies

Adults perform many mathematical tasks mentally either because the tasks are simple, or because it is quicker to work things out in the head than use pencil and paper or a calculator. Too much work with paper and pencil can inhibit students from developing the flexibility and range of strategies necessary for efficient mental work.

A central aim in this course is to develop in students the ability to add or subtract numbers mentally, and to use quick recall of multiplication facts. Many of the activities encourage students to move directly from their own strategies with apparatus to working things out in their heads.

Students should be made aware of the role of mental methods as a first resort when a calculation is necessary, and not be led to believe that there is a particular method for a particular type of work, such as vertical presentation for ‘best work’, ‘answers only’ for mental arithmetic, or ‘working out’ only in rough books or on scraps of paper.

This Teacher’s Guide presents many ideas for oral work and suggests alternatives to the standard methods to help promote the development of effective mental methods. All arithmetical problems in this course are presented horizontally to encourage students to choose and use their preferred method.

One of the most significant changes to the Cambridge Primary Mathematics Curriculum Framework for 2012 onwards is the inclusion of a specific set of objectives under the category of Mental Strategies. These objectives aim to ensure that students are encouraged to use number facts and a range of mental strategies to add, subtract, multiply and divide. The idea is that these mental strategies are developed and used across all stages, and that students will continue to use them alongside more formal written methods as appropriate.

Mental calculation is important for both school-level mathematics and daily life, not least because it is often the most effective and simplest way to get an answer. At stage 6 in the primary school, it is usually the most effective method for solving most of the problems that students are faced with. Teaching, and encouraging the use of, mental strategies helps students to realise that numbers are quantities (rather than just seeing them as separate digits). This in turn allows them to take advantage of the particular properties of the actual numbers involved in a problem and to decide which strategy lends itself best to solving it. Mental strategies also allow students to develop a good sense of equivalence in mathematics. At a basic level, this could be simply saying $5 + 6 = 5 + 5 + 1 = 11$, but it forms the basis of algebraic manipulation and the more abstract functions that students will have to deal with at higher levels.
Here are some of the reasons that researchers around the world give to support the idea that mental computation should be included at all stages in school curricula:

- Mental calculations account for more than 80% of the calculations that adults do in daily life
- Mental calculation is essential for estimation. This is an important skill because many of the calculations we do in daily life do not require an exact answer. For example, these pies cost $1.90 each, can I buy three with $5? (2 × 3 = 6, so no.) These pies are $1.90 each, I’m buying six and the seller is asking for $15, that can’t be right!
- You often need to do some mental calculation before you can use a calculator, and you need to have some idea of how big or small the answer will be to check that you have used a calculator correctly
- When students have a range of mental strategies, they are able to find the easiest way of doing calculations
- Mental strategies rely on basic number relationships and they build on counting work from earlier grades, so they provide an excellent way for students to develop good number sense.
- Many of the patterns and relationships that make up the study of mathematics are numerical, but they are too vast and numerous to learn by heart, so it makes sense to develop a concept of how these work, so that you can transfer the skills to solve previously unseen problems in creative ways. To make sense of this, you just have to think about place value and counting. Once students learn the rules for making numbers, they can read and write any number. We would not, for example, teach every single number from 10 000 to 100 000 in a rote way. Instead, we expect the students to apply their knowledge to make, read and write numbers in this range.

In the sections that follow, we will explore what it means to develop mental strategies both in theory and in the classroom. Then we will present a general approach to teaching mental calculation strategies, with some examples to show how this might work. The actual strategies themselves are dealt with in more detail in the Student’s materials and teaching guidance by topic in this Teacher’s Guide.

What are mental strategies?

Essentially mental strategies are the individual methods we use to solve problems ‘in our heads’.

As an example, try to answer this question without doing any pen and paper calculation: how many 45c tickets can you buy with $10?

Once you have an answer, think about what you did to find the answer. Did you think in any of these ways?

- You can buy two 45c tickets with 1 dollar, so you can buy about 20 with $10.
- Ten 45s are 450. 450 and 450 is 900, that’s 20 tickets. You have one dollar left, so you can get two more, 22.
- 45 is almost 50, two 50s are $1, so I can buy about 20.
Very few adults will solve this problem by doing formal long division (1000 divided by 45) in their heads. This illustrates an important point about mental strategies – they do not involve simply visualising formal algorithms in your head and solving them without writing them down. Rather, mental strategies are the ways in which we use number facts that we have learnt by heart together with the relationships that exist between numbers and operations in order to solve problems. When you are teaching mental strategies for calculation, it is therefore crucial to focus on the mental processes that students use to get to the correct answer.

Recall of number facts is an important element of mental mathematics because other strategies use and depend on these. At stage 6, students should know addition and subtraction facts to 20 by heart. They should also know multiplication facts to 100 (2 to 10 times tables). The daily mental mathematics time can be used to consolidate these facts. In general, if a student can give the answer to a known facts such as $9 \times 7$ within 2–3 seconds then you can tell that he or she has memorised and internalised it.

**Implications for classroom practice**

The Nelson International Mathematics series has the key mental strategies for each stage built into the student’s materials. There is additional support included in the Teacher’s Guide in terms of the teaching activities section for each topic as well as the sample daily mental maths activities to allow students to practise and refine their skills. In addition, we offer a series of parent cards that explain the approach and suggest how parents can support it in the home. The provision of these materials makes it easy for teachers to meet the objectives of the revised framework. However, the materials do not stand alone – your classroom methodology and the ways in which you teach, support and encourage students to use mental strategies are of utmost importance in implementing these objectives.

In a classroom where mental strategies are given their due importance, the teacher’s role would include:

- being flexible in recognising and accepting whichever strategies the students use (including allowing them to choose their own strategies as well as to work in different ways)
- using different mental strategies yourself and modelling them for the class so that they can compare them with the ones they are using
- helping students to think about their own strategies so that they can refine them and work towards more efficient strategies.

These examples demonstrate how the teacher’s behaviour and actions can support or hinder this approach.

A class is given the following subtraction: $73 – 27$.

Note that the problem is given horizontally. This is the first element of a flexible approach because it does not force the students into thinking that they have to do vertical subtraction in columns with carrying.
Here are four students’ workings.

A  \[ 73 - 27 \]
\[ +3 +3 \]
\[ 76 - 30 = 46 \]

B  \[ 73 - 27 \]
\[ 77 - 27 - 4 \]
\[ 50 - 4 \]
\[ 46 \]

C  \[ 73 - 27 = 46 \]
\[ 70 + 3 \]
\[ 27 \]
\[ 43 + 3 \]

D  \[ \begin{array}{c}
    -4 \\
    -3 \\
    -20 \\
\end{array} \]
\[ \begin{array}{c}
    46 \\
    50 \\
    53 \\
    73 \\
\end{array} \]

All the students have the correct answer but they have found it using different strategies:

- Student A has used a strategy that involved adding the same amount to each number to get numbers that are easy to subtract.
- Student B has added 4 to the first number to get a number that is easy to subtract from and then subtracted 4 again at the end using knowledge of bonds to ten.
- Student C has decomposed the larger number into 70 + 3 and then subtracted the 27 before adding the 3 back.
- Student D has used an empty number line and done the subtraction in parts, subtracting 20 first, then subtracting 3 to bridge to 50 before subtracting the last 4.

The teacher in this classroom has several options, for example:

- He or she can ask the class to put their hands up to give the answer. This focuses on the answer and ignores the processes by which the students worked the answer out. It also makes it difficult for those students who are still trying to work things out, because the quicker students put up their hands and try to get the teacher’s attention, distracting them and making it difficult to think.
- Similarly, the teacher can ignore how these students have worked and do a column subtraction to show the class how to get to the answer.

Both of the above choices are counter-productive as they do not help the students develop confidence and in fact, moving straight to the ‘old fashioned’ algorithms has been shown to undo students’ understandings of place value.

When you value mathematical thinking and reasoning, you must make time to discuss how students got to their answers. This may involve modelling their solutions on the board and having the students verbalise and explain what they did. The students will learn from these explanations and showing them different options allows them to compare their own methods and strategies with those of others and to decide whether to move to a strategy they find more efficient. Modelling solutions and explaining thinking also makes the mathematics visible to students who may not have grasped it.
When you just give students an answer, the mathematical strategies are invisible to them – they cannot see how you worked it out.

Here is the working of a fifth student.

\[ 73 - 27 \]

\[ 27 \]

\[ -4 \]

\[ 50 \]

\[ 46 \]

This student has subtracted 7 from 3 and automatically used negative numbers (even though this has not been taught to the class at this stage). She then subtracted 50 from 70 before treating the negative number like a subtraction.

The last example is a good one to emphasise the teacher’s role. In many primary classrooms (and in fact in some primary maths textbooks) students are taught ‘you cannot take a larger number away from a smaller number’ because the teacher wants them to borrow from the next place value before subtracting. However, this is an incorrect and very misleading statement (particularly as students will have to deal with integer subtraction at higher stages). The student in the last example is totally comfortable with negative numbers and she does in fact use them correctly. A flexible teacher will share this method with the class and allow them to discuss it without insisting that they use it. If some children say ‘you cannot take 7 from 3’ a flexible teacher will ask them why not, and perhaps use examples such as ‘borrowing’ money or the integer number lines that students have used for temperature to show that in some cases you can. Of course, using a calculator to find 3 – 7 will also show that you can get a negative answer.

**Teaching mental calculation strategies**

The general approach to teaching strategies for mental calculation can be seen as three steps:

1. **Introduce the strategy**
2. **Reinforce the strategy**
3. **Assess students’ mastery of the strategy**

Each of these steps is discussed in more detail below.

**Introducing the strategy**

One method of teaching a strategy is to give the class an example of a calculation for which the strategy would be useful and then to ask the students to find the answer to see whether any of them select and use the strategy. For example, you want to teach them how to multiply by multiples of ten by rewriting the multiple as \(10 \times n\).

- Start by writing the multiplication \(35 \times 40\) on the board.
- Ask the students how they could find the answer.
• If one of the students suggests viewing it as $35 \times 10 \times 4$ ask them to explain the strategy to the class with your help

• If no one suggests this, model the strategy yourself. Your modelling could include concrete materials (such as place value charts or a diagram showing that $40 = 10 \times 4$). At the demonstration phase, your modelling should include jottings to show the steps in the process.

• Discuss the thinking behind the strategy as you model it. For example, it is quite hard to multiply by 40, but it is easy to multiply by 10 and by 4, so I am going to write 40 as $10 \times 4$. I am going to multiply 35 by 10 first. Do you remember what happens to the digits when we do that? (Prompt class to say that the digits move left and we use 0 as placeholder for units.) Now I have $350 \times 4$. I’m going to work this out by doubling. Double 35 is 70, so double 350 is 700 and double 700 is 1400.

• Next, do some more examples using appropriate numbers to demonstrate the logic of the strategy and discuss when it would be useful (i.e. what numbers would it work with?). If you like, you can show the students examples of when this would not be the most useful strategy. (For example, if the calculation is $\times 50$ it may be faster to multiply by 100 and halve the answer, or if the calculation is $49 \times 40$, it may be better to use compensation strategies and work out $50 \times 40 = 2000$ and then $\frac{2000}{2} = 50$). Remind the students that the choice of strategy depends on which methods they prefer and which numbers they find easiest to work with.

**Reinforcing the strategy**

The key components for reinforcement are:

• Providing lots of similar examples to practise the strategy in isolation and develop competence in using it. You will find that the Student Book and Workbook provide pages of examples for specific strategies as they are developed.

• Getting students to talk about and explain their thinking and methods as they use them. As they become more confident in using a strategy, they may find shorter and more efficient methods.

• Allow (and in fact encourage) jottings and pen and paper workings as you develop mental methods.

The activities you use to reinforce a strategy should be varied in type and presentation, so that students do not treat it as a rote activity, and structured in ways that encourage maximum participation. You will find suggestions to help you do this in the teaching guidelines section for each topic.

When you introduce a strategy you will need to allow sufficient time for the students to explore it and become comfortable with how it works. As they become more competent in using it, you can reduce the time spent on different activities.

Once most of the class are using the strategy confidently, your role is to help them integrate the strategy with others that they use. One method of doing this is to provide activities that include a mix of calculations, some of which are not suited to the particular strategy. It is often useful to present
a mixed exercise, ask the class to look at the problems and then spend some
time discussing which strategies students think will work best for different
problems. Encourage them to identify the properties of the numbers that
suggested each strategy to them.

Assessing whether students can use the strategy

Assessing mental strategies should take a variety of forms. However, the
main aim of your classroom assessment is to see whether the students can
work efficiently and accurately by choosing an appropriate strategy rather
than to test the use of a particular strategy. So, for example, you may use
‘timed’ tests in which the students compete against themselves to recall
facts and do mental calculations over a set period (such as a test a day for
five days) in order to improve their own time and/or accuracy. You will find
examples of timed tests in the Student’s Workbook.

You can also play games to assess mental computation skills. Games such as
‘beat the calculator’ that the students play in pairs offer an opportunity for
you to observe the students as they work and to record any observations that
you make.

One method of assessing whether the students can recall facts and use
mental strategies is to gauge the time it takes them to respond to a question.
As mentioned earlier, response time for known facts should be 3 seconds or
less. With the other mental strategies, a good response time is 5–10 seconds
(depending on how difficult the problem is). Bear in mind though, that is
a goal to work towards rather than a strict guideline. When students are
beginning to use a strategy, you would allow them as much time as they need
to apply it and answer the problem.

Talking to students in small groups or one-to-one is also important for
assessing their competence, particularly if what they jot down is unclear or
incomplete. Asking questions about how they were thinking will allow you to
see whether they understand the strategy and whether or not they can use it.

Computers

The use of a computer can support students’ learning of mathematics in
a variety of ways. Obviously, your use of computers will depend on the
resources available at your school. However, it is useful to know about the
range of technologies available, as it is likely that your school will become
increasingly well resourced as time goes on.

• **CD-Roms** are disks that contain electronic files with plenty of reference
  information. They may include an array of printable materials such as
  worksheets and practice sheets, tests, resources such as graph paper,
  maps, and so on.

• **Games** enable students to apply their mathematical skills through fun,
  interactive activities. Many educational games are designed to develop
  specific skills. If you have these available at your school, try to build in the
  use of games regularly each week. Find games that reinforce the specific
  areas of mathematics in which your students need regular practice. If
  there are specific games your class is particularly keen to spend a lot of
time playing, you may want to use game time as a reward for completing
classwork or homework.
The Internet is a rich resource for teaching suggestions as well as a source of much reference material. Keep a list of maths teaching websites that you use regularly. If you find articles of particular interest, download and print them, and add them to your resource files.

The BEAM mathematics project website. (www.beam.co.uk) offers a full list of all their teaching support resources as well as a range of free resources (click on the ‘free resources’ tag). The free resources include downloadable worksheets for classroom or extension use and discussion papers and articles related to developments in primary mathematics to support professional development of maths teachers.

You can also find a wide variety of materials to support your planning, teaching and assessment of Cambridge Primary Mathematics on www.cie.org.uk.

There are interesting articles, research and activities on the Plus Magazine website of the University of Cambridge. (http://plus.maths.org).

Exploring and investigating

Primary mathematics has traditionally tended almost exclusively towards short, directed tasks which result in ‘right’ or ‘wrong’ answers. The activities in this course provide a balance between short, fairly self-contained activities and open-ended investigations that can be returned to and developed over a long period of time.

Most of the activities are designed to develop students’ awareness of the range of mathematical possibilities open to them when tackling a mathematical task. As much as possible, allow students to take control, make decisions and explore the many avenues that can arise from a simple starting point.

Even ‘dead ends’ and ‘mistakes’ provide valuable experiences, stimulating further questioning and exploration. A student freely investigating with a calculator, pressing a few ‘wrong’ keys, for example, is likely to have a better ‘feel’ for, and interest in, our number system than the student who is only allowed to use the calculator for checking the answers to ‘sums’.

Students should always be encouraged to ask ‘What if?’ and ‘Why?’ when investigating. These questions may lead to uncharted territory, new challenges, fresh understanding and the development of new skills.

Many investigations have no final solution or easily accessible generalisation for the students. Some have a simple pattern or rule which may be discovered and explained. However, many students will want to know why certain patterns repeat, and offer explanations about the rules which govern them. This is the first step towards generalisation, and teachers can encourage this by asking questions such as: ‘Why is the same number added each time?’ or ‘Can you guess what will happen next?’

The value in investigations lies in students pursuing them to the limits of their ability, and in the new skills that are acquired on the way. For some students, the early, often concrete, experimentation is enough to give them confidence, and increase their enjoyment of using already acquired skills.
There are many different ways of recording investigative work. Students should initially be allowed to explore and note their discoveries freely. Teachers may wish to intervene periodically to help them organise the results so that emerging patterns are identified and interpreted.

**Sources of investigation**

Many everyday objects can provide rich sources of investigative work. The hundred square, addition square and multiplication square all contain many fascinating patterns. Students can also explore patterns in solid and flat shapes, such as the relationships between faces, edges and vertices of 3D shapes, and the relationships between sides, corners and angles of 2D shapes.

Use investigations to enrich the introduction of new concepts. For example, you can introduce number patterns through developing number chains and introduce geometric patterns through explorations of colour arrangements on geo-boards. Students can explore the relationship between area and perimeter, and between volume and the dimensions of cuboids.

As they develop an investigative approach, help students to become systematic in the way they work. This will help them to understand the structure and formal approaches of mathematical theory.

**Mathematics in real life**

Some students may struggle to understand the relevance of mathematics in their everyday lives. This course places great emphasis on making students aware of the relevance of mathematics to their own real lives.

In this Teacher’s Guide, you will find ideas for using the student’s own environment as a stimulus for mathematical activities. The Student Book and Workbook frequently require students to look at the mathematics in the classroom, the playground and their own homes. Each set of activities and problems requires new skills and fresh understanding. Many questions are open-ended or have no exact solution, and students are asked to make predictions, generalisations and estimates, and to evaluate their own answers. Encourage this skill in all areas of the curriculum.

Students use their understanding of mathematics at home and at school, in situations such as sorting toys or books, working out the times of television programmes, making patterns, helping to prepare food and playing board or card games.

**In school**

In school, there are many opportunities for you to teach mathematics through familiar situations, so that the students experience its usefulness and appreciate the order and sense that mathematics gives to life. For example, students can identify the date each day, as well as the time at various points throughout the lesson. Registration, dinner money, timetables, sorting and putting away equipment will provide a range of relevant experience in data work, measures and shape and space as well as number.
**Play**

Students of all ages should have opportunities to play both in and out of school. This offers them the freedom to explore new situations, to make discoveries for themselves and to be creative. Unfamiliar mathematics equipment should be introduced through play, with the students exploring the functions and possibilities inherent in the materials. A good example of this is to experiment with pairs of compasses by drawing patterns and pictures before using them as mathematical instruments.

Construction kits offer students the opportunity to explore shapes and inverse operations, through building and dismantling.

**At home**

Part of the teacher’s role is to involve parents and guardians in the students’ learning. Parents need not be limited to supervising their children’s homework. There are many activities that can involve the parent actively in the child’s learning, and that can provoke mathematical discussion and language at home.

Parents can be encouraged to extend their children’s mathematical understanding through playing board and card games and by encouraging them to help with normal home activities such as cooking, gardening, cleaning and organising the home, drawing up plans and measuring when redecorating, and estimating how many or how much when shopping.

The Nelson International Mathematics scheme offers a set of parent cards that explain the approach taken in the series and suggest how parents can support it in the home. You can guide parents to these online resources, or you may like to print these out and send them home with the students.

Many of the students will also voluntarily help and encourage younger brothers and sisters in games and getting organised.

Family visits and holidays give students the opportunity to see environments different from their own, and to experience time and distance.

They are also likely to be budgeting pocket money, saving for special things and predicting how long it will take them to afford treats.

Students may have computer games that require them to use a variety of mathematical skills. They are likely to see and use a wide range of electronic equipment at home, which demands mathematical skills to be used properly.

Many students will also be responsible for their own timekeeping and have a degree of responsibility for others.

Some homes will not actively encourage girls to use construction kits, computers or calculators, and some parents will not be confident of their own mathematical skills or understanding. As a teacher, you can help a great deal by making explicit the mathematical content of everyday experiences and activities.
This book contains many suggestions for investigations, problems and research that students and parents can work on at home. Games made from suggestions in the scheme could become the core of a ‘lending library’ of games for students to take home for a period of time, to play with parents, or brothers and sisters. This would not only link home and school but also give parents and teachers a basis for discussion.

**Organisation**

**The classroom**

Each teacher will have preferences about how best to organise the available space. However, here are some useful guidelines for any classroom, irrespective of how it is arranged.

**Storage**

Always store equipment in such a way that students have easy access to it and can check it periodically. Clearly label all items and encourage students to make their own decisions about what they need.

**A mathematics centre**

This may or may not be where the equipment is stored, but it will be a part of the classroom that is bright and attractive with displays of students’ work and other mathematical stimuli. The centre is a place for students to go at odd moments in the day, to be challenged with mathematics-related questions and activities.

Questions and activities should be provided by both teachers and students for interactive problem solving, for example: ‘The answer is 15.2. What was the question?’ inviting students to write out their suggestions. A number pattern or sequence, on a series of cards organised by the students, may be ‘secretly’ altered by the teacher, and the students have to discover what has changed, and put it right.

The BEAM organisation offers a wide range of primary mathematics resources including manipulatives (place value cards, large dice, dominoes, fraction grids and spinners), games and online resources that are a useful and enriching addition to any classroom mathematics centre. You can see the full range of BEAM products on their website www.beam.co.uk or you can request a catalogue from your local Nelson Thornes representative.

**The students**

**Class teaching**

At times it is efficient to work with the class as a whole, perhaps when introducing a new topic. The course offers plenty of ideas for this kind of approach. The planned work needs to be suitable for all students, with individual needs and ability taken into account in subsequent group or individual follow-up activities.

**Group work**

You can group students in similar or mixed-ability groups, to suit the purpose of the work. This offers students the opportunity to collaborate, to discuss their work with each other and the teacher, for peer teaching to take
place and for the work to be matched to their needs. It enables the teacher to work simultaneously with a number of students and this minimises the need for repeated explanations to individuals. Group teaching is an effective form of classroom organisation for both teacher and students.

**Working individually or in pairs**

At times it may be appropriate for students to work as individuals or in pairs, to provide extra help to students who need it, or to stimulate and challenge the higher-attaining students. Working individually gives students the opportunity to concentrate on their own thinking, to develop this through investigations and problem solving, to work quietly and in private, and to experiment with materials. Students working in pairs have the opportunity to develop collaborative skills, to play games together and to share ideas in an investigation.

**Assessment and record keeping**

A significant part of a primary school teacher’s day is spent on the informal evaluation of a student’s or group’s learning, and in deciding what the next learning experiences should be.

Assessment can be both formal and informal, and can serve a number of functions. Ongoing observations and discussions with students can give teachers valuable diagnostic and formative information on which to base their teaching. Students’ written and practical work can yield valuable insight into the current stage of their conceptual development and the extent to which they are developing effective skills.

Ideally, assessment should take place throughout the year and its goal should be to support students as they learn and develop their mathematical skills. For this to happen, students need to be actively involved in both their own learning and their assessment. This means that they should understand how they are going to be assessed and how their success will be measured; they need to begin to evaluate themselves, to set targets for themselves and to reflect on their own learning, so that they become more and more self-confident in mathematics.

For assessment to support the development of learning, assessment styles must be varied and relevant to the students. The programme of assessment should present opportunities for students to talk about and demonstrate what they have learned (through oral work, worksheets, exercises, tasks, projects, tests and other activities).

**Facilitating assessment**

- Give students a variety of tasks that require them to apply different skills – for example, writing tasks, using concrete apparatus or working with money. In this way students are able to show what they know in different and exciting ways.
- Think about the most appropriate way to assess a task or activity and identify the skill that you need to assess. This will help you to decide what assessment technique is most suitable at any time.
- Develop your own observation sheets to use as students work through the activities in a topic and use this to observe and assess skills and learning.
• Watch, observe and ask students questions as they work through topics, rather than only assessing the final product. This will allow you to see where they have difficulty and to address problems that may impact on understanding.

• Use a range of different contexts for assessment (individual, peer, group, oral, written) and keep clear and well-organised records of your observations and any marks you allocate. This is not only useful for your own purposes, but is also equips you to report back to parents about their child’s progress.

**Formal testing**

From Level 3 onwards, you may want to include testing as part of the learning process. Some schools will opt for outside tests from different curriculum providers, but it is useful to introduce the idea of mental and written testing as these will be used as the students progress through school.

In this course, we provide assessment suggestions for different activities and topics in the Teacher’s Guide. Cambridge Primary provides end of stage tests, called Progression Tests, for stages 3–6. The tests are designed to allow measurement of students’ progress and identification of their strengths and weaknesses.

To help teachers prepare students for the Cambridge Progression Tests, we have provided a set of tests in the same style and format as questions in the Progression Tests.

Seven tests are provided for this stage. Tests 1–6 are intended to consolidate work as students progress through the stage. The relationship between these tests and the chapters is as follows:

Test 1: Chapters 1–3
Test 2: Chapters 4–6
Test 3: Chapters 7–9
Test 4: Chapters 10–13
Test 5: Chapters 14–17
Test 6: Chapters 18–21

The final test, ‘Stage 4 Practice Test’, covers the entire stage 4 curriculum and is intended for use as a practice for the Cambridge Progression Test.
Mental maths activities bank

You should aim to do a mental maths activity that takes about ten minutes each day.

This section contains some examples that you can use as is, or adapt to suit your own classroom. We have tried to provide a range of different types of activities (factual recall, games, grids, tables, problem solving and puzzles) to show some of the ways in which you can approach the mental maths part of the lesson. However, this is not a definitive list and some activities will appeal more to some classes and teachers than others. If you need additional ideas and suggestions, there are several useful websites for teachers which give ideas and resources. Type ‘mental maths warm ups’ into your search engine and you will be directed to a range of sites covering this topic. Many of these sites offer a range of maths activities many of which can be used with electronic whiteboards or screen projectors attached to the computer.

As you read through the activities remember:

• Most of these activities can be repeated by simply using different values. Many of them can also be adapted to make them simpler or more difficult.

• Many of these activities can be done with no resources. However, some require you to prepare grids and/or game boards or to supply students with apparatus such as dice or cards. We suggest that you keep the materials you develop and use them to build up a mental maths resource bank of your own. For example, when you prepare grids or tables with missing values, or magic squares with a wrong number, do this on card and if possible, laminate the card so that it can be re-used. Lamination also means that students can use dry-wipe markers on the cards and these can be cleaned easily. Another option is to prepare apparatus on computer and to print these out onto overhead transparency sheets. These can be stuck onto white card to make a re-usable resource that can be wiped clean.

To make it easier for you to select activities to match what you are doing and that meet your students’ needs at different times, we have organised them into six sections:

1. Place value and number sense
2. Rounding and estimating
3. Mental problem solving
4. Calculation skills
5. Calendars and time
6. Shape, space and measures
1. **Place value and number sense**

Do a range of activities in which the students have to count in given steps. Vary these according to what you are doing in class and the number range that the students are working in. Start the year by revising place value to 1000, then move onto numbers to 10 000. For example:

- Count from 999 to 1025
- Count back in twos from 2500 to 2450
- Count in tens from 4150 to 5300
- Count back in tens from 6500 to 6200
- Count in hundreds from 3245 to 5245
- Count in thousands from 1200 to 8200

Ask questions based on counting back or forwards in tens and hundreds. Some possible questions are:

- What is 10 more than 450?
- What is 10 less than 900?
- What is 100 more than 6000?
- What is 100 less than 1500?
- What is 100 less than 10 000?
- What is 100 more than 3500?
- What is 100 more than 4900?

Make up a set of questions based on a four-digit number. For example, display the number 2358 and ask students to write or say the number that has:

- five more tens
- three fewer ones
- two fewer hundreds
- three more thousands.

Do lots of activities using place value charts. For example:

Here is a place value chart showing the number 85

```
<table>
<thead>
<tr>
<th></th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>
```

What number is shown on this chart? (1011)

```
<table>
<thead>
<tr>
<th>1 000</th>
<th>2 000</th>
<th>3 000</th>
<th>4 000</th>
<th>5 000</th>
<th>6 000</th>
<th>7 000</th>
<th>8 000</th>
<th>9 000</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>200</td>
<td>300</td>
<td>400</td>
<td>500</td>
<td>600</td>
<td>700</td>
<td>800</td>
<td>900</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
<td>80</td>
<td>90</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>
```
Write the number in words. (One thousand and eleven)

If you make a large chart and laminate it, you can use wipe-dry pens to underline or circle the numbers each time. If you are using an electronic chart you can highlight the boxes you are referring to as we have above.

Let the students say each number in words, and then write it in numerals.

Tell the students to listen carefully as you say some numbers and then write them down. For example, you may read out a set of numbers such as:

- two thousand, four hundred and thirty five
- one thousand, nine hundred and three
- five thousand and ninety nine
- three thousand and seven
- five thousand, seven hundred.

Read out some numbers and ask the students to jot down the value of one particular digit. For example, what is the value of 3 in each of these numbers?

- 1366
- 435
- 1493
- 398
- 3678
- 4213
- 3908

(When you do this kind of activity, make a list of numbers in advance to make sure the numbers you use only have one digit with the value you are looking for!)

Let the students work in pairs to play the ‘10 questions’ game. Ask the students to each write down a number without showing their partner (you can select the range, for example, any number less than 5000). The students then try to guess what number their partner has written down by asking questions that can only be answered by yes or no. For example, Is it odd? Does it have three digits? Is the digit in the units larger than the one in the tens? And so on. Once the students have asked ten questions they should try to guess the number. If they can’t, their partner can give them a clue and allow them to ask two more questions before guessing again. For example, My number is between 300 and 350. It has no tens.

Write any three- or four-digit number on the board. For example: 3029

- Ask students to say the number aloud.
- Ask different students to say how many hundred thousands, ten thousands, tens, units, etc. there are.
- Point to a digit and ask students to say its value.
- Ask students to reverse the digits and say the number.
- Let students make five different numbers using digits from the given number in any order.
• Let them exchange these and say each other’s numbers aloud.
• Repeat the place value questions using numbers the students have made.

Write a selection of three- and/or four-digit numbers on cards. If you make numbers with similar digits, it makes the activity a little more demanding. Display the number cards randomly on the board. For example:

![Number cards](image-url)

• Say numbers aloud in words and ask students to come and identify the number you said. For example three thousand two hundred and forty-five.
• Let the students reverse the digits and say the numbers aloud. (If you are going to do this, be clear about what they should do with a 0 in the units place or do not use any numbers with 0 in the units place.)

Place the students in groups of two. Give them instructions such as:
• count forwards in thousands from 4000 to 10 000
• count forwards in tens from 5890 to 6020
• count backwards in hundreds from 6300 to 5200.

Give groups of students a set of four mixed digits (on cards). Instruct the groups to make:
• the smallest possible number
• the largest possible number
• the smallest possible odd/even number
• the largest possible odd/even number.

Ask the students to write down any three-digit number. Write a random set of three-digit numbers on the board.
• Let the students make number sentences using your numbers and the number they have written down using the <, > or = signs.
• Let the students use mental strategies and jottings to find the difference between the numbers in their number sentences. Spend some time talking about the strategies they suggest.

Ask the students to write a three- or four-digit number on paper. Choose one student to come to the front of the class and display his or her number. Ask, whose numbers are greater than this? Let the students display their numbers. Repeat for smaller than. Choose one student at random. Let him or her come up and stand to the left or right of the other one (depending on whether their number is smaller or greater) displaying their number. Choose other students who should come up and position themselves appropriately between the numbers already on display.
Write a list of positive and negative temperatures on the board. For example:

```
12 -3 0 -5 -4 9 5 -7 -1 1 -9
```

- Draw a blank number line on the board to represent a thermometer with
  the start and finish marked only.
- Ask the students to find the lowest temperature. Write this on the left-
  hand side of the scale. Repeat for the highest temperature, writing it on
  the right-hand side of the scale.
- Ask students to volunteer to come up to the board. Let them choose one of
  the temperatures and have them position it on the scale as accurately as
  they can. Once all the numbers have been placed, discuss whether there
  are any inaccurate placements. Let the students decide and suggest how
  to move the numbers if necessary. This activity can be adapted to work
  with decimals, fractions, mixed numbers and whole numbers.

Play bingo with the class. You will need three or four sets of 0–9 digit cards.
Let the students work in pairs to write five three- or four-digit numbers on a
grid like this one:

```
456  321  999  408  765
```

Shuffle the cards. Draw three cards at random (or four if you are making
four-digit numbers) and call out or display the digits. For example, five, six
and seven. The students use the digits to try and make the numbers on their
grid (in any order). If they are able to make one of their numbers, they can
cross it out. (So this pair could make 765 and cross it out.) The first pair of
students to cross out all their numbers can call out Bingo – they win the
game. It is useful to record the digits that you call each out each time to
resolve any disputes that may arise about numbers.

Play a find the number game with the class. Prepare a set of number cards
so that you have sets of three numbers that are 100 greater and 100 smaller
than each other. For example:

```
1324  1424  1224  
8766  8866  8666  
3455  3355  3555  
4099  4199  3999  
```

You will need enough cards to give the students one each.

Shuffle the cards and hand them out. Let the students try to find the other
two numbers in their set by asking questions (it’s more fun if they don’t just
show their numbers but have to find out what numbers other students have
by asking questions). When they find each other, let them display or write
their numbers. If some students are quick, let them extend their number
sequence in both directions.
Make a grid with one or two place decimals on it and display it for the class. For example:

<table>
<thead>
<tr>
<th>0.3</th>
<th>0.45</th>
<th>0.5</th>
<th>0.01</th>
<th>0.10</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>0.25</td>
<td>0.4</td>
<td>0.9</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Let the students draw their own blank grid and rewrite each decimal fraction as an equivalent vulgar fraction using tenths and hundredths (be aware that some students may write \( \frac{1}{4} \) for 0.25 and \( \frac{3}{4} \) for 0.75). Encourage them to explain their thinking and reasoning if they do this. You can also reverse this activity by presenting vulgar fractions and getting the students to write the decimal equivalents.

Display a number of mixed numbers for the class and let them write them in order from greatest to smallest or vice versa. Try to give some variety, including mixed numbers with the same whole number and different fractional pieces, for example \( 5\frac{1}{2}, 5\frac{3}{4} \), etc. to encourage students to compare the whole number and the fractional part to decide how to order them.

Make a chart of ten equivalent fractions including some errors. Give the students a time limit to find, write down, and correct the incorrect number sentences.

\[
\frac{1}{2} = \frac{4}{8} \quad \frac{1}{2} = \frac{6}{10} \quad \frac{1}{5} = \frac{1}{4} \quad \frac{2}{10} = \frac{20}{100} \quad \text{and so on.}
\]

You can adapt this activity to work with vulgar fractions and their decimal equivalents as well.

2. **Rounding and estimating**

Write several four-digit numbers on the board. Round each to the nearest 10 or 100 (choose one place value to round to per activity). Make sure some of the rounded values are incorrect. For example:

(rounding to the nearest 100)

- 2345 → 2300
- 2662 → 2660
- 4129 → 4200
- 3888 → 4000
- 3999 → 4000

Ask the students to find the incorrectly rounded numbers and to correct them. Repeat this using different rounded numbers and rounding to different places.

To practise and reinforce rounding off mentally, draw a grid like this one on the board. If you are going to reinforce rounding to the nearest ten, make sure the numbers all have a value other than 0 in the units place.
You can ask students to copy the grid, and have them rewrite the numbers, rounding them to a given place value as they go (for example, the nearest 10, the nearest 100). Alternatively, you can tell the class that you are going to round these numbers to the nearest ten. Then work through the grid, pointing at the numbers and asking different student to give the rounded number. Repeat this for different place values.

Play a game in which you prepare a number of ‘dartboards’ like this one. Point to a space on the dartboard and let selected, different students give the answer. Vary the game by changing what you are rounding to (centre circle), by giving the first circle of numbers, and by giving the outer circle of numbers (remember when you do this there are several options for the inner circle answers.)

Play a game in which you assign students a number (rounded to the nearest 10 or 100 depending on what you are teaching). Select a suitable range. For example, multiples of 10 from 90 to 450. The students should write their number on a card. Choose a number to start. For example, 423. Say it aloud and ask the student who has the number rounded to the nearest 10 to stand and display the rounded number. Let the class decide if he or she is correct. If the student is correct, give him or her a chance to choose any number between 90 and 450 and say it aloud. The student who has that number rounded to 10 stands up and, if correct, gets a chance to choose the next number. Try to maintain a good pace so students don’t get bored.
Ask the students to make a three-column table like this one:

<table>
<thead>
<tr>
<th>1000</th>
<th>1100</th>
<th>1200</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Read out 20 numbers in the range from 950 to 1249. As you say each number, the students should write it in the correct column to show the nearest 100. For example, if you say 1050, the students should round it to 1100 and write it in that column.

Display some three- or four-digit numbers. For example:

| 342 | 291 | 450 | 405 | 325 | 251 |

Pose questions that involve rounding and estimating and basic operations. For example:

If you multiply by 20 which numbers will give a product that is:
• less than 6000
• more than 6000
• between 6500 and 7000
• greater than 8000?

Repeat this for different multiples of ten and hundred. Adapt the numbers and the ‘answers’ for other operations. For example, if you add 520, which number will give a sum that is …? If you subtract 90 from each number, which will result in an answer that is …?

Prepare a set of money problems that involve estimating to work out whether you have enough to pay for a given number of items. For example, Priya has $50. She wants to buy 4 sets of pens at $8.50 each and 2 pencil cases at $8.99 each. Does she have enough? Vary the cost of items, using local prices and currencies where possible and also vary the amount available to spend.

Prepare a set of addition (or subtraction) problems. For example:

| 379 + 341 | 532 + 288 | 209 + 399 | 418 + 299 | 502 + 199 |

Pose questions that can be answered by rounding and estimating. For example, which sums give an answer close to 700? Which sums will give an answer greater than 600? Is 700 a good estimate for the answer to this sum? (pointing to a sum).

3. **Mental problem solving**

Test understanding of mathematical terms and vocabulary by posing worded problems to be solved mentally. For example:
• Find the number that can be increased by 30 to make 81
• What is the product of 4 and a number 3 greater than 4?
• What is the difference between 16 and double 48?
• What number do you get if you halve the product of 98 and 100?
What is the sum of 345 and double 90?

Give me a pair of three-digit numbers with a difference of 120.

If the sum of two numbers is 280 and one of the numbers is half of 180, what is the other number?

Which three numbers could have a total of 124? Are there any others?

Find four pairs of numbers with a difference of 54 (or any value).

How many lengths of 10 cm can you cut from 186 cm?

And so on.

Ask the students to write the digits 1–6 on cards or counters. Instruct them to make three two-digit numbers. They should then find the total of their three numbers. Ask them to find the smallest possible total and the largest possible total. Spend some time discussing how they worked this out.

You can repeat this with random digits from 1–9 as well.

Give the students some logic puzzles to solve. There are many examples of these and the ones you choose will depend on the ability levels in your classroom. Here are some typical examples.

Jess is 7 years older than her sister. Their combined age is 25. How old are Jess and her sister?

The sum of two numbers is 140 and there is a difference of 6 between them. What are the two numbers?

A farmer has chickens and goats in a yard. If there are 15 heads and 48 legs, how many goats are there?

Busi and Rob have six children. Each of these children have had three children of their own. How many people are in this family?

If it takes Mr Jones 4 minutes to cut a piece of pipe into two parts, how long would it take him to cut it into 5 parts? (Students may say 4 × 5 = 20 minutes. Bear in mind that to make five parts, he only needs to do 4 cuts, so the answer is 16 minutes.)

Amira has the digits 1 to 7 written on cards. How many pairs of cards add up to 8? How many groups of three card can you make that add up to 10?

Find a number between 1 and 10 that is not even and which gives a remainder of 1 when divide by 3.

Draw a series of grids like this one on the board to reinforce and practise addition and subtraction.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>45</td>
<td>19</td>
</tr>
<tr>
<td>10</td>
<td>15</td>
<td>23</td>
</tr>
<tr>
<td>50</td>
<td>21</td>
<td>15</td>
</tr>
</tbody>
</table>

Move a pointer across the grid and ask the students to work out what operation was required to get from the starting value to the end value. You can jump vertically, horizontally or diagonally and move one or two places.
For example:
From 45 one down to 15: answer minus 30
From 45 across diagonally to 10: answer minus 35
You can use this as a game in which the students work in pairs to move on a grid and answer each other. They can use a calculator to check the answers.
You can make this activity more difficult by increasing the size of the grid and extending the number range.

4. Calculation skills

Draw an empty $2 \times 5$ grid on the board. Get the students to copy it into their books.

Fill in the numbers from 1 – 10 on the grid in random order. For example:

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>6</td>
<td>1</td>
<td>5</td>
<td>q</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>8</td>
<td>4</td>
<td>10</td>
</tr>
</tbody>
</table>

Give the students a multiplier (for example $\times 3$). They then fill in the product of multiplying each number on your grid by 3.

Use number grids like these to practise mental addition and subtraction. The operations are given across and down. Students can work in any order to fill in the missing values. Choose values that suit the mental strategies you are teaching. The addition table below shows adding ten and multiples of ten. The subtraction table uses 9 and 11 as this involves subtracting 10 and then compensating by adding or subtracting 1.

<table>
<thead>
<tr>
<th>+ 30</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>+ 10</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>25</td>
</tr>
<tr>
<td>45</td>
<td>55</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>− q</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>− 11</td>
<td></td>
</tr>
<tr>
<td>59</td>
<td>48</td>
</tr>
<tr>
<td>50</td>
<td>39</td>
</tr>
</tbody>
</table>

Provide some easy magic squares and let the students work to find the missing values. Make sure they understand that the rows, columns and diagonals all have the same total. The first three examples are easy, the last two are more open ended and will require a little more thought to make them work.
Play target numbers with the class. Display a number and then ask the students to write five or more number sentences which give that result. You may also time this activity and ask the students to find as many ways as possible in the given time. (Some students may work systematically and produce a number of similar number sentences. For example, to make 100, they may write $99 + 1$, $98 + 2$, $97 + 3$ and so on. This shows quite a sophisticated understanding of bonds and complementary addition, so be sure to share ideas and methods as a class afterwards.)

Display a number of three- and/or four-digit numbers and ask the students to perform an operation on the numbers. For example, add 99 to each number (focusing on near multiples of 10) or write the number that is 7 less than each number (subtract a small number from it, with crossing hundreds).

Use 1–99 number cards to practise addition and subtraction to 100. Draw a card at random, for example 36, and point to a student. The student should say what number should be added to that to make 100. Remove that card and continue. Alternatively, let the whole class jot down the answers as you draw cards. For subtraction, say 100 minus (draw the number) and let the students work out what the answer would be.

Play five number targets using small numbers (less than 100). Display a number, for example 52, and let the students work out how to get this number adding four or five smaller numbers.

Play doubling games with the class. Either display a range of two-digit numbers, and point to them asking the students to double them. Or, display two sets of numbers that are mixed up (one set being the doubles of the other). Ask the students to match the numbers by doubling and/or halving.

Prepare a range of multiplication grids in which the students have to multiply two-digit numbers by one-digit numbers. For example:

<table>
<thead>
<tr>
<th></th>
<th>12</th>
<th>15</th>
<th>13</th>
<th>24</th>
<th>31</th>
<th>42</th>
<th>50</th>
<th>57</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>× 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>× 8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Students then work to complete these. This particular one involves doubling, and they should recognise that once they have worked out $4 \times$ each number, that they can find $8 \times$ by doubling. Such grids can also be used for division, either like this one, or by giving the solutions and asking the students to work out the divisors (the numbers in the top row).

Play a matching game using fractions of amounts. Choose a fraction, for example $\frac{1}{4}$ of … List the numbers that are $\frac{1}{4}$ of an amount down the left-hand side of the display in random order. Display the values down the right-hand side, also in random order. Let the students match up the number and say or write a number sentence, for example a quarter of 36 is 9. Or 6 is a quarter of 24. For example:

<table>
<thead>
<tr>
<th>7</th>
<th>8</th>
<th>... is $\frac{1}{4}$ of ...</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td></td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>9</td>
<td>36</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>32</td>
<td></td>
</tr>
</tbody>
</table>

Repeat this for halves, thirds, fifths, eighths and tenths of numbers. You can also set questions where you give the fraction of the amount and ask the students to say what the original amount could be. For example, $\frac{3}{5}$ of an amount is 20. What is the amount?

Display a set of measuring scales with an amount marked in millilitres (using multiples of 50). Let the students either say how much liquid you would need to add to make 1 litre (1000 ml) or how much was poured out from a litre. You can adapt this by asking them to combine amounts to make 1 litre as well.

Display the multiples of 100 from 0 to 900. Ask the students to sit in pairs and each to write down a three-digit number. Select a multiple at random, for example 200. Ask the students to add that to their number. Let the students compare answers with their partners and decide whose answer is closest to 1000. Repeat this, with pace, and allow students keep a tally of who ‘wins’ each round.

Display a number of related fractions. Let the students rewrite them in ascending or descending order. Let them explain how they decided. You can also adapt this by asking which is more: $\frac{3}{4}$ of 100 or $\frac{4}{5}$ of 100?

Prepare a series of short quizzes (15 to 20 questions) with mixed operations and mental strategies. Use these as the mental warm-up activities at intervals. Read the questions aloud and display them one by one for the class. Allow 20 to 30 seconds for the students to answer before moving on. (So, a 20 question quiz would take 10 minutes to complete.) The students can prepare a table and keep track of their results for different quizzes. Below are three sample quizzes, but bear in mind, you’d need to have taught the topics before you include questions on them.
### Sample Quiz 1
1. Write nine thousand and twenty-three in numerals.
2. What is the value of the 3 in 2038?
3. What is 456 rounded to the nearest 10.
5. 20 – 13 =
6. $4 \times 9 =$
7. How many fours in 28?
8. 137 + 9 =
9. $10 \times 45 =$
10. $450 \div 10 =$
11. $4 + [x] = 20$
12. What is half of 36?
13. What is 1/3 of 15?
14. What number is 100 less than 876?
15. Round 1455 to the nearest 100.

### Sample Quiz 2
1. Write in words the number that is 10 more than 1232.
2. How many tens are there in 4000?
3. Round 1254 to the nearest 10.
4. What is 1/5 of 40?
5. Write 0.5 as a vulgar fraction.
6. $6 \times 9 =$
7. $54 \div 6 =$
8. $81 \div [x] = 9$
9. $19 + 327 =$
10. $400 + 321 =$
11. What is the time half an hour later than 3.40?
12. How many weeks pass in 28 days?
14. Double 75.
15. $850 + [x] = 1000$

### Sample Quiz 3
1. Arrange in order from smallest to largest: $4\frac{1}{2}, 3\frac{1}{4}, 4\frac{1}{4}$
2. Write eight thousand three hundred and four in figures.
3. What is the difference between 750 and 1000?
4. How many jumps of 5 will you need to get from 0 to 80?
5. $65 + 66 =$
6. $97 - 8 =$
7. $1234 + 9 =$
8. $3267 - 8 =$
9. $143 + [x] = 200$
10. $325 - 45 =$
11. $9 \times 11 =$
12. $34 + 19 =$
13. $135 + 49 =$
14. Is 552 a multiple of 5?
15. $499 - 100 =$

---

### 5. Calendars and time
Ask questions linking fractions and periods of time. For example, you might ask: ‘How many months in:

- half a year
- a quarter of a year
- one third of a year
- three quarters of a year
- $\frac{2}{3}$ of a year.

You can extend this by asking the students to estimate how many weeks this is (multiplying by 4, or halving 52).

Use the calendar to develop counting sequences in sevens, for example: write the next three numbers in each of these sequences:

- 7, 14, 21 ___ ___
- 1, 8, 15, ___ ___
- 4, 11, 18 ___ ___
You can also ask: What would the tenth number be in each sequence (the first number times 10).

Pose questions based on current dates (you can have a calendar on display for this). For example. Starting this week, write down:

- the dates of the next four Fridays
- the date on Saturday
- the dates of the next three Tuesdays
- the date a week ago today.

Provide a blank calendar for a month with some dates filled in and let the students work on the missing ones (and only those). For example, fill in the 1st and then let the students work out the dates going down diagonally.

Prepare a table like this one for display in the classroom:

<table>
<thead>
<tr>
<th>½ hour earlier</th>
<th>Time</th>
<th>15 minutes later</th>
<th>¾ hour later</th>
<th>Time as it would appear on a digital clock</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 a.m.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12.30 p.m.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.45 p.m.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.20 a.m.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Half past two</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quarter to ten</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Point to different blocks on the table and let different students give the missing values.

Display a simple timetable, for example one showing the times of television programmes or school activities. For example:

**TV schedule**

- 4.15 Robot dolls
- 4.20 Let us talk
- 4.30 Learning about trees
- 4.45 Spelling quiz
- 5.10 Popstarz
- 5.20 Fashion buzz
- 5.45 Sports highlights
- 6.00 News and weather
Ask questions to make sure the students can read and interpret the timetable. Some possible ones include:

- Which starts earlier – Popstarz or Spelling Quiz?
- Which programme starts at quarter to five?
- Which programme ends at quarter to six?
- Which programme ends at quarter to six?
- What programme can you watch at 10 past five?
- How long are the sports highlights?
- Which programme is longest? How long is it on for?
- The news is 25 minutes long. When does it end?

6. **Shape, space and measures**

Display a number of items with their mass displayed in grams (multiples of 50). Ask questions that involve ordering, comparing, adding and subtracting the masses. For example:

- Which items weigh less than half a kilogram?
- Which items are heavier than 600 grams?
- Which two items together weight 1 kilogram?
- What is the total mass of item A and B?
- I need 1 kilogram of sugar. How much more will I need?
- What is half of this item’s mass?

Display a number of shapes with parts shaded. Ask the students to correctly name each shape based on its sides and/or angles. Also ask them to say what fraction of the shape is shaded. You can extend this to include ordering the fractions or finding and naming all the shapes with a particular fraction shaded.

Display a number of solids and ask the students to draw all their faces (for example, for a cube they would draw 6 squares).

Use the mental warm-up session as an opportunity to explore solids and shapes used in buildings. You can find pictures of homes, places of worship and/or murals and decorative patterns from a range of sources. Display these and spend some time identifying and naming the shapes used in construction, symmetry and other properties.
Display a large grid and use it to describe and identify the position of objects as well as to practise following paths. It is easy to do this using a table on the computer. For example:

<table>
<thead>
<tr>
<th></th>
<th>Bus stop</th>
<th>Petrol station</th>
<th>Station</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Post office</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Mosque</td>
<td>Market</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Library</td>
<td></td>
<td>School</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td>Soccer field</td>
</tr>
<tr>
<td>1</td>
<td>Pete’s house</td>
<td>Hoosain’s house</td>
<td>Indira’s house</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Point to items on the grid and ask the students to give the position.

Give the position of items on the grid and ask the students to say what is found there.

Give the students directions to get from one place to another. Tell them where to start, but not where they will end up. Ask them to say where they end up to check that they have followed the directions carefully.

You can also combine this activity with shape knowledge if you use polygons on the grid and ask the students to find the position of different shapes. This tests that they can name and recognise shapes and that they can use the position system on a grid.

**Using arrow cards to teach place value and partitioning**

Place value cards with a ‘point’ at one end are called arrow cards. These can be used for a number of different activities relating to place value and to partitioning of numbers. When calculating mentally, students need to be able to partition numbers, perform the necessary operation and recombine the numbers to find the result.

The templates for the arrow cards include hundreds, tens and ones (units), but they can be extended to larger numbers and into decimals. You make numbers by placing the cards on top of each other, with the smallest number on top and the arrows aligned. Aligning the arrows makes sure that the digits are all in the correct places.
843 is made by using 800 (eight hundred), 40 (forty) and 3 (three). It is important to stress to the students the names of the numbers involved, rather than just saying 8 hundreds, four tens and three units.

The following activities are examples of how the cards can be used, rather than a teaching progression.

**Objective: To combine cards to produce a given number**

Write a number on the board, demonstrate how to use the cards and then ask the students to make that number.

**Objective: To show how zero is used as a place holder**

Use two cards to illustrate how a three-digit number can be made. Stress that the zero is a place holder.
Objective: To make numbers from expanded notation

60 + 4

200 + 80 + 7

Objective: To make a number from words
Say a number, for example, two hundred and sixty three and then ask the students to make this number with their arrow cards.

"Show me how you would make four hundred and twenty two."
"How would you make three hundred and one?"

Objective: To reinforce the variety of language associated with place value
Ask questions that use different words, for example, “Show me a three-digit number in which all the digits are the same.”

“Show me a number that does not have any hundreds.”
“What about a number that does not have any tens?”

Use any opportunities that arise for students to explain why they choose that number. List out the range of numbers on the board.
1 Number and place value

<table>
<thead>
<tr>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>4Nn1 Read and write numbers up to 10 000.</td>
</tr>
<tr>
<td>Understand what each digit represents in three- and four-digit numbers and partition into thousands, hundreds, tens and units.</td>
</tr>
<tr>
<td>4Nn3 Round three-and four-digit numbers to the nearest 10 or 100.</td>
</tr>
<tr>
<td>4Nn10 Position accurately numbers up to 1000 on an empty number line or line marked off in multiples of 10 or 100.</td>
</tr>
<tr>
<td>4Nn9 Estimate where three- and four-digit numbers lie on empty 0–1000 or 0–10 000 lines.</td>
</tr>
<tr>
<td>4Nn11 Compare pairs of three-digit or four-digit numbers, using the &gt; and &lt; signs and find a number in between each pair.</td>
</tr>
</tbody>
</table>

Vocabulary

Units, tens, hundreds, thousands, digit, place value, partition, number track, number line, round up, round down, estimate, compare, smaller than, greater than

Resources needed

Number strips; 1–100 number grid; number line to 1000; 1–100 number cards; place value cards.

Mental maths activities

Choose any suitable activities from the number and place value section of the mental maths activity bank.

Concepts that may be unfamiliar in this chapter

Extending the number system into thousands

In this chapter students will build on the skills they have used with two- and three-digit numbers and extend them to working with four-digit numbers (place value, value of digits, positioning numbers, comparing numbers and rounding to tens and hundreds). Students will work with larger numbers,
round to hundreds when there is a digit in the thousands place and use different number lines and intervals to position numbers and to estimate the position of larger numbers. All of these skills need to be practised and reinforced well so that the students can transfer them to working with an even greater range of numbers as they move through later stages.

**Teaching ideas**

**Practical activities**

- Students use straws or beads to represent numbers. A 10 can be represented by a bundle of 10 straws or placing 10 beans in a cup, and a hundred by 10 bundles of 10 straws.

- Students could use number cards and counters (straws, etc.) to represent three-digit numbers.

- Students work in pairs. One student can write down a number and the other has to write it down in words.

- Students could use 100-bead strings and number lines to complete number patterns based on skip counting.

- Students could use number lines to show the position of numbers and also to help them decide how to round numbers to the nearest 10, and the nearest 100.

- Ask a group of students to stand up. Give each student a card showing a number less than 1000. Get the rest of the class to instruct the group so they stand showing the numbers in order from smallest to largest/largest to smallest. Give another student another number card and ask them to position themselves in the line. Ask students to say a number that could lie between two of the numbers in the line.

- Give students a set of place value cards or several sets of 0 to 9 digit cards. Say numbers less than 1000 and ask students to lay out the cards to show that number.

- Draw a place value chart like the one below on the board. Put numbers into the chart and use it to help students say the numbers aloud. Say a number and ask students to come up and write the number in the chart. Start with a chart using three-digit numbers and extend this to include four-digit numbers. It is important to say the number: for example ‘two thousand, three hundred and twenty-three’, so that students link place value with the counting system.

<table>
<thead>
<tr>
<th>1000</th>
<th>2000</th>
<th>3000</th>
<th>4000</th>
<th>5000</th>
<th>6000</th>
<th>7000</th>
<th>8000</th>
<th>9000</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>200</td>
<td>300</td>
<td>400</td>
<td>500</td>
<td>600</td>
<td>700</td>
<td>800</td>
<td>900</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
<td>80</td>
<td>90</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>
Ask a student to pick out up to four number cards and stick them on a board in a line for the rest of the class to see. Select another student to say aloud the number that has been made.

Give a group of students a set of up to four cards with a combination of digits. Ask them to arrange the cards to make the smallest/largest number they can. Can they make other numbers? Can they order the numbers they have created? How many different numbers can they make?

Draw four empty boxes in a row on the board. Ask students to make a copy of this on paper. Pick a card from a set of digit cards and then return it to the pack and shuffle. Each time students must select a box to put the number in. Once placed the number cannot be moved. The aim is to have created the largest number when all four boxes have been filled. Discuss with students the strategies they used to make decisions about where to place digits. Repeat, trying to make the smallest number.

Use place value cards (arrow cards) to make an ‘incomplete’ number (see pages 38 – 40 for some ideas on how to use these). For example display 1000, 40 and 3 and ask the students what card is missing for the number ‘one thousand three hundred and forty-three’. Repeat this for several numbers with two or more digits the same to reinforce the ideas of place value into the thousands.

Spend some time reminding the students how to position numbers on a number line from 0–1000 marked in increments of 100. Ask them to estimate the position of some in-between numbers. Point out that the number 550 will be half-way between 500 and 600. Show them how to work with a blank number line marked from 0 to 10 000. Again, discuss where 5000 would lie, and discuss where you would put 9000 and 9999. Give the students time to explain how they work out/estimate where a number will go. Place the number 3000 on the line. Ask them how this can help them work out where 1500 would lie and so on.

Using the Student Book and Workbook

Student Book page 5 revises place value into hundreds and expanding (partitioning) numbers into hundreds, tens and units. Work through the example at the top of the page with the class. If necessary, demonstrate some other examples of expanded form using place value cards. Let the students work on their own to complete the activities. When you are happy that they can remember the concepts, have them work through the problem solving activities and riddles on Workbook page 4.

Spend some time counting on beyond 1000 before you tackle Student Book page 6 which extends the place-value table into the thousands. Remind the students that 10 hundreds make one thousand and count on in thousands to teach the vocabulary. Make sure the students
understand that four-digit numbers have four places: thousands, hundreds, tens and units. Clearly demonstrate the value of each digit using place value cards and/or other concrete apparatus. Let students work in pairs to complete Student Book page 6. Use Workbook page 5 to check that the students can work with numbers in the thousands.

Student Book page 7 deals with expanded form. Revise expanded form as necessary and demonstrate partitioning numbers using place value cards. Then have the students work independently to complete the activities. Use Workbook page 6 to consolidate and reinforce the concepts.

Spend some time revising and working with number lines in the classroom before moving on to ordering and comparing larger numbers. Adapt the practical activities for this chapter if you need to do more practice. It is crucial that the students work out what the intervals shown on unmarked number lines represent (they will use this skill in measuring as well) before they decide what the numbers are. Remind them that if a blank number line is 10 cm long, then 5 cm is half-way and 1 cm is a tenth of the total length. Work through Student Book page 8 with the class to make sure they are able to do this. Use Workbook page 7 to assess that they can apply the skills in a slightly different context.

Revise the use of the < and > symbols. Let the students work through the activities on their own. Workbook page 8 provides additional practice.

Once you are satisfied that the students can order and compare three-digit numbers, move on to working with larger numbers. Let students work in pairs to complete Student Book page 10. Spend some time discussing their answers and methods of working, particularly for Question 3, as their answers here will show you how well they have mastered and understood the concepts linked to place value.

Student Book page 11 deals with rounding off to the nearest ten. Put a number line up on the board. Using sticky notes or index cards, write a number on each note or card and put it on the number line. Ask how we should round the number to the nearest ten – do we round up or down? Draw a dotted line along the half-way point between the tens, as shown in the number line on page 11. Demonstrate with a few examples how to round off to the nearest ten. Then let the students come up and complete some more examples. Finally, have them work through Student Book page 11 and Workbook page 9.

Work through the example on Student Book page 12 to teach the students to round the to the nearest 100. Remind them that this works in the same way as rounding to ten. Do some examples involving three- and four-digit numbers on a number line with the class before asking the students to complete Student Book page 12 and Workbook page 10.
Assessment questions to ask

- Complete the following number tracks.

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- What do each of the digits stand for in the number 1372?
- What is the value of the 4 in 1408?
- Complete this number pattern: 1610, 1615, 1620, ..., ..., ..., ....
- Write these numbers in order of increasing size: 954, 949, 994, 998, 940.
- What is 75 rounded to the nearest 10?
- What is 444 rounded to the nearest 100?
- Which number is in the units place? Does that mean we round it up or down to get to the nearest ten?
- Which number is in the tens place? Does that mean we round it up or down to get to the nearest hundred?
- Which number is in the hundreds place? Does that mean we round it up or down to get to the nearest hundred?

Common errors and misconceptions

Students may struggle to say large numbers correctly and resort to simply saying the digits of the number in order. Do not allow them to do this. Help them to move to saying numbers correctly by identifying the value of each digit and saying the numbers in expanded form.

Students may struggle to write and/or say numbers where zero is a place holder – for example, 4056, 1304, etc. If this is a problem, spend time asking students to enter numbers into a place-value chart and then say them in expanded form, emphasising where there is a zero in a column and what that means.

When comparing the magnitude of numbers, students must remember to read the digits from the left and not the right. Students may also get confused between the < and > signs. Remind them to imagine a ‘hungry mouth’ that opens towards the biggest meal.

When rounding to the nearest hundred or ten, students must remember that half-way or more (i.e. 50 or 5) should round up, otherwise they should round off without changing the digit to the left of the place they are rounding to.
2 2D shapes

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<th>Objectives</th>
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<td><strong>4Gs1</strong></td>
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<td><strong>4Gs2</strong></td>
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</table>

Vocabulary

Polygon, angle, side, triangle, square, rectangle, circle, pentagon, hexagon, heptagon, octagon, irregular, regular, right-angled

Resources needed

2D shapes, large sheets of paper; card; card rectangles; circles and squares of paper; squared paper; rulers; set squares or right-angle measures; pinboards; elastic bands or string, dotted paper.

Mental maths activities

Choose any suitable activities from the mental maths activity bank. Remember that the students will not be doing much number or calculation work in this chapter, so you may want to select calculation activities alongside any activities you do with shapes.

Concepts that may be unfamiliar in this chapter

Names and properties of quadrilaterals

Students already know that shapes can be classified and named using the number of sides they have. In this chapter they will learn that the larger group of quadrilaterals can be sub-divided into specialised shapes based on the angle properties and whether or not the sides are equal and/or parallel. Emphasise the correct names for each shape and use these when you deal with the shapes to help students get used to the terminology. Bear in mind it may take them some time to realise that these specialised shapes have common properties (for example, a rectangle is a parallelogram with...
right angles, a square is a rectangle with equal sides, therefore a square is also a parallelogram). Identifying the properties of different shapes is an important component of Euclidean geometry and students will use these properties in higher grades to work out missing angles and solve problems related to shape.

**Teaching ideas**

**Practical activities**

- Revise or introduce the names and definitions of some of the common plane shapes and associated terms, such as triangles, rectangles, squares, pentagons, hexagons, heptagons, octagons and circles.

- Put a selection of plane shapes into a feely bag. Students pull out specific shapes such as an irregular pentagon; two similar triangles; a regular octagon, and so on.

- Give groups of students a variety of regular and irregular plane shapes. They need to decide on different ways of sorting and re-sorting shapes.

- Students construct rectangles on a pinboard. Ask how they know their shapes are rectangles. Revise the properties of rectangles: four sides (quadrilateral); opposite sides equal; angles are right angles (square corners). Ask them to construct squares on their pinboards. Discuss the properties of a square: four equal sides; four right angles; and so on.

- Have the students suggest ways of drawing a rectangle (these might include tracing, measuring out, etc.) Let them practise drawing rectangles and other shapes.

- Play a ‘guess the shape’ game in pairs. Give each student a plastic or card polygon. They should not show their shape to their partner. Have them try to guess which shape their partner has by asking questions about it. The partner may only answer yes or no to questions. So they cannot ask: how many sides does it have? They can ask: does it have more than three sides?

**Using the Student Book and Workbook**

- Use Student Book page 13 to revise the names of polygons the students already know and to teach the names and properties that they don’t know or don’t remember. It is useful to ask the students to draw three of four different versions of each shape as you discuss it to consolidate the properties. Make sure the students understand that regular polygons have equal sides and angles. Bear in mind that the square is the only regular quadrilateral. (If a rhombus has equal angles, it is a square.) Let the students complete the activities on page 13 and then ask them to classify the shapes on Workbook page 11.
Use Workbook page 12 to check that the students know what a polygon is before moving onto the investigations in the Student Book. You will need pinboards or dotted paper to complete the activities on Student Book page 14 and 15. Allow time for investigation and discussion. Once students have done the practical activities, let them complete Workbook page 13 to summarise what they have found out. Remind them that tables are a very useful format for summarising properties such as these.

Work though Student Book page 16 with the class. Use the diagrams, but also show them cut out or plastic shapes in different orientations. The students must be able to recognise shapes in any position or orientation. Once they have completed the activities in the Student Book, let the students work in pairs to draw 16 different polygons. Allow them to use pinboards to model their answers before they draw them if necessary.

Spend some time looking at rectangles of different dimensions with the class. Students often struggle with the idea that a square is also a rectangle (a rectangle with equal sides). Let them investigate using pinboards or dotted paper and spend some time discussing what they find out. Use the tangram activity on Workbook page 15 as a fun method of consolidating work on shapes.

**Assessment questions to ask**

- Select a plastic shape from a bag and ask the students to name it and describe its properties.
- Can you draw me a shape with five straight sides? What do we call these shapes?
- Make up a set of instructions to tell someone how to draw a square.
- What is the difference between a pentagon and a hexagon?
- Look at these two shapes. How are they similar? How are they different?

- What are the similarities/differences between squares and rectangles?
- Is a circle a polygon? Why or why not?
- Name three different polygons in this pattern.
- What do all quadrilaterals have in common?
Common errors and misconceptions

Students may find it difficult to identify some polygons when they are differently oriented. For example, when they see a square like the one on the left, they know it’s a square, but when they see it like the one on the right, they tend to say it’s a rhombus or a kite (or incorrectly, a diamond).

It is important to work with shapes in different orientations. Lots of practice with a pinboard and cut out shapes will help with this.

Some students will get confused by the names of shapes. Try to provide links between the names and the properties to help them remember them: for example, tri – means three, a tricycle has three wheels, a triangle has three angles, and so on. Also make sure you model the correct terms, display the labels on shapes, and insist that students use the correct terms to talk about shapes.

3 Time

<table>
<thead>
<tr>
<th>3: Time</th>
<th>Student’s Book pp 18–23</th>
<th>Workbook pp 16–18</th>
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<tr>
<td><strong>Objectives</strong></td>
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<tr>
<td>4Mt1</td>
<td>Read and tell time to the nearest minute on 12-hour digital and analogue clocks.</td>
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<tr>
<td>4Mt2</td>
<td>Use a.m./p.m. and 12-hour digital clock notation.</td>
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<tr>
<td>4Mt3</td>
<td>Read simple timetables and use a calendar.</td>
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<tr>
<td>4Mt4</td>
<td>Choose units of time to measure time intervals.</td>
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</table>

Vocabulary

Analogue, digital, a.m./p.m., calendar, date, week, month, year, leap year, timetable

Resources needed

Large demonstration clock face; analogue and digital clock to show the class; card and markers for making a digital clock; a large calendar for classroom
display; a range of local timetables (bus and train times, TV schedules – but be aware that many of these use 24-hour notation and the students are only expected to work with 12-hour time, so make sure you select extracts that relate to 12-hour times).

**Mental maths activities**

Select activities from the number and/or time and calendars section of the mental maths activity bank.

**Concepts that may be unfamiliar in this chapter**

**The notation a.m. and p.m. for different times**

In the digital age that we live in, it is unlikely that students will not have seen or heard the terms a.m. and p.m. applied to time. However, this is the first time they are formally required to use this notation to distinguish between times before and after noon. The abbreviations are short for the Latin terms for ante meridiem and post meridiem which essentially mean before and after noon. It has become acceptable to write the abbreviations with or without full stops.

**Read simple timetables**

A timetable is basically a list of times with some details of what happens at each time. Students have probably already worked with class timetables and television schedules so the concept should not be completely unfamiliar to them. You will need to teach them that they can work out time intervals (how much time passes from one point to another) and the duration of events (how long something takes) using the times on the timetables. It is always useful to relate this work to everyday timetables that students may use such as bus or train timetables, tide tables, television schedules and so on.

**Teaching ideas**

**Practical activities**

Invite students to the front of the class in pairs. Ask one student to say a time and the other to demonstrate the time on a large clock face. Alternatively, ask a student to set the clock and another to tell the time.

Say events of the day to students, such as lunch time or end of school day, and ask them to set the clock face to the time when the event is likely to happen. Include events that may be different for different students or on different days, and ask students to show possible answers and explain the decisions they make.

Ask students to work out durations in the day by regularly asking questions such as: ‘What time is it now?’; ‘How much longer is it until lunch time?’, and so on.
Ask students to make up time stories – for example, ‘Ayeisha left her house at 12.34 p.m., reached the bus stop at 1.00 p.m., got off the bus at 1.23 p.m. and arrived at Sally’s house at 1.56 p.m. It took her 1 hour 22 minutes to reach Sally’s house.’ Continue with these practical activities alongside the Student Book to ensure students are confident with telling the time and finding durations in time.

Make a large class calendar on which you record important dates. These could include birthdays (if appropriate), holidays and feast days, important school events, national days and the beginning and end of school terms. Use the activity of putting events on the calendar to teach and discuss how calendars work and how to read dates from them.

If appropriate, bring some post-marked envelopes to class and ask the students to work out when each item was posted. You may need to explain that the post is stamped at the posting office when it is received. You can also set some problems around post-marks. For example, if this letter took 3 days to arrive, on what day/date did it arrive? And so on.

Let the students develop a timetable of their own to show what they do on a typical weekend day. Before they start, explain that a timetable shows the time at which an event starts. It does not give the duration as such, but you can work out how long something took by looking at when the next event starts. This can cause confusion, but dealing with it in this way often makes it easier for students when they work with other timetables.

**Using the Student Book and Workbook**

Using a large clock face, show different times and ask students to tell the time. Let them work on their own or in pairs to complete Student Book page 18. Check their answers and, if necessary, show them the times indicated in question 1 so they can see where the minute hand lies at each time.

Work through Student Book page 19 with the class to teach the correct use of a.m. and p.m. times.

Demonstrate digital times on the board. Give the students some opportunities to write given times as digital times on the board. Most students find this much easier than analogue time as they are exposed to digital watches and timers in daily life. Let the complete the table and then work through Workbook page 16 for additional practice in working with both analogue and digital versions of the same time.

Student Book page 21 is a practical activity that involves measuring as well as construction skills. You may like to complete this activity in groups. Spend some time playing with the clocks and posing and solving time problems using them.
Once you have done some introductory and practical work on calendars, work through Student Book page 22 with the class to make sure they understand the concepts and how to read a calendar. Let the students work in pairs to complete the related activities on Workbook page 17.

Use the bus timetable on Student Book page 23 to make sure the students understand how to read a simple timetable. Work though the questions orally with the class, discussing how they would find the answers. Once you have done this, let the students complete the activities in their books. Use Workbook page 18 for drawing up a personal timetable for a week’s activities.

**Assessment questions to ask**

- What time does this clock show? (analogue and digital clocks)
- Can you set this clock to show 10.53 a.m.?
- What is a quarter to three in digital time?
- Look at this TV guide. At what time does the news start? (and related questions)
- Use the calendar to find the day on which the 1st of May fell this year.
- What will the date be in a week’s time?
- How many days are there in March/February/December?
- What is a leap year?
- What was the date on the first Sunday in September?

**Common errors and misconceptions**

Many students find it very difficult to tell the time, and much of this is to do with the different ‘values’ the numbers on the clock face have, depending on whether we are looking at the hour hand or minute hand. Students need to be able to see a clock face and experiment with what happens as the hands move around it. Students need to have regular experience of telling the time, thinking into the past and future and working out how long it is since something happened or will be until something is going to happen.

Students may not immediately realise that they can move down columns on a calendar to find the date ‘a week’ or ‘three weeks’ later. It helps to model this on a class calendar as you work out problems to do with dates.

Some students find it difficult to answer questions like: what is on TV at 4.30? because only starting times are given in schedules. You will need to pose questions carefully and focus on the duration of intervals/events on timetables. Use the school timetable as an example to illustrate how this works. For example, what lesson do you have at 9.30 (choose a time in between lesson starting times for this). Students then need to work out that PE (or whatever lesson is involved) starts at 9.15 and continues to 10.00, so they will be in PE at 9.30. The actual time 9.30 may not appear on the scheduled timetable.
4 Decimals

Objectives

**4Nn4** Use decimal notation and place value for tenths and hundredths in context; e.g. order amounts of money; convert a sum of money such as $13.25 to cents, or a length such as 125 cm to metres, round a sum of money to the nearest pound.

**4Nn5** Understand decimal notation for tenths and hundredths in context, e.g. length.

**4Nn20** Understand the equivalence between one-place decimals and fractions in tenths.

**4Nn21** Understand that ½ is equivalent to 0.5 and also to 5/10.

Vocabulary

Decimal, decimal point, place value, tenths, hundredths

Resources needed

Flashcards showing a variety of decimal amounts; place value notation cards; 0–9 digit cards and a decimal point card; a metre stick or a length of card divided into tenths.

Mental maths activities

Select a range of suitable mental maths activities from the mental maths activity bank.

Concepts that may be unfamiliar in this chapter

Decimal notation and place value

This is the first time students are formally introduced to decimal notation and the concept of extending the place value table to include fractions (tenths and hundredths). This foundational learning is very important as students will work with decimals to increasing degrees as they move up the school levels. At this stage, the concepts are taught and then applied in the context of money and measurements so that students get a good sense of how and why decimal fractions are used. Students usually have a fairly well developed concept of money and how it works (no matter where they live or what currency they use) but they may not understand that a money amount such as $1.50 means the same as 150 cents or one-and-a-half dollars.
Teaching ideas

Practical activities

Mark one end of a metre stick ‘1’ and the other end ‘2’ and make sure the 10 cm divisions are clearly marked. Ask: ‘Into how many sections has the stick or strip been divided?’ (Ten)

Point to the midway division and talk about it being half-way between 1 and 2. In other words, this is $1\frac{1}{2}$. Explain that each division is one-tenth, so the first division represents ‘one and one-tenth’.

Explain that in decimals, one and one-tenth is referred to as ‘one point one’. Show the students this is written as 1.1. Students say the number. Ask them the value of each digit in the number. Stress that $1\frac{1}{2}$ is equivalent to $1\frac{5}{10}$ or 1.5. This will help students consolidate their understanding of the equivalence between a half and 0.5.

Refer to each of the other divisions up to 2. Ask the students to count in tenths (1.1, 1.2, and so on) from 1 to 2, and back again. Ask them to point to divisions on the strip for others to name and write the decimal number represented.

Students can make their own decimal number lines.

Ask a group of students to stand up. Give each student a card showing a decimal number. Get the rest of the class to instruct the group so they stand showing the numbers in order from smallest to largest/largest to smallest. Give another student another number card and ask that they position themselves in the line. Ask students to say a number that could lie between two of the numbers in the line.

Give students interlocking cubes or base-ten apparatus. They join the cubes together in tens of the same colour. Each stick of ten cubes represents a ‘one’. Refer to them as sticks or rods to help to establish their unity. Alternatively, use the base-ten apparatus, establishing each length of ten cubes as a ‘one’ and the loose cubes as ‘tenths’. Ask: ‘Into how many equal pieces is each stick divided?’ Talk about tenths.

Ask students to show you ‘one and two-tenths of a stick’. Refer to ‘one point two of a stick’ and write out the decimal. Do the same for other quantities. Include numbers less than one and discuss the use (for clarity) of the zero in the unit’s place, even when it is superfluous (for example, 0.5). Students could work in pairs with one student showing the sticks and the other saying and writing the quantity as a decimal number and vice versa.

Have the students cut out, join and mount number lines to make a bigger number line from 0 to 10 marked and numbered in tenths. They can shuffle several sets of cards 0 to 9, and draw cards in pairs.
to generate a series of two-digit decimal numbers (for example, 2.3, 3.2, 5.6, 6.5, and so on). They mark on the number line the position of each number and finally list the numbers in order.

- Use a tape measure and let the students measure their own heights in centimetres. Point out that 1 metre is divided into 100 centimetres, so each centimetre is 1/100 of a metre. Discuss how to write heights as decimal numbers in metres.

- Find some common food and drinks containers with different masses and capacities. Discuss what it means if a bottle says it holds 1.5 litres, and how much 0.5 kg is.

- Use money to demonstrate how currency uses decimal notation. Choose a price or amount such as $5.99. Demonstrate that this is 5 dollars (ones), nine ten-cent coins and nine cents. Use a place value table extended to hundredths to write the amounts. Use different amounts and ask the class to say how many whole dollars are involved and what fraction of a dollar is given after the decimal place (in hundredths). Look at amounts such as $1.50. Stress that this is 1 dollar and \( \frac{50}{100} \) of a dollar. When we work with money, it is a ‘rule’ (convention) to write the 0 on the end if there are no cents in the hundredths column. This is largely to do with the history of accounts and the ways in which receipts, etc. are printed. If you just wrote $1.5, the person could easily add 9 cents to each amount and cheat! Then discuss how you would write one dollar and five cents. Stress that 5c is \( \frac{5}{100} \) of a dollar. Use the place value table to show that you would write the 5 in the hundredths place. You have to write 0 as a place holder in the tenths column to show that there are no tenths.

**Using the Student Book and Workbook**

- Student Book page 24 deals with tenths. Workbook page 19 gives practice representing tenths in numerals and on a number line.

- Once the students have had sufficient practice representing decimals in a variety of ways, have them work through Student Book page 25 to check that they understand the concepts.

- Student Book page 26 uses decimals in the context of measurement. Spend some time counting in tenths and working with number lines before asking the students to complete the activities. Use Workbook page 20 to check they are able to order decimals and place them on a number line. The number line work is essential for the work that follows in measurement and reading scales.

- Work through the example on Student Book page 27 with the students and then have them complete the activities. Use Workbook page 21 to consolidate work on ordering decimal fractions using number lines.
Use Student Book page 28 to consolidate students’ understanding of the relationship between decimals and money amounts.

Students can work on their own to complete Student Book page 29. If they are unsure of the amounts, they can use a 100 square to help them visualise them. Workbook page 22 involves translating their understanding of decimal comparison to measures and money amounts.

Student Book page 30 gives the students a chance to apply their skills in a problem solving context (decimal scores). You may need to work through some examples and to explain how scores are given in contests such as the Olympic Games or talent shows on television.

Assessment questions to ask
• What does this digit (pointing to one digit in a decimal) represent?
• How do you say this number?
• When ordering these decimals, what did you look at first? Then what did you do?
• Which numbers do you find hardest to order? Why?
• Can you give me a number that lies between 3.2 and 3.3? How many answers do you think there are to this question?
• Can you give instructions to someone to explain how to round numbers to the nearest tenth/hundredth?
• Place these numbers in the correct position on the number line. (Give various decimals for positioning on a number line.)
• Show these decimals on an abacus. (Give various decimals for students to represent on an abacus.)
• What is one-hundredth of a dollar? What is one-hundredth of $100?
• What is 25-hundredths of a dollar? What is another way we can write this fraction? (Give other examples such as 10-hundredths of a dollar, 5-hundredths of a dollar, 50-hundredths of a dollar, and so on.)
• Draw diagrams to represent and compare each pair of decimals. (Give various pairs of decimals for students to represent and compare – for example, 0.22 and 0.3, 0.48 and 0.51, 0.19 and 0.91, and so on.)

Common errors and misconceptions
There are many common misconceptions surrounding the concept of decimals. Although at this stage students are working with numbers to one decimal place, it is useful to remember that some students see the numbers after the decimal point as a mirror of those before, thus thinking of 34.56 as ‘thirty-four point fifty-six’. This leads to incorrect comparison of numbers as if students were to compare 34.56 with 34.7, for example, they would feel 34.56 is larger as ‘fifty-six is bigger than seven’. To avoid this, it is important to use place-value charts or notation cards to emphasise the value of digits in a number and regularly practise how to say numbers correctly. Another common error relates to sequences of decimals (for example, ‘0.6, 0.7, …’). When reaching 0.9, students often follow with 0.10, the line of thinking being
‘nine tenths, then ten tenths’. Time needs to be spent with students thinking about what is meant by ten-tenths and diagrams can often help to clarify this. Using a place-value chart will show that 0.1 and 0.10 are the same.

When students work with decimals in real-life contexts (measures, money, etc.), they need to realise that a fraction is not an absolute quantity. For example, one dollar is one-hundredth of $100, and one cent is one-hundredth of one dollar. The fraction tells us about the relationship of the part to the whole.

## 5 Length, mass and capacity

| Objectives |
|-----------------|------------------------------------------------|
| 4Ml1            | Choose and use standard metric units and their abbreviations (km, m, cm, mm, kg, g, l and ml) when estimating measuring and recording length, weight and capacity. Know and use the relationships between familiar units of length, mass and capacity; know the meaning of kilo, centi and milli. |
| 4M12            | Where appropriate, use decimal notation to record measurements, e.g. 1.3 m, 0.6 kg, 1.2 l. |
| 4Ml3            | Interpret intervals/divisions on partially numbered scales and record readings accurately. |

### Vocabulary

Length, units, centimetre (cm), metre (m), kilometre (km), height, distance, capacity, millilitre (ml), litre (l), mass, scale, weight, gram (g), kilogram (kg), estimate, measure, scale, interval

### Resources needed

Metre rules; rulers; tape measures; chalk; string; any other instruments for measuring length (e.g. trundle wheel, callipers); various items for weighing (a brick, various vegetables, books and stationery, various coins); kilogram weight (such as a 1 kg bag of rice); bathroom and kitchen scales; containers with various capacities: jugs, spoons, bottles, cups, and so on; junk containers, such as empty and clean yogurt cups, milk bottles, juice bottles, bottles from hand soap and detergents, and so on; 10 ml measure such as a syringe or eyedropper; 5 ml spoon; two calibrated measuring containers (such as test tubes or small jugs).
Concepts that may be unfamiliar in this chapter

Using decimal notation to record measurements

Although the students have worked with measurements in earlier stages, they will now be expected to use different notations for measurements; this includes using a larger and smaller unit such as 3 kg and 250 grams and the equivalent decimal notation of 3.25 kg. As many of the measuring instruments that students will see in real life are likely to be digital, this notation on its own should not be problematic. However, the students need to understand that the decimal part of the measurement is a fraction of the whole unit which is the same as a number of smaller units – for example, 0.6 m is $\frac{6}{10}$ of a metre which is the same as 60 cm.

Interpreting intervals on partially numbered scales

Most measuring scales have only certain intervals marked on them. Kitchen scales may have masses marked in 100 gram intervals, or in $\frac{1}{2}$ kg intervals, bathroom scales may be marked in kilograms and $\frac{1}{4}$ kilograms, measuring tapes and rulers are generally marked in centimetres, with millimetres shown, but not labelled. Students need to be taught to count the intervals, work out how much each one represents and use that information to read off an accurate measure when it is in between the marked units. As with all measuring topics, lots of exposure to practical examples and real life measuring equipment will help the students develop this skill.

Teaching ideas

Practical activities

✍️ Ask students what units they know for measuring length. Once ‘metre’ has been offered as an answer, ask them if they know of a smaller unit. Ask students to look at their rulers and notice the smaller divisions: ‘centimetres’ and ‘millimetres’. Write these terms on the board. Ask students to suggest things that we would need to use metres, centimetres or millimetres to measure. Write 5 m 6 cm on the board. Ask students how many centimetres that is in total. Encourage students to explain how they worked it out – for example, ‘There are 100 cm in 1 m so it is 500 + 6, which is 506 cm.’ Repeat for other examples.

📏 Show the class a ruler. Point out how each centimetre is divided into millimetres. Write on the board 10 mm = 1 cm. Ask a student to come to the board and to draw a line of 5 cm, using a ruler to measure it out. Write: 5 cm = ___ mm. Then have another student come up and measure it in millimetres, and fill in the answer. Repeat with some other lengths.

✏️ Write a length on the board – for example, 5.6 cm. Ask students to use a straight edge to draw a line they think is about this length. Using a ruler, ask students to measure their lines and see how accurate their estimate was. Points could be awarded for degrees of accuracy – for
example, 2 points if you were within 5 mm, 3 points if you were within 2 mm, five points if you were within 1 mm. After ten goes, see who has the most points.

Ask students to suggest lengths that would be measured in millimetres, centimetres, metres and kilometres.

Write distances on the board – for example, ‘the length of a pencil’, ‘the distance between these two towns’, etc. Ask students to say a sensible unit to use for measuring these distances.

Ask students to suggest things that would be similar, lighter and heavier in weight than a chosen item. Ask students to suggest objects that would be weighed in kilograms or grams.

Suggest various objects and ask students to suggest a sensible unit to measure these in.

Continue with these practical activities alongside the Student Book to ensure students are confident with the metric units and measuring different quantities.

Show the students a range of containers. Hold up pairs of containers and ask them to say which holds more. Show them a litre container or the mark on a calibrated measuring jug. Remind them that there are 1000 millilitres in a litre. Have them estimate the capacity of each container before filling it with water and pouring the contents into the measuring jug to measure the capacity.

Show the students how to record capacities in millilitres and litres – for example, 1200 ml or 1 litre 200 ml. Also demonstrate how to record capacity as a decimal of a litre, such as 1.5 l. Discuss the fact that 1 ml is one-thousandth of a litre, that 100 ml is one-tenth of a litre, and so on.

Fill a standard litre measure with water and share the litre equally into two identical, unmarked containers. Ask what fraction of a litre has been poured into each container.

**Using the Student Book and Workbook**

Student Book page 31 deals with units and instruments of measurement. Show the class some different instruments that we use to measure length, such as tape measures, rulers, metre rules. If you can get hold of a trundle wheel and a pair of callipers, show the class how we use these to measure different kinds of distances. A trundle wheel has a circumference of 1 m, and clicks after each rotation. A pair of callipers is useful for very precise measurement of tiny items. Have a class discussion about what we use to measure particular kinds of lengths and distances. Use Workbook page 23 to summarise and consolidate learning about units and measuring instruments.
Once the students have done some practical measuring tasks, show them how lengths can be written in different ways (in metres and centimetres, in centimetres only or in metres and parts of metres as a decimal fraction). If necessary remind them about the work they did earlier on decimal fractions. Let the students work independently to complete the measuring tasks and activities on Student Book page 32 and Workbook page 24.

Use Student Book page 33 to revise and extend students’ understanding of capacity.

Student Book page 34 and 35 are problem solving pages that involves adding and subtracting amounts to 1000. Discuss strategies the students could use to solve the problems as necessary and allow some time to check answers and discuss methods that students used. Allow them to work in pairs if necessary to complete the problems.

Use Student Book page 36 and Workbook page 35 to consolidate and extend the students’ understandings of mass. Bear in mind that we talk about weighing an item, but we are technically finding (measuring) its mass. As students advance, they should begin to talk about the mass of objects as this is the correct terminology and it is what they will come across in science lessons as well.

Spend some time working through the example on Student Book page 37 and showing students examples of different measuring scales to reinforce the concepts. Work through the activities with the class, making sure the students are able to read and make sense of the scales. Use Workbook pages 26 and 27 for additional practice and consolidation.

Assessment questions to ask

- What would you use to measure the length of a kitten? A desk? The side of a triangle in your maths book?
- How many millimetres is 4 cm? 6 cm? 15 cm?, and so on.
- How many centimetres is 20 mm? 30 mm? 84 mm?, and so on.
- What units would you use to weigh this [object]?
- Do you think this [object] is heavier or lighter than a kilogram weight?
- Can you suggest three objects that you would weigh in kilograms or grams?
- A chair weighs 2 kg and an exercise book weighs 50 g. Can you explain to me how to add these two weights?
- If I subtract 100 g from 0.5 kg I get 0.49 kg. Do you think this is right? Where did I make any mistakes?
- Which of these two containers holds more?
- How much more fits into this container than into that one?
- How can we measure how much a container holds?
- Find three containers of different shapes that all have the same capacity – for example, three 1 l containers; three 250 ml containers, and so on.
- How many millilitres in a litre? In half a litre? In a quarter-litre?
Common errors and misconceptions

Some students may be confused about how many centimetres are equal to 1 m, and how many millimetres are equal to 1 cm. In their work on weight and capacity they learn that there are 1000 g in 1 kg, and 1000 ml in 1 l respectively, so they may fall into the trap of thinking that there are 1000 cm in 1 m, or 100 mm in 1 cm.

The correct symbol for centimetre is cm and not Cm, cM or CM. Similarly, the correct symbol for kilometre is km and not Km, kM or KM. The prefixes kilo-, centi- and milli- come up in a variety of contexts in mathematics and the sciences and it is worth getting students into the habit of writing these symbols correctly now to avoid problems in the future.

The main errors students are likely to make with work on mass and weight is in converting between units of measurement. Many of these errors come down to a misunderstanding of place value and the decimal point. Give students plenty of experience in ordering measurements, saying conversions out loud and placing measurements in a place value chart.

Students may get confused about the distinction between capacity and volume as the terms are often used interchangeably in real life contexts. ‘Capacity’ means the amount of space inside a container. In other words, it is a term we use for containers that can be filled. Make sure the students understand that the capacity of a container is how much it can hold (for example 1 litre); it may hold less than that if it is not filled.

6 Counting, adding and subtracting

<table>
<thead>
<tr>
<th>Objectives</th>
<th>Student’s Book pp 38–44</th>
<th>Workbook pp 28–30</th>
</tr>
</thead>
<tbody>
<tr>
<td>4Nn2</td>
<td>Count on and back in ones, tens, hundreds and thousands from four-digit numbers.</td>
<td></td>
</tr>
<tr>
<td>4Nn6</td>
<td>Find multiples of 10, 100, 1000 more/less than numbers of up to four digits, e.g. 3407 + 20 = 3427.</td>
<td></td>
</tr>
<tr>
<td>4Nc1</td>
<td>Derive quickly pairs of two-digit numbers with a total of 100, e.g. 72 + [ ] = 100.</td>
<td></td>
</tr>
<tr>
<td>4Nc2</td>
<td>Derive quickly pairs of multiples of 50 with a total of 1000, e.g. 850 + [ ] = 1000.</td>
<td></td>
</tr>
<tr>
<td>4Nc6</td>
<td>Add three or four small numbers, finding pairs that equal 10 or 20.</td>
<td></td>
</tr>
<tr>
<td>4Nc7</td>
<td>Add three two-digit multiples of 10, e.g. 40 + 70 + 50.</td>
<td></td>
</tr>
</tbody>
</table>
Vocabulary
Count on, count back, skip count, sum, total, multiple, add

Resources needed
Number cards; number lines; concrete apparatus as needed by the students.

Mental maths activities
Select suitable number concept, place value and calculation activities from the mental maths activity bank.

Concepts that may be unfamiliar in this chapter
The concepts of counting on and back, as well as skip-counting in groups should be familiar to the students. However, this year, they work with a higher number range, and also count in larger groups, including thousands. The counting principles established earlier and reinforced here are fundamental to developing efficient calculation strategies. As students work through this chapter, they will further refine and develop their mental strategies in adding and subtracting numbers, as well as learning more bonds (pairs of numbers that make 100 and pairs of multiples of 50 that make 1000). Although technically new to the students, this work relies and builds on what they already know about bonds to 10 and counting in fives.

Teaching ideas

Practical activities

 Spend time rehearsing counting in different groups with and without the use of a number line. For example, say a number and then go round the class counting up or down in a specified interval.

 Display the terms ‘count on’ and ‘count back’ on the board with the numerals 1, 10, 100 and 1000 below them. Choose one student to say a three- or four-digit number. Point to an instruction (say count on) and a number (say 1). Go round the class counting on in ones from the given number, keeping up a good pace. At intervals point to a new instruction and number (say count back in tens) and continue in this way for some time.

 Write a number sequence on the board. For example: 1345, 1445, 1545 … Ask the students to say how the sequence is generated. (In this case by adding 100 or counting on in hundreds.) Ask the students to say what the next number would be. You can also ask what the number before 1345 would be. Repeat for several different sequences.

 Display a 10 to 1000 grid like the one that follows (you could also reproduce this for each student if you prefer).
Choose a number (for example 330) and discuss the values of the numbers to the left and right of it (ten less, and ten more), then look at the values of the numbers above and below it (hundred less and hundred more). Spend some time looking at the digits that ‘change’ when you move left and right (i.e. the digits in the tens place) as well as the digits that change when you move up or down the grid (the digits in the hundreds place). Play some games where you draw a $3 \times 3$ section of this grid and place one number in it (you can vary the position for interest). The students have to work out what the missing numbers are.

Use the grid above to introduce the idea of finding a number that is a multiple of ten or hundred more than a given number. For example, how could you find the number that is 300 more than 390? (300 is three jumps of 100, so move down three blocks, or increase the hundreds digit by 3 to get 690). Or, what is 80 more than 140? (80 is 8 tens, add 6 tens to get to 200 and then add another two tens to get to 220.)

Once students are confident with numbers to 1000, repeat these activities for counting in hundreds and thousands using a grid from 100 to 10 000 in intervals of 100 like the one below.
Throughout the day, look for opportunities to add single-digit numbers by asking questions such as:
‘Which groups of students could we put together to make a group of 12?’
‘There are two rulers on that table, five on that one, and six on that. How many rulers are out?’
And so on.

Give each of ten students a large numeral from 0 to 9. Give instructions such as:
• ‘Find a partner to make a total of nine with you.’
• ‘Get into a group to make 15.’
• ‘Make a group that adds up to 20 and a group that totals 25.’
• And so on.

Use a range of terms involved with addition such as ‘add’, ‘total’, ‘equal to’, ‘makes’. The total of the ten digits is 45. Ask students to explain how they know that they have the right total.

A small group of students draws five numbers from a stack of numeral cards 1 to 9. They investigate: how many numbers between 20 and 30 they can make by adding the chosen numbers; how many different ways they can make a particular total using the numbers; the least amount of numbers needed to make a particular total. (The numbers can be used more than once.)

Make a simple dartboard circle divided into six segments on a large sheet of paper, each numbered with a single digit. Then, ask the students if they had three darts:
• ‘How many ways can you make a score of 15?’
• ‘How many ways can you make a total of more than 20?’
• ‘How many ways can you score less than 10?’
• And so on.

Encourage the students to work systematically and to record their combinations. Discuss whether the order in which the numbers are ‘hit’ makes any difference to the final total. Identifying all possible outcomes of throwing the three darts will link in with later work on probability.

Students spread a set of numeral cards face up in front of them. They take turns to select a card. The object of the game is to collect a set of three cards which total 15. It does not matter how many cards they have as long as three of them add up to 15. They use mental arithmetic to decide which cards they need and also which cards to take in order to block their opponent. The game could be played for other totals.
Using the Student Book and Workbook

Let the students work independently to complete Student Book page 38 and Workbook page 28. Observe them as they work to see who is still reliant on grids and who is developing a patterning strategy with understanding of place value. Assist as necessary.

Work through and discuss the addition and subtraction strategies using counting on Student Book page 39 and revise this using grids if necessary before asking the students to complete the activities. Spend some time discussing their methods to make sure they understand the ideas.

Revise addition bonds to 10 and multiples of 10 that make 100 before working through the examples on Student Book page 40. Let the students work on their own through the activities and then have them complete Workbook page 29.

Discuss how you can use the known facts to find pairs of numbers (multiples of 50 only) that make 1000. Make sure students can see and articulate the patterns by working through Student Book page 41. Let them complete the activities independently to consolidate this work.

After working through some practical activities on adding small numbers, let the students work in pairs to complete Student Book page 42 and the number chain investigations on Workbook page 30.

Use Student Book pages 43 and 44 to revise and extend addition facts and strategies. Discuss the methods that students use and establish which methods the class find most effective.

Assessment questions to ask

- What number is [choose a value] more than [choose a number]?
- How many more is 1234 than 1034? (and similar)
- What must I add to 2454 to get to 2554/2464/3454? (and similar, including subtractions)
- Decrease 4567 by 400 (and similar, including increasing).
- I have 650 grams of sugar, how much more sugar do I need to make 1000 grams? (and similar, including capacity measures)
- Which groups of students could we put together to make a group of 15? Which other groups could make the same total?
- How many groups of books could we put together to make a group of 20? Which other groups could make the same total?
- Take your numeral card. Find a group that makes a total of . . . (give various totals to be made up).

Common errors and misconceptions

Some students will struggle to count in tens when they have to cross the hundreds, and in hundreds when they have to cross the thousands. Use counting grids and charts to help them work out the answers and also to practice counting aloud in different steps to bridge the barriers.
Students may attempt to count on instead of recalling addition facts to find pairs to 100 and 1000. You can address this by discussing methods, and also by doing lots of practice of addition facts and combinations. Encourage jotting down addition facts to assist as this often makes the connection clearer for the students.

7 Symmetry

<table>
<thead>
<tr>
<th>7: Symmetry</th>
<th>Student’s Book pp 45–47</th>
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<tbody>
<tr>
<td></td>
<td>Workbook p 30</td>
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<table>
<thead>
<tr>
<th>Objectives</th>
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<tbody>
<tr>
<td>4Gs2</td>
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<tr>
<td>4Gs3</td>
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<tr>
<td>4Gs5</td>
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</tbody>
</table>

Classify polygons (including a range of quadrilaterals) using criteria such as the number of right angles, whether or not they are regular and their symmetrical properties.

Identify and sketch lines of symmetry in 2D shapes and patterns.

Find examples of shapes and symmetry in the environment and in art.

Vocabulary
Polygon, shape, line of symmetry, axis of symmetry, symmetrical

Resources needed
Mirror, paper shapes that can be folded; plastic shapes, examples of patterns that are symmetrical (wallpaper, fabric, carpets, tiles, etc.); pictures of buildings and other built elements of the environment to demonstrate symmetry properties.

Mental maths activities
Select suitable activities from the mental maths activity bank to start each lesson.

Concepts that may be unfamiliar in this chapter
It is possible to have more than one line of symmetry

Students have already worked with reflective symmetry. In this chapter they will extend their ideas and learn that shapes can have more than one axis of symmetry. This is a particularly important property of regular polygons, where the number of sides and angles is equal to the number of axes of symmetry that the shape has.
Teaching ideas

Practical activities

Show students a large symmetrical 2D shape cut out of paper, for example a circle or a rectangle. Remind students that if you fold the shape in half the fold line is called the ‘line of symmetry’. Hold a mirror over the fold line to show how the other half of the shape is reflected in the mirror. Fold the shape in a different way to show that it is symmetrical about another line. Explain that some shapes have more than one line of symmetry.

Ask students to look at shapes around the room – for example, doors, windows, floor tiles, brick patterns, ventilation grids, etc. and identify which are symmetrical and where the lines of symmetry lie. It’s useful to have a mirror to demonstrate the symmetry if students struggle to see it.

Draw three columns on the board with the headers: no line of symmetry, 1 line of symmetry and 2 or more lines of symmetry. Display various paper shapes and ask the students to say where each would fit into the table. Ask students to come up and fold the shapes to confirm their choices. Continue sorting shapes in this way, naming them and discussing their properties as you do so to revise those concepts.

Give each group a set of shapes that cannot be folded (plastic or thick card) and ask them to classify them as symmetrical or non-symmetrical. Once they have done this, ask them to sort the symmetrical shapes into those that have only 1 line of symmetry and those that have more.

Let the students do a small project to find examples of symmetry in the local environment. If they have access to digital cameras, they could record these photographically and prepare a slide show. If not, they could draw the symmetrical patterns they find and make a poster to show their designs.

Using the Student Book and Workbook

Let the students work in pairs to decide whether the shapes on Student Book page 45 are symmetrical or not. Discuss the answers with the class and let the students say how they decided.

Once you have done some practical activities with polygons, let the students work through Student Book page 46 to consolidate their understanding of the concepts.

Use Student Book page 47 and Workbook page 31 as a basis for discussing and identifying shapes and symmetry in the environment.
Assessment questions to ask
- Is this shape symmetrical? Why or why not?
- What does symmetry mean?
- What is a line of symmetry?
- Where is the line of symmetry on this shape? Is there only the one?
- What do you call this fold line if it divides the shape into two equal halves?
- Give me two examples of polygons with more than one line of symmetry?
- Draw a shape with more than one line of symmetry.

Common errors and misconceptions
Students may find it difficult to visualise what the shape will look like if folded. This means that they think parallelograms are symmetrical (because the diagonal divides the shape into two equal parts). If students make this mistake, encourage them to trace and fold shapes to check, or use a mirror to check whether the two halves fit onto each other.

8 Organising data and drawing graphs

<table>
<thead>
<tr>
<th>8: Organising data and drawing graphs</th>
<th>Student’s Book pp 48–55</th>
<th>Workbook pp 32–33</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Objectives</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4Dh1</td>
<td>Answer a question by identifying what data to collect, organising, presenting and interpreting data in tables, diagrams, tally chats, frequency tables, pictograms (symbol representing 2, 5, 10 or 20 units) and bar charts (intervals labelled in twos, fives, tens or twenties).</td>
<td></td>
</tr>
<tr>
<td>4Dh2</td>
<td>Compare the impact of representations where scales have different intervals.</td>
<td></td>
</tr>
</tbody>
</table>

Vocabulary
- Table, tally, chart, frequency, frequency table, bar chart, bar, axis, scale, pictogram, symbol, key

Resources needed
- Dice for students to use to complete Workbook page 32; graph paper as required.

Mental maths activities
- Select suitable activities from the mental maths activity bank.
Concepts that may be unfamiliar in this chapter

Scales with different intervals and the impact thereof

As students move through the stages they will be expected to work with graphs where the interval on the scale can be any number. It is important that they realise how the interval you choose can impact on the shape and message of the graph. This is important as in real life graphs may be drawn with an interval that makes them look more (or less) dramatic than they really are. The examples in the book will show the students that a bigger interval (such as twenties) for a small range of data items means that the bars are fairly short and that this makes the differences between them less obvious.

Teaching ideas

Practical activities

Obtain from the internet, a newspaper or record shop, a chart of the top ten most popular songs, CDs or albums for the current week. List them in random order on a large sheet of paper. Each student makes a tally mark against their favourite, either during the course of a day or in one session. Revise the ‘gate’ method of tallying that students used in stage 3 (4 strokes and a diagonal across to make 5). Talk about the advantages of this method when counting tallies. At the end of the tally, ask students to total the votes for each song. Discuss the fact that the number of tallies should equal the number of students who took part in the survey.

Ask them to make a chart which lists the songs in order of class popularity, with the total votes cast against each. Students may use different formats for their charts, some may list the songs horizontally – others may list them vertically. Discuss and compare the different formats. Have the students identify which are clearest and easiest to read. Introduce the term ‘frequency table’ as a table for showing the number of times different events occurred, in this case votes for different songs. Then show the students the official top ten chart and compare the order.

Make a classroom display showing different types of graph. Cut these out from newspapers or magazines. Let the students add different examples that they find at home. As each new graph is added, interpret the information with the students, even if that type of graph has not yet been formally introduced in the classroom.

Students could devise and carry out a survey of all the members of the class. They could record their data in the form of a tally chart and draw a pictogram using a suitable scale factor.
Using the Student Book and Workbook

Make sure students know how to record tallies before discussing and completing Student Book page 48 individually or as a class.

Let students work independently to complete activity 1 on Student Book page 49. Check the answers before organising students into pairs or groups to do the survey activities (2 and 3). Spend some time reporting back on these activities and discuss the methods students used to record and display their results.

Work through Student Book page 50 with the class to revise and consolidate work on bar charts. Let the students work through the activities on Student Book page 50 independently to consolidate their understanding.

Discuss the two different graphs on Student Book page 52 as a class. Make sure the students understand that both graphs show the same data and that they are both technically correct. Let the students work on their own to complete the activities on this page and on Workbook page 32.

Use Student Book page 53 to revise the concept of a pictogram. Highlight that one ice-cream cone symbol is used to represent 10 ice-creams. Discuss why the key is so important in this type of graph.

Let the students work independently to complete Student Book page 54. Discuss and compare answers as a class.

Let the students work independently to complete Student Book page 55, recording their work on the grids provided on Workbook page 33.

Assessment questions to ask

- On this bar graph, what does each bar represent?
- What does the length of the bars represent?
- What does one block/one centimetre on the bar graph show?
- Why do you think we use the four tallies with a line through to represent five?
- You will also need to ask specific, directed questions about each specific graph to make sure students can read and interpret the data.
- Show the students two graphs showing the same data, with a different scale and ask: can you explain why the bars on this graph are much longer than the bars on this graph?

Common errors and misconceptions

Students should take care when interpreting bar graphs that the height of each bar is read accurately from the frequency axis.

Students should take care when interpreting graphs with a key of one symbol representing more than one item (many-to-one correspondence). They need to be aware that one symbol can represent more than one item of data.
9 Multiplication and division facts

<table>
<thead>
<tr>
<th>Objectives</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>4Nc4</td>
<td>Know multiplication facts for 2x, 3x, 4x, 5x, 6x, 9x and 10x tables and derive division facts.</td>
</tr>
<tr>
<td>4Nc5</td>
<td>Recognise and begin to know multiples of 2, 3, 4, 5 and 10 up to the tenth multiple.</td>
</tr>
<tr>
<td>4Nc13</td>
<td>Multiply any pair of single-digit numbers together.</td>
</tr>
<tr>
<td>4Nn8</td>
<td>Recognise multiples of 5, 10 and 100 up to 1000.</td>
</tr>
<tr>
<td>4Nc25</td>
<td>Understand that multiplication and division are the inverse function of each other.</td>
</tr>
</tbody>
</table>

Vocabulary
Times, multiply, product, multiplication, tables, multiple, division, inverse

Resources needed
Counters; squared paper for grids; times table fans; times tables grids.

Mental maths activities
Choose suitable activities from the mental maths activity bank or spend the time doing different activities to help students learn their times tables. Mental activities could include using flow diagrams (number machines with operators such as × 2 or ÷ 4), or activities in which you give a number (say 20) and the students have to jot down multiplications that give 20, or a display of arrays for which the students have to give the facts.

Concepts that may be unfamiliar in this chapter
4×, 6× and 9× tables
Students should already know their times-tables facts for the 2×, 3×, 5× and 10× tables. They will now extend these to learn the 4×, 6× and 9× tables and associated division facts. They will also need to begin to use the patterns from tables in their work with multiples.

Deriving division facts from known multiplication facts
Students should remember that division is the inverse of multiplication. However, they need to extend their understanding of this concept to realise that once they know multiplication facts, they can use these to derive two related division facts.
Recognising a wider range of multiples

Previously students were only expected to recognise multiples of 2, 5 and 10. Now they will extend this to recognise the multiples of 3 and 4 as well. They only need to know up to the 10th multiple of each, so they can use their tables knowledge to derive and recognise these.

Strategies for multiplying any pair of single-digit numbers

Students need to be able to multiply any two single-digit numbers. In other words, perform any multiplication up to 9 × 9. For most of these, they can use their tables, but as they are not yet expect to know the 7× and 8× tables, they will need to develop strategies for multiplying by 7 or 8. The most useful of these is commutativity. To find 7 × 5, simply multiply 5 × 7 and use table facts to get 35. To find 8 × 9, you can use commutativity or doubling, 4 × 9 = 36, so 8 × 9 = 72. The doubling strategy would need to be used for 8 × 7, as not knowing either table prevents the students from using the commutative property. Other strategies include multiplying by factors or multiplying in parts (7 × 4 is the same as 3 × 4 + 4 × 4).

Teaching ideas

Practical activities

- Students arrange beans (counters, etc.) in sets of four, six and nine. They add up the number of beans in one set, two sets, etc. and report on their results.

- Students experiment with beads to prove to themselves that two sets of four contain the same as four sets of two. They should think of other examples like this.

- Students in pairs test each other on times tables up to 10. Students could use multiplication tables and/or times table fans to test themselves or other students on their tables.

- Arrange a ‘times table’ competition by dividing students into teams of 4 or 5 and getting them to write questions which they use to test other teams on their tables.

- Get students to make ten groups of six using counters. Ask the class how many counters are in one group of six, two groups of six, etc. Write the total down on the chalkboard each time. Ask students if they can see a pattern. Reinforce the pattern obtained by counting in sixes using a number line. Show students that, starting at zero and jumping 6 each time, they get the same pattern of numbers as when they added the sets of six together. Summarise the 6× table with a whole-class session. Repeat this for the 9× table if you want.

- Make up 16 using base-ten blocks or 16 counters. Ask whether you can divide 16 by 2. Ask the students how they know that they can divide 16 by 2. Let them work with the blocks to show how to divide the blocks into pairs. Give various larger numbers (some odd) and ask the
students if they can be divided by 2. Discuss what the class knows about multiples of 2 (they are all even, they end with 0, 2, 4, 6, or 8). Together with the class use this information to reach a rule for determining whether a number is a multiple of 2. Repeat this with other multiples.

Continue to use practical and concrete methods to reinforce multiplication and division facts alongside the Student Book and Workbook activities as you need to.

Using the Student Book and Workbook

In stage 3, the students began to learn the 4× table. Use Student Book page 56 to revise and consolidate this learning while stressing that repeated addition and subtraction can be used as methods of multiplying and subtracting. The concept of repeated subtraction is important for division and it is an idea that the students will return to as they develop their methods of dividing larger numbers.

Use Student Book page 57 alongside practical methods to revise the multiplication facts that students already know. Let the students complete the activities individually and then check their answers with a partner.

Use Student Book page 58 and Workbook page 34 to formalise and reinforce learning of the 4× table.

Spend some time working through Student Book page 59 and the activities on Workbook pages 35 and 36 to make sure students know their 6× and 9× tables and that they can multiply single-digit numbers as well as derive division facts and recognise multiples of these numbers. Use the method described in the Student Book to help the students learn their facts and encourage them to make and use the cards on an ongoing basis.

Work through Student Book page 60 with the class to make sure they know that division is the inverse of multiplication and that they can derive division facts from multiplication arrays. Let the students complete Student Book page 61 on their own. Using grids to multiply like this will help them develop their understanding of how to calculate the area of a rectangle in higher levels.

Let the students work in pairs to complete Student Book page 62.

Discuss the examples on Student Book page 63 with the class before asking the students to derive the division facts. Let them complete Workbook page 37 in pairs.

Assessment questions to ask

- What is 9 × 3? (or any other table facts)
- What is three eighths?
- Double 9.
• Is 40 a multiple of 5? How do you know?
• What is the product of 8 and 9?
• Share 36 between 4 people.
• How many lengths of 8 cm can you cut from 56 cm?
• Is 36 divisible by 8? How do you know?
• The answer to a division is 5. What could the division be?
• Starting on 21 and making three jumps of 7 along a number line, what numbers do you land on?
• How many groups of five are needed to make 40?
• What is 63 divided by 9? (and other division facts)
• Write another multiplication sentence that has the same answer as 6 × 5.
• Is 27 a multiple of 3? Is 10? (and so on)

Common errors and misconceptions
It is essential that students are familiar with their times tables. Rote learning of tables at this stage will provide students with a resource that will be useful throughout the rest of their lives. If tables are neglected at this stage students will be hindered in their progress later in their mathematics course.

10 3D shapes and nets

<table>
<thead>
<tr>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>4Gs1</td>
</tr>
<tr>
<td>4Gs4</td>
</tr>
</tbody>
</table>

Vocabulary
Vertex, corner, edge, face, cube, cuboid, prism, pyramid, cone, cylinder, tetrahedron, net

Resources needed
3D shapes; dictionaries; card; cuboid boxes; models; packets; felt-tipped pens; pencils; scissors.
Mental maths activities
Select suitable activities from the mental maths activity bank.

Concepts that may be unfamiliar in this chapter
There are no fundamentally new concepts in this chapter. Students will build on what they already know to refine and extend their understanding of 3D shapes and their properties.

Teaching ideas
Practical activities

Let the students look at and handle a variety of 3D shapes. They need to describe and identify the faces. It might help some students if they drew around the faces on paper. Discuss: whether the faces are regular (sides of equal length) or irregular (sides of unequal lengths); the number of sides and angles; the shape names; whether they are symmetrical or not, and so on.

Display a variety of cuboids. Review the properties of cuboids: six rectangular faces; right-angled vertices; opposite faces identical (congruent), and so on. Remind the students that cubes are special cuboids. Ask for examples of cubes and cuboids that we see in the environment and everyday life. Discuss the usefulness of cuboids as a building and stacking shape.

Students construct cubes and cuboids from various nets. An accurate, well-creased fold helps towards a more accurate finished product. Discuss the net, the number of faces, which edges will join and ways of making alternative nets.

Discuss with the students how they could make copies of model cubes and cuboids without measuring. They can do this by drawing around the face of the model onto a piece of card, flipping, and so on, until they have traced all six faces. Students construct the cube (gluing flaps or taping the edges) and compare it with the original.

Discuss pyramids with the class. Show pictures of the Egyptian pyramids and discuss their general features – for example, sloping faces meeting at a point, square base, four faces the same shape.

Display several examples of 3D shapes (polyhedra), including cones and pyramids with differently shaped bases. Ask the students to sort the shapes they think are pyramids and those that are not into different sets. Discuss the reasons why they think some shapes are pyramids. Remove from the sets any mis-sorted shapes. Ask the students what attributes are special to pyramids. A valuable way of doing this would be by comparing a pyramid to each of the other shapes in turn. They should understand that:
• A pyramid has a polygon base, with all the other faces triangular and meeting at a point.
• Pyramids are named after their bases – for example, square-based pyramid, hexagonal-based pyramid.
• The bases of pyramids are usually regular polygons, the most common being square or triangle.
• A regular tetrahedron is a pyramid with four equilateral triangular faces.

**Using the Student Book and Workbook**

- Work through Student Book page 64 with the class to revise the names of solids and the correct vocabulary for the parts.
- Teach the names and properties of solids using Student Book page 65. Try to show the students actual examples of each solid as well.
- Remind the students that the net is the flat ‘pattern’ that you can fold up to make a solid. Show them the net of a cuboid by slicing open a cardboard box to get its net. Let the students work on their own to complete the activity on Student Book page 66.
- Show the students a range of cubes and cuboids and discuss what the faces will look like. Then have the students complete the activities on Student Book page 67.
- Once you have done some practical work on pyramids, have the students work through Student Book page 68. Stress that a pyramid can have any polygon as a base but that the other faces will always be triangles of the same height (otherwise they won’t meet at the apex or top of the pyramid). Let the students use the net on Workbook page 38 to build their own model of a tetrahedron. If you like, you can have the students experiment with different ways of joining their tetrahedra together to make new solids.

**Assessment questions to ask**

- Show the student a solid and ask: what do you call this shape?
- Point to the parts of a solid and ask the students to say the name of each part.
- How many faces does this shape have?
- What shapes are the end faces of this solid?
- Draw me the faces of this solid.
- How could you group these shapes into sets? What are the criteria for each of your sets?
- How many faces/edges/vertices does this shape have?
- Can you describe this shape to me?
Can you draw me a net for a triangular prism/pyramid/etc.?
I am a shape with six faces. Is that enough information to make the net of the shape? What other information do you need?
How many faces does a cube have? How many faces does a cuboid have?
What is the difference between a cube and a cuboid?
How many faces does a pyramid have? What do we need to know in order to work it out?

**Common errors and misconceptions**

Some students may have difficulty with the terminology used when talking about the properties of 3D shapes. Regularly use the correct terminology with them by playing games that involve students describing shapes or listening to descriptions to identify shapes.

Students may take a while to distinguish between pyramids and prisms. Remind them that pyramids end at a point, while prisms have two flat end (parallel) faces.

**11 Negative numbers**

<table>
<thead>
<tr>
<th>11: Negative numbers</th>
<th><strong>Student’s Book</strong> pp 69–71</th>
<th><strong>Workbook</strong> pp 38</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Objectives</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4Nn13</td>
<td>Use negative numbers in context, e.g. temperature.</td>
<td></td>
</tr>
<tr>
<td>4Nn14</td>
<td>Recognise and extend number sequences formed by counting in steps of constant size, extending beyond zero when counting back.</td>
<td></td>
</tr>
</tbody>
</table>

**Vocabulary**

Negative, below zero, minus, degrees, Celsius

**Resources needed**

Number lines; thermometer; newspapers with examples of weather reports or other articles demonstrating the use of degree and/or negative numbers.

**Mental maths activities**

Select suitable activities from the mental maths activity bank or develop your own counting activities using a number line that extends below 0.
Concepts that may be unfamiliar in this chapter

The number line can be extended to show numbers below 0 (negative numbers).

Students will extend their understanding of the number system by extending beyond 0 when they are counting backwards to include negative numbers. This concept is best understood in contexts that may be familiar to the students. Temperature, particularly in areas where winter temperatures drop below freezing, is a useful context. For students who have not experienced cold temperatures, you may need to relate this to the kitchen freezer (where food is kept at low temperatures to keep it frozen). Most modern freezers have some sort of temperature display that will show negative numbers. Other contexts are depth below sea level, underground parking levels, and negative balances in the bank.

Teaching ideas

Practical activities

The easiest way for students to grasp the concept of negative numbers is by using real-life examples, such as the floors below ground level in a building. Draw a picture of a tall skyscraper building. Explain that some buildings may have basement levels underground. Now label the floors above the ground 1, 2, 3, 4, 5, and so on. Label the ground floor 0.

- Ask: ‘If the ground floor is zero, what do we call the floor just below ground?’
- Elicit or explain the answer: −1. Then ask: ‘What about the next floor down?’ (−2) and so on.

Draw a number line on the board. Model counting forwards and backwards from zero. Discuss negative numbers and how we represent them using the negative sign. Give students opportunities to come up to the board and count on or count back to given negative numbers.

Give students some inequalities to solve using positive and negative numbers. They must fill in < or > between the two numbers. Work through several examples.

Using the Student Book and Workbook

Student Book page 69 has students working with negative numbers on number lines. They can work through Workbook page 38 as additional class work.
Have a class discussion about measuring temperature. Discuss contexts where we use, record and report on temperatures. These could include: radio, TV and newspaper weather forecasts; body temperatures when people are ill; choosing holiday destinations; cooking temperatures; swimming pool temperatures. Ask the students if they know of any particular temperatures, such as the boiling and freezing points of water (100°C and 0°C) or normal body temperature (about 37°C). Discuss units of temperature, such as degrees Centigrade, Celsius and Fahrenheit. Explain that Celsius and Centigrade are different names for the same unit. Show them a thermometer and demonstrate how we read the temperature. As a class, work through some of the examples on Student Book page 70.

Then let the students complete the page individually or in pairs.

On Student Book Book page 71, students start dealing with changes in temperature. Work through the example carefully with the class. Go through the first few examples in the table as a class before letting the students complete the table individually. If necessary, work through the activities together as a class.

**Assessment questions to ask**

- What number is 1 less than zero? 2 less than zero?
- Count backwards from −1 to −10.
- Which number is less, −3 or 8?
- Which number is less, −3 or −8?, and so on.
- If the temperature is 5°C, and it drops by 10 degrees Celsius, what is the new temperature?

**Common errors and misconceptions**

You may find that students can easily continue counting once you have started a series (say, −1, −2, −3, and so on) but may need reminding that the ‘bigger’ numbers are actually ‘smaller’ when they have the minus sign in front of them. They may find the < and > signs difficult to apply as a result of this confusion. Say, for example, that students have to compare the numbers −3 and −5. Keep reminding the students of the example of the elevator going down below ground level.

You can ask:

‘Which is further down, the third floor below ground, or the fifth floor?’

−5 is under −3, so −5 < −3.
12 Fractions

Objectives

<table>
<thead>
<tr>
<th>Objective</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4Nn17</td>
<td>Order and compare two or more fractions with the same denominator (halves, quarters, thirds, fifths, eighths and tenths).</td>
</tr>
<tr>
<td>4Nn18</td>
<td>Recognise the equivalence between $\frac{1}{2}$, $\frac{4}{8}$ and $\frac{5}{10}$; $\frac{1}{4}$ and $\frac{2}{8}$; $\frac{1}{5}$ and $\frac{2}{10}$.</td>
</tr>
<tr>
<td>4Nn19</td>
<td>Use equivalence to help order fractions, e.g. $\frac{7}{10}$ and $\frac{3}{4}$.</td>
</tr>
<tr>
<td>4Nn20</td>
<td>Understand the equivalence between one-place decimals and fractions in tenths.</td>
</tr>
<tr>
<td>4Nn21</td>
<td>Understand that $\frac{1}{2}$ is equivalent to 0.5 and also to $\frac{5}{10}$.</td>
</tr>
<tr>
<td>4Nn22</td>
<td>Recognise the equivalence between the decimal fraction and vulgar fraction forms of halves, quarters and hundredths.</td>
</tr>
<tr>
<td>4Nn23</td>
<td>Recognise mixed numbers, e.g. $5\frac{3}{4}$ and order these on a number line.</td>
</tr>
<tr>
<td>4Nn24</td>
<td>Relate finding fractions to division.</td>
</tr>
<tr>
<td>4Nn25</td>
<td>Find halves, quarters, thirds, fifths, eighths and tenths of shapes and numbers.</td>
</tr>
<tr>
<td>4Nc3</td>
<td>Identify simple fractions with a total of 1, e.g. $\frac{3}{4} + \Box = 1$.</td>
</tr>
</tbody>
</table>

Vocabulary

Fraction, numerator, denominator, half, third, quarter, fifth, sixth, eighth, tenth, equivalent, mixed number.

Resources needed

Modelling clay; string; paper strips or ribbon; scissors; paste; coloured backing paper; thin card; coloured rods such as Colour Factor or Cuisenaire; coloured pencils; gummed paper squares; large number line from 0 to 3 divided into eighths; number line divided into tenths and hundredths.

Mental maths activities

Select suitable activities from the mental maths activity bank.
Concepts that may be unfamiliar in this chapter

The concepts in this chapter should be familiar to students as they have worked with vulgar fractions and mixed number previously and they have begun to understand that decimal and vulgar fractions are equivalent (Chapter 4). This year they will work with a larger range of fractions (different denominators) and use knowledge of equivalence to compare and order fractions with and without a number line.

Teaching ideas

Practical activities

Ask for an example of a shape that has been cut into two equal pieces. Talk about two halves. Emphasise the fact that the two halves are equal in size. Write the word ‘half’. Also write the symbolic representation $\frac{1}{2}$.

Introduce the terms ‘numerator’ and ‘denominator’. Discuss the fact that with numerator 1, the larger the denominator, the smaller the fraction. Show this practically by dividing shapes. When the students are thoroughly familiar with unit fractions (numerator 1) and their notation, use the same shapes to introduce non-unit fractions, such as $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{4}{5}$ and their notation.

Students could put their fraction shapes on display, together with the symbols. For instance, each of the five fifths of the shape could be labelled $\frac{1}{5}$ or they could be grouped into two pieces and three pieces labelled $\frac{2}{5}$ and $\frac{3}{5}$ respectively. Use the displays to confirm that $\frac{2}{5}$ is smaller than $\frac{3}{5}$. (And repeat for different denominators.)

Provide students with equal lengths of string, paper strips or ribbon (say, 30 cm each). They fold and cut the string, ribbon or strips to show halves, thirds, quarters, fifths, sixths, and so on. On backing paper, they mount a whole length labelled ‘1, one whole’, and underneath mount each fraction length appropriately labelled in words and symbols.

Give the students specific non-unit fractions of the lengths, such as $\frac{2}{3}$ and $\frac{4}{5}$ to fold, cut, mount and label (together with a whole length) in a similar way. Discuss which fraction is longest and which is shortest.

Have students cut out and paste circles on different coloured backing paper or thin card. They write the fraction name and symbol on each segment of each circle and then cut out the circles and segments. These can be used to show specific fractions. The fractional pieces can be put in a small plastic bag for use whenever students are working with fractions.

Use the fraction circle models that students made in earlier activities. Ask: ‘How many halves in one circle?’ ‘How many halves in two
"circles?" Establish that there are four halves in two circles. Write on the board: 2 wholes = \( \frac{4}{2} \)

Repeat this for other denominators.

Ask the students to show one-and-a-half circles and ask them how many halves that is. The students should say ‘three halves’. Explain that this is also 1 whole and 1 half. Remind them that this can be written as \( 1\frac{1}{2} \), which is called a ‘mixed number’ because it contains a mixture of whole numbers and fractions. Repeat the procedure for other mixed numbers such as \( \frac{11}{3} \), \( 2\frac{1}{4} \) and so on. Students can make their own number lines marked in eighths to show mixed fractions.

Write down two mixed numbers between 1 and 2 on the board or a sheet of paper – for example \( 1\frac{3}{5} \) or \( 1\frac{1}{4} \). Ask the students which one they think is greater. Encourage them to consider such things as whether \( \frac{1}{4} \) is greater than \( \frac{1}{5} \), how many fifths there are in \( 1\frac{3}{5} \), and how many quarters make \( 1\frac{1}{4} \). Ask the students to position each number on the number line to see whether their estimations were correct.

**Using the Student Book and Workbook**

Once you have done some introductory practical activities, work through Student Book page 72 with the class, discussing each activity as the students complete it. Then let the students work independently to complete Student Book page 73 and Workbook page 40. Check that they are able to do this before you move on.

Allow students to model the cube on Student Book page 74 if they need to. Let them work in pairs to discuss and then complete the activities on the page. Use Workbook page 41 to see whether the students can find fractions of an amount.

Work through Student Book page 75 with the class to introduce ordering and comparing fractions with the same denominators as well as equivalent fractions. Once the students are confident in this, move onto Student Book page 76 and Workbook page 42.

Use Student Book page 77 to check that students are able to interpret and work with the table of equivalent fractions and that they are able to order and compare fractions. Workbook page 43 requires them to apply these skills to represent comparisons visually and to complete number lines to show equivalence. Use Workbook page 44 to consolidate the concepts taught thus far before moving on to fractions and decimals.

Remind the students that they have already worked with decimal fractions and ask them what they remember. Work through Student Book page 78 with the class and then have the students complete Workbook page 45 as consolidation.
Work through some practical activities related to mixed numbers and then work through the example on Student Book page 79 with the class. Let the students work on their own to complete the activities. Then have them work through Workbook page 46 as an informal assessment task.

Let the students spend some time talking about how they would solve the problems on Student Book page 80 before asking them to solve them. Discuss the answers and the strategies used as a class.

Assessment questions to ask

- How many equal parts have you divided this shape into?
- What do we call each part?
- What does the denominator (number under the line in a fraction) tell us?
- What does the numerator (number above the line in a fraction) tell us?
- How many halves make one-and-a-half? Two? Three-and-a-half?
- Give me a fraction that is equivalent to $\frac{3}{5}$ but has a denominator of 10. How did you do it?

Common errors and misconceptions

Students frequently struggle with fractions. Many of these difficulties seem to arise from an over-reliance on the ‘part of a whole’ model and that this is always presented through shaded diagrams. It is important for students to see fractions as numbers that can be ordered on a number line and compared. They also need to be aware that the ‘whole’ can be a quantity and thus, for example, 12 out of 24 can be expressed as a fraction.

13 Position and directions

<table>
<thead>
<tr>
<th>13: Position and directions</th>
<th>Student’s Book pp 81–83</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Workbook pp 46–50</td>
</tr>
<tr>
<td><strong>Objectives</strong></td>
<td></td>
</tr>
<tr>
<td>4Gp1 4Gp3</td>
<td></td>
</tr>
<tr>
<td>Describe and identify the position of a square on a grid of squares where rows and columns are numbered and/or lettered.</td>
<td></td>
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<tr>
<td>Devise the directions to give to follow a given path.</td>
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</tr>
</tbody>
</table>

Vocabulary

Grid, position, coordinates, ordered pair, direction, north, south, east, west, directions
Resources

Flashcards showing North, South, West and East; a large compass on a chart with the four main directions labelled.

Mental maths activities

Choose some calculation activities from the mental maths activity bank as the students will not be doing any calculation work in this chapter.

Concepts that may be unfamiliar in this chapter

Grids using numbers only

In this chapter the students are introduced to the idea that a grid can be labelled with numbers only (rather than with letters on one axis and numbers on the other). This is important for later work using coordinates on a labelled set of axes. In algebra at the higher stages, the values are always given in the order \((x, y)\), so it is important that the students learn to give the position of an object using the horizontal value first and the vertical value second. You may want to tell the students that they can remember this by thinking – ‘I have to walk across the ground before I can climb the tree.’ Or, that A for across comes first and U for up comes last. Note that the labels at this stage are inbetween the lines on the grid as the students only have to locate a specific square; they are not finding the intersection of the lines at this stage.

Teaching ideas

Practical activities

If possible, label the rows and columns of shelves or lockers in the classroom using letters and numbers and have the students identify and point out various locations using the letters and numbers you have labelled.

Show the class a game board with coloured squares (chess or draughts) and place a counter on one square of the board. Let the students suggest how they could give the position of the square that the counter is on. This should lead into numbering the squares.

Draw a grid on the ground outside. You can either use chalk or make the lines out of string. Number the rows and columns. Put objects into some of the blocks and have the students take turns to call out instructions to each other saying how many blocks forward, backward, right or left they need to move the objects in given positions. (You will need to give them the ordered pair to identify each position.)

Teach the main compass directions. If possible, show the students a compass and let them see how it works.

Play a simple direction game. Write the letters A – I in a 3 × 3 grid and show North as up using an arrow:
Ask the students questions using directions. For example: What is the direction from G to H, C to I? and so on. In which direction do I move to get from H to E? E to H? and so on.

Label the four walls of the classroom North, South, East and West. It would be good if you could align these with real directions. Place the desks in rows with space between them. Let the students take turns to direct each other between the desks to a place they have marked. They should give direction and distance; for example, ‘Turn to the East and walk past three desks.’

**Using the Student Book and Workbook**

- Revise using letters and numbers to find position using Student Book page 81. Then show the students how this translates into numbered grids using the example. Stress that the horizontal value is always given first. Let the students work in pairs to complete the activities here and on Workbook pages 47 and 48.

- Teach the compass directions and then work through the example and first activity on Student Book page 82 with the students. Let them work in pairs to complete activity 2 and to draw and find the routes on Workbook page 49.

- Discuss the example on Student Book page 83 with the students and then let them work in pairs to complete the activities here and on Workbook pages 50 and 51. Let them check each other’s answers.

**Assessment questions to ask**

- Where is the . . . ?
- What is in block A4? and so on
- Show me where point \((x, y)\) is on your grid.
- What is at \((2, 3)\) etc.?
- Which direction is opposite (give direction)?
- Where will you end up if you move four blocks to the East?
- Can I move four blocks North from here?

**Common errors and misconceptions**

Many students will get the numbers mixed up when they work with numbered pairs (coordinates). Remind them that the horizontal number is given first by stressing that you have to walk across the ground to get to a tree or ladder before you can climb it. So we give the across numbers first and then up and down ones. This is a very important concept to get right at this level as all later work in algebraic graphs will depend on students knowing this rule (\(x\)-coordinate before \(y\)-coordinate).
Students (and indeed many adults) struggle to remember the relative position of the cardinal points. Encourage them to draw compass arrows and fill in the letters. They can use a mnemonic such as Naughty Elephants Squirt Water to remember the directions clockwise from the top of the compass.

Similarly, point out that West and East always spell WE on the compass.

## 14 Additional and subtraction strategies

<table>
<thead>
<tr>
<th>Objectives</th>
<th>Student’s Book pp 84–91</th>
<th>Workbook pp 52–55</th>
</tr>
</thead>
<tbody>
<tr>
<td>4Nc8</td>
<td>Add and subtract near multiples of 10 or 100 to or from three-digit numbers, e.g. 367 – 198 or 278 + 49.</td>
<td></td>
</tr>
<tr>
<td>4Nc9</td>
<td>Add any pair of two-digit numbers, choosing an appropriate strategy.</td>
<td></td>
</tr>
<tr>
<td>4Nc10</td>
<td>Subtract any pair of two-digit numbers, choosing an appropriate strategy.</td>
<td></td>
</tr>
<tr>
<td>4Nc11</td>
<td>Find a difference between near multiples of 100, e.g. 304 – 296.</td>
<td></td>
</tr>
<tr>
<td>4Nc12</td>
<td>Subtract a small number crossing 100, e.g. 304 – 8.</td>
<td></td>
</tr>
<tr>
<td>4Nc17</td>
<td>Add pairs of three-digit numbers.</td>
<td></td>
</tr>
<tr>
<td>4Nc18</td>
<td>Subtract a two-digit number from a three-digit number.</td>
<td></td>
</tr>
<tr>
<td>4Nc19</td>
<td>Subtract pairs of three-digit numbers.</td>
<td></td>
</tr>
</tbody>
</table>

### Vocabulary
Add, subtract, multiple, near multiple, difference, sum, total, partition, round, estimate

### Resources
Large number lines; place value cards as needed.

### Mental maths activities
Choose some suitable activities from the mental maths activity bank. You may want to do some more work on skip counting as this will help the students in their calculations.
**Concepts that may be unfamiliar in this chapter**

**Counting on in different-sized and progressively bigger chunks**

This concept is not strictly new to the students, but the aim is to develop more efficient methods of counting on by ‘chunking’ to reduce the number of ‘jumps’ and apply the mental strategies students have already mastered (such as counting in hundreds, using multiples of ten, and so on).

**Counting on or back to add and subtract**

Students have previously focused on counting back to subtract. As they develop their understanding of the operation they should begin to realise that they can also count on to find the difference between two numbers. The actual numbers involved and the way in which the problem is expressed will generally guide which method they choose.

**Teaching ideas**

**Practical activities**

You can use any of the practical adding and subtracting activities described in Chapter 6 as you work through this chapter.

**Using the Student Book and Workbook**

Work slowly and carefully through the different addition strategies on Student Book page 84 with the class, allowing time to discuss each one, suggest when you would use it and allowing students to say whether they find it easy to use or not (and why). If the students seem to need support, discuss what strategy you would use for each addition before asking them to find the answers. Let the students work on their own to find the target numbers on Workbook page 52 and give them a chance to share their strategies.

Spend some time revising rounding off to the nearest 10 before working through the examples on Student Book page 85 to show how estimating can be used to check answers. Let the students estimate and then solve the problems on their own.

Work through the different options on Student Book page 86 with the class, talking about when you would count on, and when you would count back. Let the students suggest calculations and model how you would solve them. Let the students work on their own to solve the subtractions on the page. Let them work through the number machine diagrams on Workbook page 53 to consolidate this work.

Discuss and explain each strategy shown on Student Book page 87 with the class. Again, encourage the students to say when they might choose each one and which methods they prefer. Let them work on their own to complete the activities.

Spend some time demonstrating and discussing the written strategies for adding and subtracting on Student Book page 88. Bear in mind that the students should not be forced to move to a vertical method...
until they are confident and ready to do so. Let them choose the method they prefer to do the calculations and spend some time discussing how different students worked once they have finished. Use the puzzles on Workbook page 54 as additional practice, encouraging students to do their working in their books as they need to.

Use Student Book page 89 as a fun, investigation task to consolidate methods and to give you a chance to observe students to see what method they are using.

Let the students work in pairs to complete the coded subtractions on Student Book page 90.

Student Book page 91 involves applying the learnt strategies to solve worded problems. Let the students work in pairs to discuss and solve the problems. Use Workbook page 55 as a consolidation activity.

**Assessment questions to ask**

- What is the difference between 366 and 178? 941 and 199? 253 and 187?
- How do you decide whether to use rounding off, or counting on or back?
- What is the total of 472 and 129? 588 and 259? 376 and 269?
- Find the sum of (two numbers).
- My total is 24; what two three-digit numbers could I have added to get this?
- Show me how you would add/subtract these two numbers.

**Common errors and misconceptions**

Sometimes students struggle to realise they need to subtract the smaller number from the larger number when problems are expressed in ways other than 234 – 156. This can normally be addressed by modelling solutions using base-ten blocks or talking through the calculation.

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**15 Perimeter and area**

<table>
<thead>
<tr>
<th>15: Perimeter and area</th>
<th><strong>Student’s Book</strong> pp 92–94</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Objectives</strong></td>
<td><strong>Workbook</strong> pp 56–59</td>
</tr>
</tbody>
</table>

| 4Ma1 | Draw rectangles, and measure and calculate their perimeters. |
| 4Ma2 | Understand that area is measured in square units, e.g. cm². |
| 4Ma3 | Find the area of rectilinear shapes drawn on a square grid by counting squares. |
Vocabulary
Length, width, breadth, side, perimeter, rectangle, area, square centimetre, grid

Resources needed
Plastic shapes; squared paper; rulers.

Mental maths activities
Select suitable activities from the mental maths activity bank.

Concepts that may be unfamiliar in this chapter
Perimeter
The concept of perimeter is introduced at this stage using very simple shapes (rectangles). Students have previously measured lengths and worked with centimetre and millimetre measurements, so they tend to find perimeter work fairly simple as long as they understand that they have to measure the entire distance around a shape. You can extend this concept to include measuring items in the classroom to find their perimeter.

Area and square units
The word area is used in many different ways in real life, so it’s important to spend some time making sure that the students understand that the area of a flat shape is the amount of space it takes up or covers. Tiling flat surfaces with square tiles can help the students to work out, for example, the area of their book, or the area of a floor tile in the classroom. You can also use sticky notes to demonstrate the amount of ‘square sticker space’ a shape takes up. The concept of square units is also new to the students and it needs to be dealt with in a practical way as the students do not know about square numbers or formulae for calculating the area of a square. Show them a centimetre square and explain that it has all its sides 1 centimetre long, so we call it a centimetre square (you may want to teach the students that whatever side you measure you get a centimetre, so it is a centimetre square). Make sure they realise that the grids you use are centimetre square grids and teach them the notation cm$^2$ as a shorthand way of writing the units without going into detail about powers. We have focused on square centimetres at this stage as it is easy to use grids of this size. You may want to discuss with students what a square metre would look like and perhaps draw one of these on the school grounds.

Teaching ideas
Practical activities
Display a large shape or sheet of paper. Use straws or craft sticks and ask the students how many they think will fit around the edge of the shape. Let them estimate first and then place the sticks around the outside of the shape. Explain that the distance around a shape is called the perimeter of the shape. Write perimeter = [ ] straws next to the shape. Stress that we have measured the perimeter in units called straws, but that we can also use standard units of length to measure
perimeter. Display a variety of shapes and use a tape measure or metre rule to measure their perimeter in centimetres. Write the lengths next to each side as you measure to demonstrate that you can find the perimeter by adding the lengths of the sides.

Let the students work in pairs to measure and record the perimeter of various flat shapes. You can give the students plastic shapes for this, or you can select items from the classroom, for example, a sheet of paper, a floor tile, the surface of your desk, the perimeter of a ruler and so on.

Arrange the class into groups. Hand out a number of small shapes to each group. (Try to use plastic or cardboard shapes but if this is not possible, prepare a sheet with a number of shapes on it to distribute to the class.) Let the students draw around each shape to show its perimeter on paper. Then have them measure the sides and calculate the perimeter.

Prepare some tiling activities for the students using whatever resources you have available for tiling (postage stamps, square cards, postcards, envelopes, sheets of paper, plastic shapes, etc.). Ask the students to cover the surface of a book cover, or their desk, and to say how many ‘units’ they needed to cover the area. Explain that finding out how many units we need to cover something is called finding its area.

Give the students each a sheet of centimetre squared paper (size doesn’t matter as long as they all have the same size sheet). Explain that each small square has sides of 1 cm and that they are called square centimetres. Show the students how to write \( \text{cm}^2 \) in shorthand. Work out how many square centimetres are on each sheet by counting (although some students may multiply to find the answer). Use the sheets to work out how many square centimetres it will take to cover a desk. Record the area of a desk in square centimetres on the board.

Let the students draw a number of shapes that have straight sides and only right-angles on their squared paper. Then have them exchange these and count the squares to work out the area occupied by each shape.

Hand out a clean sheet of cm-squared paper. Ask students to draw as many different shapes as possible all with an area of say 12 cm\(^2\).

Using the Student Book and Workbook

Once you have taught the concept of perimeter, consolidate the concepts by working through Student Book page 92 with the students. Once you are sure they understand the concept, have them work independently or in pairs to complete the measuring and drawing activities on Workbook pages 56 and 57.
Once you have taught the concept of area and students know what a square centimetre is, work through the example on Student Book page 93 to consolidate the concepts. Let the students complete the activity orally as a class. Once you are sure they can find the area of a shape by counting squares, ask them to complete Workbook page 58.

Let the students work independently to complete Student Book page 94 and Workbook page 59. Check their work to make sure they can find the area of any shape.

Assessment questions to ask

- What does perimeter mean?
- What units do we use to measure the perimeter of small shapes?
- What is the perimeter of this shape?
- Can you arrange these shapes in order from the smallest to largest perimeter?
- What is the area of this shape?
- Can two different shapes have the same area? Explain how.
- Arrange these shapes in order from greatest to smallest area.
- What is a square centimetre? How is it different from a centimetre?

Common errors and misconceptions

Some students may become confused and mix up the terms perimeter and area. If you give them lots of practical activities, they will normally realise that perimeter is a measure of length and area is a covering.

Students may forget to use square units for area or leave out the units altogether. Remind them that we have to say what the area is covered or filled by (in this case, square centimetres).

16 Number patterns

<table>
<thead>
<tr>
<th>16: Number patterns</th>
<th>Student’s Book pp 95–98</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Objectives</strong></td>
<td>Workbook pp 60–62</td>
</tr>
<tr>
<td>4Nn8</td>
<td>Recognise multiples of 5, 10 and 100 up to 1000.</td>
</tr>
<tr>
<td>4Nn14</td>
<td>Recognise and extend number sequences formed by counting steps of constant size, extending beyond zero when counting back.</td>
</tr>
<tr>
<td>4Nn15</td>
<td>Recognise odd and even numbers.</td>
</tr>
<tr>
<td>4Nn16</td>
<td>Make general statements about the sums and differences of odd and even numbers.</td>
</tr>
</tbody>
</table>
Vocabulary
Odd, even, multiple, sequence, sum, difference

Resources
No specific resources are needed, although you may want to prepare some counting grids or re-use the ones previously used.

Mental maths activities
Select suitable counting and/or calculating activities from the mental maths activity bank.

Concepts that may be unfamiliar in this chapter
Odd and even numbers and the patterns related to calculating with them
Students should already be able to recognise odd and even numbers (they did this in stage 2). In this chapter, they will begin to recognise what happens in general terms when you add or subtract odd and even numbers. In other words, they will learn that odd + odd gives even and even + even gives even. This is an important concept that will be revisited in algebra later on.

Teaching ideas
Practical activities
You can repeat some of the skip counting activities introduced in Chapter 6 as you work through this chapter, but the activities in the Student Book and Workbook have been developed to involve the students in practical counting and problem solving activities.

For odd and even numbers, you can play a game with dice. Stick stickers on the dice to mark the faces odd or even. Let the students play in threes to roll their dice and say what the result would be from adding. Repeat for subtracting.

Using the Student Book and Workbook
Revise the concept of odd and even numbers and spend some time identifying these before asking the students to work in groups to model the situations in Activity 2 on Student Book page 95. Spend some time discussing the patterns and why they work as a class. Then, let the students work in pairs to complete Workbook page 60 to consolidate their understanding.

Let the students work in pairs to complete the activities and investigations on Student Book page 96. Allow some time for discussion and descriptions of the patterns. Let the students complete Workbook page 61 and check their answers to make sure they can identify and describe number patterns formed in different ways.

Let the students work on their own to complete the patterning activities on Student Book page 97 but discuss their results as a class.
Revise the concept of a multiple by working through the example on Student Book page 98. Let the students work in groups to discuss the patterns and questions on the page. Use Workbook page 62 as an informal assessment task to check that the students can recognise multiples of 5, 10 and 100.

**Assessment questions to ask**
- Is this number odd or even? How can you tell?
- Solomon added an odd number to an odd number and got an odd number as an answer. Did he calculate correctly?
- Will the answer to this calculation be odd or even? (show students a calculation)
- What rule was used to make this pattern?
- Is there any other way of carrying on this pattern?
- What comes next in this pattern?
- What is a multiple?
- What can you say about all multiples of 5?
- What can you say about all multiples of 10?
- If a number is a multiple of 100, why is also a multiple of 5 and 10?

**Common errors and misconceptions**
Some students may struggle to see why a multiple of 10 is also a multiple of 5, and why a multiple of 100 is also a multiple of 5 and 10. It may help to have them mark the multiples on a grid in a systematic way. For example, count in 5s, colour all the multiples of 5. Now count in tens, colour all the multiples of ten. Look at your pattern. What happened? By discussing the overlap, students may be more able to internalise and make sense of it.

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**17 Angles**

<table>
<thead>
<tr>
<th>17: Angles</th>
<th><strong>Student’s Book</strong> pp 99–100</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Objectives</strong></td>
<td>Workbook pp 63</td>
</tr>
<tr>
<td>4Gp2</td>
<td>Know that angles are measured in degrees and that one whole turn is 360° or four right angles; compare and order angles less than 180°.</td>
</tr>
</tbody>
</table>

**Vocabulary**
Angle, right angle, turn, degree
Resources needed

Scissors; cardboard strips and split pins for making an angle measuring device.

Mental maths activities

Select suitable activities from the mental maths activity bank.

Concepts that may be unfamiliar in this chapter

Degrees as a unit of measurement for angles

Students need to learn that the size of an angle is related to the amount of turn between the arms. (And realise that the length of the arms has no impact on the size of an angle.) Clock faces are useful for demonstrating this. You may want to show the class a protractor and tell them that it is used to measure the size of angles in degrees, but the students do not need to be able to use one at this stage. Bear in mind that students may already have come across degrees Celsius as a unit of measurement for temperature. This is a different unit altogether and you may need to make this clear to them.

Teaching activities

Practical activities

Hold up a book, and show the square corner. Make a right angle with your fingers, and explain that a square corner is also known as a right angle. You can also demonstrate this with two strips of card joined with a split pin. Have the students look around the room and identify right angles. Use the same techniques (fingers, strips of card) to demonstrate angles smaller than a right angle and angles greater than a right angle.

Remind the students that one right angle is 90 degrees. Let them fold a piece of paper to make a square corner (90 degree angle). Let the students pair up to place their right angles next to each other (effectively making a half turn, or 180 degree angle). Then, have each pair team up with another pair to join their angles. Explain that 4 right angles makes a full turn, or 360 degrees.

Hand out flashcards showing different angles. Ask students to arrange themselves in a line showing the angles in order of size. Encourage students to talk about how they compared the angles.

Hold a flashcard up showing an angle. Ask students to draw a smaller/larger angle.

Using the Student Book and Workbook

Use the diagrams and teaching text on Student Book page 99 to reinforce the concept of a right angle and to show that four right angles make a complete turn of 360 degrees. Let the students identify the right angles on the clock faces.
If possible, let each student make their own tool for measuring angles following the instructions on Student Book page 100. Show them how to use the tools and revise the work done in stage 3 by getting students to measure some angles around the classroom to see which are smaller than a right angle, and which are greater. Let the students use their measuring tools to complete the activity on this page and the comparison activities on Workbook pages 63 and 64.

**Assessment questions to ask**
- Can you estimate the size of this angle? (using smaller and/or larger than 90)
- Make a right angle using your fingers/arms.
- What important tips would you give someone to help them use their measuring tool to compare angles?
- Can you label each of these angles as smaller than 90, greater than 90 and equal to 90?

**Common errors and misconceptions**
Students may get confused by the distinction between measuring length and measuring angles. Keep demonstrating angles using real-life examples: the hands of the clock; the covers of a book (when we open a book and view it side on); when we open our arms out straight to varying angles, and so on. Emphasise that an angle is only created when two lines meet at a point – the angle is the amount of turn from one line towards the other.

### 18 Multiplication

<table>
<thead>
<tr>
<th>Objectives</th>
</tr>
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<tbody>
<tr>
<td>4Nn7</td>
</tr>
<tr>
<td>4Nc14</td>
</tr>
<tr>
<td>4Nc15</td>
</tr>
<tr>
<td>4Nc16</td>
</tr>
<tr>
<td>4Nc20</td>
</tr>
<tr>
<td>4Nc21</td>
</tr>
<tr>
<td>4Nc22</td>
</tr>
</tbody>
</table>
**Vocabulary**
Multiply, divide, product, double, halve

**Resources needed**
Small cards and coloured pens for learning and showing different facts.

**Mental maths activities**
Select activities from the mental maths activities bank or use the time to revise and consolidate times table facts and learning.

**Concepts that may be unfamiliar in this chapter**
There are no fundamentally new concepts in this chapter. Students will build on known facts and apply the skills they already have to multiply and divide larger numbers. Some students will be ready to move on to more efficient compact methods of recording their work, but others may not be ready to do so yet and may need ongoing support using apparatus and strategies that they feel comfortable with.

**Teaching ideas**

**Practical activities**

- Re-use any of the practical activities from Chapter 9 as you work through this chapter.

- Revise multiplication by 10 using place value charts. Record results and make sure the students understand that to multiply by 10 we move the digits one place to the left. Next, ask what happens when you multiply by 20? Discuss the results. Explain that this is the equivalent of multiplying by $2 \times 10$. Do some examples with the class to establish the fact that to multiply by 20 you can multiply by 2 and then by 10 to get the same result. Repeat with some other multiples of 10.

- Remind students that division is the inverse of multiplication and repeat the activities you did before for division to establish the inverse rules.

- Set a problem that leads to multiplication by multiples of 10. For example, each student has 4 pencils. How many pencils would 30 students have? You can use cards or sticky notes to model this for the class:

  ![Pencil Model](image-url)
This shows that 30 is the same as \(3 \times 10\) and leads to the solution \(4 \times 3 = 12\) and \(12 \times 10 = 120\). Having found that solution, you might like to ask the class to suggest how they could find the number of pencils held by 60 students (double the answer) and 15 students (halve the answer).

Revise doubling and halving of small numbers using piles of counter or seeds. Then show the students some examples in which you halve larger numbers by partitioning them. Stick to even numbers at this stage. For example to halve 2800 you would write it as \(2000 + 800\), halve each amount to get \(1000 + 400\) and add the results to get 1400. Repeat for doubling.

**Using the Student Book and Workbook**

- Use Student Book page 101 and Workbook page 65 as an interactive and fun way of revising multiplication facts for the known times tables.

- Complete the number pattern work on Workbook page 66 to revise multiplication by 10 and its effect before you work through the examples and demonstrate how to multiply by multiples of 10 on Student Book page 102. Let the students complete the activities on their own.

- Use place value cards and tables to revise and demonstrate multiplication by 10 using the examples on Student Book page 103 and other numbers as you feel is necessary. Let the students complete the activities mentally as far as possible.

- Work through the examples multiplying by 100 on Student Book page 104. It is important to use the language 12-hundreds so that students realise 1200 is the same as 12 hundreds. Reinforce the idea using place-value cards and tables if you need to. Let the students try as far as possible to complete the activities mentally. Use the conversion activities on Workbook page 67 to consolidate multiplying by 100 in a known context.

- Once you have done some practical activities to revise doubling and halving, work through the example on Student Book page 105 carefully with the class. Encourage the use of diagrams (jottings) like the ones on this page as these can be generalised and used to find related halves as well. Let the students complete the activities on their own. Use Workbook page 68 as additional practice working up to larger numbers.

- Work through the halving examples on Student Book page 106 with the class. Continue to stress that halving is inverse of doubling. Let the students complete the activities and discuss their answers and strategies as a class before asking them to complete Workbook page 69.
Work through the example on Student Book page 107 with the class. Allow some time for discussion of alternative strategies, for example, some students may work out 15 × 9 as 15 × 10 – 15 (compensation), others may use known facts and other paths. Stress that students can use the methods they find easiest and remind them to look at the numbers to choose the best method. Let them work on their own or in pairs to complete the activities. Check them and discuss methods used before moving on. Use Workbook pages 70 and 71 as additional practice in multiplying a two-digit by a one-digit number.

Student Book page 108 is a problem solving page. Allow some time for reading and discussing what is required in the problems before asking the students to solve them.

**Assessment questions to ask**

- What is [] multiplied by 10?
- What do I get if I divide [] by 10?
- I divided a number by 10 and got an answer of []. What was the original number?
- What is one tenth of []?
- Can you tell me an easy way of multiplying by 50?
- Is multiplying by 3 and then multiplying by 10 the same as multiplying by 30? Why or why not?

**Common errors and misconceptions**

Some students may still struggle to recall multiplication and division facts, particularly for the times tables that they have only learnt this year. Allow the students to use table grids as a memory aid and reference if necessary whilst practising recall of facts on an on-going basis.

The rules for multiplying and dividing by 10 and multiples of 10 may confuse students who struggle with their understanding of place value. Use manipulatives such as place-value cards and tables and base-ten blocks to model calculations.

### 19 Using tables and diagrams to sort data

<table>
<thead>
<tr>
<th>19: Using tables and diagrams to sort data</th>
<th>Student’s Book pp 109–117</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Objectives</strong></td>
<td>Workbook pp 72–73</td>
</tr>
<tr>
<td>4Dh3</td>
<td>Use Venn diagrams or Carroll diagrams to sort data and objects using two or three criteria</td>
</tr>
</tbody>
</table>
Vocabulary
Sort, organise, Venn diagram, overlap, intersect, shared, Carroll diagram, two-way table, rules

Resources needed
Large sheets of paper; chalk; rope; logic boxes for sorting; objects to sort (such as cut outs of familiar 2D shapes in different colours or blocks).

Mental maths activities
Choose any suitable activities from the Mental warm-up activity bank.

Concepts that may be unfamiliar in this chapter
Venn diagrams with three groups
Students will have worked with Venn diagrams (and Carroll diagrams) in Stage 3. Now they will build on this to sort data using three criteria. Overlapping two sets or circles produced only one group of objects or data that was common to both sets. When you overlap three circles, you get three areas where the objects or data are common to two sets and a central section where the objects or data are common to all three sets. This can be confusing for students, so spend some time using concrete objects such as plastic shapes in different colours, or plastic numbers to build Venn diagrams themselves. Using the apparatus makes it easier for them to move items around if they are placed in the wrong groups. Venn diagrams are used at higher levels in set theory and also to show outcomes and answer questions about probability. A basic understanding of how they work can help students when they reach these stages.

Teaching ideas
Practical activities

Set up two large intersecting circles using chalk, hula hoops or rope to make the circles. Explain that you have some shapes and you are going to make two sets. Sort the shapes into the circles or intersection using particular characteristics. For example, red or small, or right-angled and red, etc. (you will need to identify characteristics that suit your resources). Let the students work out what criteria you are using to sort them, and discuss which shapes should go into the overlapping or intersecting parts of the circles (both red and small for example). Let them draw their own diagrams to represent the sorting.

Make a Venn diagram with three overlapping circles. Write criteria on each circle, for example: long hair, curly hair, black hair. Discuss what will go into each overlap stressing that the central overlap has to be long, curly, black hair. Let students decide where they will fit onto the chart. Another fun example of this is to choose film stars or singers and to make some categories that could overlap; wearing a hat, wearing glasses, wearing a necklace. The students can then
find photographs of film stars and decide where they go in this Venn diagram. Anyone in the central section would need to be wearing a hat, glasses and a necklace.

Work in the hall or outside. Prepare four pieces of paper. Take six long pieces of string of equal lengths and use the string to create a floor chart, with one of the four pieces of paper in each category. Explain that everyone must get into the correct box that describes them. Give the students a few moments to fit themselves into the correct category. Then, at random, point to individual students and ask:
- ‘Are you a girl?’
- ‘Do you wear glasses?’
- ‘Are you in the correct box?’

<table>
<thead>
<tr>
<th></th>
<th>Girl</th>
<th>Not girl</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wears glasses</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Does not wear glasses</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Allow them to answer – even if the answer is obvious; students will find it entertaining to be asked such obvious questions. Check with the class whether each individual is in the correct box. Back in the classroom, draw the diagram on the board, and write names of students in the relevant boxes. Explain that this is called a Carroll diagram, after the man who invented this kind of diagram. (They may be interested to know that this is Lewis Carroll, the author of *Alice in Wonderland.*

**Using the Student Book and Workbook**

- Revise Venn diagrams with two categories by working the examples on Student Book page 109 with the class. Let the students work in pairs to complete the activity. Use Workbook page 72 as additional practice.

- Let the students work independently to complete the activities on Student Book page 110.

- Let the students work in pairs to complete the Carroll diagram activities on Student Book page 111 and Workbook page 73.

- Once you have done some teaching and practical work sorting by three categories, work through the example on Student Book page 112 with the class. Ask questions to make sure they understand how these more complex Venn diagrams work. Let the students discuss and work through the activities on their own.

- Let the students work on their own to complete the Carroll diagram activities on Student Book page 114. Use Student Book page 115 and
116 to revise work on tables and sorting data. Let the students show and share their databases.

Use Student Book page 117 to informally assess understanding of Venn diagrams and Carroll diagrams.

**Assessment questions to ask**
- What do we call this diagram?
- Which shapes should go here?
- Why is this shape here?
- Which shapes go into this circle, but not into this one?
- Is this shape right-angled or not-right angled? Where should it go on the Carroll diagram?
And so on, related to diagrams.

**Common errors and misconceptions**
Carroll diagrams always split items into an either/or format. In other words, the criteria usually take the form ‘x’ or ‘not x’ – for example, has blue eyes/does not have blue eyes. When developing these kind of diagrams, focus on ‘x’ or ‘not x’ criteria in order to avoid the awkward situation where some elements seem to fall between categories.

### 20 Division

<table>
<thead>
<tr>
<th><strong>Objectives</strong></th>
<th><strong>Student’s Book</strong> pp 118–123</th>
<th><strong>Workbook</strong> pp 74–77</th>
</tr>
</thead>
<tbody>
<tr>
<td>4Nc23</td>
<td>Divide two-digit numbers by single-digit numbers (answers no greater than 20).</td>
<td></td>
</tr>
<tr>
<td>4Nc24</td>
<td>Decide whether to round up or down after division to give an answer to a problem.</td>
<td></td>
</tr>
<tr>
<td>4Nc25</td>
<td>Understand that multiplication and division are the inverse function of each other.</td>
<td></td>
</tr>
</tbody>
</table>

**Vocabulary**
- Divide, round up, remainder

**Resources**
- Counters; place value cards and tables.
Mental maths activities

Select suitable activities from the mental warm-up activity bank or develop your own based on multiplication and division facts.

Concepts that may be unfamiliar in this chapter

Dividing bigger two-digit numbers by single-digit numbers

Students have already dealt with division as repeated subtraction and they should be using ‘chunking’ to subtract (rather than counting in ones). Now they are expected to extend these methods to divide larger numbers. Students who are confident with their methods and who understand the division process should not have any difficulty with this.

Rounding a remainder up or down

Previously students have only been expected to write a remainder. Now they have to think about the context of the problem and decide whether or not to round up or down in the context of specific problems. For example, Pete has 18 CD to put into boxes. Each box can hold 5 CDs. How many boxes will he need? Students need to realise that in this case the answer is 4. He can’t have 3 boxes as he’ll still have 3 CDs left over. These three need a box, even if it is not full.

Teaching ideas

Practical activities

Start by informally revising the division of low range two-digit numbers by single-digit numbers. Students should be able to do this mentally, particularly when both digits are even. Discuss the connection between halving and dividing by two.

Students divide by five by quickly skip counting in fives, keeping count of the number of fives, until the number is reached.

Reinforce the equal-grouping aspect of division by modelling the division problems using number lines or coloured rods. Students draw a line and label the left-hand end ‘0’ and the right-hand end with the number they are dividing. They could also mark 10, 20, 30, and so on, along the line. They need not measure out these calibrations, as the line is really just a mental aid.

Using mental calculations and their knowledge of the tables, students skip count in equal intervals until they are just below the target number. Then they count on for the remainder.

You may also wish to use coloured rods for this work. Use tens and units rods to represent the number being divided. Use other rods to represent the divisor. Share out the divisor rods equally along the number to be divided.
When working with sharing or equal grouping, encourage the students to record in their own ways, and to compare and discuss their methods.

Using the Student Book and Workbook

Revise division as sharing using the example on Student Book page 118 and concrete apparatus if necessary. Let the students complete Workbook page 74 and spend some time discussing what they did with the remainders.

Demonstrate repeated subtraction using the examples on Student Book page 119. Discuss the shorter method of chunking and make sure the students can understand this. Repeat with additional examples if you need to. Once students have complete the activity on this page, have them work through Workbook page 75.

Remind the students that division is the inverse of multiplication and show them how to use multiplication facts to divide. Let them work through Student Book page 120 and spend some time discussing which facts they used to find the answers.

Use Student Book page 121 to revisit and reinforce the idea that you sometime have to round up or down after a division. Let the students use the methods they prefer to find the answers.

Revise multiplication by 10 and 100 using place value cards as necessary before working through the inverse operations shown on Student Book page 122. Encourage the students to do the calculations mentally as far as they can. Use the flow diagram activities on Workbook page 76 as additional practice.

Use Student Book page 123 and Workbook page 77 to consolidate and reinforce the inverse relationship between multiplication and division.

Assessment questions to ask

- What is [] divided by 10? (and similar)
- Can you explain to me the method you used to find your answer?
- In this division problem there is a remainder of 4. What should I do with it?
- Which of these calculations are correct/incorrect? What has this person done wrong? How could you help them to correct it?

Common errors and misconceptions

Students may make mistakes in calculations due to mental arithmetic errors – for example, incorrect times tables, errors in repeated subtraction. Encourage students to always check their calculations and estimate to consider whether an answer is likely to be correct.
21 Ratio and proportion

<table>
<thead>
<tr>
<th>Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>4Nc26 Begin to understand simple ideas of ratio and proportion, e.g. a picture of a dog is one fifth of the size of a real dog. The dog is 25 cm long in the picture, so it is $5 \times 25$ cm long in real life.</td>
</tr>
</tbody>
</table>

Vocabulary
Larger, smaller, fraction of

Resources needed
Photographs; models; toy animals; a slice of bread.

Mental maths activities
Select suitable activities from the mental maths activity bank.

Concepts that may be unfamiliar in this chapter
Students have not previously worked with ratio and proportion in any formal way. At this stage, you can teach them the basic ideas of proportional reasoning in terms of enlarging or reducing amounts or items without too much emphasis on the terminology. However, as always, you would use the correct terms to discuss what you are doing. For example, you may say something like, ‘When you reduce something on the photocopy machine every part of it is reduced in the same proportion.’

Teaching ideas
Practical activities

Give the students a range of toy items or models and let them estimate how many times longer or wider the real item is. Stress that the reduction is a fraction of the lengths on the real item. So, if students say ‘I think a real car is about 100 times longer than this model.’ Say, that means the model is a hundred times shorter than the car. It is $\frac{1}{100}$th of the length of the real car.

Show some pictures of real life items. In each case, let the students estimate how much shorter a given length is and then measure it and work out what they think the real life size will be.
Using the Student Book and Workbook

Show the class a real slice of bread if possible and then use the slice of bread in the picture on Student Book page 124 to teach the ideas that you have developed through the practical activities.

Let the students work in pairs to find the real life lengths of the animals shown.

Let the students discuss the problems on Student Book page 125 in groups and then have them solve them. Discuss the answers and the methods they used to find them as a class.

Remind the students that they know how to multiply by 100 before they move onto Student Book page 126. Let them measure the lengths on the plan and complete the table. Use Workbook page 78 as a fun way of dealing with enlarging and reducing shapes.

Assessment questions to ask

- This recipe is for 6 people. What do I have to do to the ingredients if I only want to make enough for 3 people? What if I want to make enough for 12 people?
- This model car is 1/100th of the length of a real car. How much longer is the real car?
- A length on picture is one fifth of the length of the real item. If the item is 13 cm long in the picture, how long is it in real life?

Common errors and misconceptions

Students may struggle to increase or decrease amounts in proportion because they don’t understand the concept of multiplicative reasoning. It is important to emphasise that doubling, halving, finding three times as much or a quarter as much all involve multiplying or dividing. Many of the ways in which we compare amounts and express change involve multiplicative reasoning, for example, this picture of a doll is a third of the height of the real doll. Students need to understand that this means the real doll is $3 \times$ taller than the one in the picture.