Representing data

Practise... 1.1 Stem-and-leaf diagrams

1 a

| 2 | 1 |
| 3 | 2 5 8 |
| 4 | 2 4 4 5 |
| 5 | 4 |
| 6 | 0 6 |

Key: 2 | 1 represents 21p

b 66 – 21 = 45p
c 44p
d 44p
e \( \frac{8}{11} \times 100 = 72.7272\ldots \%

2 a

| 0 | 8 9 9 |
| 1 | 0 2 3 4 4 7 8 9 9 |
| 2 | 0 0 1 2 2 4 5 |

Key: 2 | 4 represents 24 marks

b 25
c 18
d 17
e \( \frac{11}{19} \)

3 b

| 1 | 26 43 46 49 50 53 55 57 |
| 2 | 05 06 10 12 26 |

Key: 2 | 26 represents 2 hours 26 min

4

| 8 | 6 |
| 9 |
| 10 | 3 6 9 9 |
| 11 | 0 3 5 7 |
| 12 | 5 6 |
| 13 | 0 2 |
| 14 | 6 |

Key: 10 | 3 represents 103 minutes

This diagram is far better than the one in question 3 as it spreads the data out over a stem of seven rows rather than two. It is therefore far more informative about the distribution than the first.

5 There is no set answer but make sure the stem-and-leaf has min 23, max 65, median 44, mode of 42 and 55 (and no other) and a key.

6 a 170 or 262
b 262 doesn't but 170 does. 170 would change the median from 215 to 213.
c Mode also unaffected as there are four 209s.

7

<table>
<thead>
<tr>
<th>Max</th>
<th>Min</th>
<th>Range</th>
<th>UQ</th>
<th>LQ</th>
<th>IQR</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>Girls</td>
<td>37</td>
<td>12</td>
<td>25</td>
<td>26</td>
<td>15</td>
<td>9</td>
</tr>
<tr>
<td>Boys</td>
<td>36</td>
<td>11</td>
<td>25</td>
<td>29</td>
<td>18</td>
<td>11</td>
</tr>
</tbody>
</table>

Using the measures found girls have a smaller median, meaning that they complete the exercise quicker on average. The ranges are equal but the interquartile range of the boys is larger than that for the girls, meaning that the boys times are more variable.

8

a Year 7 Year 11

| 9 9 6 5 4 | 0 | 7 8 8 9 9 |
| 9 8 8 5 4 1 0 | 1 | 1 2 2 5 6 7 9 |
| 8 8 7 | 2 | 0 1 |

Key: 5 | 1 represents 15 tenths of a second

b Using the diagram alone, it is clear that the Year 7s have the three fastest reaction times and also the three slowest reaction times, so there is very little evidence here one way or the other.

c He should aim to collect a larger sample of data.
1 a
Freezer temperatures

b About –20 as the freezer appears to have reached more or less its intended temperature of operation.

2 a 335

b Runners’ fun run times

3 a and b
Puzzle-solving times

c The modal group for finishing time was quicker for the adults than the students. The spread of the data (range) is similar for both sets of people.

4 The peak for boys is to the left of the peak for girls indicating that the boys are faster on average. The boys have a plot on 23, which also supports this as the girls do not. The spread of boys’ times is greater.

5 a
Student attendance
b

<table>
<thead>
<tr>
<th>Day</th>
<th>Mon</th>
<th>Mon</th>
<th>Tue</th>
<th>Tue</th>
<th>Wed</th>
<th>Wed</th>
<th>Thu</th>
<th>Thu</th>
<th>Fri</th>
<th>Fri</th>
</tr>
</thead>
<tbody>
<tr>
<td>Session</td>
<td>am</td>
<td>pm</td>
<td>am</td>
<td>pm</td>
<td>am</td>
<td>pm</td>
<td>am</td>
<td>pm</td>
<td>am</td>
<td>pm</td>
</tr>
<tr>
<td>Percentage</td>
<td>84.61</td>
<td>80.77</td>
<td>93.46</td>
<td>82.69</td>
<td>97.69</td>
<td>83.85</td>
<td>96.54</td>
<td>77.31</td>
<td>71.15</td>
<td>58.46</td>
</tr>
<tr>
<td>Percentage to nearest whole number</td>
<td>85</td>
<td>81</td>
<td>93</td>
<td>83</td>
<td>98</td>
<td>84</td>
<td>97</td>
<td>77</td>
<td>71</td>
<td>58</td>
</tr>
</tbody>
</table>

Both the graph and the percentages seem to show clear evidence of worsening attendance at registration as the week progresses, supporting the principal’s thoughts.

There are many possible answers to this based on the groupings chosen.

The solution below is based on class intervals each twenty minutes wide, beginning with 1 hour 20 minutes up to (but not including) 1 hour 40 minutes and so on. Remember from Chapter 3 that you should aim for about 4–8 class intervals. The frequency polygon drawn is one example of a frequency diagram—an histogram with equal intervals could equally have been drawn to show the data (see 7.5).

Practise... 1.3 Cumulative frequency diagrams

1 a

<table>
<thead>
<tr>
<th>Height, h (cm)</th>
<th>Frequency</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 &lt; h ≤ 120</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>120 &lt; h ≤ 140</td>
<td>12</td>
<td>5 + 12 = 17</td>
</tr>
<tr>
<td>140 &lt; h ≤ 160</td>
<td>10</td>
<td>17 + 10 = 27</td>
</tr>
<tr>
<td>160 &lt; h ≤ 180</td>
<td>7</td>
<td>27 + 7 = 34</td>
</tr>
<tr>
<td>180 &lt; h ≤ 200</td>
<td>4</td>
<td>34 + 4 = 38</td>
</tr>
</tbody>
</table>

b

<table>
<thead>
<tr>
<th>Weight, w (kg)</th>
<th>Frequency</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 &lt; h ≤ 11</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>11 &lt; h ≤ 12</td>
<td>254</td>
<td>554</td>
</tr>
<tr>
<td>12 &lt; h ≤ 13</td>
<td>401</td>
<td>955</td>
</tr>
<tr>
<td>13 &lt; h ≤ 14</td>
<td>308</td>
<td>1263</td>
</tr>
<tr>
<td>14 &lt; h ≤ 15</td>
<td>126</td>
<td>1275</td>
</tr>
</tbody>
</table>

d

<table>
<thead>
<tr>
<th>Height, h (feet)</th>
<th>Frequency</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 ≤ h &lt; 150</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>150 ≤ h &lt; 200</td>
<td>15</td>
<td>16</td>
</tr>
<tr>
<td>200 ≤ h &lt; 250</td>
<td>34</td>
<td>50</td>
</tr>
<tr>
<td>250 ≤ h &lt; 300</td>
<td>46</td>
<td>96</td>
</tr>
<tr>
<td>300 ≤ h &lt; 350</td>
<td>16</td>
<td>112</td>
</tr>
<tr>
<td>350 ≤ h &lt; 400</td>
<td>9</td>
<td>121</td>
</tr>
</tbody>
</table>

c

<table>
<thead>
<tr>
<th>Time, t (seconds)</th>
<th>Frequency</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 ≤ t &lt; 30</td>
<td>43</td>
<td>43</td>
</tr>
<tr>
<td>30 ≤ t &lt; 50</td>
<td>65</td>
<td>108</td>
</tr>
<tr>
<td>50 ≤ t &lt; 70</td>
<td>72</td>
<td>180</td>
</tr>
<tr>
<td>70 ≤ t &lt; 90</td>
<td>55</td>
<td>235</td>
</tr>
</tbody>
</table>
### 3

- **a** 4.3 ounces
- **b** 3.6 ounces
- **c** 4.85 ounces
- **d** $4.85 - 3.60 = 1.25$ ounces
- **e** 2%
- **f** 11%

### 4

#### a

<table>
<thead>
<tr>
<th>Lifetime, $l$ (hours)</th>
<th>Frequency</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$50 &lt; l \leq 100$</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>$100 &lt; l \leq 150$</td>
<td>240</td>
<td>320</td>
</tr>
<tr>
<td>$150 &lt; l \leq 200$</td>
<td>390</td>
<td>710</td>
</tr>
<tr>
<td>$200 &lt; l \leq 250$</td>
<td>200</td>
<td>910</td>
</tr>
<tr>
<td>$250 &lt; l \leq 300$</td>
<td>90</td>
<td>1000</td>
</tr>
</tbody>
</table>
c  i  172 hours
   ii  135 hours
   iii  210 hours
   iv  $210 - 135 = 75$ hours
   v  $7 \times 24 = 168$ hours in a week
      from graph = 460 bulbs below 168 so
      $1000 - 460 = 540$ above
      $= 54\%$

   d  The data is grouped so you do not know any
      exact values.

5  a

<table>
<thead>
<tr>
<th>Weight, $w$ (kg)</th>
<th>Frequency</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 &lt; w \leq 1$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$0 &lt; w \leq 2$</td>
<td>17</td>
<td>18</td>
</tr>
<tr>
<td>$2 &lt; w \leq 3$</td>
<td>72</td>
<td>90</td>
</tr>
<tr>
<td>$3 &lt; w \leq 4$</td>
<td>8</td>
<td>98</td>
</tr>
<tr>
<td>$4 &lt; w \leq 5$</td>
<td>2</td>
<td>100</td>
</tr>
</tbody>
</table>

B

c  $2.8 - 2.1 = 0.7$ kg

6  a  almost certainly 50 mph
   b  A cumulative frequency graph is needed for
      the data values in the following table.

<table>
<thead>
<tr>
<th>Speed, $s$ (mph)</th>
<th>Frequency</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$55 &lt; s \leq 60$</td>
<td>27</td>
<td>27</td>
</tr>
<tr>
<td>$60 &lt; s \leq 65$</td>
<td>36</td>
<td>63</td>
</tr>
<tr>
<td>$65 &lt; s \leq 72$</td>
<td>40</td>
<td>103</td>
</tr>
<tr>
<td>$72 &lt; s \leq 80$</td>
<td>11</td>
<td>114</td>
</tr>
<tr>
<td>$80 &lt; s \leq 95$</td>
<td>6</td>
<td>120</td>
</tr>
</tbody>
</table>

Upper quartile = 67.5 mph (reading off at cumulative frequency of 80)
The speed limit is therefore higher than
the upper quartile and enforcement is not needed.

7  a  A cumulative frequency diagram is needed
      to estimate the median values in the
      following table.

<table>
<thead>
<tr>
<th>Hourly pay, $p$ (euro)</th>
<th>Frequency</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4 &lt; w \leq 5$</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>$5 &lt; w \leq 6$</td>
<td>56</td>
<td>73</td>
</tr>
<tr>
<td>$6 &lt; w \leq 7$</td>
<td>55</td>
<td>128</td>
</tr>
<tr>
<td>$7 &lt; w \leq 8$</td>
<td>101</td>
<td>229</td>
</tr>
<tr>
<td>$8 &lt; w \leq 9$</td>
<td>39</td>
<td>268</td>
</tr>
</tbody>
</table>
Median is about 7.05 euros (reading off at cumulative frequency of 134)
\[ \frac{2}{3} \times 7.05 = 4.7 \text{ euros} \]

**b** Reading off at this value, you can estimate that about 13 of these people are at (or below) minimum wage.

13 out of 268 = \( \frac{13}{268} \times 100 = 4.85\% \)

**8** The company has 146 employees and all but six have been with the company for at least 8 years.

The median length of service is 25.6 years. This is an increase of 2.6 years on the figure of 2 years ago, which was 23 years.

The interquartile range is 10% less than 2 years ago when the figure was 14.4* years.

I am pleased to be able to recommend 26 workers for long service awards as they have worked for more than 35 years.

* from graph, IQR is approx. 31.8 – 18.8 = 13 years. If this is 10% less than 2 years ago, then 2 years ago it was \( \frac{13}{0.9} = 14.444 \ldots \)

### Practise... 1.4 Box plots

**1 a** 20  
**c** 34  
**e** 45

**b** 29.5  
**d** 37  
**f** 37 – 29.5 = 7.5

**2 a**

- As it is the upper quartile, take away the lower quartile, 50 – 29 = 21

**3 a**

<table>
<thead>
<tr>
<th></th>
<th>Ayton</th>
<th>Beesby</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum age</td>
<td>28</td>
<td>2</td>
</tr>
<tr>
<td>Maximum age</td>
<td>70</td>
<td>52</td>
</tr>
<tr>
<td>Lower quartile age</td>
<td>35</td>
<td>14</td>
</tr>
<tr>
<td>Upper quartile</td>
<td>63</td>
<td>43</td>
</tr>
<tr>
<td>Median</td>
<td>56</td>
<td>28</td>
</tr>
</tbody>
</table>

**b** The population of Ayton is much older on average than Beesby as the median is significantly larger. The spread of ages is similar if based on interquartile range but more spread out in Beesby if based on range. There are no children in Ayton.

**4 a**

For box plot, minimum taken as 25 and maximum as 50. Other measures estimated from cumulative frequency graph.
c  The average time is a little slower as the median is 37.5 minutes compared to 37.1 minutes last year. The times are much less spread out as the interquartile range has come down from 9.9 to about 7.5 minutes.

5  The median waiting time is virtually the same at the main post office as the village post office, meaning average waiting times are similar. The waiting times for the main post office are less spread out than the village post office (interquartile ranges of 4 and 6 respectively).

6  a  and  ii
   b  and  iii
   c  and  i

7  Need to obtain box plot for new manager data via a cumulative frequency diagram.

<table>
<thead>
<tr>
<th>Hourly Sales (£)</th>
<th>Frequency</th>
<th>Cumulative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>£5 up to £10</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>£10 up to £15</td>
<td>23</td>
<td>34</td>
</tr>
<tr>
<td>£15 up to £20</td>
<td>63</td>
<td>97</td>
</tr>
<tr>
<td>£20 up to £25</td>
<td>2</td>
<td>99</td>
</tr>
<tr>
<td>£30 up to £35</td>
<td>1</td>
<td>100</td>
</tr>
</tbody>
</table>

Minimum is taken as £5, maximum as £35. Other measures are estimated from graph above.

Under the new manager, average sales are slightly down as can be seen from the slightly reduced median. The spread of sales is also lower as can be seen by the reduction in the interquartile range.
1 a Heights of people auditioning

b The final bar would be twice as wide but only half as tall.

2 a

<table>
<thead>
<tr>
<th>Wages, x (£)</th>
<th>Frequency</th>
<th>Class width</th>
<th>Height = frequency/class width</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 ≤ x &lt; 200</td>
<td>120</td>
<td>100</td>
<td>Height = 120 ÷ 100 = 1.2</td>
</tr>
<tr>
<td>200 ≤ x &lt; 250</td>
<td>165</td>
<td>50</td>
<td>Height = 165 ÷ 50 = 3.3</td>
</tr>
<tr>
<td>250 ≤ x &lt; 300</td>
<td>182</td>
<td>50</td>
<td>Height = 182 ÷ 50 = 3.64</td>
</tr>
<tr>
<td>300 ≤ x &lt; 350</td>
<td>197</td>
<td>50</td>
<td>Height = 197 ÷ 50 = 3.94</td>
</tr>
<tr>
<td>350 ≤ x &lt; 400</td>
<td>40</td>
<td>50</td>
<td>Height = 40 ÷ 50 = 0.8</td>
</tr>
<tr>
<td>400 ≤ x &lt; 600</td>
<td>6</td>
<td>200</td>
<td>Height = 6 ÷ 200 = 0.03</td>
</tr>
</tbody>
</table>

b 2 or 3

3 a

<table>
<thead>
<tr>
<th>Time, t (hours)</th>
<th>Frequency</th>
<th>Class Width</th>
<th>Frequency Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; t ≤ 20</td>
<td>3</td>
<td>20</td>
<td>0.15</td>
</tr>
<tr>
<td>20 &lt; t ≤ 30</td>
<td>11</td>
<td>10</td>
<td>1.1</td>
</tr>
<tr>
<td>30 &lt; t ≤ 35</td>
<td>35</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>35 &lt; t ≤ 45</td>
<td>27</td>
<td>10</td>
<td>2.7</td>
</tr>
<tr>
<td>45 &lt; t ≤ 95</td>
<td>4</td>
<td>50</td>
<td>0.08</td>
</tr>
</tbody>
</table>

b

<table>
<thead>
<tr>
<th>Speed, s (mph)</th>
<th>Frequency</th>
<th>Class width</th>
<th>Frequency density</th>
</tr>
</thead>
<tbody>
<tr>
<td>55 &lt; s ≤ 60</td>
<td>25</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>60 &lt; s ≤ 65</td>
<td>35</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>65 &lt; s ≤ 72</td>
<td>42</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>72 &lt; s ≤ 80</td>
<td>12</td>
<td>8</td>
<td>1.5</td>
</tr>
<tr>
<td>80 &lt; s ≤ 95</td>
<td>6</td>
<td>15</td>
<td>0.4</td>
</tr>
</tbody>
</table>

c For part a, estimate of median answers between 33.5 and 34 hours.
For part b estimate of median = 65 mph.
Answers

4

<table>
<thead>
<tr>
<th>Age</th>
<th>0–10</th>
<th>10–20</th>
<th>20–40</th>
<th>40–60</th>
<th>60–90–</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>4</td>
<td>7</td>
<td>15</td>
<td>11</td>
<td>3</td>
</tr>
</tbody>
</table>

5 a It is unclear where values such as 50 should go.

Depth of snow

6

<table>
<thead>
<tr>
<th>Weight, w (g)</th>
<th>Percentage</th>
<th>Class width</th>
<th>Percentage density</th>
</tr>
</thead>
<tbody>
<tr>
<td>390 &lt; w ≤ 400</td>
<td>10</td>
<td>10</td>
<td>10 / 10 = 1</td>
</tr>
<tr>
<td>401 &lt; w ≤ 402</td>
<td>26</td>
<td>1</td>
<td>26 / 1 = 26</td>
</tr>
<tr>
<td>401 &lt; w ≤ 402</td>
<td>17</td>
<td>1</td>
<td>17 / 1 = 17</td>
</tr>
<tr>
<td>402 &lt; w ≤ 405</td>
<td>33</td>
<td>3</td>
<td>33 / 3 = 11</td>
</tr>
<tr>
<td>405 &lt; w ≤ 412</td>
<td>14</td>
<td>7</td>
<td>14 / 7 = 2</td>
</tr>
</tbody>
</table>

7 a

Drink volumes

b About 38%

c This is a very high proportion under the advertised volume, strictly speaking no drinks should be under the advertised volume for trading standards purposes.
1 a 16 males so median is the mean of the 8th and 9th.

2 a 16 males so median is the mean of the 8th and 9th.

8th height is 174, 9th height is 175, mean of these is
\[
\frac{174 + 175}{2} = 174.5 \text{ cm}
\]

b i This equates to the paper arriving before 7.45 – roughly 50%
ii This equates to the paper arriving before 8.15 – roughly 93%

9 a Variations in the labelling are possible but the boundary values should be as shown.

<table>
<thead>
<tr>
<th>Weight, w grams</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 (\leq w &lt; 30)</td>
<td>20</td>
</tr>
<tr>
<td>30 (\leq w &lt; 40)</td>
<td>42</td>
</tr>
<tr>
<td>40 (\leq w &lt; 50)</td>
<td>33</td>
</tr>
<tr>
<td>50 (\leq w &lt; 65)</td>
<td>21</td>
</tr>
<tr>
<td>65 (\leq w &lt; 90)</td>
<td>5</td>
</tr>
</tbody>
</table>

b 40.91 g (2 d.p.)
c

<table>
<thead>
<tr>
<th>Time</th>
<th>Number of workers in canteen</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.00</td>
<td>0</td>
</tr>
<tr>
<td>11.00</td>
<td>10</td>
</tr>
<tr>
<td>12.00</td>
<td>18</td>
</tr>
<tr>
<td>13.00</td>
<td>38</td>
</tr>
<tr>
<td>14.00</td>
<td>25</td>
</tr>
<tr>
<td>15.00</td>
<td>18</td>
</tr>
</tbody>
</table>

d Not really. It is a time series so the structure is appropriate but the context means that the chances of a smooth change between numbers each hour is unlikely. For example, a staff break at 10.30–10.45 would not be shown by this data.

e pictogram or bar chart

4 a 11, 43, 71, 106, 151, 209, 324, 400

b Counting squares to half of the area, the median is just under 5/6 through the 30–50 class interval, which gives a median of about 30 + 16.7 = 46.7 years. This is very close to the value 47 obtained from the cumulative frequency graph.

5 Ages of passengers on train and aircraft

The average age of traveller on the train is much older than on the aircraft as shown by the much larger median.

The range of the two distributions is the same but the interquartile range for the aircraft is smaller than that for trains so the spread of the ages is smaller on the aircraft.

6 a

b Counting squares to half of the area, the median is just under 5/6 through the 30–50 class interval, which gives a median of about 30 + 16.7 = 46.7 years. This is very close to the value 47 obtained from the cumulative frequency graph.

7 a

b Counting squares to half of the area, the median is just under 5/6 through the 30–50 class interval, which gives a median of about 30 + 16.7 = 46.7 years. This is very close to the value 47 obtained from the cumulative frequency graph.
b The midpoint of the last interval is 25, not 22.5.

c

<table>
<thead>
<tr>
<th>Height, $h$ (metres)</th>
<th>Frequency</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 &lt; $h$ ≤ 10</td>
<td>48</td>
<td>48</td>
</tr>
<tr>
<td>10 &lt; $h$ ≤ 15</td>
<td>55</td>
<td>103</td>
</tr>
<tr>
<td>15 &lt; $h$ ≤ 20</td>
<td>27</td>
<td>130</td>
</tr>
<tr>
<td>20 &lt; $h$ ≤ 25</td>
<td>16</td>
<td>146</td>
</tr>
<tr>
<td>25 &lt; $h$ ≤ 30</td>
<td>4</td>
<td>150</td>
</tr>
</tbody>
</table>

d From the country park graph, estimates are lower quartile = 9 metres, median = 12.5 metres and upper quartile = 17 metres (so interquartile range = 8)

From the local park graph, estimates are lower quartile = 6.5 metres, median = 8.8 metres and upper quartile = 12 metres (so interquartile range = 5.5 metres)

e Local park trees are on average shorter as median is lower. Local park tree heights are less spread out as the interquartile range is smaller than that of the country park.
1 a i 40 ii 20
b i 25 ii

---

**Diagram Description:**
- **Marks:** 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100
- **Frequency density:** 0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5
2 Properties of circles

2.1 Angle properties of circles

1. a) \( a = 100^\circ \) (angle at centre = twice that at circumference)
   b) \( b = 90^\circ \) (angle in semicircle)
   c) \( c = 50^\circ \) (angles subtended by same arc are equal)
   d) \( d = 125^\circ \) (angle at centre = twice that at circumference)
   e) \( e = 92^\circ \) (opposite angles of cyclic quadrilateral)
   f) \( f = 42^\circ \) (angles subtended by same arc are equal)
   g) \( g = 58^\circ \) (angle in semicircle, angle sum of triangle)
   h) \( h = 112^\circ \) (angle at centre = twice that at circumference)
   i) \( i = 50^\circ \) (base angles of isosceles triangle, opposite angles of a cyclic quadrilateral)
   j) \( j = 60^\circ \) (opposite angles of cyclic quadrilateral)
   k) \( k = 18^\circ \) (angle in semicircle, angle sum of triangle)
   l) \( l = 80^\circ \); \( m = 50^\circ \) (base angles of isosceles triangle, angles subtended by same arc)
   m) \( n = 50^\circ \); \( o = 10^\circ \) (angles subtended by same arc)
   n) \( p = 50^\circ \); \( q = 80^\circ \) (angles subtended by same arc, parallel lines)
   o) \( s = 72^\circ \); \( r = 144^\circ \) (opposite angles of cyclic quadrilateral, angles subtended by same arc)

3. a) True
   b) True
   c) False (only true for cyclic quadrilaterals)

4. reflex angle \( ADC = 260^\circ \); obtuse angle \( ADC = 100^\circ \) not \( 65^\circ \)

5. angle \( OSP = 52^\circ \) (angles subtended by same arc)
   angle \( RQO = 52^\circ \) (base angles of isosceles triangle)
   angle \( QOR = 76^\circ \) (angle sum of triangle)

6. \( x = 30^\circ \); angle \( MNL = 60^\circ \)

7. a) \( x = 40^\circ \)
   b) \( y = 140^\circ \)

8. angle \( PQS = 40^\circ \) (base angles of isosceles triangle)
   angle \( SRP = angle PQS = 40^\circ \) (angles subtended by same arc)

9. angle \( POR \) reflex = \( 210^\circ \) (angle sum of circle)
   angle \( PQR = 105^\circ \) (angle at centre = twice that at circumference)
   angle \( ORQ = 40^\circ \) (angle sum of quadrilateral)

10. See proof in teacher notes for Learn 11.1.

11. a) \( a = 40^\circ \) (angles on a straight line, base angles of isosceles triangle)
    b) \( b = 20^\circ \) (angle sum of triangle, opposite angles of cyclic quadrilateral)

12. top angle of isosceles triangle = \( 46^\circ \) (angle in semicircle)
    each base angle of isosceles triangle = \( 67^\circ \);
    \( x = 67^\circ \) (angles subtended by same arc)

Practise...

2.1 Angle properties of circles

No, an isosceles trapezium cannot be a cyclic quadrilateral.
2.2 Tangents and chords

**Practise...**

1. **a** = 57° (tangents equal in length, base angles of isosceles triangle)
   **b** = 50° (radius at right angle to tangent, angle sum of triangle)
   **c** = 50° (symmetry, angle in a semicircle, angle sum of quadrilateral)
   **d** = 35° (alternate segment theorem)
   **e** = 53° (angle at centre = twice that at circumference, alternate segment)
   **f** = 44.5° (alternate segment, base angles of isosceles triangle)
   **g** = 30° (radius at right angle to tangent, angle sum of triangle)

2. **a** True  **c** False  **e** True  
   **b** True  **d** True

3. **a** 22° (symmetry)  
   **b** 136° (radius at right angle to tangent, angle sum of triangle)  
   **c** 68° (angle at centre = twice that at circumference)

4. \( \text{angle } OQP = 90° \) (radius at right angle to tangent)  
   \( \text{angle } OQR = 50° \) (base angles of isosceles triangle \( \text{QOR} \))  
   \( \text{angle } RQP = 90° - 50° = 40° \)  
   Triangle \( \text{QOR} \) is isosceles as \( \text{angle } RQP = \text{RPQ} \)

5. 13 cm

6. 18 cm  
   \( OB = 12 \text{ cm}, \ OA = 15 \text{ cm} \)  
   Using Pythagoras: \( OA^2 = OB^2 + AB^2 \) gives \( AB = 9 \text{ cm} \) so chord \( AC = 18 \text{ cm} \)

7. **a** 48° (alternate segment theorem)  
   **b** 84° (base angles of isosceles triangle, angle sum of triangle)  
   **c** 42° (cyclic quadrilateral, base angles of isosceles triangle)

8. \( \text{angle } QBO = \text{angle } QCO = 90° \) (radius at right angle to triangle)  
   \( \text{angle } PBO = 8° \) (angle sum of straight line)  
   \( x = 164° \) (isosceles triangle)  
   \( \text{angle } BOC = 140° \) (angles of quadrilateral add up to 360°)  
   \( \text{angle } AOC = 56° \) (angles in a circle add up to 360°)  
   \( y = 62° \) (base angles of isosceles triangle \( \text{AOC} \))

9. \( \text{angle } RQW = \text{angle } QSW; \text{angle } TSW = \text{angle } SQW \) (alternate segment theorem)  
   \( \text{angle } SQW = \text{angle } QSW \) (base angles of isosceles triangle)  
   so \( \text{angle } RQW = \text{angle } TSW \)

10. See proof in teacher notes for Learn 11.2.

11. Draw two chords in a circle. Draw in their perpendicular bisectors. The centre of the circle is where these cross.

12. \( \text{angle } AOB = 70° \) (angle sum of circle)  
   \( x = 35° \) (angle at centre = twice that at circumference)
Assess 2

1 a  $a = 18^\circ; b = 90^\circ; c = 72^\circ$
   b  $e = f = 50^\circ$
   c  $g = 91^\circ$
   d  $h = 132.5^\circ$
   e  $p = 41^\circ; q = 38^\circ$
   f  $a = 90^\circ; b = 55^\circ$
   g  $q = 90^\circ; b = 70^\circ$
   h  $t = 44^\circ$

2 a  $90^\circ$  b  $30^\circ$  c  $120^\circ$  d  $55^\circ$

3

4 $\angle PQR = 105^\circ$ (angle at centre = twice that at circumference)
   $\angle ORQ = 50^\circ$ (angle sum of quadrilateral)

5 a  $e = 33^\circ$
   b  $f = 58^\circ$
   c  $e = 44^\circ; f = 78^\circ; g = 58^\circ$
   d  $h = 75^\circ; t = 75^\circ; j = 30^\circ$

6 angle $ACB = 51^\circ$; angle $QAB = 51^\circ$

7 angle $CBA = 35^\circ$ (alternate segment theorem);
   angle $CAB = 90^\circ$ (angle in semicircle)
   angle $CPA = 20^\circ$ (angle sum of triangle)

8 a  $x$
   b  $180^\circ - 2x$
   c  $2x = 102^\circ$ (opposite angles of cyclic quadrilateral), $x = 51^\circ$

AQA Examination-style questions

1 a  $90^\circ$  b  i  $27^\circ$  ii  $63^\circ$  c  $52^\circ$
Practise... 3.1 One quantity as a fraction of another

1 a $\frac{1}{2}$  c $\frac{1}{3}$  e $\frac{1}{13}$  
b $\frac{1}{20}$  d $\frac{1}{6}$  f $\frac{2}{13}$

2 No; Kevin has worked out 50 as a fraction of 500, which is $\frac{1}{10}$.

50p as a fraction of £500 is 50 as a fraction of 50 000, which is $\frac{1}{100}$.

3 a i $\frac{1}{2}$  ii $\frac{1}{2}$  iii $\frac{1}{2}$  
b $\frac{1}{4}$

4 a i $\frac{1}{2}$  iii $\frac{13}{28}$  v $\frac{5}{7}$  

ii $\frac{3}{7}$  iv $\frac{9}{14}$

b 21

c The number of students passing can never be more than the number taking the test.

5 a $\frac{3}{8}$  b $\frac{5}{8}$

6 a $\frac{1}{6}$  c $\frac{7}{10}$  e 80  
b $\frac{3}{10}$  d 54

7 a $\frac{1}{10}$  b $\frac{1}{5}$  c $\frac{1}{3}$

8 a $\frac{2}{3}$  b UK  c Greece  
d No; the fractions would not be changed if the vertical axis was scaled differently.

9 48, 49, 50, 51, 52, 53 or 54

Practise... 3.2 Calculating with fractions

1 a i $\frac{59}{60}$

2 60

3 45

4 a $\frac{15}{16} \div \frac{31}{32}$  c The sum will become closer to 1.

b $\frac{991}{992}$

5 Tees R Us: £5; Getting Shirty: £5.40

6 a £18  c £18.74  
b £52.49  d 67p

7 a 4$\frac{1}{4}$ yards  b $\$63.75$

8 1$\frac{3}{7}$ seconds

9 No, not quite; total needed is $3\frac{1}{24}$ cups.

10 14$\frac{3}{4}$ yards

11 a i A is bigger as A divided by 5 is the same as B divided by 4.

ii 5, 4; 10, 8; 15, 12; 20, 16; 25, 20, etc.

(plus non-integer answers) There is an infinite number of pairs.

iii The answer is always $\frac{3}{4}$

b $\frac{8}{9}$

12 55 cm

Practise... 3.3 Rounding

1 a i 10  ii 50  iii 100  iv 5  v 0.8  
b i 13  ii 55  iii 110  
iv 4.8  v 0.84

c i 12.9  ii 54.5  iii 110  
iv 4.76  v 0.836

2 1749, 1250

3 a 80%  c 55%  e 67%  
b 90%  d 46%

4 0.4, $\frac{5}{12}$, $\frac{3}{7}$, $\frac{7}{16}$, $\frac{4}{9}$, 45%

5 The fraction of girls in the chess club is now more. The original figures gave a fraction of $\frac{3}{10}$; the new figures give a fraction of $\frac{1}{3}$.

6 a ii, iii and v  
b Any fraction in its lowest terms which has prime factors other than 2 and 5 will convert to a recurring decimal (and student’s examples).
### Practise... 3.4 Upper and lower bounds

1. a 15.5, 14.5  
   b 157.5, 156.5  
   c 100.5, 99.5

2. a 100.5 cm$^3$, 99.5 cm$^3$  
   b 15.5 cm$^3$, 14.5 cm$^3$  
   c 245.5 cm$^3$, 244.5 cm$^3$

3. 257.5 km

4. 65.75 kg

5. a i 10.75 cm, 10.25 cm  
   ii 7.25 cm, 6.75 cm  
   iii 23.75 cm, 23.25 cm  
   iv 15.25 cm, 14.75 cm

6. 37.75 g, 37.25 g.

7. Yes. Measured to the nearest gram.

### Practise... 3.5 Calculating with bounds

1. 36 cm

2. No; the upper bound of the length is 93.5 cm so the length could be, for example, 93.495 cm.

3. 7.661 m/s

4. 2.94 hours

5. a 13  
   b 11

6. Upper bound of total weight is 191.25 kg so it is not safe to use the hoist.

7. Maximum difference is 20 g, minimum difference is 18 g.

8. a and d as big as possible and b and c as small as possible.

9. Yes, 5% of 5.3 mm is less than 0.5 mm.

10. No, the tube could have a diameter as small as 4.86 mm.

11. 118.47 pounds
Assess 3

1 \(\frac{1}{5}\)

2 £1050

3 a \(\frac{7}{12}\) b \(\frac{1}{5}\)

4 a \(\frac{1}{2}\) b \(\frac{4}{7}\)

5 a 8 b 21 c 21 6 \(\frac{13}{18}\) d \(\frac{21}{20}\)

6 a e.g. i 124, 137.2 ii 0.012, 0.014792
b e.g. i 121.9, 123 ii 0.123, 0.12185

7 £49.49, £48.50

8 12 years, 364 days (or 365 if it is a leap year), 23 hours, 59 minutes, 59 seconds, ...

9 14.75 \(\leq l < 15.25\)

10 50.92 litres, but it could be as much as 51.23 litres or as little as 50.65 litres

11 8 cm

12 Just. The least weight of the sweets is 97.5 g.

13 a 277 kg b No; the weight could be as much as 352 kg.

14 6.71 m/s

15 b, d and f

16 Student’s own example

17 \(\frac{508}{625}\)

18 \(\frac{2}{3}\), \(\frac{5}{6}\), \(\frac{8}{9}\), \(\frac{10}{11}\), \(\frac{11}{12}\)

19 No. £13.50 + (6 \times £10.50) = £76.50

This is greater than £75.
4 Probability

Practise... 4.1 Mutually exclusive events

1 b, c and d
2 0.05
3 \( \frac{93}{100} \)
4 0.999
5 7/10
6 a \( \frac{5}{15} = \frac{1}{3} \)  c \( \frac{12}{15} = \frac{4}{5} \)  e \( \frac{4}{15} \)
    b \( \frac{3}{15} = \frac{1}{5} \)  d \( \frac{7}{15} \)  f \( \frac{9}{15} = \frac{3}{5} \)
7 a 0.4
    b 0.5
8 0.462
9 One dice as it is a \( \frac{1}{6} \) chance of getting a 4 with 1 dice, which is higher than the \( \frac{3}{36} \) chance of scoring 4 with two dice.
10 All red shapes are squares.

Practise... 4.2 Relative frequency

1 a 120
    b Probably not. 109 is not that far from the expected 120.
2 a 8
    b approximately 1700
3 a \( \frac{9}{40} \)
    b \( \frac{21}{40} \)
    c \( \frac{1}{4} \)
    d The additional 40 draws are unlikely to have the same set of outcomes as the first 40.
4 a i 6
    ii 22
    b 0.4
5 a 5
    b 0
    c EITHER
        Probably not, after 100 rolls there had been 11 ones compared with an expected 16 or 17.
        OR
        Possibly, as 100 is quite a large sample so the difference between 11 ones seen and the expected 16 or 17 could be significant.
6 a \( \frac{56}{270} = \frac{28}{135} \)
    b \( \frac{84}{270} = \frac{14}{45} \)

7 a \( \frac{132}{270} = \frac{22}{45} \)
    d No, it is \( \frac{108}{270} = \frac{2}{5} \)
    e the 45 for a score of 5
    b The relative frequencies after a large number of picks is very similar for the different colours, making it highly likely that there are the same (or a similar) number of each colour in the bag.
1. a No, a 3 occurring affects the probability of an odd number (makes it certain).
   b Yes, whatever you pick from the pack cannot affect the outcome for the coin.

2. Ball 1 Ball 2 Outcome Probability
   A 0.3 A AA 0.3 × 0.3 = 0.09
   B 0.7 B AB 0.3 × 0.7 = 0.21
   A 0.3 A BA 0.7 × 0.3 = 0.21
   B 0.7 B BB 0.7 × 0.7 = 0.49

3. 0.64

4. a 0.08
   b 0.48

5. a

Day 1 Day 2 Outcome Probability
   A 0.4 A AA 0.4 × 0.4 = 0.16
   N 0.6 N AN 0.4 × 0.6 = 0.24
   A 0.4 A NA 0.6 × 0.4 = 0.24
   N 0.6 N NN 0.6 × 0.6 = 0.36
   b i 0.16 ii 0.36 iii 0.48 iv 0.64

6. 0.176 (3 d.p.)

7. a

Coin 1 Coin 2 Coin 3 Outcome
   H H HHH
   H T HHT
   H H HTH
   H T HTT
   H H THH
   H T THT
   H H TTH
   T T TTT

8. a

Person 1 Person 2 Outcome Probability
   0.32 S SS 0.32 × 0.32 = 0.1024
   0.32 S SN 0.32 × 0.68 = 0.2176
   0.68 N NS 0.68 × 0.32 = 0.2176
   0.68 N NN 0.68 × 0.68 = 0.4624
   i 0.1024
   ii 0.4352
   b They will have had a similar education so are more likely to either speak a foreign language or not.
   c 0.904 (3 d.p.)

9. 0.42

10. You are multiplying probabilities that cannot be more than 1. Multiply by 1, the result is the same, multiply by a decimal, the result is smaller.
Practise... 4.4 Dependent events and conditional probability

1 a 0 b 1/5 c 1/5 d 4/5 e 1

2

<table>
<thead>
<tr>
<th>Disc 1</th>
<th>Disc 2</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>6/10</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>4/10</td>
<td>W</td>
<td>BW</td>
</tr>
<tr>
<td>4/10</td>
<td>W</td>
<td>WB</td>
</tr>
<tr>
<td>3/10</td>
<td>W</td>
<td>WW</td>
</tr>
</tbody>
</table>

a 6/10 × 5/9 = 30/90 = 1/3
b 4/10 × 3/9 = 12/90 = 2/15
c 1 − 30/90 − 12/90 = 48/90 = 8/15

3

<table>
<thead>
<tr>
<th>Pencil 1</th>
<th>Pencil 2</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>2/10</td>
<td>R</td>
<td>RR</td>
</tr>
<tr>
<td>3/11</td>
<td>R</td>
<td>RB</td>
</tr>
<tr>
<td>8/11</td>
<td>B</td>
<td>BR</td>
</tr>
<tr>
<td>7/10</td>
<td>B</td>
<td>BB</td>
</tr>
</tbody>
</table>

a 3/11 × 2/10 = 6/110 = 3/55
b 8/11 × 7/10 = 56/110 = 28/55
c 1 − 3/55 − 28/55 = 24/55

4 a 1/10 × 1/2 + 1/2 × 1/10 = 5/20 = 1/10
b 1/10 × 5/9 + 1/2 × 1/9 = 10/90 = 1/9

5 a

<table>
<thead>
<tr>
<th>Person 1</th>
<th>Person 2</th>
<th>Outcome</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.72</td>
<td>MB</td>
<td>SMB = 0.09 × 0.72 = 0.0648</td>
<td></td>
</tr>
<tr>
<td>0.09</td>
<td>S</td>
<td>N = 0.09 × 0.28 = 0.0252</td>
<td></td>
</tr>
<tr>
<td>0.28</td>
<td>N</td>
<td>SN = 0.09 × 0.26 = 0.02366</td>
<td></td>
</tr>
<tr>
<td>0.91</td>
<td>N</td>
<td>NMB = 0.91 × 0.26 = 0.2366</td>
<td></td>
</tr>
<tr>
<td>0.74</td>
<td>N</td>
<td>NN = 0.91 × 0.74 = 0.6734</td>
<td></td>
</tr>
</tbody>
</table>

b i 0.0648
ii 0.6734

6

<table>
<thead>
<tr>
<th>Ball 1</th>
<th>Ball 2</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>39/89</td>
<td>R</td>
<td>RR</td>
</tr>
<tr>
<td>40/90</td>
<td>G</td>
<td>RG</td>
</tr>
<tr>
<td>50/89</td>
<td>G</td>
<td>GR</td>
</tr>
<tr>
<td>49/89</td>
<td>B</td>
<td>GG</td>
</tr>
</tbody>
</table>

RG OR GR = > 40/90 × 50/89 + 50/90 × 40/89 = 4000/8010 = 400/801

7 6/49 × 5/48 × 4/47 × 3/46 × 2/45 × 1/44 = 1/13983816 = 0.0000000715

8 a i 85/200 × 34/199 = 7140/39800 = 357/1990
ii 85/200 × 115/199 + 115/200 × 85/199 = 19550/39800 = 391/796
b 38/70 × 37/69 = 1406/4830 = 703/2415

9 a 14/42 × 13/41 = 182/1722 = 13/123
b 26/42 × 25/41 = 650/1722 = 325/861

10 20 socks of which 12 were black
1. a. 3 or 4  
   b. around 14 or 15  
   c. around 6050

2. a. |   | 1 | 2 | 3 | 4 | 5 |
    |---|---|---|---|---|---|
    1 | 2 | 3 | 4 | 5 | 6 |
    2 | 3 | 4 | 5 | 6 | 7 |
    3 | 4 | 5 | 6 | 7 | 8 |
    4 | 5 | 6 | 7 | 8 | 9 |
    5 | 6 | 7 | 8 | 9 |10 |

   b. i. $\frac{3}{25}$  
    ii. $\frac{4}{25}$  
    iii. $\frac{2}{25}$  
    iv. 6

   c. |   | 1 | 2 | 3 | 4 | 5 |
    |---|---|---|---|---|---|
    1 | 1 | 2 | 3 | 4 | 5 |
    2 | 2 | 4 | 6 | 8 |10 |
    3 | 3 | 6 | 9 |12 |15 |
    4 | 4 | 8 |12 |16 |20 |
    5 | 5 |10 |15 |20 |25 |

   i. $\frac{3}{25}$  
    ii. $\frac{2}{25}$  
    iii. $\frac{1}{25}$  
    iv. 4

3. a. $\frac{45}{250}$  
    b. $\frac{36}{250}$  
    c. $\frac{114}{250}$  
    d. probably, as the frequencies are quite similar after a fairly large number of trials  
    e. all six frequencies around the 1000 mark (probably plus or minus 50) but not all six frequencies of exactly 1000

4. a. $\frac{1}{8}$  
    b. $\frac{1}{4}$  
    c. $\frac{9}{16}$  
    d. $\frac{41}{80}$

5. a. $0.12 \times 0.12 = 0.0144$  
    b. that successive days were independent

6. a. Day 1 | Day 2 | Outcome | Probability
    |   |   |   |   |
    | 0.42 | R | RR | 0.42 $\times$ 0.42 = 0.1764
    | 0.58 | N | RN | 0.42 $\times$ 0.58 = 0.2436
    | 0.42 | R | NR | 0.58 $\times$ 0.42 = 0.2436
    | 0.58 | N | NN | 0.58 $\times$ 0.58 = 0.3364

   b. 0.1764  
   c. Weather on consecutive days is often related so the assumption of independence which needs to be made is probably incorrect.
**Examination-style questions**

1. i. 0.2 and 0.5  
   ii. 0.1  

   b. 0.9  

   c. 0.18
Enlargements

Practise... 6.1 Enlargement and scale factor

1 a i 4 ii 3
   b i object = 4 units², image = 64 units²
      ii object = 3 units², image = 27 units²
   c i \(64 = 4^2 \times 4\)
      ii \(27 = 3^2 \times 3\)
       area of image = \((\text{scale factor})^2 \times \text{area of object}\)

2 a 2.5 cm b 45°

3 a 6 cm by 4.5 cm b 9 cm by 6.75 cm

4 a i 3 m
    ii small quadrant = 3.14 m², large quadrant = 7.07 m²

iii \(7.07 \div 3.14 = 2.25\) (2 d.p.) \(2.25 = 1.5^2\)
   b i 1.8
      ii 10.18 m² (2 d.p.)
   c 50 – 10.18 – 7.07 = 32.75 m²
   d 131 m²

5 a 1.75 b i 5.5 ii 55 cm iii 3.14 (2 d.p.)

6 a i 1 unit², 4 units², 9 units²
   ii The areas are the length of sides squared and \((\text{scale factor for length})^2\).
   b i 1 unit³, 8 units³, 27 units³
      ii The volumes are the length of sides cubed and \((\text{scale factor for length})^3\).

Practise... 6.2 Centres of enlargement

1

2 scale factor 3; centre of enlargement (0, 0)

3 a and b

4 a i and ii

b i and ii

c (9, 3), (9, 6), (15, 6) and (15, 3)
5 a and b

- a
- b

6

- c  $A'(2, -2.5), B'(4, -1.5), C'(4, 0), D'(2, -1)$
- d ii $AC = 2A'C$ or $A'C = \frac{1}{2}AC$
- e ii $BD = 2B'D$ or $B'D = \frac{1}{2}BD$
- f $A'C = \text{(scale factor for length)} \times AC$

7 a–d

- e Each hexagon is larger than the previous one. The gaps between one hexagon and the next one are getting larger. The image is always a scale factor of 1.5 of the previous image but the lengths that you multiply the scale factor by are becoming increasingly larger each time so the gaps between images are becoming larger.
- f The scale factors between the hexagons and their original, not the previous image, are increasing: 1.5, 2.25, 3.375, 5.0625...

Practise... 6.3 Negative scale factors

1

2 a–c
centre of enlargement \((-3, -2)\), scale factor \(-0.4\)

4  
\(a\) 2  
\(b\) \(DE = 4.1\) cm; \(BC = 5.8\) cm

Practise... 6.4 Similar shapes and scale factors

1  
\(a\) \(8:12\)  
\(b\) \(1:1.5\)  
\(c\) \(8.1\) cm

2  5 cm and 4.8 cm

3  
\(a\) \(1:1.5\)  
\(b\)  
\(i\) area of small trapezium = \(54\) cm\(^2\);  
area of large trapezium = \(121.5\) cm\(^2\)  
\(ii\) ratio of areas = \(4:9\)  
\(iii\) \(1:2.25\)  
\(iv\) scale factor = \(2.25\)

4  
\(a\) \(1:5\)  
\(b\) \(1:5\)  
\(c\)  
\(i\) \(0.36\pi\) cm\(^2\) and \(9\pi\) cm\(^2\)  
\(ii\) ratio of areas = \(1:25\)  
\(d\) \(1:125\)

5  
\(a\) not similar \((6 \div 3 = 2\) and \(9 \div 3 = 3)\)  
\(b\) not similar \((18 \div 12 = 1.5\) and \(32 \div 28 = 1.14...\)  
\(c\) similar  
\(d\) not similar \((12 \div 3 = 4\) and \(7.5 \div 2.5 = 3)\)

6  
\(25 \times 7 = 175\) cm\(^2\)

7  \(125 \times 4 = 500\) cm\(^3\)

8  \(216 \times 24 = 5184\) cm\(^3\)

9  \(4.096 \times 0.5 = 2.048\) litres

10  \(2 \times 14 = 28\) cm

11  \(8 \times 12.5 = 100\) cm\(^3\)

12  
\(a\) \(3\) cm  
\(b\) \(1:3:6\)
Assess 6

1

2 a and b

3 a 15:17 = 1:1.13
   b 25:27 = 1:1.08
   c Yes

4 a 30:33 = 10:11 or 1:1.1
   b side of larger triangle = 3.3 cm; height of larger triangle = 2.64 cm; base of smaller triangle = 3.6 cm
   c area of smaller triangle = 4.32 cm²; area of larger triangle = 5.2272 cm²
   d 1:1.21

5 a and b

6 a and b

7 192 cm²

---

AQA Examination-style questions

1 a (0, 1) b $-\frac{1}{2}$

2 60 cm

3 X = 125
   Y = 10 800
5

Formulæ

Practise... 5.1 Substitution and writing formulæ

1  a  112  b  168  c  \( S = 56C \)

2  a  \( T = 70x + 80y \)
   b  \( T = 90c + 20b \)
   It not a good idea to use \( C \) for the total cost
   in the formula because lower case \( c \) is being
   used already and this might get confusing.

3  \( T = 40w + 20 \)

4  a  \( xy \)  b  \( C = 200 - xy \)

5  \( C = 80x + 90y \) is the correct formula.
   \( C = 90x + 80y \) is incorrect as the tea \( x \) costs
   80p, and the coffee \( y \) costs 90p.
   \( C = 80x90y \) is incorrect as this means you
   multiply the terms, not add them together.

6  a  £3.75  b  6 pizzas  c  8 – 1.5 = 6.5 which does not divide by 0.75
   into whole numbers. Mary cannot have
   ordered \( 8 \frac{2}{3} \) pizzas.

7  a  Let \( w \) be wages, hours be \( h \) and \( r \) be rate per
   hour: \( w = hr \)

Practise... 5.2 Changing the subject of a formula

1  a  formula  d  equation  g  formula
   b  expression  e  equation  h  equation
   c  identity  f  expression

2  \( n = M - 42 \)

3  a  \( y = c - 5 \)  c  \( y = \frac{d - 24}{3} \)
   b  \( y = \frac{f - 64}{2} \)  d  \( y = \frac{j + 3k}{4} \)

4  a  \( x = \frac{s^2 - t^2}{8} \)  c  \( x = \frac{n + 19}{k} \)
   b  \( x = \frac{m + 49}{7} \)  d  \( x = y - 50 \)

5  Answer B, \( x = \frac{m + 3}{4} \), is the correct
   rearrangement of \( m = 4x - 3 \)

6  Sam is incorrect. It should be \( x = \frac{3}{y} \)

7  a  \( d = st \)  b  \( t = \frac{d}{s} \)

8  a  \( x = \sqrt{9 + b} = 3 \)  b  \( x = \sqrt{\frac{V}{ah}} \)  c  \( x = \sqrt{\frac{d + c}{g}} \)

9  No, the square root sign needs to cover the
   whole of the fraction as in: \( \sqrt{\frac{V}{ah}} \)

Assess 5

1  a  32  b  19th term

2  a  \( T = 1.8x + 1.2y \)  b  £10.20

3  a  \( P = x + 2 \)  b  \( N = 3P \) or \( N = 3(x + 2) \)

4  a  \( d = \frac{s}{2} + 2 \)  b  \( d = 10 \) inches
   c  12 feet

5  a  278  c  \( F = \frac{9(K - 273)}{5} + 32 \)
   b  86

6  \( c = \frac{1}{3}(\frac{m + 1}{4} - d) \)

7  \( y = \frac{12 - 3x}{4} \)
8. \( c = \sqrt{\frac{E}{m}} \)

9. a. \( x = \sqrt{y - 9} \)  
   b. \( x = y^2 \)
   c. \( x = \sqrt{\frac{3y - 3}{2}} \)
   d. \( x = \frac{5y - 4}{m + 2} \)

10. £20

---

**AQA Examination-style questions**

1. a. £256  
   b. 2 years  
   c. £20

2. a. 24 marks  
   b. 20 marks

© Nelson Thornes 2012
7

Trigonometry 1

Practise... 7.1 Calculating the side of a right-angled triangle using trigonometry

1 a  

\[ \text{opp} \quad \text{hyp} \]

\[ \angle B \quad 70^\circ \]

\[ \text{adj} \]

b

\[ \text{hyp} \quad \text{adj} \]

\[ \angle E \quad 50^\circ \]

\[ \text{opp} \]

2 a 5.0 cm  

b 6.5 cm

c 5.7 cm

d 8.1 cm

e 8.8 cm

f 4.5 cm

3 a 9.4 cm  

b 8.1 cm

c 15.0 cm

d 9.7 cm

e 7.1 cm

f 18.3 cm

4 \[ AE = 15.4 \text{ cm (3 s.f.)} \]

5 4.78 m (to the nearest cm)

6 5.20 m (to the nearest cm)

7 4.61 m (to the nearest cm)

8 62 cm (2 s.f.)

9 a 44.5 cm  

b 83.6 cm

c 128 cm

Practise... 7.2 Calculating angles using trigonometry

1 a 40.5°  

b 45.6°  

c 40.7°  

d 76.5°

\[ \text{a} = 53.8^\circ \]

\[ \text{b} = 34.1^\circ \]

\[ \text{c} = 26.1^\circ \]

\[ \text{d} = 50.2^\circ \]

2 a 45.6°  

b 52.1°  

c 47.2°

d 43.7°

e 42.8°

\[ \text{f} = 49.5^\circ \]

3 a

\[ 8 \text{ cm} \quad 17.0 \text{ cm} \]

\[ 62^\circ \quad 28^\circ \]

\[ 15.0 \text{ cm} \]

b

\[ 11.3 \text{ cm} \quad 35.5^\circ \]

\[ 9.2 \text{ cm} \]

\[ 6.6 \text{ cm} \]

c

\[ 9.7 \text{ cm} \quad 64^\circ \]

\[ 8.5 \text{ cm} \]

\[ 9.7 \text{ cm} \]

4 \[ ABD = 43.4^\circ \]

5 a 303.7°  

b 34.4 km

6 a 41°  

b 4.1°

7 60°  

\[ 50^\circ \]

\[ \sin a = \cos (90^\circ - a) \]
## Practise... 7.3 Trigonometry in three dimensions

1. \(3.98\) m  
2. \(ABC = 59.7^\circ\)  
3. a \(DEF = 32.0^\circ\) b \(BFH = 26.6^\circ\)  
4. a \(BAC = 19.2^\circ\) b \(BAD = 27.3^\circ\)  
5. \(3.15\) m

### Assess 7

1. a \(5.4\) cm; b \(7.9\) cm; c \(7.3\) cm  
2. a \(62.3^\circ\); b \(33.1^\circ\)  
3. \(25.5\) km  
4. a \(ACE = 63.4^\circ\) b \(BAD = 53.1^\circ\)

### AQA Examination-style questions

1. \(\sin y = \frac{4}{5}\)  
2. a \(15.3\) or \(15.26\) cm b \(19.1^\circ\)
8 Percentages and ratios

Practise... 8.1 Increasing or decreasing by a percentage

1 a 350 m  c 22.8 miles  e £78.08
  b 136 kg  d 33 litres  f £67.50

2 a £249.69  b £141.67  c £93.99

3 £2092.80

4 a 4000  b 2250
  c i (Note: values may be slightly different if students use rounded values at each stage.)

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of bacteria</td>
<td>1000</td>
<td>1250</td>
<td>1563</td>
<td>1953</td>
<td>2441</td>
<td>3052</td>
</tr>
</tbody>
</table>

ii Graph of bacteria population against time

iii exponential

5 a i 4232  iii 2172
   ii 3295  iv 943

b Number of trout

6 a £5062.50  b Value of car

£16 000

£5062.50

O  

4

Time (years)

7 a i £19456  ii £19923  iii £24085 (to nearest £)
  b Unlikely to get the same rise every year for 10 years, may move jobs etc.

8 a 145 g (to nearest g)
  b 3 more days

9 a 45%
  b No, it will take just 4 years because 1.2^4 = 2.0736

10 The area of Arctic polar ice cap will vary from 2.7 million km² to 5.8 million km² (1 d.p.)

11 Assuming a 13.1% increase in each decade gives the following predictions:

<table>
<thead>
<tr>
<th>Year</th>
<th>2020</th>
<th>2030</th>
<th>2040</th>
<th>2050</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population (billions)</td>
<td>7.8</td>
<td>8.8</td>
<td>10.0</td>
<td>11.3</td>
</tr>
</tbody>
</table>

Rate of increase is not likely to remain constant because of, for example, population control measures or lack of food.

12 25% reduction

Practise... 8.2 Writing one quantity as a percentage of another

1 a 45.7%  b 54.3%

2 14.5%

3 76.8%

4 14.7%
5 22.4%
6 a Greg has divided by the new cost instead of the original cost.
   b 14.3%
7 No, since 0.7 × 0.9 = 0.63, the % reduction is 37%
8 a i 25% increase
   ii 56.3% increase
   b Yes, the percentage increase in the perimeter and area of a square rug is the same as the rectangular rug.
9 Reducing the contents gives 6.67% more income. This will be the more profitable option (assuming the extra packaging costs are not too high).
10 Teacher: Maths % increase \(\frac{16}{46} \times 100 = 34.8\%
   
   
   Science % increase \(\frac{18}{53} \times 100 = 33.9\%\), so he has done better in maths.
   
   Paul just looked at the differences:
   
   Maths = 62 - 46 = 16%
   
   Science = 71 - 53 = 18%
   
   He should have compared the differences with the original marks.

**Practise...** 8.3 Using ratios and proportion

1 a 15 buckets of sand
   b 2 buckets of cement
   c 5 buckets of cement and 25 buckets of sand
2 a 16
   b 9
   c \(1\frac{1}{3}\)
   d \(\frac{3}{4}\)
   e \(x = 15\) and \(y = 20\)
3 a \(£125\)
   b 16 hours
   c All hours worked are at the same rate of pay
4 a 330 euros
   b \(£84\)
5 a 312 km
   b 393 miles
6 a i \(\frac{4}{9}\)
   ii \(\frac{1}{3}\)
   iii \(\frac{2}{9}\)
   b \(\frac{4}{9} + \frac{1}{3} + \frac{2}{9} = 1\)
7 62.5%
8 a 6 km
   b No, the map distance should be doubled to give 24 cm.
9 1 : 8 is steeper. It is equivalent to 12.5%
10 a 3 : 2
   b 45 cm by 30 cm (same ratio)
11 a 5
   b 6
12 a Small bottle gives 300 ml for £1.99 in 2 bottles, equivalent to \(66.33\ldots\)p per 100 ml.
   
   Regular bottle gives 300 ml for £2.39 in 1 bottle, equivalent to \(79.66\ldots\)p per 100 ml.
   
   Large bottle gives 400 ml for £2.88 in 1 bottle, equivalent to \(72\)p per 100 ml.
   
   The small bottles give the best value.
   
   b Someone may want to buy a different size because it may be more convenient to have the shampoo in one bottle, rather than two. Or they may want more than 300 ml.
13 19 parent volunteers are needed.

**Practise...** 8.4 Direct and indirect proportion

1 a \(y = 4x^2\)
   b \[\text{Graph}\]
2 a \(TV = 240\)
   b 20
3 a \(b = \frac{30}{h}\) (\(k = 30\))
   b Three pairs of values for \(b\) and \(h\) satisfying \(bh = 30\)
4 a \(x = \frac{4.8}{\sqrt{y}}\)
   b 9.6
   c 16
5 £6.75
6  a  $y \propto x$  with $k = \pi$
  b  $y \propto x^2$  with $k = \pi$
  c  $y \propto \frac{1}{x}$  with $k = 200$
  d  $y \propto \frac{1}{x^2}$  with $k = 250$
  e  $y \propto x$  with $k = 0.5$
  f  $y \propto \frac{1}{x}$  with $k = 10000$

7  a  neither  c  $y \propto x$
  b  neither  d  $y \propto \frac{1}{x}$

8  a  ![Graph of BMI vs h](image)
  b  ![Graph of BMI vs W](image)

9  a  i  $P = 8.75h$  where $P$ is the amount paid to Peter in £ and $h$ hours is the time he works.
    ii  $C = 10h$  where $C$ is the amount paid to Carly in £ and $h$ hours is the time she works.

10  a  ![Graph of Cost vs Call time](image)
     i  £4.50
     ii  40 minutes more
  c  i  shown on graph
     ii  No (with variety of possible reasons, e.g. not straight line through (0, 0), doubling the time does not double the cost)

11  z is inversely proportional to $x$.

---

**Assess 8**

1  94%
2  80%
3  a  4:5  b  1:1.25
4  flour 450g, butter 225g, sugar 150g, fruit 180g
5  10.3% (to 1 d.p.)
6  £467.64
7  a  large (5.995p/ml whereas the small bottle costs 7.192p/ml)
    b  May want smaller bottle for holiday or to fit in cupboard; may not have enough money for the large bottle, etc.
8  a  £18040
    b  No. $0.82 \times 0.88 = 0.7216$ so the value has fallen by 27.84%
9  a  6900

10  a  $P = \frac{180}{Q}$
    b  18
    c  18
    d  ![Graph of P vs Q](image)
  c  This shows exponential growth. In the long run, people will get better. Then there will be fewer left to infect so the number will eventually fall.

11  a  $y = 20x$ matches graph Q.
    b  $y = \frac{20}{x}$ matches graph P.
    c  $y = 20 \times 1.5^x$ matches graph R.
1. Repeatedly multiplying by 0.89 gives: 222,500, 198,025, 176,242, 156,856, 139,601, 124,245, 110,578, 98,415, 87,589, 77,954, 69,379, or trying powers e.g. $250,000 \times 0.89^{10} = 77,954$ and $250,000 \times 0.89^{11} = 69,379$

Both methods show that it takes 11 years.

2. 40 days
9 Quadratic equations and algebraic proof

Practise... 9.1 Factorising quadratic expressions

1 \((a + 5)(a + 4)\)
2 \((b + 6)(b + 4)\)
3 \((c - 6)(c - 3)\)
4 \((d - 3)(d - 4)\)
5 \((e + 10)(e - 2)\)
6 \((f + 7)(f - 3)\)
7 \((m - 5)(m + 3)\)
8 \((n - 5)(n + 4)\)
9 \((p + 3)(p + 3)\)
10 \((q - 2)(q - 2)\)
11 \((r - 2)(r + 2)\)
12 \((v - 7)(v + 7)\)
13 \((w - 8)(w + 3)\)
14 \((x - 12)(x + 2)\)
15 \((y + 6)(y - 5)\)
16 \((t - 8)(t + 4)\)
17 \((h + 9)(h - 2)\)
18 \((k - 11)(k - 2)\)
19 \((x + 21)(x + 2)\)
20 \((y + 9)(y - 7)\)
21 \(a\) \((x - y)(x - 4y)\)
\(c\) \((a - b)(a + b)\)
\(b\) \((p - 3q)(p + 4q)\)
\(d\) \((2c + 5d)(2c - 5d)\)
22 \(a\) \(a^2 - b^2 = (a + b)(a - b)\) so \(31^2 - 30^2\)
\(= (31 - 30)(31 + 30) = 1 \times 61 = 61\)
\(b\) \(a^2 - b^2 = (a + b)(a - b)\) so \(57^2 - 43^2\)
\(= (57 - 43)(57 + 43) = 14 \times 100 = 1400\)
23 \((x + a)(x + b) = x^2 + ax + bx + ab = x^2 + (a + b)x + ab\)
\(a + b = k\)
\(ab = 12\) so possible combinations are \(1, 12, 2, 6, 3, 4, 4, 3, 6, 2, 12, 1\)
These combinations of \(a\) and \(b\) when added together give values of \(k\) as \(13, 8, 7\).
Thus there are three possible values of \(k\).
24 \((y + c)(y + d) = y^2 + (c + d)x + cd\)
\(c + d = n\)
\(cd = -15\) so possible combinations are \(-1, -15, -3 and 5, -5 and 3 and -15 and 1\).
These combinations of \(c\) and \(d\) when added together give values of \(n\) as \(14, 2, -2 and -14\).
Thus there are four possible values of \(n\).

Practise... 9.2 Factorising harder quadratic expressions

1 \((2a + 1)(a + 3)\)
2 \((3b + 2)(b + 1)\)
3 \((2c + 1)(c - 3)\)
4 \((5d - 7)(d + 1)\)
5 \((7e + 5)(e - 1)\)
6 \((3f - 2)(f + 1)\)
7 \((2x + 1)(x - 4)\)
8 \((3y - 2)(y + 3)\)
9 \((5m + 4)(m - 3)\)
10 \((3n - 1)(n - 9)\)
11 \((5p + 3)(p + 2)\)
12 \((2q + 3)(q + 6)\)
13 \((2t - 3)(2t - 3)\) or \((2t - 3)^2\)
14 \((3u + 5)(3u + 5)\) or \((3u + 5)^2\)
15 \((2v + 1)(2v + 3)\)
16 \((4w - 1)(w - 5)\)
17 \((2x - 3)(2x + 3)\)
18 \((3y - 5)(3y + 5)\)
19 \(2(x + 1)(x + 4)\)
20 \(3(y - 1)(y - 2)\)
Practise... 9.3 Simplifying algebraic fractions

1 \( \frac{c + 1}{4} \)
2 \( \frac{y + 2}{3} \)
3 \( \frac{w - 2}{5} \)
4 \( \frac{t + 9}{t + 7} \)
5 \( \frac{x - 1}{x + 3} \)
6 \( \frac{y + 1}{y - 3} \)
7 \( \frac{x + 4}{x + 1} \)

Practise... 9.4 Solving quadratic equations by factorising

1 \( m = 3 \) or \( m = -2 \)
2 \( n = -1 \) or \( n = 2.5 \)
3 \( x = 4 \) or \( x = 1 \)
4 \( y = 5 \) or \( y = -3 \)
5 \( b = -12 \) or \( b = 7 \)
6 \( c = 4 \) or \( c = -3 \)
7 \( p = 2 \) or \( p = -7 \)
8 \( q = 3 \) or \( q = 9 \)
9 \( t = -10 \) or \( t = 10 \)
10 \( a = -7 \) or \( a = 0 \)
11 \( b = 4 \) or \( b = 0 \)
12 \( x = 5 \) or \( x = -\frac{1}{2} \)
13 \( y = -3 \) or \( y = \frac{1}{3} \)
14 \( z = -\frac{3}{5} \) or \( z = -2 \)
15 \( t = 2 \) or \( t = \frac{-7}{5} \) or \( -1.4 \)
16 \( p = \frac{1}{2} \) or \( p = -7 \)
17 \( p = 15 \)
18 \( q = 1 \)
19 Area of the rectangle \( PQRS = x(x - 5) \) cm
Area of the triangle \( PTU = 6 \times 4 \) cm = 12 cm
So the shaded area = \( (x(x - 5) - 12) \) cm = \( x^2 - 5x - 12 = 164 \) so \( x^2 - 5x - 176 = 0 \)
So \( x = 16 \) or \( -11 \)
As lengths cannot be negative, the length of the rectangle is 16 cm.
20 a Technically it has two solutions which are \( x = 7 \) repeated/twice. This is because it factorises to give \( (x - 7)(x - 7) \) and both of these brackets give the answer \( x = 7 \).
b Student’s own quadratic equation that only has one answer, i.e. of the form \( (x + a)^2 \) or \( (x - a)^2 \).
21 \( x^2 + 2x - 99 = 0 \)
So Anna started with either \(-11 \) or \( 9 \).
22 \( y(y - 5) = 234 \) so \( y^2 - 5y - 234 = 0 \)
So \( y = -13 \) or \( 18 \)
As lengths cannot be negative, the length of the garden is 18 m.
### Practise... 9.5 Solving equations with fractions

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( x = 2 ) \text{ or } ( x = -3 )</td>
</tr>
<tr>
<td>2</td>
<td>( x = 6 )</td>
</tr>
<tr>
<td>3</td>
<td>( x = 2 ) \text{ or } ( x = -3 )</td>
</tr>
<tr>
<td>4</td>
<td>( x = -4 ) \text{ or } ( x = 6 )</td>
</tr>
<tr>
<td>5</td>
<td>( x = 4 ) \text{ or } ( x = -7 )</td>
</tr>
</tbody>
</table>
| 6 | a \( \frac{60}{x} + \frac{60}{x + 20} = 2 \frac{1}{2} \)  
   b \( 40 \text{ km/h and } 60 \text{ km/h} \) |
| 7 | a \( \frac{45}{x} \); Ben: \( \frac{45}{x + 4} \)  
   b \( \frac{45}{x} - \frac{45}{x + 4} = 4 \)  
   c \( x = 5 \) so Shaun played 5 games. |

### Practise... 9.6 Solving quadratic equations by completing the square

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( a = 3 )</td>
</tr>
<tr>
<td>2</td>
<td>( b = -41 )</td>
</tr>
<tr>
<td>3</td>
<td>( x = -1 \pm \sqrt{2} )</td>
</tr>
<tr>
<td>4</td>
<td>( y = 3 \pm \sqrt{8} )</td>
</tr>
<tr>
<td>5</td>
<td>( z = -4 \pm \sqrt{18} )</td>
</tr>
<tr>
<td>6</td>
<td>( t = 2 \pm \sqrt{12} = (2 \pm 2 \sqrt{3}) )</td>
</tr>
<tr>
<td>7</td>
<td>( p = -6 \pm \sqrt{6} )</td>
</tr>
<tr>
<td>8</td>
<td>( q = -2 \pm \sqrt{2} )</td>
</tr>
<tr>
<td>9</td>
<td>( u = 4 \pm \sqrt{13} )</td>
</tr>
<tr>
<td>10</td>
<td>( v = 10 \pm \sqrt{8} )</td>
</tr>
<tr>
<td>11</td>
<td>( a = 1 \pm \sqrt{5} )</td>
</tr>
<tr>
<td>12</td>
<td>( x = 3 \pm \sqrt{38} )</td>
</tr>
</tbody>
</table>
| 13 | \( 25 = n^2 \) so \( n = \pm 5 \)  
   Choose +5 to give positive middle term  
   \((x + 5)^2 = x^2 + 10x + 25\), so \( m = 10 \) |
| 14 | \( 121 = q^2 \) so \( q = \pm 11 \)  
   Choose −11 to give negative middle term  
   \((x - 11)^2 = x^2 - 22x + 121\), so \( p = 22 \) |
| 15 | \((2x + a)^2 + b = 4x^2 + 4ax + a^2 + b\)  
   so \( 4ax = 12x\) so \( a = 3 \)  
   \(a^2 + b = 3\) so \( b = -6 \) |
| 16 | You cannot solve the equation because if you try to solve it using the perfect square method, when you rearrange the equation you get  
   \((y + \frac{3}{2})^2 + 2.75 = 0\)  
   This means that \((y + \frac{3}{2})^2 = -2.75\) which results in an imaginary number \(\sqrt{-2.75}\) |

### Practise... 9.7 Solving quadratic equations by using the quadratic formula

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
</table>
| 1 | a \( x = -1.35 \) \text{ or } \( x = -6.65 \)  
   b \( z = 1.19 \) \text{ or } \( z = -4.19 \)  
   c \( x = 2.78 \) \text{ or } \( x = 0.72 \)  
   d \( y = 3.46 \) \text{ or } \( y = -0.46 \)  
   e \( x = 0.29 \) \text{ or } \( x = -2.29 \)  
   f \( p = 1.39 \) \text{ or } \( p = 0.36 \)  
   g \( x = 3.10 \) \text{ or } \( x = -3.77 \)  
   h \( x = -3.56 \) \text{ or } \( x = 0.56 \)  
   i \( x = 1.15 \) \text{ or } \( x = -0.35 \) |
| 2 | \( x = 0.69 \) \text{ or } \( x = -2.19 \) |
| 3 | \( x = -1.41 \) \text{ or } \( x = -2.84 \) |
| 4 | a \( x = 0.815 \) \text{ (3 d.p.)} \text{ (dismiss negative solution because } x \text{ is a length)}  
   b \( \text{height} = 9.815 \) \text{ (3 d.p.)} |
| 5 | a \( t = 0.40 \text{ seconds or } t = 4.09 \text{ seconds} \)  
   b \( \text{The ball could get to 8 m above the point from which it was thrown on its way up, then again on its way down.} \) |
| 6 | a \((6 - \pi)x^2 + 17x - 15 = 0\)  
   b \( x = 0.780 \) \text{ (3 d.p.)}  
   c \( \text{width} = 3.34 \text{ m, length} = 6.56 \text{ m} \) |
| 7 | The equation cannot be solved because \( b^2 - 4ac \) is negative and has no square roots. |
| 8 | \( x^2 + 3x - 2 = 0 \) |
**9.8 Solving quadratic equations graphically**

### 1. a

<table>
<thead>
<tr>
<th>$x$</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^2$</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>25</td>
</tr>
</tbody>
</table>

### 1. b

\[ y = x^2 - 2x + 8 \]

### 1. c

- $x = -5$
- $x = 2$
- $x = 4$

### 1. d

- $x = -2$ or $x = 4$

### 2. a

<table>
<thead>
<tr>
<th>$x$</th>
<th>-3</th>
<th>-2</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>10</td>
<td>5</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

### 2. b

\[ y = x^2 + 1 \]

### 2. c

- $(0, 1)$
- $(1.75, -1.125)$
- $(1.75, -3.125)$

### 2. d

- $x = 0.15$

### 3. a

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2x^2$</td>
<td>0</td>
<td>0.5</td>
<td>2</td>
<td>4.5</td>
<td>8</td>
<td>12.5</td>
<td>18</td>
</tr>
<tr>
<td>$-7x$</td>
<td>0</td>
<td>-3.5</td>
<td>-7</td>
<td>-10.5</td>
<td>-14</td>
<td>-17.5</td>
<td>-21</td>
</tr>
<tr>
<td>$+5$</td>
<td>+5</td>
<td>+5</td>
<td>+5</td>
<td>+5</td>
<td>+5</td>
<td>+5</td>
<td>+5</td>
</tr>
<tr>
<td>$y$</td>
<td>5</td>
<td>2</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

### 3. b

\[ y = 2x^2 - 7x + 5 \]

### 3. c

- $(1.75, -1.125)$
- $(1.75, -3.125)$

### 3. d

- $x = 0.15$

### 4. a

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2x^2$</td>
<td>0</td>
<td>0.5</td>
<td>2</td>
<td>4.5</td>
<td>8</td>
<td>12.5</td>
<td>18</td>
</tr>
<tr>
<td>$-7x$</td>
<td>0</td>
<td>-3.5</td>
<td>-7</td>
<td>-10.5</td>
<td>-14</td>
<td>-17.5</td>
<td>-21</td>
</tr>
<tr>
<td>$+5$</td>
<td>+5</td>
<td>+5</td>
<td>+5</td>
<td>+5</td>
<td>+5</td>
<td>+5</td>
<td>+5</td>
</tr>
<tr>
<td>$y$</td>
<td>3</td>
<td>0</td>
<td>-2</td>
<td>-3</td>
<td>-3</td>
<td>-2</td>
<td>0</td>
</tr>
</tbody>
</table>

### 4. b

- When $x = 1.75$, $y = 2(1.75)^2 - 7(1.75) + 3 = -3.125$
- $(1.75, -3.125)$

### 4. c

- $x = 1$ or $x = 2.5$
- $x = 0.5$ or $x = 3$

### 4. d

- The graph of $y = 2x^2 - 7x + 3$ does not intersect the line $x = -4$
5 a

<table>
<thead>
<tr>
<th>x</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4x</td>
<td>-4</td>
<td>0</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>16</td>
<td>20</td>
<td>24</td>
<td>28</td>
<td>32</td>
<td>36</td>
</tr>
<tr>
<td>2x</td>
<td>-0.5</td>
<td>0</td>
<td>0.5</td>
<td>-2</td>
<td>-4.5</td>
<td>-8</td>
<td>-12.5</td>
<td>-18</td>
<td>-24.5</td>
<td>-32</td>
<td>-40.5</td>
</tr>
<tr>
<td>y</td>
<td>-3</td>
<td>1</td>
<td>4.5</td>
<td>7</td>
<td>8.5</td>
<td>9</td>
<td>8.5</td>
<td>7</td>
<td>4.5</td>
<td>1</td>
<td>-3.5</td>
</tr>
</tbody>
</table>

b \( y = 1 + 4x - \frac{1}{2}x^2 \)

c \( x = 4 \)
d i \( x = 2 \) or \( x = 6 \)
ii \( x = -0.25 \) or \( x = 8.25 \)
e Both solutions add up to 8

6 a \( 0.5x^2 + 1.5x = 5 \)
b \( x^2 + 3x - 10 = 0 \)
c

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-10</td>
<td>-8.25</td>
<td>-6</td>
<td>-3.25</td>
<td>0</td>
<td>3.75</td>
<td>8</td>
</tr>
</tbody>
</table>

7 a

<table>
<thead>
<tr>
<th>r</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.6</td>
<td>6.3</td>
<td>14.1</td>
<td>25.1</td>
<td>39.3</td>
</tr>
</tbody>
</table>

b \( A = \frac{1}{2} \pi r^2 \)

c i \( 3.5 \text{ m}^2 \)
ii \( 23 \text{ m}^2 \)
d i \( 4.4 \text{ m} \)
ii \( 3.2 \text{ m} \)

Practise... 9.9 Algebraic proof

1 \( 5(2x + 5) + 11(x - 2) \)
\( 10x + 25 + 11x - 2^2 \)
\( 21x + 3 \)
\( 3(7x + 1) \)

2 \( 11(x - 1) + 4(4x + 14) \)
\( 11x - 11 + 4x + 56 \)
\( 15x + 45 \)
\( 15(x + 3) \)

3 \( a = 2 \)
\( b = 1 \)

4 An odd number may be written as \( 2a + 1 \).
The sum of two consecutive odd numbers may be written as \( (2a + 1) + ((2a + 1) + 2) = 4a + 4 = 4(a + 1) \) As any whole number multiplied by 2 is an even number \( 4(a + 1) \) must be even.
So the sum of two consecutive odd numbers is always an even number.

5 The sum of three consecutive numbers may be written as \( a + (a + 1) + (a + 2) = 3a + 3 \) Three times the middle number is \( 3(a + 1) \) which also equals \( 3a + 3 \).
So the sum of three consecutive numbers is equal to three times the middle number.
6 Tasha is incorrect because \((a + b)^2 = a^2 + 2ab + b^2\) so unless \(a\) or \(b\) is \(-1\), there will always be \(ab\) terms. Tasha is therefore only correct if \(a\) or \(b\) is \(0\) or both \(a\) and \(b\) are \(0\).

7 \(\begin{array}{c|cc}
\hline
n & n + 1 & n + 9 \\
\hline
n + 8 & n + 9 & \\
\hline
\end{array}\)

\(b\) \(n(n + 9) = n^2 + 9n\)
\((n + 1)(n + 8) = n^2 + n + 8n + 8 = n^2 + 9n + 8\) which is \(8\) more than \(n(n + 9)\).
So the difference of the products of the diagonals is \(8\).

8 An even number may be written as \(2a\)
The product of three consecutive numbers may be written as \((2a)(2a + 2)(2a + 4) = 8a^3 + 24a^2 + 16a = 8(a^3 + 3a^2 + 2a)\)
So the product of three consecutive even numbers is a multiple of \(8\).

9 The sum of the squares of three consecutive numbers may be written as
\[a^2 + (a + 1)^2 + (a + 2)^2 = 3a^2 + 6a + 5\]
If you take \(2\) from this answer you get \(3a^2 + 6a + 3 = 3(a^2 + 2a + 1)\)
So if you add the squares of three consecutive numbers and then subtract two, the answer is a multiple of \(3\).

10 \[\frac{n + 1}{n} - \frac{n}{n + 1} = \frac{(n + 1)(n + 1) - n^2}{n(n + 1)} = \frac{n^2 + 2n + 1 - n^2}{n(n + 1)} = \frac{2n + 1}{n(n + 1)}\]

11 \(a\) \(1, 3, 6\)
\(b\) The sum of two consecutive triangle numbers may be written as
\[\frac{n(n + 1)}{2} + \frac{(n + 1)(n + 1 + 1)}{2} = \frac{n^2 + n + 2n^2 + 3n + 2}{2} = \frac{n^2 + 2n + 1}{2} = (n + 1)^2\]
So the sum of any two consecutive triangle numbers is always a square number.

12 Let the digits of the three-digit number you pick be \(a, b\) and \(c\) where \(c < a\)
Reverse the three-digit number and take the lesser away from the greater:

\[
\begin{array}{c|cc}
\text{Hundreds} & \text{Tens} & \text{Units} \\
\hline
a & b & c \\
\hline
? & ? & ? \\
\end{array}
\]

In the first row/first three-digit number borrow one ten from \(b\) and give it to \(c\) to make it bigger than \(a\):

\[
\begin{array}{c|cc}
\text{Hundreds} & \text{Tens} & \text{Units} \\
\hline
a - 1 & b & c \\
\hline
? & ? & ? \\
\end{array}
\]

Then borrow one from \(a\) for \((b - 1)\) to make it bigger than \(b\):

\[
\begin{array}{c|cc}
\text{Hundreds} & \text{Tens} & \text{Units} \\
\hline
a - 1 & b + 9 & c \\
\hline
? & ? & ? \\
\end{array}
\]

Now reverse this number and add it to the number you started with:

\[
\begin{array}{c|cc}
\text{Hundreds} & \text{Tens} & \text{Units} \\
\hline
a - 1 & b + 9 & c + 10 - a \\
\hline
9 & 18 & 9 \\
\end{array}
\]
Take the \(10\) in the Tens column into the Hundreds column and the answer is \(1089\).
7 \ a \ x = -\frac{3}{5} \\
\ b \ x = 5, x = -6 \\
8 \ a \ a = 2 or a = -6 \ f \ f = 8 or f = -5 \\
\ b \ b = -7 or b = -1 \ g \ k = -1 or k = 1 \\
\ c \ c = 8 or c = 1 \ h \ m = 16 or m = -2 \\
\ d \ d = -3 or d = 2 \ i \ n = 1 or \frac{3}{2} (2.5) \\
\ e \ e = 0 or e = 5 \ j \ p = -2 or p = \frac{2}{3} \\
9 \ p = 6 \\
10 \ q = 24 \\
11 \ a \ x = -0.84 \ or \ x = -7.16 \\
\ b \ x = 1.18 \ or \ x = -0.43 \\
\ c \ x = 5.54 \ or \ x = -0.54 \\
\ d \ x = -0.55 \ or \ x = -1.45 \\
\ e \ x = 2.33 \ or \ x = -1 \\
12 \ a = 21 \\
13 \ The \ sum \ of \ any \ two \ consecutive \ terms \ may \ be \ written \ as \ \frac{n(n - 1)}{2} + \frac{(n + 1)(n + 1) - 1}{2} \\
\ = \frac{n^2 - n + n^2 + n}{2} = \frac{2n^2}{2} = n^2 \\
So \ the \ sum \ of \ any \ two \ consecutive \ terms \ is \ always \ a \ square \ number. \\
14 \ a \ -5 \pm \sqrt{21} \ c \ 8 \pm \sqrt{52} (= 8 \pm 4 \sqrt{13}) \\
\ b \ 2 \pm \sqrt{3} \ d \ -3 \pm \sqrt{17} \\
15 \ (x - b)^2 = x^2 - 2bx + b^2, b^2 = 49 \ so \ b = 7 \\
x^2 - ax + 49 = (x - 7)^2 \ so \ a = 14 \\
16 \ x = 4 or \frac{7}{8} \\
17 \ x = -\frac{1}{3} \\
18 \ (y + 7)(y - 3) = 56 \ so \ y^2 + 4y - 77 = 0 \\
y = 7 or -11 \\
As \ lengths \ cannot \ be \ negative, \ y \ must \ be \ 7 \ cm. \\
So \ the \ length \ of \ the \ rectangle \ is \ 14 \ cm. \\
19 \ 6 \times x \times (x + 10) = 450 \\
6x^2 + 60x - 450 = 0 \\
x^2 + 10x - 75 = 0 \\
x = -15 \ or \ 5 \\
As \ lengths \ cannot \ be \ negative, \ x \ must \ be \ 5 \ cm. \\
So \ the \ width \ of \ the \ cuboid \ is \ 5 \ cm. \\
\ AQA \ Examination-style \ questions \\
1 \ a \\
\ x \\
\ b \ y = x^2 - x - 5 \\
\ c \ The \ solution \ is \ where \ the \ graph \ crosses \ the \ x-axis. \\
2 \ a \ area \ covered \ by \ grass = \frac{3}{8} \times 80 = 48 \ m^2 \\
\ (10 - x)(8 - x) = 48 \\
80 - x^2 - 18x = 48 \\
x^2 + 18x - 32 = 0 \\
\ b \ x = 2 \ or \ x = 16 \\
As \ the \ width \ of \ the \ grass \ is \ 8 - x, \ x = 16 \ cannot \ be \ correct. \ So \ the \ width \ of \ the \ path \ must \ be \ 2 \ m. \\
3 \ -3 \pm \sqrt{3^2 - 4 \times 2 \times -7} = -3 \pm \sqrt{65} \\
so \ x = 1.27 \ and \ x = -2.77 \\
4 \ a \ i \ This \ shows \ two \ consecutive \ numbers \ multiplied \ together. \ This \ means \ that \ it \\
shows \ an \ even \ number \ multiplied \ by \ an \ odd \ number, \ so \ the \ product \ is \ even. \\
ii \ 2 \ multiplied \ by \ n \ is \ always \ even \ so \ 2n + 1 \ is \ always \ odd. \\
b \ 4n^2 + 2n + 2n + 1 = 4n^2 + 4n + 1 \\
c \ An \ odd \ number \ may \ be \ written \ as \ 2a + 1 \\
The \ square \ of \ two \ odd \ numbers \ may \ be \\
written \ as \ (2a + 1)^2 = 4a^2 + 4a + 1 \\
Take \ away \ 1 \ from \ this \ and \ you \ get \ 4a^2 + \\
4a = 4a(a + 1) \ which \ is \ 4 \times \ even \ number \\
which \ is \ thus \ a \ multiple \ of \ 8. \\
So \ the \ square \ of \ any \ odd \ number \ is \ always \ 1 \\
more \ than \ a \ multiple \ of \ 8. 
© Nelson Thornes 2012
10 Simultaneous equations

Practise... 10.1 Solving simultaneous equations by elimination

1 a $x = 2, y = 2$ d $e = 4, f = 1$
   b $x = \frac{5}{2}, y = \frac{3}{4}$ e $p = -1, r = 3$
   c $a = 5, b = 1$ f $x = 4, y = 2$
2 a $s = 3, r = 4$ d $e = 2.84, f = -0.82$
   b $x = 8, y = -2$ e $p = 2, r = 2$
   c $a = -1, b = 3$ f $x = -1, y = -2$
3 a $e = -7, f = -4$ d $i = 5, j = -2$
   b $v = -3, w = -9$ e $g = 1, h = -2$
   c $x = 2, y = 3$ f $x = -1, y = 2\frac{1}{2}$
4 a $2x + y = 180^\circ$ and $x = \frac{y}{2} - 6^\circ$
   These give the two equations to be solved:
   $2x + y = 180$ and $2x - y = -12$
   b $x = 42^\circ, y = 96^\circ$

Practise... 10.2 Solving simultaneous equations by substitution

1 a $x = 3, y = 5$ c $p = 2, r = -1$
   b $c = 5, d = 4$ d $v = 1.5, w = 1$
2 a $m = 5, n = 2$ c $x = 2, y = 5$
   b $x = -1, y = 10$ d $e = 12, f = 2$
3 a $x + 3y = 90^\circ$
   x = y + 30^\circ
   b $x = 45^\circ$ and $y = 15^\circ$
4 a $x - y = 24^\circ$
   $x + y = 76^\circ$
   x = 50^\circ and $y = 26^\circ$
   b The bearing of $B$ from $A$ is $76^\circ$.
5 a $\frac{x}{y} = \frac{4}{5}$

Practise... 10.3 Solving simultaneous linear equations graphically

1 a $x = 4, y = 2$ d $x = 3, y = \frac{1}{2}$
   b $x = \frac{2}{3}, y = \frac{4}{3}$ e $x = \frac{1}{2}, y = \frac{1}{2}$
   c $x = 3, y = 1$ f $x = 4, y = 1\frac{1}{2}$
2 a $x = -2, y = 1$ d $x = -\frac{1}{2}, y = 1$
   b $x = 1, y = -3$ e $x = 2, y = -1$
   c $x = -2, y = -4$ f $x = -\frac{1}{2}, y = -\frac{1}{2}$

3

An adult meal costs £12 and a child meal costs £6.
Coffee costs £3 and a hot chocolate costs £3.50.

10.4 Solving simultaneous equations algebraically, where one is linear and one is quadratic

1.a \[ x = -1.25, y = 6.25 \] and \[ x = 2, y = 16 \]
   b \[ c = \frac{1}{4}, d = -\frac{7}{8} \] and \[ c = -2, d = 6 \]
   c \[ g = 4, h = 2 \] and \[ g = 49, h = 7 \]
   d no solutions

2.a \[ m = 2.38, n = 0.62 \] and \[ m = 4.62, n = -1.62 \]
   b \[ e = 0.59, f = 3.41 \] and \[ e = 3.10, f = 22.90 \]
   c \[ x = 2.19, y = 8.56 \] and \[ x = -0.69, y = -0.06 \]
   d \[ a = 0.77, b = 4.09 \] and \[ a = 0.37, b = 2.48 \]

10.5 Solving simultaneous equations graphically, where one is linear and one is quadratic

1.a \[ x = 1, y = 4 \] and \[ x = 3, y = 12 \]
   b \[ x = 3, y = 1 \] and \[ x = -4, y = 8 \]
   c \[ x = -1, y = 0 \] and \[ x = 5, y = 6 \]
   d \[ x = 0, y = 9 \] and \[ x = 4, y = 5 \]
   e \[ x = -\frac{1}{2}, y = 1\frac{3}{4} \] and \[ x = 2, y = 3 \]

2.a

b i \[ x = 1 \] and \[ x = 4 \]
   ii \[ x = 2 \] and \[ x = 3 \]
   iii \[ x = 1 \] and \[ x = 5 \]
3 a

b i \( x = -1 \) and \( x = 5 \)

ii \( x = 2 \)

iii \( x = -2 \) and \( x = 3 \)

4 a

b i \( x = 0 \) and \( x = 6 \)

ii \( x = 1 \) and \( x = 4 \)

c The line \( y = 10 \) does not intersect the curve.

5 The line meets the curve when
\[ 3x^2 - 5x + 2 = x - 1 \]
\[ 3x^2 - 6x + 3 = 0 \]
\[ x^2 - 2x + 1 = 0 \]

\( a = 1, b = -2, c = 1 \) so \( b^2 - 4ac = 4 - 4 = 0 \)
This means there is only one solution, so the line is a tangent to the curve.

6 Lines cross when \( 2x^2 - 5x + 3 = x + 11 \)
\[ 2x^2 - 6x - 8 = 0 \]
\[ x^2 - 3x - 4 = 0 \) as required

\( A = (-1, 10) \) and \( B = (4, 15) \)
Assess 10

1 a \( x = 3, y = 1 \)  
   c \( x = 5, y = -2 \) 

b \( x = 4, y = -1 \)  
   d \( x = \frac{1}{2}, y = -1 \) 

2 Vanilla smoothie costs £1.75 and strawberry smoothie costs £2.25.

3 The mother is 30 years old and her son is 10 years old.

4 a \( x = \frac{1}{2}, y = \frac{1}{2} \)  
   \( x = 3, y = 18 \)  

b \( x = 5, y = 25 \)  
   \( x = -1 \) and \( y = 1 \) 

b \( x = -1, y = 4 \)  
   \( x = -4, y = 7 \)  

b \( x = 3.81, y = 11.91 \) and  
   \( x = -1.31, y = 9.34 \) 

5 \( x = 1 \) and \( y = 3 \) 

6 The expensive carpet is £35 per square metre and the cheaper carpet is £25 per square metre.
There are two possible triangles. The given angle must be between two known sides in order for the triangle to be unique.

6 Shorter sides are 6 cm.

7 Student’s accurate drawing

8 a

b perimeter = 285 m

9 a Sleep Inn

b Actual distances are:
4.4 km between Sleep Inn and Stay Well Inn
3.25 km between Stay Well Inn and Lie Inn
5.45 km between Lie Inn and Sleep Inn

10 Student’s accurate drawing

11 1.2 m + 3 m = 4.2 m so using a scale drawing the height is only 4.2 m. This means Patrick cannot keep his boat in the garage.
1. Student's accurate drawing.

2. The angle bisectors all meet at one point.

3. The perpendicular bisectors all meet at one point.

4. The angle bisectors all meet at one point.

5. a and b

6. The angle bisectors all meet at one point.

7. Student's accurate drawing. The shape is called a triangular prism.

8. The angle bisectors all meet at one point.
10 Student's accurate drawing. The third side is 6 cm.

11 Student's accurate drawing. The width of the base is 87 mm.

The cupboard will fit under the stairs. The doors will not open because there needs to be a further 60 cm width at the height 1.8 m.

Practise... 11.3 Similar shapes

1 A, B and F are similar to each other. C and G are similar to each other. D and I are similar to each other. E and H are similar to each other.

2 sides QP and CA; sides QR and CB; sides PR and AB

3 a angle A = 180° - (30° + 40°) = 110°
   angle Q = 180° - (30° + 40°) = 110°
   Both triangles have the same angles, so they are similar.
   b sides AC and QR; sides AB and QP; sides BC and PR

4 a side = 6 cm; angle = 60°
   b side = 6 cm
   c angle = 80°; side = 1.6 cm
   d angle x = 120°; angle y = 110°;
     side a = 3.84 cm; side b = 2.67 cm

5 a x = 80 cm; y = 19.2 cm
   b a = 4.9 cm; b = 5.5 cm; x = 74°

6 a x = 10.5 cm  c x = 8 cm
   b x = 23.65 cm  d x = 24 cm; y = 24 cm

Practise... 11.4 Congruent shapes

1 a Yes: SSS  d No  g Yes: ASA
   b Yes: SAS  e No  h Yes: RHS
   c No  f Yes: ASA  i Yes: ASA
   j Yes: ASA (even though a right-angled triangle, only the hypotenuse is known)

2 a i 81°  ii 50°  iii 49°  iv 81°  b AB

3 Jen is correct. Triangle ABC is congruent to triangle DFE.

4 BC = 4 cm; AC = 9 cm
5  a  angle $BAC = angle DEC$ (alternate angles are equal)
angle $ABC = angle EDC$ (alternate angles are equal)
$AB = ED$ (given)
triangle $ABC = triangle EDC$ (ASA)

b  $FH = JH$ (given)
$GH = IH$ (given)
age $FHG = angle JHI$ (vertically opposite angles are equal)
triangle $FHG = triangle JHI$ (SAS)

c  angle $LMK = angle PMN = 90^\circ$ (given)
$LM = PM$ (given)
$KN = MN$ (given)
triangle $LMK = triangle PMN$ (RHS)

6  $AB = CD$ (opposite sides in a rectangle are equal)
$BD = BD$ (common to both triangles)
$A = C = 90^\circ$ (angle at vertex of a rectangle is $90^\circ$)
triangle $ABD = triangle CDB$ (RHS)

7  $EF = CD$ (sides of a regular hexagon are equal)
$ED = ED$ (common to both triangles)
age $FED = angle EDC$ (interior angles of a regular hexagon are equal)
triangle $DEF = triangle CDE$ (SAS)

8  $PA = PB$ (given in method)
$AC = BC$ (given in method)
$PC$ is common to both triangles.
triangle $APC = triangle BPC$ (SSS)
age $APC = angle BPC$ (corresponding angles in congruent triangles)
age $APC + angle BPC = 180^\circ$ (angles on a straight line sum to $180^\circ$)
So, $angle APC = angle BPC = 90^\circ$

9  Consider triangles $APC$ and $BPC$:
$PA = PB$ (given in method)
$AC = BC$ (given in method)
$PC$ is common to both triangles.
triangle $APC = triangle BPC$ (SSS)
age $APC = angle BPC$ (corresponding angles in congruent triangles)

Consider triangles $APD$ and $BPD$:
age $APD = angle BPD$ (corresponding angles in congruent triangles)
$PA = PB$ (given in method)
$PD$ is common to both triangles.
triangle $APD = triangle BPD$ (SAS)
age $ADP = angle BDP$ (corresponding angles in congruent triangles)
age $ADP + angle BDP = 180^\circ$ (angles on a straight line sum to $180^\circ$)
So, $angle ADP = angle BDP = 90^\circ$

10  Consider triangle $ACD$ and triangle $BCD$:
$AC = BC$ (given in method)
$AD = BD$ (given in method)
$DC$ is common to both triangles.
triangle $ACD = triangle BCD$ (SSS)
Therefore, $angle ACD = angle BCD$ (corresponding angles in congruent triangles)

Consider triangle $ACE$ and triangle $BCE$:
$AC = BC$ (given in method)
$EC$ is common to both triangles.
age $ACE = angle BCE$ (corresponding angles in congruent triangles $ACD$ and $BCD$)
Therefore, triangle $ACE = triangle BCE$ (SSS)
age $AE = BE$ (corresponding angles in congruent triangles)
Therefore, $CD$ bisects $AB$.
age $AEC = angle BEC$ (corresponding angles in congruent triangles)
age $AEC + angle BEC = 180^\circ$ (angles on a straight line sum to $180^\circ$)
Therefore, $angle AEC = angle BEC = 90^\circ$
Therefore, $CD$ is perpendicular to $AB$.
So, $CD$ is the perpendicular bisector of $AB$. 
1 Student’s accurate drawing

2 a There are two possible triangles. The given angle must be between two known sides in order for the triangle to be unique.

b

3

4 \( x = 4.5\) cm

5 a \( \angle AEB = \angle DEC \) (vertically opposite angles are equal)
\( \angle BAE = \angle CDE \) (alternate angles are equal)
\( \angle ABE = \angle DCE \) (alternate angles are equal)
As all three pairs of angles in the triangles are equal the triangles are similar.

b \( x = 5\) cm; \( y = 12\) cm

6 \( x = 38.5\) cm; \( y = 8\) cm

7 \( OA \) and \( OB \) are both radii and therefore equal.
\( \angle TAO \) and \( \angle TBO \) are both right angles because they are tangents to a circle.
\( OT \) is common to both triangles \( TAO \) and \( TBO \).
Therefore triangles \( TAO \) and \( TBO \) have two sides and an angle in common, and must be congruent.

8 \( \angle PXB = \angle AXQ \) (vertically opposite angles)
\( X \) is the midpoint of both lines, so \( AX = BX \) and \( PX = QX \)
Therefore triangles \( AXQ \) and \( BXP \) have two sides and an angle in common, and must be congruent.

9 \( AP = PQ \) and \( PR = PB \) (sides of equilateral triangles)
\( \angle APR = 60^\circ + \angle QPR \)
\( \angle QPB = 60^\circ + \angle QPR \)
\( = \angle APR \)
Therefore triangles \( APR \) and \( QPB \) have two sides and an angle in common, and must be congruent.

\( \angle PCD = \angle RCB = (90^\circ - \angle BCP) \)
\( DC = BC \)
\( PC = RC \)

The triangles are congruent. (SAS)
Therefore, \( DP = BR \)
12 Loci

Practise... 12.1 Constructing loci

1. Arrangement A is probably the best because it has the smallest area on the left (the driver's side) that is not covered.

2. Hope is correct. The ball will bounce and change direction suddenly. The ball will follow a series of curves.

3. Kat is correct. Alice's outer corners are further away than required. Becky's straight edges at the corners are too near.

4. 

5. 

6. a–c

7. a–c Student should draw a triangle and two perpendicular bisectors.

8. Student should draw three perpendicular bisectors. The regions are as shown:

9. a and b

   c  The house must go in the shaded area and will just fit.
11 a and b

reception outside

reception inside

12 Eight sprinklers easily cover the lawn. The diagram below shows that seven should just about do it.
3 Border
Flower bed
Lawn

4 a–d

5

6 a–c Student’s drawing of a triangle and two perpendicular bisectors.

AQA Examination-style questions

1 Scale: 1 cm represents 10 km

2 Scale: 1 cm represents 10 km
13 Cubic, circular and exponential functions

**Practise...** 13.1 Cubic functions

1  a  $(-1, 5), (2.5, -17)$
   b  $x = -2, x = 0, x = 4$
2  a  $x = -0.5$  b  $x = 2.2$  c  $x = 2.7$
3  a  $x = -1.8, x = -0.4, x = 1.3$
   b  $x = -2, x = 0, x = 1$
   c  $x = -2.1$

<table>
<thead>
<tr>
<th>$x$</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^3$</td>
<td>-27</td>
<td>-8</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>8</td>
<td>27</td>
<td>64</td>
<td>125</td>
</tr>
<tr>
<td>$-3x^2$</td>
<td>-27</td>
<td>-12</td>
<td>-3</td>
<td>0</td>
<td>-3</td>
<td>-12</td>
<td>-27</td>
<td>-48</td>
<td>-75</td>
</tr>
<tr>
<td>+ 4</td>
<td>+4</td>
<td>+4</td>
<td>+4</td>
<td>+4</td>
<td>+4</td>
<td>+4</td>
<td>+4</td>
<td>+4</td>
<td>+4</td>
</tr>
<tr>
<td>$y$</td>
<td>-50</td>
<td>-16</td>
<td>0</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>4</td>
<td>20</td>
<td>54</td>
</tr>
</tbody>
</table>

**Practise...** 13.2 Reciprocal functions

1  a  i  $x = -1.5$
   ii  $x = 1.5$
   iii  $x = 6$
   b  A solution for $\frac{3}{x} = 0$ cannot be found because it is undefined.

A theme park rides has more than two turns whereas a cubic function has only two.
2 a

<table>
<thead>
<tr>
<th>x</th>
<th>-20</th>
<th>-15</th>
<th>-10</th>
<th>-5</th>
<th>-2</th>
<th>-1</th>
<th>-0.1</th>
<th>0</th>
<th>0.1</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-0.2</td>
<td>-0.27</td>
<td>-0.4</td>
<td>-0.8</td>
<td>-2</td>
<td>-4</td>
<td>-40</td>
<td>40</td>
<td>1</td>
<td>2</td>
<td>0.8</td>
<td>0.4</td>
<td>0.27</td>
<td>0.2</td>
<td></td>
</tr>
</tbody>
</table>

b

y = \frac{4}{x}

b

y = \frac{1}{x}

y = \frac{2}{x}

y = \frac{3}{x}

c i x = 1

ii x = -1.3

iii x = 0.2

iv x = -0.13

d 5.1 cm

e correct statements: i, iii and iv; incorrect statement: ii

f Both have reciprocal shape. Becky’s graph is further from the axes.

3 a

a = \frac{46}{b}

b An area cannot be negative.

c

\frac{d}{b} = 46

4 a

b As the numerator increases, the graphs move further from the axes.
Practise... 13.3 Circular functions

1 a

<table>
<thead>
<tr>
<th>x</th>
<th>−360°</th>
<th>−270°</th>
<th>−180°</th>
<th>−90°</th>
<th>−45°</th>
</tr>
</thead>
<tbody>
<tr>
<td>sinx</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>−1</td>
<td>−0.71</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>0°</th>
<th>45°</th>
<th>90°</th>
<th>180°</th>
<th>270°</th>
<th>360°</th>
</tr>
</thead>
<tbody>
<tr>
<td>sinx</td>
<td>0</td>
<td>0.71</td>
<td>1</td>
<td>0</td>
<td>−1</td>
<td>0</td>
</tr>
</tbody>
</table>

b

(c) i −330°, −210°, 30°, 150°

ii −352°, −188°, 8°, 172°

iii −307°, −233°, 53°, 127°

iv −315°, −225°, 45°, 135°

v −281°, −259°, 79°, 101°

2 a

−314°, −46°, 46°, 314°

b −240°, −120°, 120°, 240°

c −270°, −90°, 90°, 270°

d −360°, 0°, 360°

e −180°, 180°

f i −315°, −45°, 315°

ii −260°, −100°, 260°

iii −300°, −60°, 300°

iv −330°, −30°, 330°

v −346°, −14°, 346°

vi −225°, −135°, 135°

vii −198°, −162°, 162°

viii −286°, −74°, 74°

ix −352°, −8°, 8°

b

(c) i true

ii false

iii false

iv true

v false

vi false

vii false

viii true

Practise... 13.4 Exponential functions

1 a

i 2.2

ii 11

b

i x = 1.3

ii x = 1.43

iii x = 1.86

2 a

<table>
<thead>
<tr>
<th>x</th>
<th>−2</th>
<th>−1</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0.1</td>
<td>0.3</td>
<td>1</td>
<td>1.7</td>
<td>3</td>
<td>5.2</td>
<td>9</td>
</tr>
</tbody>
</table>

b

(c) i y = 3x

ii y = −3x

iii y = 2x

iv y = −2x

v y = 0.5x

vi y = −0.5x

vii y = 0.1x

viii y = −0.1x

© Nelson Thornes 2012
Practise… 13.5 Graph recognition and graphs of loci

1 a C
   b A
   c D
   d B

2 a B
   b D
   c C
   d A

3 Chris is correct
   Anna is wrong – her graph would have to pass through (0, 4)
   Bindia is wrong, since when \( x = 1 \),
   \( y = 1^3 - 4(1) = -3 \)
   Dave is wrong – his graph would also pass through (0, 4)

4 a S
   b P
   c R
   d Q

5 a

b (3.4, 3.7) and (−1, −4.9)
Assess 13

1 a

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>x^3</td>
<td>27</td>
<td>8</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>-8</td>
<td>-27</td>
</tr>
<tr>
<td>-5</td>
<td>-5</td>
<td>-5</td>
<td>-5</td>
<td>-5</td>
<td>-5</td>
<td>-5</td>
<td>-5</td>
</tr>
<tr>
<td>y</td>
<td>22</td>
<td>3</td>
<td>-4</td>
<td>-5</td>
<td>-6</td>
<td>-13</td>
<td>-32</td>
</tr>
</tbody>
</table>

b

\[ y = x^3 - 3x - 2 \]

2 a i x = 1.2 ii x = 2 iii x = -12

b The value is undefined.

3 a i x = -296°, -244°, 64°, 116°
   ii x = -150°, -30°, 210°, 330°
   iii x = -348.5°, -191.5°, 11.5°, 168.5°
   iv x = -270°, 90°
   v x = -90°, 270°

b i -315°, -225°, 135°
   ii -280°, -260°, 100°
   iii -340°, -200°, 160°
   iv -300°, -240°, 120°
   v -135°, -45°, 315°
   vi -45°, -135°, 225°

4 x = -2, x = 2, x = 3

5 a i x = 0.5 ii x = 8 iii x = 2.83
   b i x = -0.5 ii x = 0 iii x = 1.16

6 a

\[ x = -157.5°, x = -67.5°, x = 22.5°, \\
    x = 112.5°, x = 202.5° \]

7 a

<table>
<thead>
<tr>
<th>x</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-54</td>
<td>-20</td>
<td>-4</td>
<td>0</td>
<td>-2</td>
<td>-4</td>
<td>0</td>
<td>16</td>
<td>50</td>
</tr>
</tbody>
</table>

b

\[ y = x^3 - 3x - 2 \]

8 A y = -2x  4
   B y = -\frac{5}{x}  1
   C y = \cos 3x  2
   D y = 2x^3  3

9 a smooth curve drawn from given values
   b completed table of values taken from the
      student's curve
   c exponential
   d i value from student's graph
      ii value from student's graph
      iii value from student's graph
**AQA**

**Examination-style questions**

1 a

<p>| | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>-2</td>
<td>-1.5</td>
<td>-1</td>
<td>-0.5</td>
<td>0</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>y</td>
<td>-6</td>
<td>-1.875</td>
<td>0</td>
<td>0.375</td>
<td>0</td>
<td>-0.375</td>
<td>0</td>
</tr>
</tbody>
</table>

b

![Graph of y = x^3 - x](image-url)

1 2 3 4

12 14 16 18 20

-4 -2 0 2 4

x

C x = 1.8
## 14 Trigonometry 2

### Practise... 14.1 The sine rule

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a</td>
<td>7.5 cm</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>10.5 cm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a</td>
<td>70.5° or 109.5°</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>46.1°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>44.4°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>6.2 cm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>53.4°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>5.4 km from X; 5.1 km from Y</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>127.6°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>32 m (This is an estimate, so giving the answer to the nearest metre is more than adequate.)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Practise... 14.2 The cosine rule

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a</td>
<td>6.3 cm</td>
<td>b</td>
<td>8.9 cm</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>7.3 cm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a</td>
<td>76.7°</td>
<td>b</td>
<td>53.8°</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>107.6°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>7.4 cm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>75.9°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>a</td>
<td>7.2 cm</td>
<td>b</td>
<td>5.7 cm</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>47.3°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>BEC is the shorter path by about 28.2 m.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>5.3 m</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Practise... 14.3 Finding the area of a triangle using trigonometry

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a</td>
<td>28.4 cm²</td>
<td>b</td>
<td>42.4 cm²</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>16.4 cm²</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a</td>
<td>29.7 cm²</td>
<td>b</td>
<td>22.2 cm²</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a</td>
<td>41.8° or 138.2°</td>
<td>b</td>
<td>6.2 cm</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>7803 m²</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>22.8 cm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Assess 14

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a</td>
<td>6.4 cm</td>
<td>c</td>
<td>10.8 cm</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>45.3°</td>
<td>d</td>
<td>49.0°</td>
</tr>
<tr>
<td>2</td>
<td>a</td>
<td>7.74 kg</td>
<td>b</td>
<td>198.4 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td>132.8°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>31.6°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>45.1 cm²</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>45°</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### AQA Examination-style questions

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>13.9 cm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>a</td>
<td>13.9 cm</td>
<td>b</td>
<td>40.3°</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>256.2 m</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
15 Vectors

1 a (2 0) (0 5) (0 -2)  
b (3 1) (2 -2) (-3 6) (-1) 

2 \( \vec{CD} = \vec{KL} = \vec{OP} = \left( \begin{array}{c} 3 \\ 2 \end{array} \right) \)  
\( \vec{HG} = \vec{IJ} = \left( \begin{array}{c} -2 \\ 3 \end{array} \right) \)

3 a i (4 -2), iii (4 -2) and iv (10 -5) are parallel to \( \vec{AB} \).
   b i (4 -2) = 2\( \vec{AB} \), iii (4 -2) = -2\( \vec{AB} \) and iv (10 -5) = -5\( \vec{AB} \)

4 a

b \( \vec{AC} = \left( \begin{array}{c} 4 \\ -2 \end{array} \right) \)

5 a True because \( \left( \begin{array}{c} 5x \\ -10y \end{array} \right) = 2.5 \left( \begin{array}{c} 2x \\ -4y \end{array} \right) \)
   b True because \( \left( \begin{array}{c} 0.2x \\ 1.2y \end{array} \right) = -5 \left( \begin{array}{c} -x \\ -6y \end{array} \right) \)
   c False. The second vector is not a multiple of the first.

6 a i

b hexagon
Practise... 15.2 Vector geometry

1 a a + b  d –c – b  
b –d – c  e a = –b – c – d  
c d + a

2 a b – a  b d – 2c  c 5f – 3e

3 a a = \overrightarrow{OA} = \overrightarrow{CB} = \overrightarrow{EF} = \overrightarrow{DO}  
b b = \overrightarrow{AB} = \overrightarrow{OC} = \overrightarrow{ED} = \overrightarrow{FO}  
c \overrightarrow{DC} = \overrightarrow{OB} = \overrightarrow{EO} = \overrightarrow{FA} = a + b

4 a \overrightarrow{NM} = c – b  
b \overrightarrow{AB} = 3b – 3c = –3(c – b)  
c They are parallel because \overrightarrow{AB} is a multiple of \overrightarrow{NM}.

5 a i 2a  v a + b  
   ii 2a + b  vi a  
   iii a – b  vii a + b  
   iv \overrightarrow{v} \overrightarrow{b}  
b \overrightarrow{AE} = \overrightarrow{BC} = a + b  
   Therefore they are equal and parallel.  
   \overrightarrow{AB} = \overrightarrow{EC} = a  
   Therefore they are equal and parallel.  
   Therefore \overrightarrow{AECB} is a parallelogram.

6 a i a + b  iv c  
   ii –b + c  v –a – b + c  
   iii b – a  vi b – a  
   b \overrightarrow{CF} = a + b

7 a i –a + 1\overrightarrow{b}  v \frac{1}{3}(a + b)  
   ii –b + a  vi \frac{1}{2} \overrightarrow{b}  
   iii \frac{1}{2}(–b + a)  vii \frac{1}{6}(–2a + b)  
   iv \frac{1}{2}(a + b)  
b \overrightarrow{YS} = –a + \frac{1}{2} \overrightarrow{b} \text{ and } \overrightarrow{RS} = \frac{1}{6}(–2a + b)  
   (–a + \frac{1}{2}b) : \frac{1}{6}(–2a + b)  
   (–a + \frac{1}{2}b) : 2 \times \frac{1}{6}(–a + \frac{1}{2}b)  
   1 : \frac{2}{6}  
YS : RS = 3 : 1
**Assess 15**

1. a $\left( \begin{array}{c} 1 \\ 1 \end{array} \right)$  
   b $\left( \begin{array}{c} 4 \\ 0 \end{array} \right)$  
   c $\left( \begin{array}{c} -5 \\ 6 \end{array} \right)$

2. $A = I = \left( \begin{array}{c} 2 \\ 0 \end{array} \right)$  
   $C = E = \left( \begin{array}{c} 3 \\ 2 \end{array} \right)$  
   $B = F = \left( \begin{array}{c} 1 \\ -3 \end{array} \right)$

3. a

4. The vectors for parts b, d and e are parallel to $\overrightarrow{AB}$.
   
   b $\left( \begin{array}{c} 4 \\ -6 \end{array} \right)$  
   d $\left( \begin{array}{c} -2 \\ 3 \end{array} \right)$  
   e $\left( \begin{array}{c} -2 \\ -3 \end{array} \right)$

   and e $\left( \begin{array}{c} 20 \\ -30 \end{array} \right) = 10 \left( \begin{array}{c} 2 \\ -3 \end{array} \right)$

5. a $-3b$  
   c $2a - 3b$  
   b $-3b + 2a$  
   d $-6b + 2a$

6. a $\overrightarrow{AB} = 2b$  
   b $\overrightarrow{CD} = -4c + 2d$

7. a $b - a$  
   b $\overrightarrow{MC} = 2a$  
   $\overrightarrow{NB} = 2b$  
   c $1:2$  
   d They are similar triangles.

8. a $\overrightarrow{MB} = a$  
   b $\overrightarrow{BO} = \frac{9}{5}b$  
   c $\overrightarrow{MO} = a + \frac{9}{5}b$  
   d $\overrightarrow{NO} = 2a + \frac{1}{5}b$

---

**Examination-style questions**

1. a $-2a + 2b$  
   b $\overrightarrow{PQ} = \overrightarrow{PA} + \overrightarrow{AQ}$  
      $= a + \frac{3}{2}b - \frac{1}{2}a$  
      $= \frac{1}{2}a + \frac{3}{2}b$

2. c $\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$  
      $= 2a + b - a$  
      $= a + b$  
   d $\overrightarrow{OM}$ is a multiple of $\overrightarrow{PQ}$. Therefore $\overrightarrow{OM}$ and $\overrightarrow{PQ}$ are parallel and OPQM is a trapezium.
16

Area and volume 2

Practise... 16.1 Arcs and sectors

1. a. 8.7 cm  
   b. 31.4 cm  
   c. 31.6 cm

2. a. 21.8 cm²  
   b. 157.1 cm²  
   c. 105.8 cm²

3. a. 15.7 cm  
   b. 10.5 cm  
   c. 11.6 cm

4. 6°

5. a. 26.2 cm²  
   b. 240.8 cm²  
   c. 1.8 m²

6. a. 41.1 cm²  
   b. 23.7 cm²  
   c. 8722 cm²

7. a. 21.5 cm²  
   b. 85.8 cm²  
   c. 57.1 cm²

8. perimeter = 20.9 cm; area = 30.7 cm²

9. radius of base of cone = \( \frac{R}{2} \)
   
   \[ h = \sqrt{R^2 - \left(\frac{R}{2}\right)^2} = \frac{R}{2} \sqrt{3} \]

10. £105.41

11. 2.59 m

Practise... 16.2 Volumes and surface areas of pyramids, cones and spheres

1. a. 523.6 cm³  
   b. 2094.4 cm³  
   c. 144 cm³

2. 68.3 cm²

3. a. volume of small cone = 513.1 cm³; volume of large cone = 4105.0 cm³;  
   b. \( \frac{1}{8} \)

4. a. 1:3  
   b. 1:9  
   c. 1:27

5. a. i. 500 cm³  
   ii. 192.4 cm³  
   b. 5.6 cm

6. 10.4 cm

7. 2.4 cm

8. 226.7 cm³

9. cone  
   flattened cone

   \[ 2\pi = \frac{x}{360} \times 2\pi l, \text{ so } \frac{x}{360} = \frac{r}{l} \]

   Area = \( \frac{x}{360} \times \pi l^2 = \frac{r}{l} \times \pi l^2 = \pi rl \)

10. 9.1 litres

11. a. 0.949 m³  
   b. 4.46 m²

Assess 16

1. a. 12.2 cm²  
   b. 263 cm²  
   c. 35.2 cm²

2. 72°

3. They have the same volume.

4. 16 m

5. 300000 m³

6. area = 2.33 cm²; perimeter = 6.14 cm

7. 628 ball bearings; 0.29 cm³

8. a. \( \frac{27}{125} \)  
   b. 167 cm³

9. 21900 m³

10. 622 litres

AQA Examination-style questions

1. a. 5 cm  
   b. 9.6 cm
## Transforming functions

### 1. a

1. Graph i
   - **a** \( y = x \)
   - **b** \( A: y = x + 5 \)
   - **c** translations

2. Graph ii
   - **a** \( y = x^2 \)
   - **b** \( A: y = x^2 + 4 \)
   - **c** translations

3. Graph iii
   - **a** i \( D: y = 2 \sin x \)
   - **b** A: translation with vector \( \begin{pmatrix} 0 \\ 1 \end{pmatrix} \)
   - **c** translations

4. Graph iv
   - **a** \( y = 3x \)
   - **b** \( A: y = 3 \sin x + 1 \)
   - **c** translations

5. Graph v
   - **a** i \( y = \sin x \)  
   - **b** A: translation with vector \( \begin{pmatrix} 0 \\ 1 \end{pmatrix} \)
   - **c** translations

6. Graph vi
   - **a** \( y = \cos x \)  
   - **b** B: translation with vector \( \begin{pmatrix} 0 \\ 2 \end{pmatrix} \)
   - **c** translations

### 2. Graph i

- **a** \( y = x \)
- **b** \( A: y = x + 5 \)
- **c** translations

### 3. Graph ii

- **a** i \( D: y = 2 \sin x \)
- **b** A: translation with vector \( \begin{pmatrix} 0 \\ 1 \end{pmatrix} \)
- **c** translations

### 4. Graph iii

- **a** \( y = 3x \)
- **b** \( A: y = 3 \sin x + 1 \)
- **c** translations

### 5. Graph iv

- **a** \( y = 3x \)
- **b** \( A: y = 3 \sin x + 1 \)
- **c** translations

### 6. Equations on diagrams in parts b and c

- **b** \( A: y = x^2 + 4 \)
- **c** translations

- **c** \( A: y = x^2 + 4 \)
- **c** translations

- **c** \( A: y = x^2 + 4 \)
- **c** translations
5 a  i  \( y = \frac{1}{2} + 1 \)  

ii  \( y = \frac{1}{2} + 2 \)  

iii  \( y = \frac{1}{2} - 1 \)  

b  i–iii translation  

iv one-way stretch in \( y \)-direction, scale factor 2  

v one-way stretch in \( y \)-direction, scale factor \( \frac{1}{2} \)  

vi reflection in \( x \)-axis  

6 a  i reflection in \( x \)-axis  

ii one-way stretch in \( y \)-direction, scale factor 2  

\[ y = \frac{2}{x} \]
iii  translation vector \( \begin{pmatrix} 0 \\ 1 \end{pmatrix} \)

\[ y = f(x-1) \]

b i reflection in x-axis

\[ y = -f(x) \]

ii one-way stretch in y-direction, scale factor 2

\[ y = 2f(x) \]

iii translation vector \( \begin{pmatrix} 0 \\ 1 \end{pmatrix} \)

\[ y = f(x) \]

7 Faisal should choose Hamish because the graph has been stretched in the y-direction by scale factor 2, then translated with vector \( \begin{pmatrix} 0 \\ 1 \end{pmatrix} \).

James is not correct. \( y = 3\cos x \) is a one-way stretch in the y-direction with scale factor 3 so the graph has a minimum value of −3, not −1.

Ken is not correct. \( y = \cos x + 2 \) is always greater than 0 and the graph given has the same negative y-value.

8 a One-way stretch in the y-direction with scale factor \( a \) followed by a translation with vector \( \begin{pmatrix} 0 \\ b \end{pmatrix} \).

b Student's own check

9 \[ y = a(x - b)^2 + c \]

Student's own results

Changing \( b \) and \( c \) will give translations by vector \( \begin{pmatrix} b \\ c \end{pmatrix} \).

Changing \( a \) will give a one-way stretch, scale factor \( a \) in the y-direction.
1 a translation \[ \begin{pmatrix} 0 \\ 1 \end{pmatrix} \]
1 b translation \[ \begin{pmatrix} 0 \\ 1 \end{pmatrix} \]

1 c i translation with vector \[ \begin{pmatrix} 0 \\ 2 \end{pmatrix} \]

2 a \[ y = \frac{1}{2}x + 2 \]
2 b A: \[ y = f(x) + 3 \]
B: \[ y = f(x) - 3 \]
C: \[ y = f(x) - 7 \]
2 c A: \[ y = \frac{1}{2}x + 5 \]
B: \[ y = \frac{1}{2}x - 1 \]
C: \[ y = \frac{1}{2}x - 5 \]

3 No, Hamilton is not correct. \[ y = f(x - 1) \] is a translation by vector \[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \]