Answers

1 Angles and areas

Practise... 1.1 Angles and parallel lines

1 \( x = 135^\circ \) (corresponding angles)  
  \( z = 140^\circ \) (corresponding angles)  
  \( q = 95^\circ \) (corresponding angles)  
  \( r = 80^\circ \) (corresponding angles, or angles on a straight line = 180°)  
  \( s = 100^\circ \) (corresponding angles, or angles on a straight line = 180°)

2 \( x = 105^\circ \) (allied angles)  
  \( y = 83^\circ \) (alternate angles)  
  \( z = 142^\circ \) (allied angles)

3 \( a = 72^\circ \) (alternate angles)  
  \( b = 72^\circ \) (vertically opposite angles or corresponding angles)  
  \( c = 115^\circ \) (corresponding angles)  
  \( d = 65^\circ \) (angles on a straight line)  
  \( e = 115^\circ \) (vertically opposite angles or alternate angles with c)

\[ \begin{align*}
  f &= 88^\circ \text{ (vertically opposite angles or corresponding angles)} \\
  g &= 60^\circ \text{ (allied angles)} \\
  h &= 60^\circ \text{ (vertically opposite angles or corresponding angles)} \\
  i &= 88^\circ \text{ (corresponding angles)}
\end{align*} \]

4 \( a \) Yes, corresponding angles are equal.  
  \( b \) Yes, allied angles add up to 180°.

5 \( a \) \( a = 105^\circ, b = 68^\circ, c = 112^\circ \)  
  \( b \) \( d = 137^\circ \)  
  \( c \) \( p = 85^\circ, q = 85^\circ, r = 85^\circ, s = 95^\circ \)

6 \( a \) \( x = 50^\circ, y = 50^\circ \)  
  \( b \) \( x = 61^\circ, y = 58^\circ \)

Practise... 1.2 Bearings

1 \( a \) 115°  
  \( b \) 029°  
  \( c \) 270°  
  \( d \) 286°  
  \( e \) 009°  
  \( f \) 228°

2 Student’s accurate diagrams

3 258°

4 070°

5 Student’s accurate drawing of a bearing of 282° for a distance of 9.2 km

6 \( a \) \( H_1 \)  
  \( b \) 080°  
  \( c \) 205°  
  \( d \) 212°

Practise... 1.3 Area of parallelograms, trapeziums and triangles

1 \( a \) 96 cm²  
  \( b \) 450 mm²  
  \( c \) 43.32 cm²  
  \( d \) 26 mm²

2 \( a \) 24 cm²  
  \( b \) 11.52 m²  
  \( c \) 21 cm²  
  \( d \) 45 cm²

3 The parallelogram has the largest area. (area of rectangle = 36 cm²; area of triangle = 24 cm²; area of parallelogram = 40 cm²)

4 13.5 cm

5 \( a \) Leanne  
  \( b \) Kieran has not divided by 2. Javed has found the perimeter. Megan has not used the perpendicular height.

6 \( a \) \( P = 22.4\text{ m}; P = 31.6\text{ cm}; P = 66\text{ mm}\)  
  \( b \) \( A = 27\text{ m}^2; A = 36\text{ cm}^2; A = 247.5\text{ mm}^2\)

7 28 cm

8 \( a \) 58.8 mm²  
  \( b \) 16.5 m²  
  \( c \) 3186 mm²  
  \( d \) 12.04 cm²

9 \( a \) 24 cm²  
  \( b \) 64.5 cm²

10 \( h = 3\text{ cm}\)

11 10 cm by 14 cm; 55.4% is wasted.

12 Yes, it will cost him £2.88.

13 \( x = 9 \) (cm)
### Practise... 1.4 Circumference and area of a circle

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<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
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<tbody>
<tr>
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<td>31.4 cm</td>
<td>7.5 cm</td>
<td>25.1 m</td>
<td>109.3 cm</td>
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<td>47.1 mm</td>
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<td>201.1 mm</td>
<td>54.0 cm</td>
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<td>2</td>
<td>7.5 cm</td>
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<td>25.1 m</td>
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<td>54.0 cm</td>
<td>201.1 mm</td>
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<td>3</td>
<td>20 cm</td>
<td>25.1 m</td>
<td>201.1 mm</td>
<td>54.0 cm</td>
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<tr>
<td>4</td>
<td>50.27 m²</td>
<td>951.15 cm²</td>
<td>3216.99 mm²</td>
<td>298.65 cm²</td>
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<tr>
<td>5</td>
<td>78.54 cm²</td>
<td>4.52 cm²</td>
<td>176.71 mm²</td>
<td>298.65 cm²</td>
</tr>
<tr>
<td>6</td>
<td>P = 46.3 cm; A = 127.2 cm²</td>
<td>P = 24.7 cm; A = 36.2 cm²</td>
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### Practise... 1.5 Compound shapes

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<tr>
<td>1</td>
<td>65 cm²</td>
<td>363 mm²</td>
<td>51 cm²</td>
<td>28 m²</td>
<td>100 m²</td>
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<tr>
<td></td>
<td>54.76 m²</td>
<td>28 m²</td>
<td>100 m²</td>
<td></td>
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<tr>
<td>2</td>
<td>i 39.8 cm</td>
<td>ii 63.5 cm²</td>
<td>i 42.8 cm</td>
<td>ii 80 cm²</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ii 80 cm²</td>
<td>i 48 cm</td>
<td>ii 138 cm²</td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ii 92 cm²</td>
<td>i 35.2 cm</td>
<td></td>
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<tr>
<td>3</td>
<td>28.9 cm²</td>
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<tr>
<td>4</td>
<td>320 cm²</td>
<td>39.62 m²</td>
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<tr>
<td>5</td>
<td>40 cm² is required.</td>
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<tr>
<td>6</td>
<td>No, she needs 19.75 m² whereas two tins cover 19 m².</td>
<td>£55.93</td>
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### Assess 1

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<th>e</th>
<th>f</th>
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<tbody>
<tr>
<td>1</td>
<td>72° (corresponding angles)</td>
<td>108° (angles on a straight line = 180°)</td>
<td>105° (alternate or vertically opposite angles)</td>
<td>68° (angles on a straight line and corresponding angles)</td>
<td>18°</td>
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<tr>
<td>2</td>
<td>064°</td>
<td>192°</td>
<td>314°</td>
<td></td>
<td></td>
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<tr>
<td>3</td>
<td>244°</td>
<td>012°</td>
<td>134°</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>4</td>
<td>x = 39°; bearing of C from B = 309°</td>
<td></td>
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<tr>
<td>5</td>
<td>24 cm²</td>
<td>60 cm²</td>
<td>24 cm²</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>6</td>
<td>C = 49 m</td>
<td>A = 191.1 m²</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>C = 27 cm</td>
<td>A = 58.09 cm²</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>C = 94.2 mm</td>
<td>A = 706.9 mm²</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>C = 207.3 cm</td>
<td>A = 3421.2 cm²</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>7</td>
<td>909.6 cm²</td>
<td></td>
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<tr>
<td>8</td>
<td>No, corresponding angle to 52° would give angles on a straight line = 126° + 52° = 178° This should equal 180°.</td>
<td></td>
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<tr>
<td>9</td>
<td>Yes, he needs 7964 m² and he has bought 80 × 100 m² = 8000 m²</td>
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### AQA Examination-style questions

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>128° (corresponding angles)</td>
<td>95° (alternate angles and then angles on a straight line gives y = 180° – 85°)</td>
</tr>
<tr>
<td>2</td>
<td>157 cm² (3 s.f.)</td>
<td></td>
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<tr>
<td>3</td>
<td>48 cm²</td>
<td></td>
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2
Prime factors

**Practise...** 2.1 Factors and multiples

1  a  9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99, 108. Digit sums = 9  
   b  153, 207, 378, 3789  
   c  Student's own checking  
   d  digit sum of 9  
   e  even number with digit sum of 9  

2  a  627 is not even and does not end in 0 or 5.  
   b  A number is only a factor if it divides into 627 a whole number of times.  
   c  
   \[
   \begin{array}{c|c|c|c|c} 
   627 \div & 1 & 3 & 11 & 19 \\
   \hline 
   \text{gives} & 627 & 209 & 57 & 33 \\
   \end{array}
   
   \]  
   d  627 does not divide by any other numbers up to \( \sqrt{627} \) (25)  

3  a  1, 3  
   b  1, 2, 4  
   c  1, 7  
   e  1, 5  
   d  1, 2, 3, 4, 6, 12  
   f  1, 2, 3, 6  

4  a  true  
   b  false  
   c  false  
   d  true  

5  1, 2, 7, 14  

6  a  6  
   b  8  
   c  12  
   d  25  
   e  2  

7  a  Any two numbers with HCF of 7 (e.g. 35 and 42).  
   b  Any three numbers with HCF 15 (e.g. 15, 30 and 45).  

8  a  24  
   b  45  
   c  60  
   d  30  
   e  72  
   f  84

9  No, 12 is the HCF of 24 and 60, not the LCM.  

10 a  96  
   b  300  
   c  300  

11 Any two numbers that have a common factor other than 1, e.g. 6 and 8 with LCM = 24  
   (which is not \( 6 \times 8 \)).  

12 a  9  
   b  14  
   c  15

13 a  i  156  
   b  84  
   c  15

14 a  3  
   b  42

15 12 days (assuming 12-hour clock) or 24 days  
   (assuming 24 hour clock)

16 a  28 is the next perfect number because  
   \[ 1 + 2 + 4 + 7 + 14 = 28 \]  
   b  7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 19, 21, 22, 23, 25, 26 and 27 are deficient numbers.  
   12, 18, 20 and 24 are abundant numbers.  
   Note: Perfect numbers end in either 28 or 6 preceded by an odd number. They are very special numbers and only a few are known.  
   After 28, the next perfect numbers are 496 then 8128 then 33550336.  
   c  Their only other factor is 1 (apart from themselves).

**Practise...** 2.2 Prime numbers and prime factors

1  23, 29, 31, 37

2  51 is not prime; it has four factors (1, 3, 17, 51).  
   55 is not prime; it has four factors (1, 5, 11, 55).  
   57 is not prime; it has four factors (1, 3, 19, 57).

3  a  83 + 89 = 172  
   b  89 − 83 = 6

4  a  2 × 7  
   b  3 × 11  
   c  5 × 13  
   d  7 × 13

5  a  \( 2^2 \times 3 \)  
   b  \( 2^2 \times 3^2 \)  
   c  \( 2^2 \times 3 \times 7 \)  
   d  \( 2^2 \times 3 \)  
   e  \( 2^2 \times 3 \times 11 \)  
   f  \( 2^2 \times 3^2 \)  
   g  \( 2^3 \times 3 \)  
   h  \( 2^3 \times 5 \times 13 \)

6  a  \( 30 = 2 \times 3 \times 5 \)  
   b  i  \( 60 = 2^2 \times 3 \times 5 \)  
   ii  \( 90 = 2 \times 3^2 \times 5 \)  
   iii  \( 210 = 2 \times 3 \times 5 \times 7 \)  
   iv  \( 300 = 2^2 \times 3 \times 5^2 \)  
   v  \( 750 = 2 \times 3 \times 5^3 \)

7  a  i  \( 270 = 2 \times 3^3 \times 5 \)  
   ii  \( 230 = 2 \times 3^2 \times 5^2 \)  
   b  i  \( 2 \times 3^2 \times 5 = 90 \)  
   ii  \( 2 \times 3^2 \times 5^2 = 1350 \)

8  a  i  \( 2 \times 3 \times 7 \)  
   ii  \( 2^2 \times 3 \times 5 \)  
   iii  \( 2^3 \times 3 \)  
   b  i  \( 2 \times 3 = 6 \)  
   ii  \( 2^3 \times 3 \times 5 \times 7 = 2520 \)

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<tr>
<td>2^n - 1</td>
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<td>3</td>
<td>7</td>
<td>15</td>
<td>31</td>
<td>63</td>
<td>127</td>
<td>255</td>
<td>511</td>
<td>1023</td>
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<td>Prime</td>
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<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
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Note: The Mersenne numbers in the table are prime when \( n \) is prime, but this is not always the case. For example, when \( n = 11 \), the Mersenne number is 2047 which equals \( 23 \times 89 \).

10 1895 is a multiple of 5; 2356 is an even number; 5739 is a multiple of 3. (3457 is prime.)

11 a i 1080 = \( 2^3 \times 3^3 \times 5 \)
   ii 1800 = \( 2^3 \times 3^2 \times 5^2 \)
   iii 8100 = \( 2^2 \times 3^4 \times 5^2 \)
   b HCF = \( 2^2 \times 3^2 \times 5^1 \) LCM = \( 2^3 \times 3^4 \times 5^2 \)

12 72

Assess 2

1 1, 2, 3, 4, 6, 12
2 a 21 or 35 b 97
3 a 8 b 48
4 \( 2^3 \times 7^2 \)
5 36 is not a multiple of 7; 42 is not a square number; 49 is not a multiple of 6.

13 a i \( 3 \times 37 \) ii \( 11 \times 29 \) iii \( 13 \times 59 \)
   b \( 2 \) is the only even prime number.
   c Try primes up to the square root. Look at the units digit and find numbers that have a product with this units digit.

14 a 7 b 30 c 73 d Student’s own work

15 Possibilities include:
   a i \( 8 = 3 + 5 \) ii \( 20 = 3 + 17 \) iii \( 42 = 5 + 37 \)
   b i \( 12 = 2 + 3 + 7 \) ii \( 25 = 3 + 5 + 17 \) iv \( 99 = 3 + 7 + 89 \)

AQA Examination-style questions

1 84 seconds
2 \( 3 \times 7 = 21 \)

10 a 41, 43, 47, 53, 61, 71, 83 b 41

13 a i \( 3 \times 37 \) ii \( 11 \times 29 \) iii \( 13 \times 59 \)
   b 2 is the only even prime number.
   c Try primes up to the square root. Look at the units digit and find numbers that have a product with this units digit.

14 a 7 b 30 c 73 d Student’s own work

15 Possibilities include:
   a i \( 8 = 3 + 5 \) ii \( 20 = 3 + 17 \) iii \( 42 = 5 + 37 \)
   b i \( 12 = 2 + 3 + 7 \) ii \( 25 = 3 + 5 + 17 \) iv \( 99 = 3 + 7 + 89 \)

AQA Examination-style questions

1 84 seconds
2 \( 3 \times 7 = 21 \)
Collecting data

Practise... 3.1 Types of data

1

<table>
<thead>
<tr>
<th>Person</th>
<th>Qualitative</th>
<th>Quantitative</th>
<th>Discrete</th>
<th>Continuous</th>
<th>Primary</th>
<th>Secondary</th>
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<tr>
<td>Nat</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
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<tr>
<td>Prita</td>
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2  a discrete  d discrete  g continuous  
b discrete  e continuous  h continuous  
c continuous  f discrete  i discrete

3  a A sample is a part of the population that it is hoped has the same features as the population.
b to save time and money
   Items may be ‘used up’ when sampled so this does not want to be too large scale or items are wasted. Also, it is time consuming and possibly expensive to take a particularly large sample.

4 (many possible answers)
   a the colour/style/type of pattern
   b number of goals scored/league position/match attendance
   c speed/time to leave Earth's atmosphere/age of astronauts
   d your mark/your answers/your feelings when doing it
   e life expectancy/types of rat food/average length it might grow to

5 Answers depend upon the information obtained from newspaper or internet.

6 How: a sample from the production line needs to be taken across several time periods – it is not sufficient to simply test, say, the first 20 bulbs produced one morning. May make mention of use of random sampling in answer.

   Why: Testing will use up the light bulbs so testing the whole population would mean there were no light bulbs left (which would leave everyone in the dark).

Practise... 3.2 Data collection methods

1 When rewriting the questions, there are many possible options.
   a Too few groups and no box for an answer of 1 hour.
      Rewrite response boxes with groups.
      Less than 1 hour/1 or more but less than 2/2 or more but less than 3/3 or more.

   b only two of many choices
      It is probably best to rewrite this as an open question asking for their own answer as there are so many possible options.

   c Most people will do more than one thing so the question should probably ask which they do most.
      Which of these do you do most of in your leisure time? (tick one box)
      Then offer same choices plus the choice of ‘other’.

   d A biased question leading people into saying ‘yes’. Options could also be better.
      Do you like football? Yes I love it/Yes I quite like it/No, I am not keen/No, I hate it!

   e This is a personal question that should be avoided unless essential. If it had to be asked, additional options should be offered.
      less than £10 000/£10 000 to £19 999/£20 000 to £29 999/£30 000 to £39 999/£40 000 or more

   f The question needs a time scale so that everyone is not left to judge how often ‘rarely’ and ‘sometimes’ is. Also, anyone who never goes cannot answer.
      How often do you go to the cinema in a typical month?
      Never/Once/Twice/Three times/More than three times
g  Too black and white, it would be more useful to find out how much/often.
How often do you travel by taxi?
Every day/More than once a week/More than once a month/Less than once a month/Never. (In some ways, some of these options overlap but this is a frequently used type of scale.)

h  Opinion of question writer is in the question.
What do you think of dogs?
Love them/Like them a little/Do not particularly like them/Totally dislike them/Don’t know

2 a  data logging (3i below)
b  online questionnaire (email questionnaire) (3ii below)
c  controlled experiment (3iii below)
d  face-to-face (or personal) interview (3iv below)
e  observation (3v below)

3 a  i  Does not need a person to be employed to carry out this check.
   ii  He can do it at his own convenience.
   iii  He is collecting the data himself so will be content it is accurate.
   iv  Data is obtained straight away.
   v  should be accurate and complete
b  i  may break down
   ii  He may decide not to do it/make up answers.
   iii  time consuming work
   iv  Annie may not like interviews or be in a rush.
   v  Students may not behave normally if being watched and recorded.

4 to check that the questions in a questionnaire give the type of response desired
to check that questions are understood
to check the way that data is collected works

5 a  i  Are you married? Yes ☐ No ☐
   ii  Which is your favourite holiday destination?
       UK ☐ France ☐ Spain ☐
       USA ☐ Greece ☐
       Other ☐ (please state where) ___________________

b  i  What is your gender?
    Male ☐    Female ☐

       How old are you?
       Under 10 ☐ 10–14 ☐ 15–19 ☐
       20–29 ☐ 30–39 ☐ 40+ ☐
   iv  Do you like Wayne Rooney?
       Yes ☐ No ☐
       Don’t know ☐
   c  Does someone in your family buy a newspaper?
       Yes ☐ No ☐
       Don’t know ☐

   If yes, how much does it cost?
       Under 30p ☐ 30p–39p ☐
       40p–49p ☐ 50p or more ☐
       Don’t know ☐

Practise... 3.3 Organising data

1 a  25   c  15
   b  41   d  observation

2 | Sheep | Cattle | Pigs |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>80</td>
<td>170</td>
</tr>
<tr>
<td>Female</td>
<td>50</td>
<td>70</td>
</tr>
</tbody>
</table>
3  a  i

<table>
<thead>
<tr>
<th></th>
<th>Apple</th>
<th>Strawberry</th>
<th>Banana</th>
<th>Orange</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y10 girl</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y10 boy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ii

<table>
<thead>
<tr>
<th></th>
<th>Black</th>
<th>Brown</th>
<th>Blonde</th>
<th>Red</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Female</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

iii

<table>
<thead>
<tr>
<th></th>
<th>Full (100%)</th>
<th>Quite full (75%)</th>
<th>Half full (50%)</th>
<th>Nearly empty (25%)</th>
<th>Empty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Morning</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Evening</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b  i  A student may not have a single favourite or may not like fruit at all.

ii  Many people do not have hair of one colour or what one person may call brown, another may judge to be red. Also in observation situations, it is often reported that it can be difficult to judge whether a person is male or female at times.

iii  judging the ‘fullness’ of a bus as it passes you and categorising the fullness in the first place.

4  Weather | Morning | Afternoon
---|---------|---------
Sunshine |         |         |
Cloudy (dry) | | |
Raining | | |
Snow / sleet / hail | | |
Other | | |

5  a  7 April to 5 June would cost £124 + £124 + £89 + £0 = £337, which is OK.

6 June to 21 July would cost £168 + £168 + £120 + £12 = £468, which is OK.

22 July to 5 Sept would cost £215 + £215 + £199 + £50 = £679, which is too much.

So they could go on their holiday any time between 7 April and 21 July.

b  Yes, he is right. If they had £700, this would cover 2 × £337 = £674 and they could have 2 weeks between 7 April and 5 June.

Practise... 3.4 Sampling methods

1  Sample depends on the random numbers used.

2  a  Year 10 = 15
Year 11 = 15
Year 12 = 11
Year 13 = 9

b  Give each student a number and then, using a calculator or tables, obtain random numbers. Match the random numbers to the students in the list.

3  a  Management = 2, Sales = 26, Security = 5, Office = 7

b  Give the office staff numbers from 01 to 35. Use calculators or the random numbers table to obtain seven different values between 01 and 35. Match these values to the office staff.

4  Social Science = 8 (rounded from 8.205 . . .)
The Arts = 3 (rounded from 2.852 . . .)
Science = 6 (rounded from 6.382 . . .)
Sport and Leisure = 3 (rounded from 2.561 . . .)

5  a  to get a fair proportion from each age group, as children in different age groups will probably have different views

b  by gender or perhaps by household income

c  age 5–7 = 18 (rounded from 18.075 . . .)
age 8–10 = 25 (rounded from 24.648 . . .)
age 11–13 = 31 (rounded from 31.221 . . .)
age 14–16 = 26 (rounded from 26.056 . . .)
6. Answers will vary according to the random numbers used.

7. Year 7 = 23.394 before rounding
   Year 8 = 21.189 before rounding
   Year 9 = 19.271 before rounding
   Year 10 = 18.696 before rounding
   Year 11 = 17.450 before rounding

   If these are all rounded to nearest integer, the total would be 99. Round the Year 11 value up, not down as this was nearest to rounding up initially out of those rounded down.

   Final sample Year 7 = 23, Year 8 = 21, Year 9 = 19, Year 10 = 19, Year 11 = 18

8. Use a stratified random sample.

   800 components should be from Machine A, 700 from Machine B and 500 from Machine C.

For each machine, try to get the output numbered in some way so that a random selection can be obtained using random numbers. Alternatively, perhaps a sample could be taken at particular time intervals.

9. Number of members

<table>
<thead>
<tr>
<th>Adults</th>
<th>Children</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>1323</td>
</tr>
<tr>
<td>Female</td>
<td>3003</td>
</tr>
</tbody>
</table>

   Number in sample of 200

<table>
<thead>
<tr>
<th>Adults</th>
<th>Children</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>45 (44.893)</td>
</tr>
<tr>
<td>Female</td>
<td>102 (101.900)</td>
</tr>
</tbody>
</table>

Assess 3

1. a) 24
   c) \( \frac{19}{40} \)
   b) 27
   d) 50%

2. a)

<table>
<thead>
<tr>
<th>Chocolate</th>
<th>Sweets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>24</td>
</tr>
<tr>
<td>Female</td>
<td>16</td>
</tr>
</tbody>
</table>

   b) 20
   d) \( \frac{16}{40} = \frac{2}{5} \)
   c) 70

3. a) continuous
   b) discrete
   c) discrete
   d) continuous
   e) discrete
   g) discrete
   f) continuous

4. a) i) It would be very difficult to remember accurately.
   ii) need to offer choices regarding how often they are watched
   b) i) This is biased with words our and improved.
      Rate the fruit juice you have just had.
      Excellent/Good/Average/Poor/Terrible
   ii) A personal question. What is your monthly salary? (Again could probably offer choices.)
   iii) A leading question. Give your opinion on the new bypass (with choices).
   iv) A leading question. Do you prefer smoking or non-smoking areas?
   v) No time period is specified. How many showers do you have in a typical week?
      (Options possible again.)

5. a) Take a sample of 20–40 sheep from throughout the farm and weigh them, finding the average.

   b) Ask random people from the town, using a phone, postal or face-to-face interview.
   c) Question a sample of students from across the school in different classes.
   d) Measure the hand spans of a sample of students from your school, including all ages and genders.
   e) Take a sample of villagers and ask them. Perhaps go from door to door to ensure a variety of the homes within the village.
   f) Phone, postal or email survey with a carefully chosen and unbiased sample.
   g) Use a data-logging machine to keep a full record. This should enable an accurate result for this data.

6. Write a hypothesis such as ‘this shop is the cheapest in town for fruit and vegetables’.

   Collect data by finding a sample of fruit and vegetables from this shop and from other fruit and vegetable sellers in the town.

   For each selected item, calculate measures of average and spread, and show prices in simple diagrams such as bar charts and pictograms.

   Interpret the data, the measures and the diagrams, and decide whether they support the hypothesis or not.

7. Answer will depend upon random numbers chosen.

8. a) A random sample would not be appropriate as it is not likely to give sufficient weighting to management, office and sales staff.
b The sample needs to be chosen so that the proportions represent those of the population as a whole.

<table>
<thead>
<tr>
<th>Occupation</th>
<th>Management</th>
<th>Office</th>
<th>Sales</th>
<th>Shop Floor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>10</td>
<td>15</td>
<td>30</td>
<td>145</td>
</tr>
<tr>
<td>Fraction</td>
<td>10/200</td>
<td>15/200</td>
<td>30/200</td>
<td>145/200</td>
</tr>
<tr>
<td>For a sample size of 20</td>
<td>( \frac{10}{200} \times 20 = \frac{15}{200} \times 20 = \frac{30}{200} \times 20 = \frac{145}{200} \times 20 = )</td>
<td>1 worker</td>
<td>1.5 workers</td>
<td>3 workers</td>
</tr>
<tr>
<td>For a sample size of 35</td>
<td>( \frac{10}{200} \times 35 = \frac{15}{200} \times 35 = \frac{30}{200} \times 35 = \frac{145}{200} \times 35 = )</td>
<td>1.75 workers</td>
<td>2.625 workers</td>
<td>5.25 workers</td>
</tr>
</tbody>
</table>

In the case of the sample of 20, it is important to round 1.5 and 14.5 for sampling purposes. If both numbers are rounded up, the sample size is 21. You are told the sample size is 20 so you need to round one up and one down to get the right total; it doesn’t matter which.

**AQA Examination-style questions**

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>10–24</th>
<th>25–44</th>
<th>45–60</th>
<th>61+</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of members</td>
<td>150</td>
<td>198</td>
<td>132</td>
<td>120</td>
</tr>
<tr>
<td>Number in sample</td>
<td>25</td>
<td>33</td>
<td>22</td>
<td>20</td>
</tr>
</tbody>
</table>
### Fractions and decimals

#### Practise... 4.1 Adding and subtracting fractions

1. **a** \( \frac{11}{20} \)  
   **b** \( \frac{1}{20} \)  
   **c** \( \frac{31}{24} \)  
   **d** \( \frac{7}{24} \)  

2. She has added the numerators and added the denominators.

3. \( \frac{1}{20} \)

4. D is the odd one out: \( \frac{3}{4} + \frac{1}{4} + \frac{1}{16} \)

5. **a** \( \frac{1}{2} \)  
   **b** \( 2 \frac{1}{4} \)  
   **c** 3  
   **d** \( 3 \frac{3}{4} \)

#### Practise... 4.2 Multiplying and dividing fractions

1. **a** \( \frac{3}{5} \)  
   **c** \( \frac{5}{24} \)  
   **e** \( \frac{7}{2} \)  
   **g** \( \frac{6}{4} \)  
   **b** \( 1\frac{1}{5} \)  
   **d** \( 1\frac{1}{6} \)  
   **f** \( 20\frac{5}{6} \)  
   **h** \( 3\frac{5}{8} \)  

2. **a** A number divided by itself is 1.  
   **b** \( 1\frac{1}{2} \) is multiplied by \( \frac{2}{3} \) then divided by \( \frac{2}{3} \) so answer is \( 1\frac{1}{2} \).  
   **c** \( 1\frac{1}{2} \) is multiplied by \( \frac{2}{3} \) then by the reciprocal of \( \frac{2}{3} \) so answer is \( 1\frac{1}{2} \).

3. **a** i \( 1\frac{1}{5} \)  
   ii \( \frac{8}{5} \)  
   **b** They are the reciprocals of each other.  
   **c** Student’s own fractions  
   **d** Yes

4. **a** i \( \frac{1}{2} \)  
   ii \( \frac{1}{8} \)  
   iii \( \frac{1}{10} \)  
   iv \( \frac{1}{20} \)  
   v \( \frac{1}{100} \)  
   **b** i \( 3 \)  
   ii \( 7 \)  
   iii \( 9 \)  
   iv \( 15 \)  
   v \( 50 \)  
   **c** i \( 1\frac{1}{2} \)  
   ii \( 1\frac{1}{4} \)  
   iii \( 1\frac{1}{5} \)  
   iv \( \frac{3}{2} \)  
   v \( \frac{6}{3} \)  
   **d** i \( 10 \)  
   ii \( 1\frac{1}{2} \)  
   iii \( 1\frac{1}{5} \)  
   iv \( \frac{33}{3} \)  
   v \( \frac{6}{3} \)  
   **e** i \( \frac{5}{6} \)  
   ii \( \frac{5}{13} \)  
   iii \( \frac{8}{29} \)  
   iv \( \frac{8}{6} \)  
   v \( \frac{5}{14} \)  
   **f** i \( -1 \)  
   ii \( -\frac{1}{3} \)  
   iii \( -4 \)  
   iv \( -\frac{21}{2} \)  
   v \( -\frac{7}{30} \)

#### Practise... 4.3 One quantity as a fraction of another

1. **a** \( \frac{3}{5} \)  
   **c** \( \frac{2}{5} \)  
   **e** \( \frac{1}{4} \)  
   **b** \( \frac{1}{9} \)  
   **d** \( \frac{1}{10} \)  
   **f** \( \frac{1}{6} \)  

2. **a** \( \frac{7}{10} \)  
   **b** \( \frac{3}{10} \)  

3. **a** \( \frac{3}{5} \)  
   **b** Mr Howes

4. \( \frac{2}{5} \)

#### Practise... 4.4 Fractions as decimals

1. Fraction A: \( \frac{3}{5} \) will convert to a terminating decimal.

2. \( \frac{2}{3} \)

3. **a** 0.111... , 0.222... , 0.333... 
   **b** 0.444... , 0.555... \( = 0.999... \)

4. \( \frac{3}{5}, \frac{5}{8}, \frac{3}{4}, \frac{7}{9} \)

5. **a** \( \frac{7}{20} \)  
   **c** \( \frac{1}{4} \)  
   **e** \( \frac{9}{40} \)  
   **b** \( \frac{1}{8} \)  
   **d** \( \frac{1}{250} \)  
   **f** \( \frac{2}{9} \)  

6. **a** \( \frac{1}{3} \)  
   **b** \( \frac{2}{11} \)  
   **c** 1  
   **d** \( \frac{35}{111} \)
7 a i Prime factors are 2 and 5.
  ii Prime factors include numbers other than 2 and 5.
Fractions in their lowest terms with denominators that are multiples of 2 and 5 only convert to terminating decimals.
b i 25 and ii 100 produce terminating decimals

**Practise... 4.5 Dividing quantities in given ratios**

1 £2250 Drama, £3250 Art
2 a i £50, £50
  ii £33.33, £66.67
  iii £25, £75
  iv £20, £80
  v £16.67, £83.33
b They are easier if the total number of parts is a divisor of 100.
3 a £20, £30, £50
  b £10, £40, £50

**Practise... 4.6 Rounding**

1 a 12.57 c 67.90 e 0.00
  b 0.00 d 0.57
2 a No, it is 500000
   The zeros are needed to show the size of the number.
b No, it should be 15.6
   A zero on the end would indicate that the number was rounded to 4 s.f.
3 a 90 c 20 e 0.6
  b 1000 d 0.2
4 a 60 × 4 = 240 c 30 × 1000 = 30000
  b 0.6 × 100 = 60 d 10 ÷ 2 = 5
5 a 1950 b 24.6 c 129 d 0.00953
6 a 13 c 68 e 0.0048
  b 0.0039 d 0.57
   0.568 is the same when rounded to two decimal places and two significant figures.
7 Student’s own numbers that are the same when rounded to one significant figure as when rounded to one decimal place, e.g. 0.53, 0.245, 0.9234
8 a 154.9999… b 145
9 Student’s own numbers that are 0.01 when rounded to one significant figure, e.g. 0.006, 0.01234, 0.009
10 Student’s own answers as follows.
   In left-hand box: any number between 0.35 and 0.45 (not inclusive of 0.45)
   In right-hand box: any number between 0.435 and 0.445 (not inclusive of 0.445)
   In the overlap between the boxes any number between 0.35 and 0.45 (not inclusive of 0.45)
11 a £367.17 b 400 m c 367
Practise... 4.7 Decimal calculations

1 a 42  b 103.76  c 10  d 37.7

2 a 7.92  b 5.34  c 6  d 0.292

3 a i 1.5  iii 150  v 15 000
   ii 15  iv 1500

   b Each divisor is a tenth of the size of the previous one.
   c Each answer is ten times the previous one.
      The smaller the divisor is, the larger the answer.

4 a i 15 000  iii 150  v 1.50
   ii 1500  iv 15

   b Each multiplier is a tenth of the size of the previous one.
   c Each answer is a tenth of the answer before because the multiplier is one tenth the size.

5 a 8.21 × 5.6 = 45.976
   b 8.21 × 0.56 = 4.5976
   c 8.21 × 0.056 = 0.45976
   d 8.21 × 0.0056 = 0.045976

6 a 42.6 ÷ 40 = 1.065
   b 42.6 ÷ 0.4 = 106.5
   c 426 ÷ 0.4 = 1065
   d 426 ÷ 0.004 = 10650

7 £13.24

8 645

9 Not quite. The carpet would cost £250.80

Assess 4

1 a \( \frac{2}{3} \)  b \( \frac{4}{9} \)  c \( \frac{1}{10} \)  d \( \frac{1}{10} \)

2 \( \frac{11}{40} \)

3 \( \frac{5}{31} \)

4 one and a half yards

5 \( 2\frac{3}{5} \times 1\frac{1}{2} \)
   Multiplying by a number bigger than 1 will increase the fraction; dividing it by a number bigger than 1 will decrease it.

6 \( \frac{3}{5}, \frac{2}{3}, \frac{2}{5}, \frac{8}{9}, \frac{9}{10} \)

7 A \( \frac{4}{7} \), C \( \frac{1}{12} \) and D \( \frac{3}{11} \) will convert to recurring decimals.

8 a \( \frac{1}{8} \)  b \( -\frac{1}{3} \)  c \( 2\frac{1}{2} \)  d \( -2 \)  e \( \frac{4}{9} \)

9 11.4 pints

10 285

11 £22.50

12 D 0.055 is not 0.05 to one significant figure.

13 61 miles

14 167 miles

15 a \( \frac{49}{200} \)  b \( \frac{11}{20} \)  c \( \frac{159}{200} \)  d \( \frac{143}{250} \)  e \( \frac{909}{2000} \)

16 a 1.2 = 1.
   b 0.1 is not defined.

17 a false  c false  e false
   b true  d true

18 a \( \frac{5}{9} \)  b \( \frac{10}{99} \)  c \( \frac{208}{333} \)  d \( 1\frac{3}{11} \)  e \( 2\frac{41}{333} \)

AQA Examination-style questions

1 240

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Graphs of linear functions

Practise...

5.1 Drawing straight-line graphs

1 a

\[ y = x + 2 \]

b (0, 2)

2 a

\[ y = 3x - 1 \]

b (-2/3, -3)

3 a

\[ y = \frac{1}{2}x \]

c (0, 1)

4 a and b

No, this line would not go through the point (7, 4) because if you substitute these numbers into the equation of the line it does not work: \(4 \neq \frac{1}{2} \times 7\)

5 a and b

The lines cross at the origin, coordinates (0, 0).
Practise...  5.2 Gradients of straight-line graphs

1  a  gradient = 3, intercept = (0, −3)  
b  gradient = −2, intercept = (0, −1)

2  The gradient of the line $y = 5 − 2x$ is −2 and 
    the gradient of the line $y = 5 − 4x$ is −4 so Jo is 
    wrong to say they are the same. One is double 
    the other.

3  a  Line A has a gradient of 2 because the 
    increase in $y$ is double the increase in $x$.  
b  Line C has a gradient of 1 because the 
    increase in $y$ is equal to the increase in $x$.  
c  Line D has a negative gradient because the 
    $y$-values decrease as the $x$-values increase.

4  a  gradient $= \frac{40}{10} = 4$  
b  gradient $= \frac{−15}{3} = −5$

5  a  $y = 3x + 6$  
c  $2y = −x + 6$ so $y = \frac{−x}{2} + 3$

6  a  $(0, 4)$  
    gradient $= 9$  
b  $(0, 2)$  
    gradient $= 5$  
c  $(0, 3)$  
    gradient $= 1$  
d  $(0, −5)$  
    gradient $= −2$  
e  $(0, −3.5)$  
    gradient $= 3$  
f  $(0, 9)$  
    gradient $= −4$

7  The equation of the line is $y = 3x − 5$ so the 
    gradient is 3.  
    The gradient $= \frac{PQ}{RQ}$ so $3 = \frac{PQ}{3}$ so $PQ = 9$ and 
    the length of $PQ = 9$ units.

8  The gradient of the line $5x + 2y = 9$ is 
    $\frac{−5}{2} = −2.5$  
    The gradient of the line $4y = 3 − 10x$ is 
    $−\frac{10}{4} = −2.5$
The lines are all parallel. The equations all have the same number preceding the $x$ term. This means they have the same gradient so they must be parallel.

10. **a** This is $y = 4 - x$ (iii)  
   **c** This is $y = 3x + 8$ (iv)  
   **b** This is $y = 3x$ (i)  
   **d** This is $y = -2x$ (ii)
8. Graph showing points T, U, and W.

V is (3, 1).

Practise... 5.4 Lines through two given points and parallel lines

1. \( y = 4x - 5 \)
2. \( y = 3x + 2 \)
3. \( 2y = 11 - x \)
4. \( 4y = x + 26 \)
5. Anthony is wrong because for two lines to be parallel, their gradients must be equal. For these two lines, \( y = 2x \) has a gradient of 2 and \( y = 4 - 2x \) has a gradient of -2 so they are not parallel.
6. \( y = 4x \) and \( x = \frac{y}{4} + 2 \) are parallel, with gradients of 4.
   \( y = 1 + 3x \) and \( 2y - 6x = 5 \) are parallel, with gradients of 3.
7. \( x + y = 8 \) and \( y = 3 - x \) are parallel, with gradients of -1.
8. \( y = 3x - 2 \)
9. \( y + x = -5 \) or \( y = -x - 5 \) or \( y + x + 5 = 0 \)
10. \( 2y = 10 - 3x \)
11. \( a \) The line \( BC \) has equation \( 3y = x + 5 \)
    \( b \) \( M \) has coordinates (2, 4). \( N \) has coordinates (5, 5). The line \( MN \) has equation \( 3y = x + 10 \)
    \( c \) \( BC \) has gradient \( \frac{1}{3} \) and \( MN \) has gradient \( \frac{1}{3} \) so the lines \( BC \) and \( MN \) are parallel.

Assess 5
3 a (1, 0), (4, 4) and (7, 8) lie on the line 4x − 3y = 4
Putting (1, 0) into the equation gives an answer of (4 × 1) − (3 × 0) = 4 ✓
Putting (0, 1) into the equation gives an answer of (4 × 0) − (3 × 1) = −3 ✗
Putting (3, 4) into the equation gives an answer of (4 × 3) − (3 × 4) = 0 ✗
Putting (4, 4) into the equation gives an answer of (4 × 4) − (3 × 4) = 4 ✓
Putting (7, 8) into the equation gives an answer of (4 × 7) − (3 × 8) = 4 ✓

b A is on line 3y = 4x + 7 and 4y − 3x = 0
B is on line y + x = 0 and 3y = 4x + 7
C is on line y = 6 and y + x = 3
D is on line 4y − 3x = 0 and y + 5x = 23
E is on line y + x = 3 and y + 5x = 23

4 a y = 5x − 6
   b y = −x + 4

5 a y = −6x + 2
   b y = 3x − \frac{5}{2} or y = 3x − 2.5

6 y = 9 − x has gradient = −1
   x + y = 4 has gradient = −1
   So y = 9 − x and x + y = 4 are parallel.

7 a gradient of AB = 2
   b (−1, 1)

8 (3, 7)

9 3y = 8x − 5

10 2y = 3x − 4 and 3x − 2y = 1

11 3y = −4x − 1 or 3y + 4x = −1
    or 3y + 4x + 1 = 0

AQA Examination-style questions

1 a y = 3x
   b y = 3x + 6
   c Any valid point, e.g. they are the same/equal gradient; they are parallel; the gradients are positive; both gradients are 3. Do not accept ‘the lines are the same distance apart’.

2 y = 2x + 3
   b y = 3x − 2
   c Any valid point, e.g. they are the same/equal gradient; they are parallel; the gradients are positive; both gradients are 3. Do not accept ‘the lines are the same distance apart’.

3 y = 2x − 1
   b y = 2x + 1
   c Any valid point, e.g. they are the same/equal gradient; they are parallel; the gradients are positive; both gradients are 2. Do not accept ‘the lines are the same distance apart’.

4 y = −x + 3
   b y = −x + 5
   c Any valid point, e.g. they are the same/equal gradient; they are parallel; the gradients are positive; both gradients are −1. Do not accept ‘the lines are the same distance apart’.

5 y = 2x + 1
   b y = 2x − 2
   c Any valid point, e.g. they are the same/equal gradient; they are parallel; the gradients are positive; both gradients are 2. Do not accept ‘the lines are the same distance apart’.

6 y = 3x + 2
   b y = 3x − 2
   c Any valid point, e.g. they are the same/equal gradient; they are parallel; the gradients are positive; both gradients are 3. Do not accept ‘the lines are the same distance apart’.

7 Gradient of AB = 2
   b (−1, 1)
   c Any valid point, e.g. they are the same/equal gradient; they are parallel; the gradients are positive; both gradients are 2. Do not accept ‘the lines are the same distance apart’.

8 Gradient of AB = 3
   b (−1, 1)
   c Any valid point, e.g. they are the same/equal gradient; they are parallel; the gradients are positive; both gradients are 3. Do not accept ‘the lines are the same distance apart’.

9 Gradient of AB = 1
   b (−1, 1)
   c Any valid point, e.g. they are the same/equal gradient; they are parallel; the gradients are positive; both gradients are 1. Do not accept ‘the lines are the same distance apart’.

10 Gradient of AB = −2
    c Any valid point, e.g. they are the same/equal gradient; they are parallel; the gradients are positive; both gradients are −2. Do not accept ‘the lines are the same distance apart’.

11 Gradient of AB = 0
    c Any valid point, e.g. they are the same/equal gradient; they are parallel; the gradients are positive; both gradients are 0. Do not accept ‘the lines are the same distance apart’.
6 Working with symbols

**Practise...** 6.1 Expanding brackets and collecting like terms

1 a \(3(x + 4) = 3x + 12\)

b \(5(y - 2) = 5y - 10\)

c \(8(2 - c) = 16 - 8c\)

d \(3(2p + 5) = 6p + 15\)

e \(5(5d - 1) = 25d - 5\)

f \(7(2 - 2f) = 14 - 14f\)

g \(3(10v + 7) = 30v + 21\)

h \(11(7 + 3m) = 77 + 33m\)

i \(3a(a - 2) = 3a^2 - 6a\)

j \(2d(3 - 5d) = 6d - 10d^2\)

k \(\frac{1}{2}(t - 4) = \frac{t^2}{2} - 2t\)

l \(6(t - 2) = 3t^2 - 12t\)

2 a Tom: \(4f(2f - 3) = 8f^2 - 12f\)

Jade: \(6f(f - 2) + 2f^3 = 6f^2 - 12f + 2f^3 = 8f^2 - 12f\)

b \(-4f\)

3 a \(3(5a + 1) = 15a + 3\)

b \(2(b - 5) = 2b - 10\)

c \(6(4c + 3) = 24c + 18\)

d \(1.5d(d - 2) = 1.5d^2 - 3d\)

e \(2.5e(10 + e) = 25e + 2.5e^2\)

f \(5.5f(3 - f) = 16.5f^2 - 5.5f^2\)

4 a \(n\) is multiplied by 5 as well as 2.

b i \(5(n - 8) = 5n - 40\)

ii \(3(10 - n) + 3n = 10\)

c i \(5(n + 2) - 10 = 5n + 10 - 10 = 5n\)

ii \(3n - 2(n - 1) = 3n - 2n + 2 = n + 2\)

iii \(5(n + 1) + 2(3 - n) = 5n + 5 + 6 - 2n = 3n + 11\)

iv \(7(3 - 2n) + 10n = 21 - 14n + 10n = 21 - 4n\)

v \(3(2 + 3n) - 2(3 + 4n) = 6 + 9n - 6 - 8n = n\)

vi \(2(n + 1) + 5(n - 2) - 4n = 2n + 2 + 5n - 10 - 4n = 3n - 8\)

5 a \(6(n + m) + 5n = 11n + 6m\)

b \(4(3 - x) + 10(x - 2) = 6x - 8\)

c \(10(b + c) - 5b = 5b + 10c\)

d \(k(12 + k) - 12(k - 11) = k^2 - 132\)

e \(3.5(3 + t) - t = 10.5 - 2.5t\)

f \(3a \times 2a + 2a \times a = 8a^2\)

6 a \(2(a + 4) + 3(a - 1) = 5(a + 1)\)

b \(5(1 - b) - 3(b - 3) = 2(7 + 4b)\)

c \(2(x + 1) - 5(x + 4) = -3(x + 6)\)

d \(4(2x - 1) + 2(x - 3) = 10(x - 1)\)

e \(5(1 - x) + 2(4x + 5) = 3(x + 5)\)

7 a \(x = 5\)

b \(x = 3.5\)

c \(x = 3.0\)

8 \(2y(3y - 6) = 6y(y - 2)\)

\(6y^2 - 12y\)

\(6y^2 - 12y\)

9

\[\begin{array}{c|c}
25y^2 & \hline
5y & 5y \\
\end{array}\]

\[\begin{array}{c|c|c}
2x - 6 & \hline
2 & x - 3 \\
\end{array}\]

\[\begin{array}{c|c|c}
2x & \hline
2x & 10x \\
\end{array}\]

10 a Student’s own expressions correctly simplified

b i \(2(2x + 1) - 5(x + 2) = 4x^2 + 2x - 5x - 10 = 4x^2 - 3x - 10\)

ii \(4x(3x + 1) - x(2 - x) = 12x^2 - 4x - 2x + x^2 = 13x^2 - 6x\)

iii \(5x(x - 1) + 2(3 - x) = 5x^2 - 5x + 6 - 2x = 5x^2 - 7x + 6\)
### Practise... 6.2 Factorising expressions

1. a) \(4(2c + 1)\)
   b) \(3(4d - 5)\)
   c) \(10(2 - p)\)
   d) \(6(4 + 3k)\)
   e) \(x(20 + x)\)
   f) \(y(y - 5)\)

2. a) \(6x - 21 = 3(2x - 7)\)
   b) \(20x^2 - 25x = 5x(4x - 5)\)
   c) \(7p^2 - 5p = p(7p - 5)\)
   d) \(100r^2 + 125t = 25(4t + 5)\)
   e) \(2kq - 3kr = k(2q - 3r)\)

3. a) Sides are \(fg\) and \((5 + 8g)\)
   b) Sides are \(3x\) and \((x + 7y^2)\)
   c) Sides are \(2pq^2\) and \((3p - q)\)
   d) Sides are \(5abc\) and \((ac + 3b)\)

4. Sue is correct because she has factorised fully whereas Chris has not factorise fully.

5. a) \(3ap - 9p = 3p(a - 3)\) right
   b) \(12f^2 - 18f = 3f(4f - 6)\) wrong: \(6f(2f - 3)\)
   c) \(36 - 4t^2 + 12t = 4(9 - t^2 + 3t)\) right
   d) \(5x^2y - 20x^2y = 5xy(3xy - 4x)\) wrong: \(15x^3y (3y - 4)\)
   e) \(55k - 44klm^2 = 11k(5 - 4lm^2)\) right

### Practise... 6.3 Multiplying two brackets together

1. a) \((a + 2)(a + 5) = a^2 + 2a + 5a + 10\)
   b) \((b + 3)(b + 7) = b^2 + 3b + 7b + 21\)
   c) \((c - 1)(c + 4) = c^2 - c + 4c - 4\)
   d) \((d + 11)(d - 2) = d^2 + 11d - 2d - 22\)
   e) \((e - 2)(e - 5) = e^2 - 2e - 5e + 10\)
   f) \((4 - f)(9 - f) = 36 - 9f - 4f + f^2\)
   g) \((k + 3)(k - 3) = k^2 + 3k - 3k - 9\)
   h) \((5 - m)(5 + m) = 25 - 5m + 5m - m^2\)
   i) \((p + 1)^2 = (p + 1)(p + 1) = p^2 + p + p + 1\)
   j) \((q - 5)^2 = (q - 5)(q - 5)\)

2. a) \((r + 7)(2r + 1) = 2r^2 + 14r + r + 7\)
   b) \((t + 4)(3t - 2) = 3t^2 + 12t - 2t - 8\)
   c) \((2u - 3)(u - 4) = 2u^2 - 3u - 8u + 12\)
   d) \((5r + 2)(2r + 3) = 10r^2 + 4v + 15v + 6\)
   e) \((3w - 1)(2w + 1) = 6w^2 - 2w + 3w - 1\)
   f) \((4x - 1)(2x - 5) = 8x^2 - 2x - 20x + 5\)
   g) \((3y + 1)(3y - 1) = 9y^2 + 3y - 3y - 1\)

3. a) \((5 - 2)(5 + 2) = 25 - 10z + 10z - 4z^2\)
   b) \((3p + 1)^2 = (3p + 1)(3p + 1)\)
   c) \((2q - 5)^2 = (2q - 5)(2q - 5)\)
5 \[(4x + 3)(x - 1) - (x - 3)(4x + 1)\]
\[= (4x^2 + 3x - 4x - 3) - (4x^2 - 12x + x - 3)\]
\[= 4x^2 + 3x - 4x - 3 - 4x^2 + 12x - x + 3\]
\[= 4x^2 - 4x^2 + 3x + 12x - x - 4x + 3 + 3\]
\[= 10x\]

6 a \[(5a + b)(a - 3b) = 5a^2 + ba - 15ab - 3b^2\]
\[= 5a^2 - 14ab - 3b^2\]

b \[(2c - 7d)(c - 2d) = 2c^2 - 7dc - 4cd + 14d^2\]
\[= 2c^2 - 11cd + 14d^2\]

c \[(4p - q)(4p + q) = 16p^2 - 4pq + 4pq - q^2\]
\[= 16p^2 - q^2\]

d \[(2x - 3y)^2 = (2x - 3y)(2x - 3y)\]
\[= 4x^2 - 6xy - 6yx + 9y^2\]
\[= 4x^2 - 12xy + 9y^2\]

e \[(m + 7n)(m - 7n) = m^2 + 7nm - 7nm - 49n^2\]
\[= m^2 - 49n^2\]

f \[(3e + 2f)(2e - 3f) = 6e^2 + 4ef - 9ef - 6e^2\]
\[= 6e^2 - 5ef - 6e^2\]

7 a i Area of ABED \[= 5(y - 2)\]
\[= 5y - 10\]

ii Area of BCFE \[= (x - 5)(y - 2)\]
\[= xy - 5y - 2x + 10\]

iii Area of DEHG \[= 2 \times 5 = 10\]

iv Area of EFH \[= 2(x - 5) = 2x - 10\]

b Area of ACIG \[= 5y - 10 + xy - 5y - 2x\]
\[+ 10 + 10 + 2x - 10\]
\[= xy + 5y - 5y - 2x + 2x\]
\[= 10 + 10 + 10 - 10\]
\[= xy\]

8 \[(3x + 1)^2 = (3x + 1)(3x + 1)\]
\[= 9x^2 + 3x + 3x + 1 = 9x^2 + 6x + 1\]

Louise has not added up the x terms correctly.

9 Ed is wrong because there is a minus sign in both brackets so the y term in the solution must be negative.

\[(2y - 7)(3y - 1) = 6y^2 - 21y - 2y + 7\]
\[= 6y^2 - 23y + 7\]
Assess 6

1 a 7a - 14  
   b 6b - 3  
   c 4c + 4d  
   d 5e - 15f  
   e 2mn + 4mp

2 a 3(a - 4)  
   b 2(b + 5)  
   c 3(3 - 2c)  
   d 7(2 - d)  
   e 2(2p - q + 4r)

3 a 3a + 6 + 2a - 2 = 5a + 4  
   b 3b - 6 + 3b + 15 = 6b + 9  
   c 6c + 8 - 3c - 12 = 3c - 4  
   d 5x + 5 + 6x + 42 = 11x + 47  
   e 12y - 18 - 12y + 8 = -10  
   f 8w - 8 + 4w = 12w  
   g 2x + 2y + 10x + 5y = 12x + 7y  
   h 12p - 3q + 2p - 6q = 14p - 9q  
   i 10a + 15b - 6a - 8b = 4a + 7b  
   j 4m - 12n - 3m + 5n = m - 7n

4 \(5(x + 1) + 2(5 - x) = 5x + 5 + 10 - 2x\)
   \(= 5x - 2x + 5 + 10\)
   \(= 3x + 15 = 3(x + 5)\)

5 a \((a + 1)(a + 3) = a^2 + a + 3a + 3\)
   \(= a^2 + 4a + 3\)
   b \((b - 2)(b + 8) = b^2 - 2b + 8b - 16\)
   \(= b^2 + 6b - 16\)
   c \((c - 6)(c + 6) = c^2 - 6c + 6c - 36\)
   \(= c^2 - 36\)
   d \((d - 7)(d - 4) = d^2 - 7d - 4d + 28\)
   \(= d^2 - 11d + 28\)
   e \((e - 4)^2 = (e - 4)(e - 4)\)
   \(= e^2 - 4e - 4e + 16\)
   \(= e^2 - 8e + 16\)
   f \((f + 2)(f - 1) = f^2 + 2f - f - 2\)
   \(= f^2 + f - 2\)
   g \((h - 9)(h - 2) = h^2 - 9h - 2h + 18\)
   \(= h^2 - 11h + 18\)
   h \((m + 2)(m - 2) = m^2 + 2m - 2m - 4\)
   \(= m^2 - 4\)
   i \((2x + 1)(x + 4) = 2x^2 + x + 8x + 4\)
   \(= 2x^2 + 9x + 4\)
   j \((3y + 4)(y - 2) = 3y^2 + 4y - 6y - 8\)
   \(= 3y^2 - 2y - 8\)

k \((5z - 3)(z + 1) = 5z^2 - 3z + 5z - 3\)
   \(= 5z^2 + 2z - 3\)
   l \((2t + 3)(3t - 2) = 6t^2 + 9t - 4t - 6\)
   \(= 6t^2 + 5t - 6\)
   m \((2w + 3)^2 = (2w + 3)(2w + 3)\)
   \(= 4w^2 + 6w + 6w + 9\)
   \(= 4w^2 + 12w + 9\)

6 a \(\frac{3xy}{6x} = \frac{2y}{2}\)
   b \(\frac{pqr}{qpr} = \frac{p}{r}\)
   c \(\frac{6t}{2t - 4} = \frac{3t}{t - 2}\)
   d \(\frac{y + 5}{2y + 10} = \frac{y + 5}{2(y + 5)}\)
   \(= \frac{1}{2}\)
   e \(\frac{y^2 - 3y}{5y} = \frac{y(y - 3)}{y} = y - 3\)
   f \(\frac{3c - 9}{2c - 6} = \frac{3(c - 3)}{2(c - 3)} = \frac{1}{2} or \frac{3}{2}\)
   g \(\frac{(x + 2)(x - 7)}{(x - 1)(x + 2)} - \frac{(x - 1)(x - 2)}{x - 7} = \frac{x - 7}{x - 1}\)

7 a \(\frac{x}{8} + \frac{3x}{4} = \frac{x + 6x}{8}\)
   \(= \frac{7x}{8}\)
   b \(\frac{5y}{8} - \frac{y}{4} = \frac{5y - 2y}{8}\)
   \(= \frac{3y}{8}\)
   c \(\frac{2x}{9} + \frac{z}{6} = \frac{4z + 2x}{18}\)
   \(= \frac{7z}{18}\)
   d \(\frac{t}{2} - \frac{3t}{16} = \frac{8t - 3t}{16}\)
   \(= \frac{5t}{16}\)
   e \(\frac{9a}{10} - \frac{8a}{15} = \frac{27a - 16a}{30}\)
   \(= \frac{11a}{30}\)
   f \(\frac{7c}{12} + \frac{4c}{9} = \frac{21c + 16c}{36}\)
   \(= \frac{37c}{36}\)

8 a \((h + 1)(h - 3) = h^2 - 2h - 3\)
   b \(\frac{1}{2}(d - 1)(3d + 8) = \frac{1}{2}(3d^2 + 5d - 8)\)
   or \(1.5d^2 + 2.5d - 4\)
   \(= \frac{(3d^2 + 5d - 8)}{2}\)

9 a \(i \frac{x^2 + 4x + 3 + x^2 + 3x - 4}{x^2 + 7x - 1}\)
   \(= \frac{(2x^2 + 7x - 1)}{2}\)
   \(= (t + 4)(t - 3) - (t + 3)(t - 4) = t^2 + t - 12 - t^2 + t + 12 = 2t\)
   \(= (n + 1)^2 - n(n + 2) = n^2 + 2n + 1 - n^2 - 2n = 1\)

10 \(2n(n + 1) = 4n^2 + 2n which is even for any value of n\)
## 7 Percentages

### Practise... 7.1 Percentages, fractions and decimals

<p>| | | | | | | | | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>1</td>
<td>a</td>
<td>4%, $\frac{1}{9}$, 0.4, $\frac{4}{9}$</td>
<td>b</td>
<td>$\frac{62}{29}$, $\frac{16}{25}$, $\frac{13}{20}$, 66%, $\frac{2}{3}$, 0.67</td>
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<tr>
<td>2</td>
<td>a</td>
<td>38%</td>
<td>b</td>
<td>$\frac{7}{9}$</td>
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<tr>
<td>3</td>
<td></td>
<td>£125 000</td>
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<tr>
<td>4</td>
<td>a</td>
<td>84 kg</td>
<td>b</td>
<td>5.4 million</td>
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<tr>
<td></td>
<td>ii</td>
<td>30 m</td>
<td>v</td>
<td>1.8 litres</td>
<td>vi</td>
<td>36000</td>
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<td></td>
<td>iii</td>
<td>36000</td>
<td>vii</td>
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<td>7</td>
<td>a and b</td>
<td>£70</td>
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<td>b</td>
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<tr>
<td>9</td>
<td>a</td>
<td>£28</td>
<td>b</td>
<td>£4.20</td>
<td>c</td>
<td>£2.94</td>
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<tr>
<td>10</td>
<td>a</td>
<td>12 800</td>
<td>b</td>
<td>$\frac{1}{10}$</td>
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<tr>
<td>11</td>
<td></td>
<td>No. Karen is finding one eighth which is equivalent to 12.5%, not 8%; Sanjay is finding 80%, not 8%</td>
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<tr>
<td>12</td>
<td>$\frac{3}{5}$ of £2 million = £1.6 million</td>
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<tr>
<td></td>
<td>45% of £4 million = £1.8 million so this is bigger</td>
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<td>13</td>
<td>$\frac{1}{10}$</td>
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<td>14</td>
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<td>£10</td>
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<tr>
<td>15</td>
<td>a</td>
<td>£18 000</td>
<td>b</td>
<td>£360</td>
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<tr>
<td>16</td>
<td>a</td>
<td>Sue gets £40 000, Rowan gets £12 000 and Terry gets £8 000.</td>
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<tr>
<td></td>
<td>b</td>
<td>£40 000 + £12 000 + £8 000 = £60 000</td>
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### Practise... 7.2 Increasing or decreasing an amount by a percentage

|   |   |   |   |   |
|---|---|---|---|
| 1 | a | £600 | d | 90 km |
|   | b | 1050 kg | e | 84 000 |
|   | c | £455 | f | £14.72 |
| 2 |   | £12 |
| 3 | a and b | £540 |
| 4 |   | £37.80 |
| 5 | a |   |   |   |
|   | To increase by | 10% | 20% | 90% |
|   | Multiply by | 1.1 | 1.2 | 1.9 |
|   | To decrease by | 10% | 20% | 90% |
|   | Multiply by | 0.9 | 0.8 | 0.1 |
| b  | i | £440 | iii | 380 g | v | 4000 |
|   | ii | 6000 | iv | £360 | vi | 20 g |
| 6 |   | £7 |
| 7 | a | £65.80 | b | £344.40 | c | £31.49 |
| 8 |   | £12 |
| 9 |   | £6615 |
| 10 |   | £1690 |
| 11 | a | She has reduced the price by 20% not 2% |
|   | b | £34.30 |
| 12 |   | £14 580 |
| 13 | a | 0.8x + 4 = 32 |
|   | b | £35 |
| 14 |   | Playshop price = £79.90. This is cheaper than Arkos by 5p. |
| 15 | Item | Manager's price |
|    | Shirt | £15.99 |
|    | Skirt | £32.99 |
|    | Shorts | £9.99 |
|    | Trousers | £46.99 |
|    | Jacket | £61.99 |
| 16 |   | Option 2: reducing the quantity by 20% gives 25% more bags, so this will make more profit (assuming that packaging costs are not large). |
**Practise... 7.3 Successive percentages**

1. a) 296  
   b) 222  
   c) $1480 \times 0.2 \times 0.75$  
   d) 1110

2. 3024

3. £2.70

4. 1170

5. £110 124

6. £25 823

7. £10.33 (nearest penny)

8. 42.5%

9. a) Katie has just added the percentage rises instead of using multipliers.

b) $1.12 \times 1.18 = 1.18 \times 1.12 = 1.3216$
   Both give the same increase but of 32.16% not 30%

10. a) 12.5% (to 3 s.f.)  
    b) 32.2% (to 3 s.f.)

11. a) 61.2%  
    b) Saturday  
    c) cheapest on Saturday, but may not have any left

12. $1.3 \times 0.75 = 0.975$
    The shop makes a loss of 2.5%.

13. 20% reduction

**Practise... 7.4 Compound interest**

1. £661.50

2. £5105.82

3. a) i) £2676.45  
    ii) £3581.70  
    b) i) £676.45  
    ii) £905.25  
    iii) The interest is compounded.

4. £4009.74

5. No. Carmen will have more than 20% more because interest will be paid on the interest. $1.04^2 = 1.2166\ldots$ so Carmen will have nearly 22% more.

6. £32.23 more

7. a) After year | Value (nearest £) | 1 | 2 | 3 | 4
    | After year | 5 | 6 | 7 | 8
    | Value (nearest £) | 2589 | £2071 | £1657 | £1325

b) Assuming constant rate of decrease, population will halve in about 80 years and be less than 10 000 by the end of the century.

13. Ruby (interest is Emerald £585, Ruby £587.65, Sapphire £584.35)

14. After 10 yrs | 20 yrs | 30 yrs | 40 yrs | 50 yrs

| Population | 20240 | 18621 | 17131 | 15761 | 14500
| After 60 yrs | 70 yrs | 80 yrs | 90 yrs | 100 yrs
| Population | 13340 | 12273 | 11291 | 10388 | 9557

Assuming constant rate of decrease, population will halve in about 80 years and be less than 10 000 by the end of the century.

15. 14%
**Practise... 7.5 Writing one quantity as a percentage of another**

1. a) 65.4% (to 3 s.f.)  
   b) 34.6% (to 3 s.f.)
2. 12.9% (to 3 s.f.)
3. a) 12.5%  
   b) 2.5%  
   c) 5%  
   d) 17%  
   e) 14%  
   f) 74%
4. a) 17.1% (to 3 s.f.)  
   b) 82.9% (to 3 s.f.)
5. a) 56.3% (to 3 s.f.)  
   b) 43.7% (to 3 s.f.)
6. a) No, 10 is the frequency, not the percentage.  
   b) Blonde 33.3%, Brown 43.3%, Black 16.7%, Red 6.7%
7. 70.8% (to 3 s.f.)

**Practise... 7.6 Finding a percentage increase and decrease**

1. 3.81% (to 3 s.f.)
2. 10.5% (to 3 s.f.)
3. a) Fall in male workers = 36.9% (to 3 s.f.), less than that in female workers = 54.9% (to 3 s.f.)
   b) 41.8% (to 3 s.f.)
4. a) i) 9.09% (to 3 s.f.)  
   b) ii) 33.7% (to 3 s.f.)
5. a) She has divided £30 by £150 instead of by £120.
   b) 25%
6. Males increased by 10.9% and females by 12.8% (to 3 s.f.).
7. 66.7% (to 3 s.f.).
8. 20%
9. a) 50%  
   b) 100%  
   c) 400%
10. Yes, she is correct with the explanation.
11. | Drink | % change |
    |-------|--------|
    | Fruit juice | +21.4% |
    | Low calorie soft drinks | +14.9% |
    | Other soft drinks | -15.1% |
    | Beverages (e.g. tea, coffee) | No change |
    | Alcoholic drinks | +1.2% |
12. a) 2001–2006 (2.5% increase)  
   b) could be answered in a variety of ways

**Practise... 7.7 Reverse percentages**

1. a) £8.95  
   b) £35
2. a) £1250  
   b) £375
3. 125
4. £2.66
5. Sumira would pay less at Compsave. (PC Perfect charges £885.15 including VAT.)
6. a) The 10% saving is based on the full price and £3150 is not the full price (only 90%).
   b) Kev has saved £350.
7. £5000
8. VAT = £1373.89 (total amount = £9224.69)
9. £9506
1 Test A

<table>
<thead>
<tr>
<th>Test</th>
<th>Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>65%</td>
</tr>
<tr>
<td>B</td>
<td>60%</td>
</tr>
</tbody>
</table>

2 a 20.1% (to 3 s.f.)
   b i Some students read more than one of the categories.
   ii 86%  iii 30.4%

3 Girls did better.
   Boys 68%, Girls 70%

4 20%

5 £1622.40

6 60% of 135 = 81 marks, so she needs 49 on the second paper.

7 189

8 a 13498  b 14 hours (50736)

9 60mph

10 £295

11 Increase in English:
   75 - 53 = 22
   \( \frac{22}{53} \times 100\% = 41.5\% \)
Increase in maths:
   63 - 41 = 22
   \( \frac{22}{41} \times 100\% = 53.7\% \)

AQA Examination-style questions

1 £2000 \times 1.072^{10} = £4008.46
   The money is doubled

2 a 12%  b £930

3 75%
8 Statistical measures

**Practise... 8.1 Frequency distributions**

1. a \( \frac{1045}{43} \approx 24.30 \) (2 d.p.)
b. 30
c. 30
d. \( 40 - 0 = 40 \)
e. \( 30 - 15 = 15 \)
f. \( \frac{20}{43} \times 100 = 46.51 \) (2 d.p.)

2. a i. 96 ii. 176
b. \( \frac{88}{324} = 27.16\% \) (2 d.p.)
c. 70 – 20 = 50 mph
  i. 60 mph
  ii. 50 mph
  iii. \( \frac{15240}{324} = 47.04 \) mph (2 d.p.)

3. a. 3.47  
   c. 6  
   e. \( \frac{20}{100} = \frac{1}{5} \)
b. 3  
d. 5

4. a. 1
   b. 1.5
   c. The median, as you cannot get a person in a car.

5. Values for other country
   
   mode = 1  
   median = 1  
   mean = 1.36
   
   These measures are all smaller than the equivalent value for USA.
   
   Conclude that there are more rumbles of thunder per minute in a storm in USA.

6. Many possible answers

7. a

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>35</td>
</tr>
<tr>
<td>4</td>
<td>65</td>
</tr>
<tr>
<td>6</td>
<td>19</td>
</tr>
<tr>
<td>8</td>
<td>value &gt; 65</td>
</tr>
</tbody>
</table>

b

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>35</td>
</tr>
<tr>
<td>4</td>
<td>65</td>
</tr>
<tr>
<td>6</td>
<td>19</td>
</tr>
<tr>
<td>8</td>
<td>value 0–80</td>
</tr>
</tbody>
</table>

c

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>35</td>
</tr>
<tr>
<td>4</td>
<td>65</td>
</tr>
<tr>
<td>6</td>
<td>19</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

d

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>35</td>
</tr>
<tr>
<td>4</td>
<td>65</td>
</tr>
<tr>
<td>6</td>
<td>19</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

8. Some of the measures that may be calculated are given in the tables below.

<table>
<thead>
<tr>
<th>Measure</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Town centre</td>
</tr>
<tr>
<td>Mode</td>
<td>2</td>
</tr>
<tr>
<td>Median</td>
<td>2</td>
</tr>
<tr>
<td>Mean (2 d.p.)</td>
<td>2.06</td>
</tr>
<tr>
<td>IQR</td>
<td>0</td>
</tr>
</tbody>
</table>

Percentage of bedrooms of each size:

<table>
<thead>
<tr>
<th>Number of bedrooms</th>
<th>% in town sample</th>
<th>% in village sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>22.9</td>
<td>8.6</td>
</tr>
<tr>
<td>2</td>
<td>54.3</td>
<td>25.7</td>
</tr>
<tr>
<td>3</td>
<td>17.1</td>
<td>28.6</td>
</tr>
<tr>
<td>4</td>
<td>5.7</td>
<td>22.9</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>14.3</td>
</tr>
</tbody>
</table>

Averages suggest fewer bedrooms in a town house than in a village house on average.

Percentages show a much greater percentage for the houses with a small number of bedrooms in a town compared to a village. Many similar comments are possible.

9. a. Big Bus: mode = 0, median = 5, mean = 5.75 minutes, IQR = 10, range = 35
   Super Express: mode = 5, median = 12.5, mean = 10.7 minutes, IQR = 10, range = 20
   
   Super Express is, on average, more minutes late than Big Bus but Big Bus has a larger range of late times indicating that it can be much later than Super Express when it is late.

b. A bus that is on average a little late is unlikely to cause a problem. However, if a bus was extremely late (as indicated as a possibility by a larger range) that would be a problem.

10. The mean of \( x \) is 5.
    So the total (fx) must be 500.
    The median of \( x \) is 4.
    So the 50th and 51st values must be 4.
    The mode of \( x \) is 3.
    So 3 must have the highest frequency.
    The range of \( x \) is 7.
    So the highest take away the lowest must be 7.
    
    One of many possible distributions for \( x \) is therefore:

<table>
<thead>
<tr>
<th>( x )</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>24</td>
</tr>
<tr>
<td>8</td>
<td>14</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
</tr>
</tbody>
</table>
**Practise... 8.2 Grouped frequency distributions**

1 **a** \(2 \leq t < 4\)
   **b** \(2 \leq t < 4\)
   **c** \(\frac{23}{34} \times 100\% = 5.88235\%\)  
   \(= 6\% \text{ (1 s.f.)}\)

   **d**

<table>
<thead>
<tr>
<th>Time, (t) (minutes)</th>
<th>Frequency</th>
<th>Midpoint, (x)</th>
<th>(fx)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0 \leq t &lt; 2)</td>
<td>8</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>(2 \leq t &lt; 4)</td>
<td>14</td>
<td>3</td>
<td>42</td>
</tr>
<tr>
<td>(4 \leq t &lt; 6)</td>
<td>6</td>
<td>5</td>
<td>30</td>
</tr>
<tr>
<td>(6 \leq t &lt; 8)</td>
<td>4</td>
<td>7</td>
<td>28</td>
</tr>
<tr>
<td>(8 \leq t &lt; 10)</td>
<td>2</td>
<td>9</td>
<td>18</td>
</tr>
</tbody>
</table>

   \(126 \div 34 = 3.71\) minutes (2 d.p.)
   **e** About 10 minutes

2 **a** 15 – 19
   **b**

<table>
<thead>
<tr>
<th>Reading score</th>
<th>Frequency</th>
<th>Midpoint</th>
<th>(fx)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–4</td>
<td>15</td>
<td>2</td>
<td>30</td>
</tr>
<tr>
<td>5–9</td>
<td>60</td>
<td>7</td>
<td>420</td>
</tr>
<tr>
<td>10–14</td>
<td>125</td>
<td>12</td>
<td>1500</td>
</tr>
<tr>
<td>15–19</td>
<td>260</td>
<td>17</td>
<td>4420</td>
</tr>
<tr>
<td>20–24</td>
<td>250</td>
<td>22</td>
<td>5500</td>
</tr>
<tr>
<td>25–29</td>
<td>200</td>
<td>27</td>
<td>5400</td>
</tr>
<tr>
<td>30–34</td>
<td>90</td>
<td>32</td>
<td>2880</td>
</tr>
</tbody>
</table>

   \(\frac{20150}{1000} = 20.15\)

3 44 grams

4 **a** \(£100 \leq x < £150\)
   **b** \(£150 \leq x < £200\)
   **c** \(\frac{5600}{40} = £140\)
   **d** Probably the median as there are two quite isolated high values, which will make the mean high.

5 **a** \(25280 \div 1000 = 25.280\) g
   **b** No, the frequencies immediately show that some packets are underweight.
   **c** \(20 + (733/2) = 386.5\) out of 1000 packets are under the 25 g. This is 38.65%.
   **d** 38.65% of 3000 000 = 1 159 500

6 Mean for Machine A = \(30.07 \div 100 = 0.3007\) mm
   Mean for Machine B = \(29.07 \div 100 = 0.2907\) mm
   Mean for machine A is 0.01 mm higher than the mean for Machine B.
   The range of the two machines is similar as far as it is possible to tell from grouped data.
   Machine A is producing paper much closer to the desired thickness.

7 **a** 1.105\(t\)
   **b** The data are grouped so we do not know any of the exact values.

8 **a** \(20 \leq t < 30\)
   **b** It is not possible to calculate a midpoint.
   **c** \(50 \leq t < 80\) (as a midpoint of 65 has been used by Yasmina)

**Assess 8**

1 **a** Two very high numbers will make the mean large whereas the middle of the ordered data is only 4.
   Alternative: mean of 4 would be a total of 48 for 12 numbers, 33 and 37 alone much higher than this.
   **b** Two very low numbers will make mean small whereas the middle of the ordered data is 25.
   **c** Where data has isolated numbers of completely different size to the majority of the numbers, the median is preferable to the mean.

2 **a** One possible answer: 3 4 4 5 4 and 4 4 4 9 with ranges 2 and 5 respectively.
   **b** One possible answer: 2 3 4 5 4 and 3 3 3 3 6 with medians 4 and 3 respectively.

3 **a** 6
   **b** 5
   **c**

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
<th>(fx)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>19</td>
<td>19</td>
</tr>
<tr>
<td>2</td>
<td>26</td>
<td>52</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
<td>72</td>
</tr>
<tr>
<td>4</td>
<td>25</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>22</td>
<td>110</td>
</tr>
<tr>
<td>6</td>
<td>33</td>
<td>198</td>
</tr>
<tr>
<td>7</td>
<td>21</td>
<td>147</td>
</tr>
<tr>
<td>8</td>
<td>30</td>
<td>240</td>
</tr>
<tr>
<td>Total</td>
<td>200</td>
<td>938</td>
</tr>
</tbody>
</table>

\(\frac{938}{200} = 4.69\)

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4  
\[ a \] 16  \[ c \] 11  \[ e \] 1  
\[ b \] 12  \[ d \] 46  \[ f \] \( \frac{58}{46} = 1.35 \) (2 d.p.)

5  
\[ a \] 2  \[ c \] \( \frac{143}{36} = 3.97 \) (2 d.p.)  
\[ b \] 4  \[ d \] Mode is the lowest value, which is probably not helpful. Mean is not one of the data values, which is not helpful. Median is best measure here.

6

<table>
<thead>
<tr>
<th>Number of people in a household ( x )</th>
<th>Number of households ( f )</th>
<th>( fx )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>9 \times 1 = 9</td>
</tr>
<tr>
<td>2</td>
<td>19</td>
<td>19 \times 2 = 38</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>9 \times 3 = 27</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>8 \times 4 = 32</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>4 \times 5 = 20</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1 \times 6 = 6</td>
</tr>
<tr>
<td>Total</td>
<td>50</td>
<td>132</td>
</tr>
</tbody>
</table>

\[ a \] mean = \( \frac{132}{50} = 2.64 \)  
median = 2  
mode = 2  
range = 5  
\[ b \] Median or mode would be the best average to use as the mean gives a decimal answer and you cannot have 2.64 people.

7  
\[ a \] 5–9 years  
\[ b \]

<table>
<thead>
<tr>
<th>Number of years' service</th>
<th>Number of teachers ( f )</th>
<th>Midpoint ( x )</th>
<th>( fx )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–4</td>
<td>11</td>
<td>2</td>
<td>2 \times 11 = 22</td>
</tr>
<tr>
<td>5–9</td>
<td>15</td>
<td>7</td>
<td>7 \times 15 = 105</td>
</tr>
<tr>
<td>10–14</td>
<td>4</td>
<td>12</td>
<td>12 \times 4 = 48</td>
</tr>
<tr>
<td>15–19</td>
<td>10</td>
<td>17</td>
<td>17 \times 10 = 170</td>
</tr>
<tr>
<td>20–24</td>
<td>6</td>
<td>22</td>
<td>22 \times 6 = 132</td>
</tr>
<tr>
<td>25–29</td>
<td>4</td>
<td>27</td>
<td>27 \times 4 = 108</td>
</tr>
<tr>
<td>Total</td>
<td>50</td>
<td></td>
<td>585</td>
</tr>
</tbody>
</table>

\[ \frac{585}{50} = 11.7 \]

8  
\[ a \]

<table>
<thead>
<tr>
<th>Height, ( h ) (cm)</th>
<th>Frequency</th>
<th>Midpoint ( x )</th>
<th>( fx )</th>
</tr>
</thead>
<tbody>
<tr>
<td>149.5 ( \leq h ) &lt; 154.5</td>
<td>4</td>
<td>152</td>
<td>608</td>
</tr>
<tr>
<td>154.5 ( \leq h ) &lt; 159.5</td>
<td>21</td>
<td>157</td>
<td>3297</td>
</tr>
<tr>
<td>159.5 ( \leq h ) &lt; 164.5</td>
<td>18</td>
<td>162</td>
<td>2916</td>
</tr>
<tr>
<td>164.5 ( \leq h ) &lt; 169.5</td>
<td>7</td>
<td>167</td>
<td>1169</td>
</tr>
<tr>
<td>Total</td>
<td>50</td>
<td></td>
<td>7990</td>
</tr>
</tbody>
</table>

\[ \text{mean} = \frac{7990}{50} = 159.8 \]

9

<table>
<thead>
<tr>
<th>Length, ( x ) (minutes)</th>
<th>Frequency</th>
<th>Midpoint</th>
<th>( fx )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 ( \leq x &lt; 20 )</td>
<td>7</td>
<td>15</td>
<td>105</td>
</tr>
<tr>
<td>20 ( \leq x &lt; 20 )</td>
<td>8</td>
<td>25</td>
<td>200</td>
</tr>
<tr>
<td>30 ( \leq x &lt; 20 )</td>
<td>16</td>
<td>35</td>
<td>560</td>
</tr>
<tr>
<td>40 ( \leq x &lt; 20 )</td>
<td>6</td>
<td>45</td>
<td>270</td>
</tr>
<tr>
<td>50 ( \leq x &lt; 20 )</td>
<td>2</td>
<td>55</td>
<td>110</td>
</tr>
<tr>
<td>Total</td>
<td>39</td>
<td></td>
<td>1245</td>
</tr>
</tbody>
</table>

\[ \text{mean for 1900} = \frac{1245}{39} \]
\[ = 31.923076 \]
\[ \text{mean for 2008 is} 45\% \text{ higher} = 1.45 \times 31.923076 \]
\[ = 46.28846 \]
\[ = 46.29 \text{ minutes} \]

10  
\[ a \]

<table>
<thead>
<tr>
<th>Length, ( l ) (cm)</th>
<th>Tally</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 ( \leq l &lt; 7 )</td>
<td>iiii l</td>
<td>6</td>
</tr>
<tr>
<td>7 ( \leq l &lt; 9 )</td>
<td>iiii l</td>
<td>7</td>
</tr>
<tr>
<td>9 ( \leq l &lt; 11 )</td>
<td>iiii l</td>
<td>6</td>
</tr>
<tr>
<td>11 ( \leq l &lt; 13 )</td>
<td>iiii l</td>
<td>6</td>
</tr>
<tr>
<td>13 ( \leq l &lt; 15 )</td>
<td>iiii l</td>
<td>4</td>
</tr>
<tr>
<td>15 ( \leq l &lt; 17 )</td>
<td>i</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>30</td>
</tr>
</tbody>
</table>

\[ \[ b \] 9.87 \text{ cm} (2 \text{ d.p.}) \]
\[ \[ c \] 9.74 \text{ cm} (2 \text{ d.p.}) \]
\[ \[ d \] because the midpoints were used for each group instead of the exact values \]

11  
Clearly there are many ways to determine why one of the data sets would be the odd one out. Some of the more likely responses (linked to measures from this section) are:

Set A because it has a mean below 2.3, the other two have a mean above 2.3.

Set B because the other two have a range of 3 but set B has a range of 4.

Set B because the other two have a median that is an integer (A = 2, C = 3) B’s is 2.5.

Set C because the other two have a mode of 2 but set C has a mode of 3.

Set C because the other two have an inter-quartile range of 1 but C has an IQR of 3.
12 In games Tom won:

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
<th>Tom $f$</th>
<th>Sara $f$</th>
<th>Tom $fx$</th>
<th>Sara $fx$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>7</td>
<td>66</td>
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<tr>
<td>2</td>
<td>11</td>
<td>10</td>
<td>22</td>
<td>20</td>
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<td>3</td>
<td>8</td>
<td>12</td>
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<td>36</td>
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<tr>
<td>4</td>
<td>8</td>
<td>11</td>
<td>32</td>
<td>44</td>
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</tr>
<tr>
<td>5</td>
<td>14</td>
<td>8</td>
<td>70</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>12</td>
<td>9</td>
<td>72</td>
<td>54</td>
<td></td>
</tr>
<tr>
<td>Totals</td>
<td>62</td>
<td>57</td>
<td>229</td>
<td>213</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
<th>Tom $f$</th>
<th>Sara $f$</th>
<th>Tom $fx$</th>
<th>Sara $fx$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>8</td>
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<tr>
<td>2</td>
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<td>7</td>
<td>56</td>
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<td></td>
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<tr>
<td>3</td>
<td>11</td>
<td>9</td>
<td>99</td>
<td>81</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>10</td>
<td>70</td>
<td>40</td>
<td></td>
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<tr>
<td>5</td>
<td>8</td>
<td>8</td>
<td>64</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>14</td>
<td>140</td>
<td>84</td>
<td></td>
</tr>
<tr>
<td>Totals</td>
<td>55</td>
<td>56</td>
<td>188</td>
<td>213</td>
<td></td>
</tr>
</tbody>
</table>

Tom

<table>
<thead>
<tr>
<th>Mean</th>
<th>Mode</th>
<th>Median</th>
<th>Upper quartile</th>
<th>Lower quartile</th>
<th>Inter-quartile range</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.69</td>
<td>5</td>
<td>5</td>
<td>2</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Sara

<table>
<thead>
<tr>
<th>Mean</th>
<th>Mode</th>
<th>Median</th>
<th>Upper quartile</th>
<th>Lower quartile</th>
<th>Inter-quartile range</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.53</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

In games Sara won:

The means (and other measures of average) show that consistently the person whose mean is higher seems to win the games more often. The hypothesis is supported by the evidence. (There are little or no differences between the interquartile ranges – they are there for information – the hypothesis was about the average anyway.)

AQA Examination-style questions

1

<table>
<thead>
<tr>
<th>Height, $h$ (cm)</th>
<th>Number of students</th>
<th>Midpoint $m$</th>
<th>$fx$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$140 \leq h \leq 144$</td>
<td>4</td>
<td>142</td>
<td>568</td>
</tr>
<tr>
<td>$144 &lt; h \leq 148$</td>
<td>5</td>
<td>146</td>
<td>730</td>
</tr>
<tr>
<td>$148 &lt; h \leq 152$</td>
<td>8</td>
<td>150</td>
<td>1200</td>
</tr>
<tr>
<td>$152 &lt; h \leq 156$</td>
<td>7</td>
<td>154</td>
<td>1078</td>
</tr>
<tr>
<td>$156 &lt; h \leq 160$</td>
<td>5</td>
<td>158</td>
<td>790</td>
</tr>
<tr>
<td>$160 &lt; h \leq 164$</td>
<td>1</td>
<td>162</td>
<td>162</td>
</tr>
</tbody>
</table>

Total = 30

Estimate of mean $= \frac{4528}{30} = 150.93$ (2 d.p.)
1 a  + 3  d  + 1.5  g  + 3 or \( \times \frac{1}{3} \)
  b  + 4  e  - 5  h  \( \times 10 \)
  c  \( \times 2 \)  f  \( \times 1.5 \)

2 Any answer where the numbers increase by + 6 each time, e.g.
  0, 6, 12, 18, …
  1, 7, 13, 19, …
  50, 56, 62, 68, …
  100, 106, 112, 118, …
  -10, -4, 2, 8, …

3 a  4, 5, 6, 7, 8
  b  1.5, 2.5, 3.5, 4.5, 5.5
  c  2, 7, 12, 17, 22
  d  4, 7, 12, 19, 28
  e  -4, -1, 4, 11, 20
  f  4, 16, 36, 64, 100
  g  \( \frac{1}{2}, 2, 3, 4, 5, \frac{12}{13}, \frac{13}{14}, \frac{14}{15}, \frac{15}{16} \)

4 No, because the first term in the sequence \( n + 4 \) would be 5.

5 a  No, because 31 + 1 = 32 which does not divide by 3.
  b  No, because the 10th term is \( (10 \times 3) - 1 = 29 \) and the 20th term is \( (20 \times 3) - 1 = 59 \) which is not double 29.

6

<table>
<thead>
<tr>
<th>Pattern ((n))</th>
<th>Diagram</th>
<th>Number of matchsticks ((m))</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td></td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>11</td>
</tr>
</tbody>
</table>

a  ‘They are all odd numbers’ or ‘They all go up in twos’.

b  \( m = 2n + 1 \)

c  The tenth pattern will have \( 2 \times 10 + 1 = 21 \) matchsticks.

d  43, because the number of matchsticks is going up by two each time.

7 a  3n - n
  b  5n - 5
  c  6n + 2
  d  \( \frac{n}{2n + 3} \)
  e  -2n + 25 or 25 - 2n
  f  110 - 5n
  g  4n - 9
  h  2.5n + 1.5
  i  8n - 13
  j  \( \frac{2n}{6n + 1} \)

8 \( 4n - 3 \)

9 The sequence is 1, 4, 7… so the \( n \)th term is \( 3n - 2 \) and the 100th pattern will have 298 cubes.

10

<table>
<thead>
<tr>
<th>11111 x 11111</th>
<th>12345321</th>
</tr>
</thead>
<tbody>
<tr>
<td>111111 x 11111</td>
<td>12345654321</td>
</tr>
</tbody>
</table>

No, the pattern will break down after 12345678987654321.

11 a  \( n^2 \)
  b  \( n^2 + 1 \)
  c  \( 2n^2 \)
  d  \( n^3 \)
  e  \( n^3 - 1 \)
  f  \( 10^n \)

12 a  \( n \times (n + 1) \)
  b  \( \frac{n + 1}{n + 2} \)
  d  0.1n
  e  \( 0.11n \)
  c  \( n \times (n + 1) \times (n + 4) \)

13 a  The number of white tiles is 1, 2, 4, 8, … so the \( n \)th term is \( 2^{n - 1} \)
  b  The number of red tiles is 0, 2, 5, 8, 13, … which is difficult to assign but as the total number of squares is \( n^2 \), the number of red tiles must be \( n^2 - 2^{n - 1} \)

14 a  3n + 1
  b  31

15 a  3n + 3
  b  24 m
  i  10.5 m

16 The sequence continues 45, 31, 14, 15, 16, 17, 18, 19, 20, 11, 12, 13, 14, 15, … which is a repeating sequence. The 10th term is 11, the 20th term is 11, the 30th term is 11 and so the 100th term is 11.
Assess 9

1  a  3, 5, 7; 11; 51; 101
   b  3, 8, 13; 23; 123; 248
   c  2, 5, 10; 26; 626; 2501

2  a  30
   b  $3n + 15 = 3(n + 5)$ so every term must be divisible by 3

3  a  34, 32, 30, 28
   b  The term-to-term difference is $-2$ and the terms are even so eventually the sequence will get to ... 4, 2, 0, $-2$, ...

4  | Side of square (cm) | 1 | 2 | 3 | 4 | n |
   | Area of square (cm²) | 1 | 4 | 9 | 16 | $n^2$

5  a  $\frac{n}{(4n - 1)}$
   b  The $n$th term of the sequence is $2n \times (4n - 1)$ which equals $8n^2 - 2n$.

6  a  Student’s drawing of a seven- and an eight-sided polygon with all diagonals drawn in.

b  | Number of sides | 4 | 5 | 6 | 7 | 8 |
   | Number of diagonals | 2 | 5 | 9 | 14 | 20 |

c  i  27  ii  35  iii  44  iv  54

d  $\frac{n^2 - 3n}{2}$ or $\frac{n(n-3)}{2}$

e  i  90  ii  170  iii  1175  iv  4850

7  For a term to occur in both sequences, $4n - 5 = 2n + 8$, so $2n = 3$ and $n = 1.5$
As the $n$th term is always a whole number, $n$ could never be 1.5, so Jo is correct.

8  2, 4, 6, 8. Yes. If the first term is $a$ and the difference is $d$, the sequence is written $a, a + d,$ $a + 2d, a + 3d, \ldots$ If the first term is $d$, then the 2nd term is $d + d = 2d$ and the 4th term is $d + 3d = 4d$ so the 4th term is twice the 2nd term.

9  a  $10^{a}, 10^{b}, 10^{c}, 10^{d}$
   b  $\frac{3}{10}$

10 Largest number = 54

AQA Examination-style questions

1  a  $[(p - 1)^2 - 1]^2 = (p^2 - 2p + 1 - 1)^2 = [p(p - 2)]^2$
   b  $(x - 1)^2 = p$ so $x = \sqrt{p + 1}$

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10 Ratio and proportion

10.1 Finding and simplifying ratios

1 a 1:2 e 1:6 i 3:2 m 3:5
b 1:3 f 1:7 j 1:4 n 3:2
c 1:4 g 3:1 k 1:8 o 1:3
d 1:5 h 4:3 l 3:8 p 1:4

2 a 1:2 d 8:1 g 3:20
b 1:3 e 1:20 h 16:3
c 2:1 f 11:50

3 e.g. 2:4, 3:6, 10:20
The second number is twice the first.

4 a 2:6 c 7:21 e a:3a
b 5:15 d 1200:3600

5 1:2.5 0.2:0.5 2:5 3:7.5

6 She is adding the same number to both
numbers in the ratio instead of multiplying
them both by the same thing.

7 1:10

8 a 6 b 7

10.2 Dividing quantities in given ratios

1 a 50, 100 d 2 litres, 4 litres
b 100, 200 e £0.50, £1
0.5 litres, 1 litre
c £1.50, £3

2 a 30, 120; 60, 240; £0.90, £3.60; 1.2 litres,
4.8 litres; £0.30, £1.20; 0.3 litres, 1.2 litres
b 60, 90; 120, 180; £1.80, £2.70; 2.4 litres,
3.6 litres; 60p, 90p
c 45, 105; 90, 210; £1.35, £3.15; 1.8 litres,
4.2 litres; 45p, 105p; 0.45 litres, 1.05 litres
d 15, 30, 105; 30, 60, 210; 60p, £1.20, £3.15;
0.6 litres, 1.2 litres, 4.2 litres; 15p, 30p,
£1.05; 0.15 litres, 0.3 litres, 1.05 litres

3 £210

4 162 degrees

5 a 50 g, 75 g, 75 g, 40 g, 30 g, 65 g
b chicken sandwich, grilled salmon, yoghurt
(whole milk), milk
c 400 g
d taco chips, bread

6 a copper 950 g, tin 40 g, zinc 10 g
b copper 9.5 kg, tin 400 g, zinc 100 g
c copper 475 g, tin 20 g, zinc 5 g
d 0.06 g

7 Leena £51 851.85, Kate £18 148.15

8 a 375, 375; 400, 500; 800, 1000; 612, 714;
602, 582
b School E; this has the smallest number of
girls for one boy.
**Practise... 10.3 The unitary method**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1125 g</td>
</tr>
<tr>
<td>2</td>
<td>1.5 hours</td>
</tr>
<tr>
<td>3</td>
<td>3 kg</td>
</tr>
<tr>
<td>4</td>
<td>£114.75</td>
</tr>
<tr>
<td>5</td>
<td>400 g flour, 100 g butter, 125 g cheese</td>
</tr>
<tr>
<td>6</td>
<td>15 minutes. A piece of music is the same length no matter how many people are playing it!</td>
</tr>
<tr>
<td>7</td>
<td>a 317 miles  b 32 miles  c 3 hrs 57 minutes</td>
</tr>
<tr>
<td>8</td>
<td>a 41.3 litres  b 97 miles  c that the rate of consumption of diesel is constant</td>
</tr>
<tr>
<td>9</td>
<td>large size</td>
</tr>
<tr>
<td>10</td>
<td>a £2.85, £2.20, £2.08  b 25.50 kg  c £72.68  d It is cheaper per kilogram. It is heavy to carry, difficult to store, not easy to use, etc.</td>
</tr>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>a 900 newtons  b 60 newtons  c 1950 newtons  d 6 : 1 : 15</td>
</tr>
<tr>
<td>12</td>
<td>a £37.50</td>
</tr>
</tbody>
</table>

**Assess 10**

<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>1</td>
<td>1 : 18</td>
</tr>
<tr>
<td>2</td>
<td>3 : 7</td>
</tr>
<tr>
<td>3</td>
<td>a i 3 : 4  ii 1 : 3  b i 2 : 3  ii 7 : 10  iii 100 : 1  iv 1 : 8  v 5 : 7</td>
</tr>
<tr>
<td>4</td>
<td>a 400 g  b 375 ml</td>
</tr>
<tr>
<td>5</td>
<td>a the same in each  b 24</td>
</tr>
<tr>
<td>6</td>
<td>a £4, £8  b £2, £10  c £1.71/£10.29</td>
</tr>
<tr>
<td>7</td>
<td>15</td>
</tr>
<tr>
<td>8</td>
<td>£1350</td>
</tr>
<tr>
<td>9</td>
<td>‘Lowerpay’</td>
</tr>
<tr>
<td>10</td>
<td>a 7 seconds  b 16 seconds</td>
</tr>
<tr>
<td>11</td>
<td>a Cuba  b Cuba 20468, Israel 10 140, Italy 107 413, Nigeria 203 558, Tanzania 55 772, Thailand 109 666, UK 109 478, USA 494 999</td>
</tr>
<tr>
<td>12</td>
<td>a 3.331  b Two 2.5 litre cans (£27.96)</td>
</tr>
</tbody>
</table>

**Examination-style questions**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>195</td>
</tr>
</tbody>
</table>
### Practise... 11.1 Volume of a prism

1. a) 28 cm³  
   b) 40 cm³

2. a) i) 46 000 cm³  
    ii) 230 000 cm³  
    iii) 90 000 cm³  
    b) i) 30 m²  
    ii) 7.5 m²

3. 6 m

4. a) 176 cm³  
    b) 145 cm³

5. a) i) 8 000 000 cm³  
    ii) 3 200 000 cm³  
    iii) 765 000 cm³  
    b) i) 2.36 m³  
    ii) 56 m³  
    iii) 0.4731 m³

6. a) 540 mm³  
    b) 165.76 cm²  
    c) 96 m³

7. a) 1583.4 cm³  
    b) 7093.5 m³  
    c) 9621.1 mm³

8. 4 cm

9. a) i) 50 cm²  
    ii) 350 cm³  
    b) i) 79.2 m²  
    ii) 319.2 m³  
    c) i) 96.25 cm²  
    ii) 770 m³

10. 4.9 m²

11. 662.625 cm³

12. 1696.46 cm³

13. 18 hours and 35 minutes

14. 6.107 litres are needed. Susie has 6.6 litres so she is correct.

15. Student’s three prisms that each has a volume of 90 cm³.

### Practise... 11.2 Surface area of a prism

1. 386 cm²

2. a) 252 m²  
    b) 2 520 000 cm²

3. a) 294 cm²  
    b) 600 cm²  
    c) 174.96 cm²

4. a) 136 cm²  
    b) 1232 mm²  
    c) 133.6 m²

5. a) 487.5 mm²  
    b) 210.88 cm²

6. a) 527.8 cm²  
    b) 1688.9 m²  
    c) 1099.6 m²

7. a) 791.7 cm²  
    b) 282.7 m²

8. £3.40

9. Yes, 45 + 171 + 564 = 780 cm² (= 0.078 m²)

10. h = 3r

### Assess 11

1. 7500 cm²

2. a) 288 cm²  
    b) 3320 mm²  
    c) 112 m²  
    d) 998.2 cm²

3. a) 150 cm³  
    b) 150 000 mm³

4. a) 7200 m³  
    b) 420 m³

5. a) 288 cm²  
    b) 1152 cm²  
    c) 112 m²  
    d) 998.2 cm²  
    e) none of these  
    f) length  
    g) volume  
    h) none of these

6. 120.2 m³

7. a) area  
    b) length  
    c) length  
    d) none of these

8. £3.40

9. Yes, 45 + 171 + 564 = 780 cm² (= 0.078 m²)

10. h = 3r

### AQA Examination-style questions

1. 1560 cm³

2. a) 66.48 cm²  
    b) $2 \times \pi \times 4.6 \times 25 + 2 \times 66.48 = 855.5 cm²$

3. a) 34 cm³

4. a) 50 cm²  
    b) 0.55 litre

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**12 Real-life graphs**

**Practise...**

12.1 Distance–time graphs

1. a 6 km  
   b 15 min  
   c Sam is walking fastest on his way home. His average speed is 6 km/h.

2. a 15 km  
   b i G and H  ii 4 km  iii 5 min  
   c i B and C  ii 1 km  iii 30 min  
   d A–B average speed = 18 km/h  
   B–C average speed = 2 km/h  
   C–D average speed = 24 km/h  
   D–E average speed = 6 km/h  
   E–F average speed = 0 km/h  
   F–G average speed = 1 km/h  
   G–H average speed = 48 km/h  
   H–I average speed = 10 km/h  
   e 7.5 km/h

3. a i 10.15 am  ii 30 min  iii 75 miles  
   b B  
   c 10 miles  
   d A–B average speed = 40 mph  
   B–C average speed = 65 mph  
   C–D average speed = 0 mph  
   D–E average speed = 65 mph  
   E–F average speed = 20 mph  
   e 50 mph

4. 46.2 mph

5. a Frances walked because this is the slowest journey. Henri cycled because this is the next slowest. Fay is likely to have taken the train because this is the quickest journey. John is likely to have taken the bus because this is quite a short distance but a relatively slow average speed.

6. a Various answers are possible. For example, Hamish could have fallen off at 0.5 hours and then had a break and set off again at a slower pace to recover. Or, he could have fallen off at 3.5 hours and then ridden straight back again.
   b Between 3 and 3.5 hours.
   c Hamish cycles 5 km in half an hour, then stops for half an hour. He then cycles at a slower pace, covering 2 km in 2 hours. Between 3 and 3.5 hours he picks up speed, covering a distance of 6 km. Hamish then turns back and cycles 11 km between 4 and 5 hours.
   d 4.8 km/h
   e Various answers are possible. For example, in five hours he needs to go a total of 25 km so extending the furthest point to 12.5 would make the average speed 5 km/h.
   f Various answers are possible. For example, he could have walked back after the fall, which would have taken longer, extending the time axis past the 5 hour point. Or he could have got a lift in a car making the return journey much quicker.

**Practise...**

12.2 Other real-life graphs

1. a 90°C  
   b 10 min (allow 10–13 min)  
   c Jo is incorrect because the tea is cooling more slowly all the time. It will in fact never freeze because it will not get below room temperature, which is most likely to be above freezing. **However**, it is difficult to use a curve to make predictions. There is no information as to what happens to the tea after 30 min have elapsed so the same pattern may not continue.

2. b Frances’s average speed = 2 km/h  
   Henri’s average speed = 12 km/h  
   Fay’s average speed = 36 km/h  
   John’s average speed = 16 km/h

3. a i 10.15 am  ii 30 min  iii 75 miles  
   b B  
   c 10 miles  
   d A–B average speed = 40 mph  
   B–C average speed = 65 mph  
   C–D average speed = 0 mph  
   D–E average speed = 65 mph  
   E–F average speed = 20 mph  
   e 50 mph

4. 46.2 mph

5. a Frances walked because this is the slowest journey. Henri cycled because this is the next slowest. Fay is likely to have taken the train because this is the quickest journey. John is likely to have taken the bus because this is quite a short distance but a relatively slow average speed.

6. a Various answers are possible. For example, Hamish could have fallen off at 0.5 hours and then had a break and set off again at a slower pace to recover. Or, he could have fallen off at 3.5 hours and then ridden straight back again.
   b Between 3 and 3.5 hours.
   c Hamish cycles 5 km in half an hour, then stops for half an hour. He then cycles at a slower pace, covering 2 km in 2 hours. Between 3 and 3.5 hours he picks up speed, covering a distance of 6 km. Hamish then turns back and cycles 11 km between 4 and 5 hours.
   d 4.8 km/h
   e Various answers are possible. For example, in five hours he needs to go a total of 25 km so extending the furthest point to 12.5 would make the average speed 5 km/h.
   f Various answers are possible. For example, he could have walked back after the fall, which would have taken longer, extending the time axis past the 5 hour point. Or he could have got a lift in a car making the return journey much quicker.
2 a John has misread the key to the graph and has given the wrong variable. It is actually the power that is at its highest at 4000 rpm.

b Various answers are possible. The graph shows torque increasing steeply at first and then levelling out and then decreasing slowly. The torque starts at about 34 kW for 1000 rpm with the maximum torque being about 59 kW.

c From the graph it does not seem to be impossible that the graph might continue to predict values for 7000 rpm. However, the curves are not ‘predictable’ because a pattern is difficult to discern. Past performance is no guarantee of future performance.

(Also, look at car rev. counters and you will see that they rarely go up as high as 7000 rpm.)

3 1 goes with B
   2 goes with C
   3 goes with F
   4 goes with G
   5 goes with A
   6 goes with E
   7 goes with H
   8 goes with D

4 a and b

Assess 12

1 a 10.20am  
   b i 11.40am    ii 20 min  
   c 2.6 km/h (2 s.f.)  
   d 12.40pm and 1.20pm  
   e 7 km  
   f 2.33 km/h (3 s.f.)

2 a Beckie runs at a constant speed for 35 minutes, covering 3 miles in that time. Sophie runs at a constant speed for 15 minutes, pauses for 10 minutes and then runs at a constant speed for 20 minutes, also covering 3 miles in total.

b Sophie is resting between A and B.

c 4 mph

3 22 metres per second

4 a 11:45–12:00 and 20:00
   b 10.4 m
   c 2.4 m
   d 7 h 45 min
   e i Any time after 16:00 and before 17:00
        ii Any time after 08:00 and before 12:00
   f i She won’t be able to land her boat after 14:30 so she has two and a half hours.
        ii If she is later than 14:30 she will have to wait until 17:30 before she can land her boat.

5 a 17 h
   b 7 h
   c 13 h 48 min

6 a i The graph of the thinking distance is a straight line which shows that the thinking distance increases in direct proportion to the increase in speed.
   ii The graph of the stopping distance is a curved line which shows that the stopping distance increases more rapidly as the speed increases.

b i 12 m    ii 16.5 m    iii 23 m    iv 36 m

c Fred is incorrect because the stopping distances do not increase linearly so they cannot be simply a multiple of each other.

7 around £60
### 13 Indices and standard index form

#### Practise... 13.1 Rules of indices

<table>
<thead>
<tr>
<th></th>
<th>1 a 25</th>
<th>c 990</th>
<th>e 13</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>72</td>
<td>d 10</td>
<td>f 15</td>
</tr>
<tr>
<td></td>
<td>2 a 49</td>
<td>e 8</td>
<td>i 81</td>
</tr>
<tr>
<td>b</td>
<td>16</td>
<td>f 10000</td>
<td>j 64</td>
</tr>
<tr>
<td>c</td>
<td>121</td>
<td>g 1</td>
<td>k 100000</td>
</tr>
<tr>
<td>d</td>
<td>9</td>
<td>h 32</td>
<td>l -128</td>
</tr>
<tr>
<td></td>
<td>3 a 81</td>
<td>d 5</td>
<td>g 1</td>
</tr>
<tr>
<td>b</td>
<td>-32</td>
<td>e 4096</td>
<td>h 1</td>
</tr>
<tr>
<td>c</td>
<td>-81</td>
<td>f 1</td>
<td>i 1/3</td>
</tr>
<tr>
<td>j</td>
<td>1/8</td>
<td>m 1923</td>
<td>p 99000000</td>
</tr>
<tr>
<td>k</td>
<td>1/100</td>
<td>n 100</td>
<td></td>
</tr>
<tr>
<td>l</td>
<td>1/80</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4 a 5^8</td>
<td>c 4^4</td>
<td>e 3^-3</td>
</tr>
<tr>
<td>b</td>
<td>12^{11}</td>
<td>d 7^5</td>
<td>f 9^{10}</td>
</tr>
<tr>
<td></td>
<td>5 a False: 6^2 = 6 × 6 = 36</td>
<td>b True: 1^3 = 1 × 1 × 1 = 1</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>False: 1^-1 = 1/1 = 1</td>
<td>d False: 16^-1 = 1/16 = 1/4</td>
<td></td>
</tr>
<tr>
<td>e</td>
<td>True: 2^{10}/4^5 = 2^{10} = 1</td>
<td>f False: 3^4 + 3^3 = 81 + 243 = 324 whereas 3^5 = 19683</td>
<td></td>
</tr>
<tr>
<td>g</td>
<td>True: 10^{50} × 10^{50} = 10^{100}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>h</td>
<td>True: (-216)^{1/3} = 3√-216 = -6</td>
<td>i False: 1000000^0 = 1</td>
<td></td>
</tr>
</tbody>
</table>

#### Practise... 13.2 Standard index form

<table>
<thead>
<tr>
<th></th>
<th>1 a 4.2 × 10^{3}</th>
<th>c 1.5 × 10^{5}</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>5.9 × 10^{8}</td>
<td>f 8 × 10^{-4}</td>
</tr>
<tr>
<td>c</td>
<td>7.001 × 10^{8}</td>
<td>g 1.3 × 10^{-2}</td>
</tr>
<tr>
<td>d</td>
<td>8.6 × 10^{9}</td>
<td>h 1.78 × 10^{-7}</td>
</tr>
<tr>
<td></td>
<td>2 a 400000</td>
<td>e 0.9</td>
</tr>
<tr>
<td>b</td>
<td>600</td>
<td>f 0.000475</td>
</tr>
<tr>
<td>c</td>
<td>7005</td>
<td>g 0.0000000000999</td>
</tr>
<tr>
<td>d</td>
<td>34.01</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 9.10938 × 10^{-31} kilograms</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4 2.16 × 10^{5}</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5 a 8 × 10^{11}</td>
<td>e 1.25 × 10^{6}</td>
</tr>
<tr>
<td>b</td>
<td>9.9 × 10^{10}</td>
<td>f 2.25 × 10^{14}</td>
</tr>
<tr>
<td>c</td>
<td>9 × 10^{16}</td>
<td>g 2.5 × 10^{-7}</td>
</tr>
<tr>
<td>d</td>
<td>1.5 × 10^{15}</td>
<td>h 2 × 10^{1}</td>
</tr>
<tr>
<td></td>
<td>6 a 9 × 10^{12}</td>
<td>f 2.5 × 10^{-7}</td>
</tr>
<tr>
<td>b</td>
<td>1.6 × 10^{15}</td>
<td>g 2 × 10^{8}</td>
</tr>
<tr>
<td>c</td>
<td>8.4 × 10^{12}</td>
<td>h 3 × 10^{13}</td>
</tr>
<tr>
<td>d</td>
<td>2.821 × 10^{13}</td>
<td>i 5 × 10^{-10}</td>
</tr>
<tr>
<td>e</td>
<td>2.25 × 10^{14}</td>
<td>j 4 × 10^{-9}</td>
</tr>
<tr>
<td></td>
<td>7 a 80</td>
<td>c 400.2</td>
</tr>
<tr>
<td>b</td>
<td>2000</td>
<td>d 399.8</td>
</tr>
<tr>
<td></td>
<td>8 4.6 × 10^{23}km</td>
<td></td>
</tr>
<tr>
<td></td>
<td>9 1.8144 × 10^{14}m</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10 3.3 × 10^{5}</td>
<td></td>
</tr>
<tr>
<td></td>
<td>11 Mercury, Mars, Venus, Earth, Neptune, Uranus, Saturn, Jupiter</td>
<td></td>
</tr>
<tr>
<td></td>
<td>12 Ali is correct. (4 × 10^{6}) + (2 × 10^{4}) = (6 × 10^{4})</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Indices are only added when numbers are multiplied together, not when the numbers are added as here.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>13 4.25 × 10^{3}</td>
<td></td>
</tr>
<tr>
<td></td>
<td>14 9.6 × 10^{-2}cm</td>
<td></td>
</tr>
</tbody>
</table>
Assess 13

1 a Gareth is right. Negative numbers have no square roots.
   b No, Livia is wrong. Numbers only have one cube root.

2 a 48  c 56  e 69  g 212  i 51
   b 118  d 76  f 102  h 161  j 20

3 a 144  b 25  c 1296  d 100

4 a 35  b 2^{11}  c They are equal.

5 The correct statement is: The sum of the squares of two odd numbers is always even.
The reason is that:

- an odd \times odd = odd so all odd numbers squared are odd
- an odd + odd = even so when you add two odd numbers you get an even number.

6 1:21.4 (3 s.f.)

7 a \frac{1}{5}  c \frac{1}{12}  e \frac{1}{4}  g 8
   b 1  d \frac{1}{9}  f 1

8 a 9  d 1  g 8
   b 15  e 16  h \frac{1}{3}
   c \frac{1}{3}  f 16  i \frac{1}{25}

AQA Examination-style questions

1 a = 2 and b = 5 (or a = 5 and b = 2)

2 a 36 is not between 1 and 10.
   b 6.55 (3 s.f.)
14 Properties of polygons

Practise... 14.1 Properties of quadrilaterals

1 a square, rhombus  
   b square, rectangle, parallelogram, rhombus, trapezium  
   c square, kite, rhombus  
   d trapezium

2 Barry is wrong. These angles add up to 370° and not 360°.

3

<table>
<thead>
<tr>
<th>Shape</th>
<th>Are the diagonals equal? (Yes/No)</th>
<th>Do the diagonals bisect each other? (Yes/No/Sometimes)</th>
<th>Do the diagonals cross at right angles? (Yes/No)</th>
<th>Do the diagonals bisect the angles of the quadrilateral? (Yes/No/Sometimes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Kite</td>
<td>No</td>
<td>Sometimes</td>
<td>Yes</td>
<td>Sometimes</td>
</tr>
<tr>
<td>Parallelogram</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Trapezium</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Isosceles</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Trapezium</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Rectangle</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Rhombus</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

4 Rajesh could have drawn a square, an isosceles trapezium or a rectangle.

5 No, the diagonals of a rectangle do not bisect the corner angles.

6 58°

7  
   a = 90° (diagonals of a rhombus bisect at right angles)  
   b = 32° (diagonals of a rhombus bisect the angles)  
   c = 58° (angles in a triangle add up to 180°)  
   d = 122° (angles on a straight line)  
   e = 38° (rectangle has equal diagonals, diagonals bisect each other, isosceles triangle)  
   f = 104° (angles in a triangle add up to 180°)

8  
   a = 65° (opposite angles of parallelogram)  
   b = 115° (interior angles, parallel lines)  
   c = 65° (corresponding angles)  
   d = 25° (angles of a triangle)  
   e = 130° (isosceles triangle)  
   f = 25° (alternate angles)

9  
   Sam: 76°, 2 × 60°, 164°; Tess: 2 × 76°, 60°, 148°; Matt: 76°, 60°, 2 × 112°

10 55°

Practise... 14.2 Angle properties of polygons

1 101°

2  
   a = (4 × 180°) − 610° = 110°  
   b = 180° − 135° = 45°

3 6

4 144° − 140° = 4°

5 No, it should be (n − 2) × 180° = 4 × 180°

6 (8 − 2) × 180° = 6 × 180° = 3 × 360°
7 Regular hexagon: draw six lines radiating from the centre of the circle, each $60^\circ$ apart. Join the points at which the lines touch the circle.
Regular nonagon: draw nine lines radiating from the centre of the circle, each $40^\circ$ apart. Join the points at which the lines touch the circle, as above.

8 a $108^\circ$  d $36^\circ$  g $90^\circ$
b $120^\circ$  e $72^\circ$
c $132^\circ$  f $30^\circ$

9 $36^\circ$

10 An acute interior angle has an obtuse exterior angle. More than three obtuse exterior angles is impossible because the sum would be greater than $360^\circ$.

Assess 14

1 Possible polygons are a, b, c, e, g, h and i.

2 a $a = 112^\circ$
b $b = 50^\circ$; $c = 40^\circ$
c $d = 69^\circ$; $e = 41^\circ$; $f = 33^\circ$
d $g = 47^\circ$; $h = 90^\circ$; $i = 43^\circ$; $j = 47^\circ$
e $k = 110^\circ$; $l = 50^\circ$
f $m = 129^\circ$; $n = 100^\circ$
g $p = 128^\circ$
h $q = 60^\circ$; $r = 150^\circ$
i $s = 88^\circ$; $t = 92^\circ$
j $u = 135^\circ$; $v = 45^\circ$

3 square, hexagon and equilateral triangle

4 Sophie is correct. $40^\circ$ divides into $360^\circ$ to give 9, so it is a nonagon.

5 a $90^\circ$  b $176^\circ$  c $15840^\circ$

6 a $100^\circ$  b $180^\circ$

7 a $36^\circ$  b $10^\circ$

11 With seven (or more) angles, and only three (maximum) acute angles, it is impossible to alternate the acute and obtuse angles. Therefore there must be two adjacent obtuse angles.

12 Consider the centre hole as a polygon surrounded by pentagons. The interior angle of a regular pentagon is $108^\circ$. So the interior angle of the centre hole is: $360^\circ - 2 \times 108^\circ = 144^\circ$
The exterior angle of the centre hole is: $180^\circ - 144^\circ = 36^\circ$. The hole has $360^\circ \div 36^\circ = 10$ sides

13 a $135^\circ$  b No, a square is left between them.
15 Equations and inequalities

**Practise... 15.1 Equations where the unknown \((x)\) appears on both sides**

1. \(a\ x = 6\)  \(e\ p = 3.5\)  \(i\ c = \frac{1}{2}\)
2. \(b\ y = 7\)  \(f\ q = 2\)  \(j\ d = 7\)
3. \(c\ z = 5\)  \(g\ a = -1\)  \(k\ e = 3\)
4. \(d\ t = 1.5\)  \(h\ b = -2\)  \(l\ f = -2\)

2. No. Jared should have added \(4x\) to both sides of the equation but instead he has taken away \(4x\) from \(9x\).

3. No. Ella has taken \(2\) from \(6\) instead of \(6\) from \(2\). She should have got \(-4\) not \(4\). She has also taken \(y\) from \(5y\) instead of adding \(y\) to \(5y\).

4. Dean has taken \(4\) from \(11\) instead of adding \(11\) to \(4\).

5. Rick has added the \(y\) terms incorrectly, getting \(y\) instead of \(5y\).

6. 27
7. 14
8. \(33 - 8 = 25\)
   If \(b = 11\), \(25\) would have to equal \(19 - 2b\)
   which means \(25\) would have to equal \(19 - 22\)
   which it does not.

9. \(9 - 20 = 29\)
   If \(c = -4\), \(29\) would have to equal \(6c + 13\)
   which means \(29\) would have to equal \(-24 + 13\)
   which it does not.

**Practise... 15.2 Equations with brackets**

1. \(a\ x = 8\)  \(c\ z = 10\)  \(e\ c = 1.5\)
2. \(b\ y = 12\)  \(d\ b = 2\)
3. \(a\ p = 0.5\)  \(e\ b = -10\)  \(i\ f = -2\)
4. \(b\ q = 5\)  \(f\ c = 4.5\)  \(j\ x = -1\)
5. \(c\ t = 1.5\)  \(g\ d = 0.5\)  \(k\ y = 2\)
6. \(d\ a = -11\)  \(h\ e = -3\)

7. \(z = -1.5\)

**Practise... 15.3 Equations with fractions**

1. \(a\ x = 18\)  \(f\ p = 3\)  \(k\ p = 17\)
2. \(b\ y = 20\)  \(g\ q = 2.5\)  \(l\ q = 6\)
3. \(c\ z = 12\)  \(h\ x = 8\)  \(m\ x = 7.5\)
4. \(d\ b = 14\)  \(i\ y = 11\)  \(n\ y = 8\)
5. \(e\ c = -18\)  \(j\ z = 6\)

2. Gary is correct.

3. \(a\ x = 1\)  \(c\ z = 9\)  \(e\ b = 3\)
4. \(b\ y = 16.5\)  \(d\ a = 11\)  \(f\ c = 5\)

**Practise... 15.4 Inequalities and the number line**

1. \(a\ x \geq -5\)  \(c\ -2 < z \leq 3\)
2. \(b\ y < 2\)  \(d\ t \leq -3\) or \(t \geq 0\)

2. \(a\)

   -5  -4  -3  -2  -1  0  1  2  3  4  5  \(x\)

\(b\)

\(-5\ -4\ -3\ -2\ -1\ 0\ 1\ 2\ 3\ 4\ 5\ \(x\)\)
It is incorrect to write $2 < x < -6$ because it means $x$ is a number greater than 2 and less than $-6$ and there are no numbers which are greater than 2 and less than $-6$.

4 £6.50, £7.00, £7.50, £8.00, £8.50, £9.00, £9.50

5

The possible heights cannot be listed as there are too many possibilities due to incrementation, i.e. height is a continuous variable.

Practise... 15.5 Solving inequalities

1 a $x \geq 2$  c $z < -3$  e $q > 2$
b $y \leq 4.5$  d $p > -2$  f $t > 8$

2 2

3 -3

4 a $1, 2, 3$  c $-2, -1, 0, 1, 2$
b $-1, 0, 1, 2, 3$  d $-1, 0, 1$

5 a $u > 1$  c $w > 5.5$  e $y \leq 2.5$
b $v < -1$  d $x < -3$  f $z \geq \frac{3}{5}$

6 $x = 1$ and $y = 1$
 $x = 1$ and $y = 2$
 $x = 1$ and $y = 3$
 $x = 2$ and $y = 1$
 $x = 2$ and $y = 2$
 $x = 3$ and $y = 1$

7 Let $x =$ Asif’s age now.
$x + 9 \geq 2(x - 14)$, leading to $x \leq 37$
Asif is now 37 years of age.

Practise... 15.6 Inequalities and graphs
2 a $x < 2$  c $y \leq -3$  e $y < x$
b $y > 4$  d $x \geq 1$  f $x + y < 3$

3 a

b (3, 3), (4, 3) and (4, 4)

4 a i $y = -x$  ii $y = x + 2$  iii $y = -x - 5$
b i $y < -x$  ii $y > x + 2$  iii $y > -x - 5$

5 a

b No, there are only five points in the region: (1, 1), (2, 2), (2, 3) and (3, 2). Dean has probably included the points that lie on the dotted lines, which is incorrect.

6 a $x + y \leq 4$ and $y \geq 3x$
b $x + y \leq 4$ and $y \leq 3x$
c $y \leq -2x$
d $x + y \leq 4, y \geq -2x$ and $y \leq 3x$
e $x + y \leq 4, y \geq 3x$ and $y \geq -2x$
Assess 15

1 a $p = 3$    c $m = 2.5$    e $u = 6$
  b $q = 2$    d $n = -7$    f $t = 3$

2 a $-3 < x \leq 3$    b $x < 1$

3 a $v = 21$    c $x = 3$
  b $w = 5$    d $t = \frac{3}{4}$

4 a $y = 16$    b $z = -6$    c $x = 20$

5 $-2, -1, 0, 1, 2, 3$

6 a $a \geq 2$    b $b < -2$

7 6

8 $y < -5$

9 a $x = -1.5$    b $y = 4$

10 a

b $(0, -1), (1, -1), (1, 0), (2, 0), (3, 0), (1, 1)$

11 a $x + 2y < 7, x > 0, y > 0$
  b $y$

The possible combinations as shown by this graph are:
1 $\times \ 2 \text{ coin and } 1 \times \ 1 \text{ coin}$
2 $\times \ 2 \text{ coin and } 2 \times \ 1 \text{ coins}$
3 $\times \ 2 \text{ coin and } 3 \times \ 1 \text{ coins}$
4 $\times \ 2 \text{ coin and } 4 \times \ 1 \text{ coins}$
5 $\times \ 2 \text{ coins and } 1 \times \ 1 \text{ coin}$
6 $\times \ 2 \text{ coins and } 2 \times \ 1 \text{ coins}$

<table>
<thead>
<tr>
<th>Number of £2 coins</th>
<th>Number of £1 coins</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

AQA Examination-style questions

1 $x + (x + 6) + 4(x + 6) = 120$
   $x + x + 6 + 4x + 24 = 120$

   $6x + 30 = 120$
   $So \ x = 15$
## 16 Trial and improvement

### Practise... 16.1 Trial and improvement

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a</td>
<td>2.0</td>
<td>b</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>4.1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a</td>
<td>4.58</td>
<td>b</td>
</tr>
<tr>
<td>4</td>
<td>a</td>
<td>2.91</td>
<td>b</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>-3.91</td>
<td></td>
</tr>
</tbody>
</table>

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td></td>
<td>3.618 and 1.382</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>2.33</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>a</td>
<td>4.32</td>
<td>b</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>5.0</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>0.78 cm</td>
<td></td>
</tr>
</tbody>
</table>

### Assess 16

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.93</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-1.80 and -7.65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3.124 cm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>a</td>
<td>3.23</td>
<td>b</td>
</tr>
<tr>
<td>5</td>
<td>a</td>
<td>3.83</td>
<td>b</td>
</tr>
</tbody>
</table>

### AQA Examination-style questions

#### 1 2.4

<table>
<thead>
<tr>
<th>x</th>
<th>8x - x³</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>-3</td>
</tr>
<tr>
<td>2.1</td>
<td>7.539</td>
</tr>
<tr>
<td>2.2</td>
<td>6.952</td>
</tr>
<tr>
<td>2.3</td>
<td>6.233</td>
</tr>
<tr>
<td>2.4</td>
<td>5.376</td>
</tr>
<tr>
<td>2.5</td>
<td>4.375</td>
</tr>
<tr>
<td>2.6</td>
<td>3.224</td>
</tr>
<tr>
<td>2.7</td>
<td>1.917</td>
</tr>
<tr>
<td>2.8</td>
<td>0.448</td>
</tr>
<tr>
<td>2.9</td>
<td>-1.189</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>8x - x³</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.41</td>
<td>5.28...</td>
</tr>
<tr>
<td>2.42</td>
<td>5.18...</td>
</tr>
<tr>
<td>2.43</td>
<td>5.09...</td>
</tr>
<tr>
<td>2.44</td>
<td>4.99...</td>
</tr>
<tr>
<td>2.45</td>
<td>4.89...</td>
</tr>
<tr>
<td>2.46</td>
<td>4.79...</td>
</tr>
<tr>
<td>2.47</td>
<td>4.69...</td>
</tr>
<tr>
<td>2.48</td>
<td>4.58...</td>
</tr>
<tr>
<td>2.49</td>
<td>4.48</td>
</tr>
</tbody>
</table>

#### 2 a (0, -8)

b correct calculation of a value from
2 ≤ x ≤ 2.49 (x = 2.4 gives -1.376 all values to 1 d.p. or better, allow truncation)
correct calculation of a value for 2.5 ≤ x < 3 (x = 2.5 gives 0.125)
correct calculation for x = 2.45 to 2.49...
x = 2.45 gives -0.643875
x = 2.5

#### 3 x = 3.7
### 17 Scatter graphs

**Practise... 17.1 Interpreting scatter graphs**

**1 a**

i positive correlation  
ii negative correlation  
iii positive correlation – probably strong positive correlation  
iv positive correlation  
v negative correlation  

**b**

i The higher the number of hours of sunshine, the higher the income for sales of iced drinks.  
ii The higher the number of cars on the road, the lower the average speed.  
iii The higher the distance travelled, the higher the amount of petrol used.  
iv The higher the number of bedrooms, the higher the price of the house.  
v The higher the number of hours of sunshine, the lower the income for sales of umbrellas.

**2 a**

i strong positive correlation  
ii no correlation  
iii weak positive correlation  
iv no correlation  
v strong negative correlation  

**b**

i The higher the rainfall, the heavier the weight of apples.  
ii There is no correlation.  
iii The higher the number of caps, the higher the cost of the footballer.  
iv There is no correlation.  
v The older the car, the lower the cost.

**3 a**

![Age against arm span graph]

b The scatter graph shows positive correlation.

c The older the person, the bigger the arm span.

**4 a**

![Sunshine and rainfall graph]

b The scatter graph shows strong negative correlation.

c The greater the amount of sunshine, the less the amount of rainfall.

**5 a**

various answers, e.g. age of car against value (strong negative)

**b**

various answers, e.g. age of the car against mileage (strong positive)

**c**

various answers, e.g. age of car against number of passengers (no correlation)

**6 a**

From 0 to 20, the shoe size increases with age, so there is positive correlation.

**b**

Feet stop growing after about 20 years of age.

**7 a**

![Calories and fat graph]

b There is a strong positive correlation.

c The relation does not hold for milkshakes where there is a high calorie value in terms of fat content. This value is called a rogue value as it does not fit in with the other data.
Practise... 17.2 Lines of best fit

1 a

Rainfall and sunbeds

<table>
<thead>
<tr>
<th>Amount of rainfall (mm)</th>
<th>Number of beds sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>400</td>
</tr>
<tr>
<td>2</td>
<td>380</td>
</tr>
<tr>
<td>4</td>
<td>360</td>
</tr>
<tr>
<td>6</td>
<td>340</td>
</tr>
<tr>
<td>8</td>
<td>320</td>
</tr>
<tr>
<td>10</td>
<td>300</td>
</tr>
<tr>
<td>12</td>
<td>280</td>
</tr>
<tr>
<td>14</td>
<td>260</td>
</tr>
<tr>
<td>16</td>
<td>240</td>
</tr>
<tr>
<td>18</td>
<td>220</td>
</tr>
<tr>
<td>20</td>
<td>200</td>
</tr>
<tr>
<td>22</td>
<td>180</td>
</tr>
<tr>
<td>24</td>
<td>160</td>
</tr>
<tr>
<td>26</td>
<td>140</td>
</tr>
<tr>
<td>28</td>
<td>120</td>
</tr>
<tr>
<td>30</td>
<td>100</td>
</tr>
</tbody>
</table>

b i 260
ii 9.4

2 a

Value of car against age

<table>
<thead>
<tr>
<th>Age of car (years)</th>
<th>Value of car (£)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6000</td>
</tr>
<tr>
<td>2</td>
<td>5000</td>
</tr>
<tr>
<td>4</td>
<td>4000</td>
</tr>
<tr>
<td>6</td>
<td>3000</td>
</tr>
<tr>
<td>8</td>
<td>2000</td>
</tr>
<tr>
<td>10</td>
<td>1000</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
</tr>
</tbody>
</table>

b i £1800
ii 3.2 years old

3 a

Temperature and number of visitors

<table>
<thead>
<tr>
<th>Number of visitors</th>
<th>Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>200</td>
<td>25</td>
</tr>
<tr>
<td>400</td>
<td>20</td>
</tr>
<tr>
<td>600</td>
<td>15</td>
</tr>
<tr>
<td>800</td>
<td>10</td>
</tr>
<tr>
<td>1000</td>
<td>5</td>
</tr>
</tbody>
</table>

b i 280
ii 22.5°C
c Outliers circled on graph.

4 a

Distances and temperature

<table>
<thead>
<tr>
<th>Distance from equator (°)</th>
<th>Average temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>10</td>
<td>25</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>30</td>
<td>15</td>
</tr>
<tr>
<td>40</td>
<td>10</td>
</tr>
<tr>
<td>50</td>
<td>5</td>
</tr>
</tbody>
</table>

The temperature decreases as the distance from equator increases.
b The average temperature for Dubai is 27°C.
c Other factors affecting temperatures may include time of the year, height above sea level, humidity and so on.

5 a

House price in 2006 (£ thousand)

<table>
<thead>
<tr>
<th></th>
<th>170</th>
<th>175</th>
<th>190</th>
<th>200</th>
<th>160</th>
<th>185</th>
</tr>
</thead>
</table>
| House price in 2012 (£ thousand)

|                | 225 | 235 | 264 | 284 | 205 | 255 |

b Rashid is not correct. Reading off the line of best fit gives about £245 000.
c £196 000
d £198 000
e These results are both at the edges of the graph so neither result is likely to be more reliable.

6 a

Engine size (cc)

<table>
<thead>
<tr>
<th>Top speed (kph)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
</tr>
<tr>
<td>100</td>
</tr>
<tr>
<td>150</td>
</tr>
<tr>
<td>200</td>
</tr>
<tr>
<td>250</td>
</tr>
<tr>
<td>300</td>
</tr>
</tbody>
</table>

Engine size (cc)

As engine size increases, top speed also increases showing a positive correlation.
c i 460 cc ii 1000 cc
d Part i is likely to be a better estimate as the information is in the main body of the graph. Therefore, this result is probably more reliable. Part ii is at the edges of the graph so is probably less reliable.

Assess 17

1 Marks of 8 students in history and geography

The scatter graph shows strong positive correlation.

2 a Test marks against TV hours watched

According to the graph, the more the students watch TV the less they score in the test, which demonstrates a negative correlation between the two sets of data.

b Student ‘1’ and Student ‘4’, do not seem to fit the trend. They both watched a reasonably low amount of TV but scored low marks.

3 a

There would seem to be no correlation between the two sets of data.

4 a and b

5 a 7 metres

b Strong positive correlation
d i 8.5 m

ii The estimate is beyond the last value marked on the graph so it may not be very reliable.
6  a  Number of pages = 140 and weight = 96 g is a rogue value. It is possible that the two figures were interchanged as number of pages = 96 and weight = 140 g is a likely value.

b  Weight of books against number of pages

![Graph showing correlation between number of pages and weight of books]

Yes, Adnan’s hypothesis is correct; the data shows a positive correlation: the more pages, the heavier the book.

AQA Examination-style questions

1  a and b

![Graph showing correlation between sunshine hours and temperature]

c  27°C
d  strong positive correlation
e  Hours of sunshine and temperature were only recorded for 7 days, not the whole month. The rest of June may have had very different weather.

2  a  strong positive correlation with an outlier
   b  i  4 books, 7 kg
   ii  No effect because all other points already indicate a strong positive correlation.
18 Reflections, rotations and translations

Practise... 18.1 Reflection

1  a  (0, 0), (1, 4) (−2, −1)

b  i  (1, 4), (2, 0), (4, −1)
   ii  (3, 4), (4, 0), (6, −1)
   iii (−2, −1), (0, −2), (1, −6)
   iv (−2, −3), (0, −4), (1, −8)

2  a

b  i  (1, 4), (2, 0), (4, −1)
   ii  (3, 4), (4, 0), (6, −1)
   iii (−2, −1), (0, −2), (1, −6)
   iv (−2, −3), (0, −4), (1, −8)

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e

\[ x = 2 \]

f

\[ x = -1 \]

3

a \( x = 1 \)
b \( y = 1 \)
c \( x = 0.5 \)
d \( y = -1 \)

4

a–c Student’s own drawings

5

a and b

c parallelogram
d See diagram above: \((-1, 1), (-3, 0), (-3, 2), (-1, 3)\)
e See diagram above: \((1, -1), (3, 0), (3, -2), (1, -3)\)
f There is a relationship between the coordinates. A reflection in the line \( y = x \) reverses the coordinates. A reflection in the line \( y = -x \) reverses the coordinates and changes their sign.

6

a and d

b trapezium
c \((0, 0), (0, 4), (3, 2), (3, 1)\)

7

a \((-3, 1), (-1, 4), (3, 2), (4, -2), (-1, -4)\)
b \((-1, 3), (-4, 1), (-2, -3), (2, -4), (4, 1)\)

8

a \(A(-3, 3), B(3, 3)\)
b and c

d square
e Any line that does not begin and end on the line \( y = x \) or \( y = -x \)

9

a

i \((2, 0), (4, 3), (5, 1)\)
ii \((-1, -2), (4, -3), (4, -2)\)
iii \((2.5, -1.5), (3.5, 0.5), (1, -1)\)

b

i \((-2, 0), (-4, -3), (-5, -1)\)
ii \((1, 2), (-4, 3), (-4, 2)\)
iii \((-2.5, 1.5), (-3.5, -0.5), (-1, 1)\)

10

a a reflection in the line \( y = x \)
b \(A: (1, 3), (-1, 7), (-4, -2); A': (3, 1), (7, -1), (-2, -4)\)
c The coordinates are reversed.
d \((3, 2), (-2, -1), (-4, 0), (-2, 2)\)
e

i The \( y \)-coordinates remain the same.
The \( x \)-coordinates change sign.
ii The \( x \)-coordinates remain the same.
The \( y \)-coordinates change sign and add 6.
iii The \( y \)-coordinates remain the same.
The \( x \)-coordinates change sign and subtract 10.
Practise... 18.2 Rotation

1  

a  

i  Shape R:  

[Diagram of Shape R]

ii  Shape S:  

[Diagram of Shape S]

iii  Shape T:  

[Diagram of Shape T]

iv  Shape U:  

[Diagram of Shape U]
v Shape E:

vi Shape F:

vii Shape G:

c i Shape G:

ii Shape H:

iii Shape I:

iv Shape J:
v Shape K:

vi Shape L:

4 a 135° clockwise or 225° anticlockwise
   b 45° clockwise or 315° anticlockwise
   c 225° clockwise or 135° anticlockwise
   d 72° clockwise or 288° anticlockwise

5 a rotation of 90° clockwise about O
   b rotation of 180° about O
   c rotation of 90° anticlockwise about O
   d rotation of 180° about O
   e rotation of 90° clockwise about (2, -2)
   f rotation of 90° anticlockwise about (2, 6)
   g rotation of 90° clockwise about (-2, -1)
   h rotation of 180° about (0, 5)
   i rotation of 180° about (4, 2)
   j rotation of 90° anticlockwise about O

6 B

7
9. a The arrows have rotational symmetry but the house in the logo does not.  
b  i order 3 (ignoring any background colouring)  
   ii $120^\circ$  
c Student's own design

Practise... 18.3 Translation

1. a

b  

c

d

e
2 a \((-2, 4)\)  
   b \((4, 3)\)
   c \((-6, 0)\)  
   d \((-4, -5)\)
   e \((-3, 2)\)  
   f \((1, 5)\)

3 a i \((3, 6)\) ii \((-6, -1)\) iii \((-9, -7)\) iv \((9, 7)\)
   b They are opposites.

4 a \((2, 5)\)  
   b \((-11, 5)\)  
   c \((-3, 7)\)  
   d \((12, -7)\)

5 a-d

7 Student’s own design

8 a C E B D F A X
   b \((-1, -5, 11, -14, 11, -10, 8, -12, 8)\)
   c Student’s own vector route
1 a–e

\[ \text{f reflection in the line } x = 5 \]

\[ \text{g } Q, R \text{ and } S \text{ are congruent.} \]

2 a–d

\[ \text{e reflection in the line } x = 0 \]

3 a–c

\[ \text{Q, S and R: the sides and angles are the same.} \]

\[ \text{b } Q: \text{ reflection in } y = 1 \]

\[ S: \text{ translation by vector } \left( \frac{3}{2}, -2 \right) \]

\[ R: \text{ rotation } 90^\circ \text{ anticlockwise around the point } (-1, 2) \]

4 a–d

\[ \text{d rotation } 180^\circ \text{ about the point } (4, 0) \]

\[ \text{e vector translation } \left( \frac{4}{2}, 0 \right) \]

5 a clockwise rotation through $90^\circ$ or

\[ \text{b reflection in } x = -0.5 \]

\[ \text{c rotation } 180^\circ \text{ about } (0, 1) \]

\[ \text{d } (-1, 2) \]

6 Monique is right: the length of the sides and

\[ \text{the angles of the triangle are not equal so the} \]

\[ \text{triangles are not congruent and this is not a} \]

\[ \text{translation.} \]

7 a Q, S and R: the sides and angles are the same.

\[ \text{b } Q: \text{ reflection in } y = 1 \]

\[ S: \text{ translation by vector } \left( \frac{3}{2}, -2 \right) \]

\[ R: \text{ rotation } 90^\circ \text{ anticlockwise around the point } (-\frac{3}{2}, 3) \]

8 a, b Student’s own diagrams
1. **a and b**

2. **a** (0,0), (−2, 0), (−2, 6)
   **b** (0, 0), (0, 2), (6, 2)

3. **a and b**

4. **x = 2**

5. **a and c**

6. **a** reflection in the line $y = -1$
   **b** and **c**

7. **a**

8. **a** $B$ and $D$
   **b** $B$ and $D$. The sides and angles are the same length.
1  a  clockwise rotation about the origin through 270° (or anticlockwise through 90°)

b

\[
\begin{array}{cccccc}
& & & & & \\
\text{ } & & & & & \\
\text{ } & & & & & \\
\text{ } & & & & & \\
\text{ } & & & & & \\
\text{ } & & & & & \\
\text{ } & & & & & \\
\end{array}
\]

Examiner's tip: Examination-style questions
19 Measures

Practise... 19.1 Accuracy of measurement

1 a 13 cm  c 50 m  e 44 kg  
   b 81 m  d 7 g  f 47 l
2 a Student's own values that round to 15 cm, e.g. 14.77 cm, 15.023 cm  
   b 14.5  
   c 15.5
3 2.5 metres and 3.5 metres
4 No, some could be 7.6 cm and others could be 8.3 cm, or any other lengths that round to 8 cm to the nearest cm.

5 a 17.5 km  b 18.5 km
6 42.5 g, 43.5 g
7 No, the van may be anything up to 2.5 tonnes.
8 a, c and e
9 5 kg (The heaviest each type of book could be is 3.5 kg and 1.5 kg.)
10 Dan's maximum possible weight is 38.5 kg if the three weights given are to the nearest kilogram.

Practise... 19.2 Compound measures

1 a 25 m/s  
   b 7.14 m/s (2 d.p.)  
   c 115 mph
2 a 0.5 h  
   b 0.25 h  
   c 4.75 h
3 a 2 hours 30 minutes  
   b 3 hours 15 minutes  
   c 1 hour 45 minutes
4 a 68 mph  
   b 52.9 mph
5 a 300 cm  
   b 20 minutes
6 a 48 minutes  
   b 40 minutes
7 a 5 km  
   b 165 km  
   c 37.5 km
8 a 42.5 mpg  
   b 9.41 gallons  
   c Yes, she can travel 637.5 miles if she manages 42.5 mpg all the time.
9 4.56 people per km²
10 7.6 g/cm³
11 96.5 g
12 a lead (11.0)  
   c iron (7.8)
   b gold (19.3)  
   d copper (9.0)
13 a 250 cm³  
   b 4.825 kg  
   c £187 692.50
14 1.5 mm
15 7.29 km/h (2 d.p.)
16 a No, he needs to use 15.8 gallons of fuel.  
   b No, he still needs 16 gallons of fuel.
17 a Mary's average speed = \frac{\text{total distance}}{\text{total time}}  
   = \frac{(2 + 2)}{(\frac{2}{3} + \frac{2}{3})} = \frac{4}{0.68571428...} = 5.833... \text{ mph}
   b 1\frac{1}{2} miles at 5 mph and 2\frac{1}{2} miles at 7 mph. This gives 20 minutes at each of the two speeds, a total of 4 miles in 40 minutes, i.e. 6 mph.

Assess 19

1 32.9 mph
2 22.67 mph (2 d.p.)
3 4.17 g/cm³
4 1.1025 kg
5 18.5°C and 17.5°C

AQA Examination-style questions

1 a 140 km  
   b 100 km/h

6 £32.50 (assuming no wastage)
7 57.5 mpg
8 a 55.5 kg  
   b i 8 hours 20 minutes  
   ii 32.85 kg
9 11.16 cm

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20

3-D shapes, coordinates and graphs

Practise... 20.1 Plans and elevations

1

F

S

2

3

a Jane's view:

b Chim's view:

c Charlie's view:

d Plan view:

4 a The line across the top of the bureau should not be in the middle. The lines across the desk should be dotted.

b

1 cm

3 cm

4 cm

2 cm

4 cm

8 cm

5 a i C ii B iii A iv D

b

6

a

b

c

7 a sphere

8

a

b

c

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**Practise... 20.2 Coordinates in 3-D**

1. \( B(2, 1, 1), C(2, 3, 1) \)
2. In any order: \((0, 1, 0), (3, 4, 0), (0, 4, 0), (0, 1, 3), (3, 1, 3), (3, 4, 3), (0, 4, 3)\)
3. \( A(1, 1, 0), B(1, 1, 1), C(4, 1, 1), D(3, 2, 2), E(3, 2, 3), F(2, 3, 3), G(3, 4, 2), H(3, 4, 1) \)
4. \( A(0, 1, 0), B(0, 1, 3), C(4, 1, 3), D(4, 1, 0), E(4, 6, 0), F(4, 6, 3) \)

**Practise... 20.3 Graphs**

1. a

<table>
<thead>
<tr>
<th>Units</th>
<th>0</th>
<th>500</th>
<th>1000</th>
<th>1500</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost (£)</td>
<td>8.5</td>
<td>61</td>
<td>113.5</td>
<td>166</td>
<td>218.5</td>
</tr>
</tbody>
</table>

b and c

- For anyone using more than about 600 units, Betta-supplies is cheaper.

2. a \( £9.60 \)  
   b \( 12p \)  
   c \( £21.60 \)

d

<table>
<thead>
<tr>
<th>Number of pens</th>
<th>100</th>
<th>200</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost (£)</td>
<td>9</td>
<td>18</td>
<td>27</td>
</tr>
</tbody>
</table>

**Graphs**

- For 150 pens, Cheapo pens are cheaper by \( £1.50 \).
- 220 pens for \( £19.80 \)
- 158 is the first number of pens to make a profit (assuming they sell all the pens they buy).
- 9p, \( £2 \) profit (assuming they sell all 300 pens)

3. a \( 120 \text{ km/h} \)
   b \( 80 \text{ km/h} \)
   c No, it is stationary. It stays 10 km from Manchester.
   d You cannot tell whether it is going down hill; the negative gradient means it is getting closer to Manchester.
   e \( 96 \text{ km/h} \)

4. a

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>0</td>
<td>-0.75</td>
<td>-1</td>
<td>-0.75</td>
<td>0</td>
</tr>
</tbody>
</table>
c Xavier is wrong. The lowest point is halfway along, but only 1 m below the end A.

5 a \( y = 4x^2 + 6x \)

b

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>0</td>
<td>10</td>
<td>28</td>
<td>54</td>
<td>88</td>
<td>130</td>
</tr>
</tbody>
</table>

c

6 a

\( 18 \text{ m}^2 \)  

1 8.6 m (3 s.f.)

b i after 6.5 years  

ii after 10 years about £5200

7 a

b 3 seconds. It is the maximum height.

8 Student’s own graph, which should plot the following points (crucially where \( h \) is 25):

<table>
<thead>
<tr>
<th>( x )</th>
<th>10</th>
<th>13</th>
<th>20</th>
<th>30</th>
<th>37</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h )</td>
<td>20</td>
<td>25</td>
<td>30</td>
<td>30</td>
<td>25</td>
<td>20</td>
</tr>
</tbody>
</table>

width available: \( 37 - 13 = 24 \text{ metres} \)

9 a

<table>
<thead>
<tr>
<th>( V )</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C )</td>
<td>1040</td>
<td>772.5</td>
<td>695</td>
<td>708.5</td>
<td>780</td>
</tr>
</tbody>
</table>

b 21.5 km/h

10 10 sides

11 There are two possibilities: either 15 (with a longest side of 40) or 20 (with a longest side of 30).
Assess 20

1a

<table>
<thead>
<tr>
<th>Linda</th>
<th>Laura</th>
<th>Lynne</th>
<th>Lucy</th>
</tr>
</thead>
</table>

1b

Plan view

2 Plan

A  B  C  D

3 \( P(2, 0, 1), Q(3, 1, 1), R(3, 3, 2), S(2, 3, 3), T(0, 2, 2) \)

4 a 46.5 m  b 5 m  c 20 m
d It was thrown by someone so it was 2 m above the ground to start with.

5 a \( y = x(x + 1) \)
   b

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
x & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
y & 0 & 2.5 & 6.25 & 12.5 & 21.875 & 35.9375 \\
\hline
\end{array}
\]

\( x \) \( y \)

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
0 & 2.5 & 5 & 7.5 & 10 & 12.5 \\
\hline
\end{array}
\]

\( x \) \( y \)

1.24 pm

Examiner’s tip

Examination-style questions

1 Plan view

2 a 2pm  b 1.24pm

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{Plan view} & \text{Side elevation} & \text{Front elevation} \\
\hline
\text{Plan view} & \text{Side elevation} & \text{Front elevation} \\
\hline
\end{array}
\]
## 21 Pythagoras’ theorem

### Practise… 21.1 Pythagoras’ theorem

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a</td>
<td>13 cm</td>
<td>b</td>
<td>10 cm</td>
<td>c</td>
</tr>
<tr>
<td>2</td>
<td>a</td>
<td>15.7 cm</td>
<td>b</td>
<td>3.1 m</td>
<td>c</td>
</tr>
<tr>
<td>3</td>
<td>a</td>
<td>9 cm</td>
<td>b</td>
<td>35.3 cm</td>
<td>c</td>
</tr>
<tr>
<td>4</td>
<td>a</td>
<td>Yes</td>
<td>b</td>
<td>No</td>
<td>c</td>
</tr>
<tr>
<td>5</td>
<td>a</td>
<td>11.2 cm</td>
<td>c</td>
<td>25.6 cm</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>11.7 cm</td>
<td>d</td>
<td>11.3 cm</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Sarah has added instead of subtracting. Ravi has not found the square root of 435. The correct answer is 20.86 cm.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>4.5 cm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>60 cm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>a</td>
<td>15.8 cm</td>
<td>b</td>
<td>9.3 mm</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>a</td>
<td>5 units</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>5 units</td>
<td>ii</td>
<td>7.81 units</td>
<td>iii</td>
</tr>
<tr>
<td>11</td>
<td>2.5 km</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Practise… 21.2 Pythagoras’ theorem in three dimensions

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a</td>
<td>13 cm</td>
<td>b</td>
<td>8.1 cm</td>
<td>c</td>
</tr>
<tr>
<td>2</td>
<td>3.07 m</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>a</td>
<td>21.2 cm</td>
<td>b</td>
<td>14.6 cm</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>a</td>
<td>33.5 cm</td>
<td>b</td>
<td>45 cm</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>18 cm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Assess 21

<p>| | | | | | |</p>
<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a</td>
<td>24 cm</td>
<td>c</td>
<td>34.8 m</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>8.9 cm</td>
<td>d</td>
<td>8.7 mm</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>13 cm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>47 m</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>a</td>
<td>9.08 m</td>
<td>b</td>
<td>11.84 m</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>3.6 km</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### AQA Examination-style questions

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.5 cm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4.7 m</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>64 cm</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>12 cm²</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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22 Surds

22.1 Simplifying surds

1. a. Irrational: \(\sqrt{32} = \sqrt{4 \times 8} = \sqrt{4} \times \sqrt{8} = 4\sqrt{2}\)
   
b. Rational: \(\sqrt{16} = 4\)

c. Rational because this is a fraction with integers as both denominator and numerator.

d. Rational: \(\frac{\sqrt{4}}{11} = \frac{2}{11}\)

e. Irrational: \(\frac{4}{\sqrt{11}} = \frac{4\sqrt{11}}{11}\)

2. a. \(\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}\)

3. a. \(\sqrt{40} = \sqrt{4 \times 10} = 2\sqrt{10}\)
   
b. \(\sqrt{90} = \sqrt{9 \times 10} = 3\sqrt{10}\)
   
c. \(\sqrt{98} = \sqrt{2 \times 49} = 7\sqrt{2}\)
   
d. \(5\sqrt{3} + 3\sqrt{3} = 8\sqrt{3}\)
   
e. \(7\sqrt{5} - 2\sqrt{5} = 5\sqrt{5}\)

4. a. \(\sqrt{10} + \sqrt{15} \neq \sqrt{25}\)
   
b. \((\sqrt{2} + \sqrt{3}) \times \sqrt{5} \neq \sqrt{5} \times \sqrt{5}\)

5. Any three irrational numbers between 1 and 2, e.g. \(\sqrt{2}, \sqrt{3}\) and \(\sqrt{5}\)

6. \(\sqrt{2} \times \sqrt{3} \times \sqrt{4} \times \sqrt{5} \times \sqrt{6} = \sqrt{720} = \sqrt{144} \times \sqrt{5} = 12\sqrt{5}\)

7. a. \(\sqrt{30} \text{ cm} \times \sqrt{15} \text{ cm}\)
   
b. \(15\sqrt{2} \text{ cm}^2\)

8. a. Any two irrational numbers that multiply together to make a rational number, e.g. \(\sqrt{2} \times \sqrt{8} = 4\)
   
b. Any two irrational numbers that, when divided, give an answer of 2, e.g. \(2\sqrt{2} \div \sqrt{2}\)

9. \(\sqrt{5} \times \sqrt{3} = \sqrt{15}\)

10. a. \((\sqrt{12} + \sqrt{3})^2 + (\sqrt{12} - \sqrt{3})^2\)
   
   \(= (\sqrt{12} + \sqrt{3}) \times (\sqrt{12} + \sqrt{3}) + (\sqrt{12} - \sqrt{3}) \times (\sqrt{12} - \sqrt{3})\)
   
   \(= 12 + 6 + 6 + 3 + 12 - 6 - 6 + 3\)
   
   \(= 24 + 6\)
   
   \(= 30\)

   b. Expand brackets as in a to reach to reach 2a + 2b = rational number. So a and b can be any pair of rational numbers, e.g. 7 and 2 or 3 and 4.

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### 22.2 Rationalising the Denominator

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
<th>h</th>
<th>i</th>
<th>j</th>
<th>k</th>
<th>l</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \frac{\sqrt{7}}{\sqrt{3}} = \frac{\sqrt{7} \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{\sqrt{21}}{3} )</td>
<td>( 3 \times \frac{\sqrt{3}}{2 \sqrt{3}} = 3 \times \frac{\sqrt{3} \times \sqrt{3}}{2 \times 3} = \frac{3 \sqrt{3}}{6} = \frac{\sqrt{3}}{2} )</td>
<td>( \frac{7 \sqrt{2} - \sqrt{7}}{5 + \sqrt{7} \times 5 - \sqrt{7}} = \frac{7(5 - \sqrt{7})}{25 + 5 \sqrt{7} - 5 \sqrt{7} - 7} = \frac{7(5 - \sqrt{7})}{18} )</td>
<td>( \frac{\sqrt{5}}{\sqrt{35}} = \frac{\sqrt{5} \times \sqrt{35}}{\sqrt{35} \times \sqrt{35}} = \frac{\sqrt{5} \times \sqrt{5} \times \sqrt{7}}{35} = \frac{\sqrt{5} \times \sqrt{7}}{35} = \frac{\sqrt{35}}{35} )</td>
<td>( \frac{7}{\sqrt{47}} = \frac{7 \sqrt{47}}{\sqrt{47} \times \sqrt{47}} = \frac{28 \sqrt{7}}{16 \times 7} = \frac{\sqrt{7}}{4} )</td>
<td>( \frac{3}{3 - \sqrt{3}} = \frac{3 \times 3 + \sqrt{3}}{3 - \sqrt{3} \times 3 + \sqrt{3}} = \frac{3(3 + \sqrt{3})}{9 - 3 \sqrt{3} + 3 \sqrt{3} - 3} = \frac{3(3 + \sqrt{3})}{6} = \frac{3 + \sqrt{3}}{2} )</td>
<td>( \frac{5}{\sqrt{10}} = \frac{5 \sqrt{10}}{\sqrt{10} \times \sqrt{10}} = \frac{5 \sqrt{10}}{10} = \frac{\sqrt{10}}{2} )</td>
<td>( \frac{4}{2 + \sqrt{3}} = \frac{4 \times 2 - \sqrt{3}}{2 + \sqrt{3} \times 2 - \sqrt{3}} = \frac{4(2 - \sqrt{3})}{4 + 2 \sqrt{3} - 2 \sqrt{3} - 3} = \frac{4(2 - \sqrt{3})}{2} = 8 - 4 \sqrt{3} )</td>
<td>( \frac{\sqrt{3}}{3 - \sqrt{3}} = \frac{\sqrt{3} \times 3 + \sqrt{3}}{3 - \sqrt{3} \times 3 + \sqrt{3}} = \frac{3 \sqrt{3} + 3}{9 - 3 \sqrt{3} + 3 \sqrt{3} - 3} = \frac{3(1 + \sqrt{3})}{6} = \frac{1 + \sqrt{3}}{2} )</td>
<td>( \frac{30}{\sqrt{15}} = \frac{30 \sqrt{15}}{\sqrt{15} \times \sqrt{15}} = \frac{30 \sqrt{15}}{15} = \frac{3 \sqrt{15}}{3} = \sqrt{15} )</td>
<td>( \frac{5}{2 - \sqrt{3}} = \frac{5 \times 2 + \sqrt{3}}{2 - \sqrt{3} \times 2 + \sqrt{3}} = \frac{10 + \sqrt{3}}{4 - 2 \sqrt{3} + 2 \sqrt{3} - 3} = \frac{10 + \sqrt{3}}{2} = 5 + \frac{\sqrt{3}}{2} )</td>
<td>( \frac{\sqrt{2}}{2 + \sqrt{2}} = \frac{\sqrt{2} \times 2 - \sqrt{2}}{2 + \sqrt{2} \times 2 - \sqrt{2}} = \frac{2(\sqrt{2} - 1)}{4 + 2 \sqrt{2} - 2 \sqrt{2} - 2} = \frac{2(\sqrt{2} - 1)}{2} = \sqrt{2} - 1 )</td>
</tr>
<tr>
<td>2</td>
<td>Angela's mistake: ( \frac{1}{13} = \frac{1 \times \sqrt{3}}{13 \times \sqrt{3}} )</td>
<td>Marco's mistake: ( \frac{\sqrt{6}}{3/2} = \frac{\sqrt{3}}{3} ), not ( \frac{\sqrt{2}}{2} )</td>
<td>Correct answer: ( \frac{\sqrt{6}}{18} = \frac{1 \times \sqrt{3}}{18 \times \sqrt{3}} = \frac{\sqrt{3}}{3} )</td>
<td>( \frac{2}{3 + \sqrt{5}} = \frac{2 \times 3 - \sqrt{5}}{3 + \sqrt{5} \times 3 - \sqrt{5}} = \frac{2(3 - \sqrt{5})}{9 + 3 \sqrt{5} - 3 \sqrt{5} - 5} = \frac{2(3 - \sqrt{5})}{4} = \frac{3 - \sqrt{5}}{2} )</td>
<td>( 5 \sqrt{2} ) m</td>
<td>50 ( \sqrt{2} ) m²</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>( \frac{1}{2} + \frac{3}{\sqrt{5}} = \frac{\sqrt{5} + 6}{2 \sqrt{5} \times \sqrt{5}} = \frac{\sqrt{5} + 6}{2 \sqrt{5} \times \sqrt{5}} = \frac{\sqrt{5} + 6}{10} = \frac{5 + 6 \sqrt{5}}{10} )</td>
<td>( \frac{3}{2 \sqrt{3}} - \frac{2}{3 \sqrt{2}} = \frac{(3 \times 3 \sqrt{2}) - (2 \times 2 \sqrt{3})}{2 \sqrt{3} \times 3 \sqrt{2}} = \frac{9 \sqrt{2} - 4 \sqrt{3}}{6 \sqrt{6}} = \frac{9 \sqrt{2} - 4 \sqrt{3} \times \sqrt{6}}{6 \sqrt{6}} = \frac{9 \sqrt{12} - 4 \sqrt{18}}{36} = \frac{(9 \sqrt{4} \times \sqrt{3}) - (4 \times 3 \times \sqrt{2})}{36} = \frac{18 \sqrt{3} - 12 \sqrt{2}}{36} = \frac{3 \sqrt{3} - 2 \sqrt{2}}{6} )</td>
<td>( \frac{3}{2 \sqrt{3}} = \frac{3(2 + \sqrt{3})}{3(2 + \sqrt{3})} )</td>
<td>( x = 3(2 + \sqrt{3})(2 - \sqrt{3}) = 12 + 6 \sqrt{3} - 6 \sqrt{3} - 9 = x = 3 )</td>
<td>( \sqrt{20} - 10 )</td>
<td>( \frac{\sqrt{20} - 10}{\sqrt{5}} = \frac{\sqrt{100} - 10 \sqrt{5}}{\sqrt{5}} = \frac{10(1 - \sqrt{5})}{5} = 2(1 - \sqrt{5}) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Assess 22

1 a \(\sqrt{7}\) is irrational because it does not simplify any further.
   b \(\sqrt{16}\) is rational (\(= 4\))
   c \(\frac{\sqrt{16}}{7}\) is rational (\(= \frac{4}{7}\))
   d \(\sqrt{16}\) is irrational (\(= \frac{4}{\sqrt{7}}\))
   e \(\frac{\sqrt{63}}{\sqrt{7}}\) is rational (\(= \frac{\sqrt{9} \times \sqrt{7}}{\sqrt{7}} = 3\))
2 a \(2\sqrt{2}\) b \(3\sqrt{7}\) c \(7\sqrt{2}\) d \(12\sqrt{3}\) e \(10\sqrt{2}\)
3 a \(\sqrt{12}\) b \(\sqrt{80}\) c \(\sqrt{175}\) d \(\sqrt{44}\) e \(\sqrt{200}\)

4 a \(\sqrt{2}\) c \(\frac{2\sqrt{5}}{5}\) e \(7(\sqrt{3} + \sqrt{2})\)
   b \(\frac{3\sqrt{7}}{7}\) d \(5(\sqrt{2} - 1)\) f \(\frac{\sqrt{3}(2 - \sqrt{2})}{3}\)
5 a \(2 + 3\sqrt{2} + 3\sqrt{2} + 9 = 11 + 6\sqrt{2}\)
   b \(5 - 3\sqrt{5} - 3\sqrt{5} + 9 = 14 - 6\sqrt{5}\)
   c \(2 - \sqrt{2} + 2\sqrt{2} - 2 = \sqrt{2}\) (or \(0 + \sqrt{2}\))
   d \(3 + \sqrt{15} - \sqrt{15} - 5 = -2\)
   e \(-43\)
6 \(\frac{\sqrt{12}}{\sqrt{2} - \sqrt{8}} = \frac{\sqrt{12} (\sqrt{2} + \sqrt{8})}{(\sqrt{2} - \sqrt{8})(\sqrt{2} + \sqrt{8})} = \frac{\sqrt{12} \times 3\sqrt{2}}{2 - 8}\)
   \(= \frac{6\sqrt{6}}{-6} = -\sqrt{6}\)

AQA Examination-style questions

1 \(6\pi\sqrt{2}\)

2 \(\frac{\sqrt{x} \times 50}{\sqrt{5}} = 4\sqrt{5}\)
   \(\sqrt{x} \times 50 = 20\)
   \(\sqrt{x} = \frac{2}{5}\)
   \(x = \frac{4}{25}\)