2 Probability

Consider

Fruit machines, also known as ‘one-armed bandits’, are a popular gambling game. You can win money by playing on fruit machines, but it is more likely that you will lose money. You put a coin in the machine and pull the lever. This makes three drums rotate quickly, then slow down and finally stop. Each drum has pictures of several fruits. When the drums stop three fruits are shown in the centre of the display. Depending on what they are there may be a prize.

Max has been interested in fruit machines, but he has never played them. He would win a prize on the machine he is looking at if cherries appear on all three drums, or lemons show on all three drums. Before he takes the plunge, he would like to assess the risk of losing money. To do this, he needs to know how to find the probability of combined events such as

a getting either a cherry or a lemon on the first drum
b getting a cherry on both the first and the second drum
c getting a cherry on all three drums.

There are 10 different types of fruit on each drum. Can you give Max the answers he needs?

Consider also

An ordinary unbiased dice is rolled. What is the probability of throwing a prime number or an even number?

You should be able to solve these problems after you have worked through this chapter.

Class discussion

In the fruit machine problem above, a and b involve two events, but in different ways.

‘Getting either a cherry or a lemon on the first drum’ involves either one or the other event occurring, whereas ‘getting a cherry on both the first and the second drum’ involves both events happening. Each sentence below describes a situation where two events are involved. Discuss what the events are and whether they fall into the ‘either … or’ category or into the ‘both … and’ category.

1 An ordinary six-sided dice is rolled and scores five or six.
2 Two dice are rolled and a double six is scored.
3 Tim picks a box from a lucky dip, some boxes contain a prize and the others are empty.
4 The England cricket captain tosses a coin to find out who has the choice to bat or to field.
Mutually exclusive events
When an ordinary dice is rolled, it is possible to score either a five or a six. It is not possible to score both a five and a six. Such events are called mutually exclusive.

Independent events
When two ordinary dice are rolled, it is possible to score a six on the first dice and a six on the second dice. Also the score obtained on the second dice is not affected in any way by the score on the first dice. Such events, where both can happen but each has no influence on the occurrence or otherwise of the other, are called independent events.

Not all events are independent, as this example shows. There are two green sweets and two red sweets in a bag. Mary takes one of these sweets then Tom takes one. If Mary’s sweet is red, there is only one out of three ways in which Tom can choose a red sweet. But if Mary’s sweet is green, there are two out of three ways in which Tom can choose a red sweet. So the probability that the second sweet is red depends on the colour of the first sweet taken.

Exercise 2a

Decide whether the events described are ‘mutually exclusive’, ‘independent’ or ‘dependent’.

1 Mona and Clive each buy a ticket for a raffle and one of them wins first prize.
2 Two coins are tossed.
   a The first coin lands heads up or tails up.
   b Both coins land head up.
3 A 10 pence coin is tossed and a dice is rolled.
   a The coin lands head up and an even number is scored on the dice.
   b A three or a six is scored on the dice.
4 A blue bag and a red bag each contain a large number of coins, some of which are counterfeit. One coin is selected at random from each bag.
   a The coin taken from the blue bag is counterfeit or not counterfeit.
   b Both coins are counterfeit.
5 Hartfield Airport has 100 scheduled flights due to depart on Saturday.
   a Two or three flights are cancelled.
   b One flight is cancelled because the plane is faulty, and another flight is cancelled because of a hurricane at its destination.
6 A box contains 6 blue pens and 3 red pens. One pen is removed at random.
   a The pen is put back, then a pen is removed again.
   b The pen is not put back, and another pen is removed.
Adding probabilities

If we select a card at random from a pack of 52, the probability of drawing an ace is \( \frac{4}{52} \), and the probability of drawing a black king is \( \frac{2}{52} \).

Now drawing either an ace or a black king involves two events that are mutually exclusive, since it is impossible to draw one card which is both an ace and a black king.

There are 4 aces and 2 black kings, so if we want to find the probability of drawing either an ace or a black king there are 6 cards that we would count as ‘successful’, therefore

\[
P(\text{ace or a black king}) = \frac{6}{52}
\]

Remember: The probability that an event \( A \) happens is \( P(A) \), where

\[
P(A) = \frac{\text{the number of ways in which } A \text{ can occur}}{\text{the total number of equally likely outcomes}}
\]

\[
P(\text{ace}) = \frac{4}{52}, \text{ and } P(\text{black king}) = \frac{2}{52}
\]

Since \( \frac{6}{52} = \frac{4}{52} + \frac{2}{52} \), it follows that

\[
P(\text{ace or black king}) = P(\text{ace}) + P(\text{black king})
\]

Now consider the probability of scoring 5 or 6 when one dice is rolled.

\[
P(\text{score 5 or 6}) = \frac{2}{6}
\]

From one roll of a dice, a score of 5 and a score of 6 are mutually exclusive where

\[
P(\text{score 5}) = \frac{1}{6} \text{ and } P(\text{score 6}) = \frac{1}{6}
\]

\[
P(\text{score 5 or 6}) = \frac{2}{6} = \frac{1}{6} + \frac{1}{6} = P(\text{score 5}) + P(\text{score 6})
\]

From these examples we see that

**If \( A \) and \( B \) are mutually exclusive events, then \( P(A \text{ or } B) = P(A) + P(B) \)**

Now consider the probability of scoring either 1 or 2 or 3 or 4 or 5 or 6 when one dice is rolled.

These events are mutually exclusive and they cover all the possible outcomes. The set of all possible outcomes is called **exhaustive**.

Now \( P(\text{score 1 or 2 or 3 or 4 or 5 or 6}) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{6}{6} = 1 \)

This illustrates the general rule that

**The sum of the probabilities of an exhaustive set of mutually exclusive outcomes is 1.**

For example, a bag contains some black discs, some red discs and some white discs. When one disc is removed at random, the outcome is either black, red or white. These outcomes are mutually exclusive and exhaustive.

Given that the probability that the disc is black is \( \frac{1}{3} \) and the probability that it is white is \( \frac{1}{5} \), then we can use the fact above to find the probability that the disc is red. That is

\[
P(\text{black}) + P(\text{red}) + P(\text{white}) = 1
\]

giving \( \frac{1}{3} + P(\text{red}) + \frac{1}{5} = 1 \)

Therefore \( P(\text{red}) = 1 - \left( \frac{1}{3} + \frac{1}{5} \right) = 1 - \frac{8}{15} = \frac{7}{15} \)
1. A card is drawn at random from an ordinary pack of 52.
   What is the probability that the card is
   a) a red ace
   b) a black king
   c) a red ace or a black king?

2. Gemma rolls an ordinary dice once. What is the probability the number shown is
   a) 2
   b) 3 or 4
   c) 2, 3 or 4?

3. A card is drawn at random from the 12 court cards (jacks, queens and kings). What is the probability that the card is
   a) a black jack
   b) a red queen
   c) either a black jack or a red queen?

4. Graham is looking for his house key. The probability that it is in a pocket is \( \frac{5}{9} \), while the probability that it is in the car is \( \frac{1}{13} \).
   What is the probability that
   a) the key is either in a pocket or in the car
   b) the key is somewhere else?

5. When Mrs George goes shopping, the probability that she returns by bus is \( \frac{3}{7} \), in a taxi \( \frac{1}{7} \), and on foot \( \frac{5}{14} \).
   What is the probability that she returns
   a) by bus or taxi
   b) by bus or on foot
   c) by none of these ways?

6. Jo has a bag containing discs of four different colours. One disc is removed at random. The table shows the probabilities of choosing three of the four colours.

<table>
<thead>
<tr>
<th>Colour</th>
<th>red</th>
<th>white</th>
<th>blue</th>
<th>pink</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>( \frac{2}{7} )</td>
<td>( \frac{2}{9} )</td>
<td>( \frac{1}{4} )</td>
<td></td>
</tr>
</tbody>
</table>

   Jo removes one disc at random. What is the probability that this disc is
   a) red or white
   b) white or blue
   c) red, white or blue
   d) pink?

7. Rajev has a pack of playing cards with some cards missing. There are 45 cards in the pack. He knows that all the clubs and hearts are in his pack. One card is drawn at random from the pack.
   What is the probability that this card is not a club or a heart?

8. Maarit rolls an ordinary dice. What is the probability that the number on the dice is
   a) an even number
   b) a prime number
   c) either even or prime?
   Your answer to part c should not be the sum of the answers to parts a and b. Why not?
**Multiplication of probabilities**

When a coin is tossed and a dice is rolled, we can use a table to list all the possible outcomes. **Note:** All the possible outcomes of an experiment is called a **possibility space**.

<table>
<thead>
<tr>
<th>Dice</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coin</td>
<td>H</td>
<td>H, 1</td>
<td>H, 2</td>
<td>H, 3</td>
<td>H, 4</td>
<td>H, 5</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>T, 1</td>
<td>T, 2</td>
<td>T, 3</td>
<td>T, 4</td>
<td>T, 5</td>
</tr>
</tbody>
</table>

From the table we can see that

\[ P(\text{a head and an even number}) = \frac{3}{12} = \frac{1}{4} \]

Now a head from one toss of the coin and an even number from one throw of the dice are independent events, where

\[ P(\text{a head}) = \frac{1}{2} \quad \text{and} \quad P(\text{an even number}) = \frac{3}{6} = \frac{1}{2} \]

But

\[ P(\text{a head and an even number}) = \frac{1}{4} = \frac{1}{2} \times \frac{1}{2} = P(\text{a head}) \times P(\text{an even number}) \]

This example illustrates that

**If A and B are independent events, then** \( P(A \text{ and } B) = P(A) \times P(B) \)**

---

**Exercise 2c**

1. Two coins are tossed. What is the probability that they both land head up?
2. Two dice are tossed. Find the probability of getting a double six.
3. Peter has two tubes of Smarties. Each tube contains 10 red Smarties and 30 Smarties of other colours. Peter takes one Smartie, chosen at random, from each tube. Find
   - the probability that a red Smartie is taken from a tube
   - the probability that a Smartie other than a red one is taken from a tube
   - the probability that both Smarties removed are not red.
4. The probability that Heather will win the girls’ 100 m is \( \frac{2}{5} \), and the probability that Colin will win the boys’ 100 m is \( \frac{3}{5} \). What is the probability that
   - both of them will win their events
   - neither of them will win their event?
5. A mother has an equal chance of giving birth to a boy or a girl. Jane plans to have two children.
   - What is the probability that the first is a girl?
   - What is the probability that both are boys?
   - What is the probability that neither is a boy?
6. The probability that Eve will have to wait before she can cross Westgate Street is \( \frac{1}{3} \), and the probability that she will be able to cross High Street without waiting is \( \frac{1}{4} \).

What is the probability that

a. she does not have to wait to cross Westgate Street
b. she has to wait to cross High Street
c. she can cross both streets without waiting?

7. A bag contains 3 red sweets and 2 green sweets. Camilla takes one sweet at random and eats it. She then takes another sweet, also at random.

a. Make a possibility table to show the possible combinations of colours of the two sweets, and use it to find the probability that both sweets removed are red.
b. Explain why, in this case, the multiplication rule does not give the correct answer to part a.

Exercise 2d

Some of the events described are mutually exclusive and some are independent.

1. A red dice and a blue dice are rolled. Find the probability of getting

   a. a 5 or a 6 on the red dice
   b. a 1 or a 2 on the blue dice
   c. a 2 on both dice
   d. an even number on both dice.

2. A card is drawn at random from an ordinary pack of 52 playing cards. What is the probability that the card is

   a. a 2
   b. a red ace
   c. a 2 or a red ace?

3. When Kim goes to Weightwatchers the probability that she returns on foot is \( \frac{2}{3} \), by bus \( \frac{1}{6} \) and in a friend’s car \( \frac{1}{6} \).

   What is the probability that she returns

   a. by bus or in a friends’ car
   b. on foot or by bus?

4. The probability that Sam will complete the 5000 km race is 0.9, and the probability that Mike will complete it is 0.6. What is the probability that both Sam and Mike will complete the 5000 km race?

5. A pack of cards is cut, reshuffled and cut again. What is the probability that

   a. the first card cut is an ace or a king
   b. the second card cut is an ace or a king
   c. both cards cut are aces?
Tree diagrams

When two coins are tossed, one possible outcome is a head and a tail. This outcome involves two events, but they do not fit neatly into the ‘either … or’ category, or the ‘both … and’ category. This is because a head and a tail can be obtained by getting

- either a head on the first coin and a tail on the second,
- or a tail on the first coin and a head on the second.

So getting a head and a tail when two coins are tossed involves a mixture of independent and mutually exclusive events, and we need an organised approach to deal with such a combination. One such approach is to draw up a table showing all the equally likely outcomes, but this method cannot be used if all the possible outcomes are not equally likely such as the possible outcomes when two people take a driving test.

Now suppose that three coins are tossed and we want the probability of getting two heads and a tail. Three events are involved here, so we cannot use a table to list all the outcomes because a table can only cope with two events.

These examples show that we need a different way of listing outcomes and finding probabilities.

Suppose that we have two discs, a red one marked A on one side and B on the other, and a blue one marked E on one side and F on the other.

Tossing the red disc, the probability that we get A is $\frac{1}{2}$ and the probability that we get B is also $\frac{1}{2}$. This information is shown in the diagram.

```
  Red disc
 /     \
A 1/2  B 1/2
```

Suppose that the red disc shows A and we go on to toss the blue disc. The probability of getting E is $\frac{1}{2}$ and the probability of getting F is $\frac{1}{2}$.

We can add this information to the diagram.

```
  Red disc
 /     \
A 1/2  B 1/2
   /
  E 1/2
 /     \
F 1/2
```

We complete the diagram by considering what the probabilities are if the red disc shows a B before we toss the blue disc.

Diagrams like this are called tree diagrams or probability trees. To use the tree diagram to find the probability that we get first an A and then an E, follow the path from left to right for an A on the first branch and an E on the second. The two probabilities we find there are $\frac{1}{2}$ and $\frac{1}{2}$. The blue disc landing showing E is independent of the letter obtained on the red disc, so we multiply the probabilities together to get $\frac{1}{4}$.

To find the probability that we get a B on the red disc and an F on the blue one, follow the B and F path and multiply the probabilities, i.e.

$$P(B \text{ and } F) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Generally, we multiply probabilities when we follow a path along branches.

**Exercise 2e**

**Worked example**

A coin is tossed and a dice is thrown. Find the probability that

- the coin lands head up and the dice does not show a six
  $$P(H \text{ and not 6}) = \frac{1}{2} \times \frac{5}{6} = \frac{5}{12}$$
- the coin lands tail up and the dice shows a six.
  $$P(T \text{ and 6}) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$

There are only two possible outcomes when the coin is tossed, so we need two ‘branches’ to show these. There are six possible outcomes when the dice is thrown, but we only need to consider these in two groups: throwing a six or not throwing a six, so we need only two branches.
1. The probability that Mark gets to work on time is \( \frac{7}{8} \) and the probability that he leaves work on time is \( \frac{3}{5} \).
   a. Find the probability that he does not leave work on time.
   b. Copy and complete the given probability tree.
   c. What is the probability that Mark gets to work on time but does not leave on time?
   d. What is the probability that Mark is late for work but leaves on time?

2. When a drawing pin falls to the ground the probability that it lands point up is 0.2
   a. Find the probability that a pin does not land point up.
   b. Copy and complete the tree diagram.
   c. Find the probability that both drawing pins land point up.
   d. Both drawing pins land point down.

3. The first of two boxes of tennis balls contains one white and two yellow balls; the second box contains three yellow and two lime green balls. A ball is taken at random from each box.
   a. Copy and complete the tree diagram.
   b. Both balls are yellow.
   c. One is white and one is lime green.

4. Two soldiers fire at a target. The probability that Becker hits the target is 0.5, and the probability that Crossley does not hit the target is 0.3. Becker fires at the target first, then Crossley fires.
   Draw a tree diagram to show the possibilities and use it to find the probability that
   a. Both Becker and Crossley hit the target
   b. Neither hits the target
   c. Becker hits the target but Crossley misses
   d. Crossley hits the target but Becker misses.
The probability that my bus has to wait at the traffic lights in the morning on the way to school is \( \frac{1}{5} \).

Draw a probability tree to show the possibilities that the bus has to wait, or can drive through the traffic lights on two consecutive mornings. Find the probability that, on two consecutive mornings,

a. has to wait at the lights on both occasions
b. does not have to wait on either morning
c. has to wait on just one morning.

If a dice is rolled what is the probability of getting

i. a six
   ii. a number other than six?

Two dice, one red and the other blue, are rolled. Draw a tree diagram to show the possibilities of getting a six or not getting a six on each dice. Find the probability that
1. Both dice show sixes
2. The red dice gives a six, but the blue dice does not
3. The blue dice gives a six, but the red dice does not
4. The probability that just one six appears.

For each of the remaining questions, draw a probability tree to illustrate the given information.

7. In a group of six girls, four have blond hair and two have black hair. Of five boys, two have blond and three have black hair. One boy and one girl are picked at random. What is the probability that, of the two students picked, one has blond hair and one has black hair?

8. In a class of 20, four are left-handed. In a second class of 24, six are left-handed. One student is chosen at random from each class. What is the probability that one of the students is left-handed and one is not?

9. Derek and Alexis keep changing their minds about whether to send Christmas cards to each other. In any one year, the probability that Derek sends a card is \( \frac{3}{4} \) and that Alexis sends one is \( \frac{5}{6} \). Find the probability that next year
   a. they both send cards
   b. only one of them sends a card
   c. neither sends a card.
   What should the three answers add up to and why?

10. Three unbiased coins are tossed, one after the other. Find the probability that
   a. three heads appear
   b. three tails appear
   c. two heads and one tail appear in any order.

11. The weather forecast gives the probability that it will rain on Saturday as 0.07 and the probability that it will not rain on Sunday as 0.89.
   a. On which of these two days is it more likely to rain, and why?
   b. Copy and complete this tree diagram.
c  Use your tree diagram to find the probability that it will rain on
   i  both days
   ii  just one of the days.

The probability that it will rain on Monday is 0.3. Add more branches to your tree to include Monday.

d  Use your new tree to find the probability that it will rain on
   i  none of the three days
   ii  at least one of the three days.

12  A coin is tossed three times. Use the tree diagram drawn for question 10 to find the probability of getting
   a  a head and two tails
   b  exactly one tail
   c  at least one head
   d  at least two heads.

13  In a group of 120 girls, 24 have blue eyes, 48 have hazel eyes, 36 have green eyes and the remainder have brown eyes. All the girls have either long hair or short hair, and the probability that a given girl has long hair is 0.25. The probability that a girl has freckles is 0.65. Assume that each attribute is independent of the others. What is the probability that a girl chosen at random from this group has
   a  brown eyes, freckles and short hair
   b  long hair, no freckles and either blue or green eyes?

Venn diagrams

Venn diagrams can help find the number of times that two events can occur when those events are not mutually exclusive.

Consider, for example, a class of 30 students.

- Mr Edwards asked them if they had a calculator with them. 20 students put up a hand.
- Mr Edwards then asked them if they had a protractor with them. 16 students put up a hand.
- There were 6 students who had neither a calculator nor a protractor.

Before we can answer questions such as ‘What is the probability that a student chosen at random from the class has both a calculator and a protractor?’, we need to find out how many of them have both.

Having a calculator or having a protractor are not mutually exclusive, because there will be some students who have both. This is the number of students in the intersection of the sets {students with a calculator} and {students with a protractor}.

We do not know how many students have both, so we will use $x$ as that number.

We can now use a Venn diagram to illustrate this information. We cannot list the students, because we do not know their names so we will use the number of students in each set.
We know that the number in the overlap of the circles representing {students with a calculator} and {students with a protractor} is $x$.

We also know that the number in the set {students with a calculator} is 20.

Therefore the number in the part of the circle representing {calculators but not protractors} is $20 - x$.

Similarly the number in the part of the circle representing {protractors but not calculators} is $16 - x$.

Outside the two circles is the number who had neither a calculator nor protractor.

We know that there are 30 students in the class so we can form the equation

$$(20 - x) + x + (16 - x) + 6 = 30$$

Solving this equation gives $42 - x = 30$

giving $x = 12$

Now we can give the probability that a student has both a calculator and a protractor as $\frac{12}{30} = \frac{2}{5}$

**Exercise 2f**

1. Use the Venn diagram above to find the probability that one student chosen at random from the class
   a. has a calculator but not a protractor
   b. has a calculator and/or a protractor.

2. The Venn diagram shows how many students in a class of 30 own a mobile phone and a tablet.
   a. How many students do not own either a mobile phone or a tablet?
   What is the probability that one of these student chosen at random
   b. owns a tablet but not a mobile phone
   c. owns a mobile phone?
3. 100 adults were asked how they paid for goods bought in a shop. Some said they used a credit card, some said they paid cash, and some said they used both. Some adults used other means to pay for their goods.

Some of these results are shown in the Venn diagram.

```
<table>
<thead>
<tr>
<th>Credit Card</th>
<th>Cash</th>
</tr>
</thead>
<tbody>
<tr>
<td>55</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>
```

a. Copy and complete the Venn diagram.
b. How many adults paid for goods without using a credit card or cash?
c. What is the probability that one of these adults chosen at random only used cash?
d. What is the probability that one of these adults chosen at random never used cash?

4. In a squad of 35 cricketers, 20 said that they could bat and 8 said that they could bat and bowl. Show this information on a Venn diagram. How many more were willing to bowl than to bat?

5. In a group of 24 children, each had a dog or a cat or both. 18 kept a dog and 5 of these also kept a cat.

Show this information on a Venn diagram, and hence find the probability that one of these children chosen at random kept a cat, only a dog, just one of these as a pet.

6. A group of 50 television addicts were asked if they watched sports programmes and nature programmes. Their replies revealed that 21 watched both sports and nature programmes, but 9 watched nature programmes only. Show this information on a Venn diagram, and use it to find the probability that one of these people
   a. watched sports programmes
   b. did not watch nature programmes
   c. watched either sports or nature programmes but not both.

7. In a youth club, 35 teenagers said that they went to football matches, discos or both. Of the 22 who said they went to football matches, 12 said they also went to discos. A further 10 teenagers said they did not go to either. Show this information on a Venn diagram.

   a. How many went to football matches or discos, but not to both?
   b. One of this group of teenagers is chosen at random. What is the probability that the teenager went to discos but not to football matches?
8 There are 28 students in a form, all of whom take history or geography or both. 14 take history, 5 of whom also take geography.
   a  Show this information on a Venn diagram.

One student is chosen at random, what is the probability that the student takes
   b  geography
   c  history but not geography
   d  just one of these subjects?

9 The Venn diagram shows how many students in a class of 32 kept goldfish ($G$), budgerigars ($B$) or both.

Use the Venn diagram to find the number of students who kept both goldfish and budgerigars.

One student is chosen at random. What is the probability that the student
   a  did not have a budgerigar
   b  had at least one of these pets?

10 The passengers on a coach were questioned about the newspapers and weekly magazines they bought.

3 bought both a daily newspaper and a weekly magazine.
15 bought a daily newspaper. 8 bought a weekly magazine.
8 did not buy either a daily paper or a weekly magazine.

Show this information on a Venn diagram.
   a  How many passengers were there on the coach?
   b  What is the probability that one of the passengers, chosen at random, bought a daily newspaper, a weekly magazine or both?

11 One evening all 78 members of a youth club were asked whether they liked swimming ($S$) and/or dancing ($D$). It was found that 34 liked swimming, 41 liked dancing and 8 liked neither. Show this information on a Venn diagram. Use the diagram to find how many were swimmers and dancers.

What is the probability that one of the members, chosen at random, is
   a  a swimmer but not a dancer
   b  a dancer or a swimmer but not both?
During April, 36 cars were taken to a testing station for a road worthiness certificate. The results showed that 17 cars passed the test, 10 had defective brakes, and 13 had defective lights. Show this information on a Venn diagram.

One of these cars is chosen at random, what is the probability that it

a) failed the test
b) had both defects
c) had exactly one defect?

Write down the members of the set of all possible outcomes when an ordinary, unbiased, six-sided dice is rolled.

b) Write down the members of the set of outcomes that are prime numbers, and the members of the set of outcomes that are even numbers.

c) The dice from a is rolled once. What is the probability that it scores a number that is neither prime nor even?

List the members of the set, S, of whole numbers from 1 to 16 inclusive.

b) List the set, A, of numbers that are factors of 12 and the set, B, of numbers that are factors of 16.

c) Show the members of all three sets in a Venn diagram.

d) One number is chosen at random from the set S. What is the probability that the number is
   i) a factor of both 12 and 16
   ii) a factor of neither 12 nor 16?

The Venn diagram shows the number of students taking geography (G), history (H) and accounts (A) in a class of 43. Every student takes at least one of these subjects.

a) Write down an expression, in terms of x, for the number of students who take history.

b) Write down an equation, in terms of x, which shows all the information given.

c) Find the probability that one of these students, chosen at random
   i) takes geography only
   ii) takes accounts.
Mixed questions

The next exercise contains mixed problems on probability. Some of the questions can be answered directly from the basic definition of probability, and some can be answered using the sum and product rules. Draw a tree diagram, or a possibility table, or a Venn diagram only when you think it is needed.

Exercise 2g

1. A letter is picked at random from the word CATASTROPHE. Find the probability that
   a. the letter is a vowel
   b. the letter is A or T.

2. A knitting wool sample card has 1 green, 1 black, 4 blue and 2 red samples. If one sample is picked at random, what is the probability that it is
   a. yellow
   b. black, green, red or blue?

3. The scores on a four-sided spinner are 1, 2, 3 or 4. On a second four-sided spinner the scores are 5, 6, 7 or 8. If the two are spun, find the probability that
   a. the score on both spinners is odd
   b. the score on both spinners is even
   c. the score on neither spinner is prime.

4. A sector is chosen at random from each circle. What is the probability that
   a. both sectors picked are blue
   b. both sectors picked are white
   c. one is blue and the other not?

5. There are two bags. The first contains 2 white and 3 black marbles, and the second contains 1 red and 2 blue marbles. Two marbles are drawn, one from each bag. Find the probability that
   a. a white and a blue marble are drawn
   b. a black and a red marble are drawn
   c. neither a white marble nor a red marble is drawn.
6 In a game of skittles the probability that Ted scores more than 5 is \( \frac{2}{7} \), and the probability that George scores more than 5 is \( \frac{2}{9} \). Ted goes first followed by George. Use a probability tree to find the probability that

a both Ted and George score more than 5
b Ted scores more than 5, but George does not
c both score 5 or less
d one scores more than 5, but the other does not.

7 Mr Aziz sells vegetables from a market stall. One morning he makes a note of the sales of carrots and yams to the first 50 customers.

- 25 bought carrots.
- 36 bought yams.
- 12 bought neither carrots nor yams.

a Find the number of customers who bought carrots and yams.
b If one customer from the list is chosen at random, what is the probability that the customer bought carrots but not yams?

Consider again
Max would win on the fruit machine if cherries appear on all three drums or lemons appear on all three drums. Before he plays he would like to assess the risk of losing money.

To do this, he needs to know how to find the probability of combined events such as

a getting either a cherry or a lemon on the first drum
b getting a cherry on both the first and the second drum
c getting a cherry on all three drums.

There are 10 different types of fruit on each drum. Can you now give Max the answers he needs?

Consider also
An ordinary unbiased dice is rolled. What is the probability of throwing a prime number or an even number?

If you need some help, go to the STP website.
An agricultural society wishes to hold a two-day show in September at one of four possible venues. The table shows the number of days it rained each week at the four different places, over four years.

<table>
<thead>
<tr>
<th>Venue</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Week 2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Week 3</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Week 4</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

a) Investigate the probability that it will be dry at venue A on any one day during the first week of September.
b) What about a day during the second, third or fourth week?
c) Repeat parts a and b for the other three venues.
d) What is the probability that it will be dry on two consecutive days at venue A during the first week in September?
e) Repeat part d for other weeks and other venues.
f) What is the probability that at venue A, on two consecutive days during the first week in September, it will be
   i) dry on the first day but not on the second
   ii) dry on one of the two days but not on the other?
g) Repeat part f for other weeks and venues.
h) Which week and venue would you recommend to the organisers? Justify your answer.
   Would they be certain to get at least one dry day?
i) Compare the chance that it will be dry for two consecutive days during the driest week at the chosen venue with the wettest week anywhere else.