### Consider

A **formula** is a general instruction for finding one quantity in terms of other quantities. You might, for example, see this instruction for cooking a joint of beef in a microwave oven:

- Allow 6 minutes per 400 g.

If you know the weight of the joint of beef, this formula can be used to calculate the cooking time. The formula does, however, assume that you can work out how many 400 g there are in the weight of your joint.

We could try to overcome this problem by expressing the formula more fully in words, for example

1. Find the weight of your joint in kilograms.
2. Multiply this number by 1000 and divide the result by 400.
3. Then multiply the answer by 6 to give the cooking time in minutes.

This is lengthy, and probably not much clearer.

We can overcome this problem by using letters for the unknown numbers.

Can you find a formula using letters and symbols for the time in minutes needed to cook a joint of beef in a microwave oven?

*You should be able to solve this problem after you have worked through this chapter.*

Provided you understand the conventions used, algebra is a powerful tool for expressing general instructions in a short form.

### Class discussion

Discuss what is wrong about each of the following statements.

1. The length of the room is 2.
2. The length of the room is \( l \) metres and \( l = 2 \) metres.
3. There are \( n \) cans on the shelf and \( n = 5 \) cans.
4. The length of the room is \( l \) and \( l = 2 \).
5. Now write each of the statements in questions 1 to 4 so that they make sense.
Constructing a formula

Discussion of the examples above shows that when we use letters, it is important that we are clear about what the letters stand for. It is sensible to keep to the convention of using letters for unknown numbers, for example

If there were an unknown number of cans on a shelf, we could use $a$ for the number. We can then say that there are $a$ cans on the shelf, that is, $a$ is the number of cans.

If we use $a$ for the cans as well, we would have to talk about $a$ on the shelf. This is not a clear way to describe the situation.

Similarly, if a length is unknown, that is, it is an unknown number of units, we use a letter for the unknown number only; the letter does not include the units, for example

We can then say that the length is $b$ cm, rather than talk about the length being $b$.

If we stick to this convention, any expression involving letters is a relationship between numbers only, and the ordinary rules of arithmetic and algebra apply.

The other convention we use when working with letters is that when two letters are multiplied together, or a number is multiplied by a letter, we omit the multiplication symbol, for example

- we write $a \times b$ as $ab$
- and $3 \times a$ as $3a$

Exercise 10a

Worked example

The perimeter of this square is $P$ cm.

Write down a formula for $P$.

$P = l + l + l + l$

$P = 4l$ (collecting like terms)

In each of the following diagrams, the perimeter is $P$ cm.

Write down a formula for $P$ starting with ‘$P =$’.
I buy \( x \) kg of apples and \( y \) kg of pears. Write down a formula for \( W \) if \( W \) kg is the weight of fruit that I have bought.

If \( l \) m is the length of a rectangle and \( b \) m is the breadth, write down a formula for \( P \) if the perimeter of the rectangle is \( P \) m.

I start a game with \( N \) marbles and lose \( L \) marbles. Write down a formula for the number, \( T \), of marbles that I finish with.

Peaches cost \( n \) pence each. Write down a formula for \( N \) if the cost of 10 peaches is \( N \) pence.

Oranges cost \( x \) pence each and I buy \( n \) of these oranges. Write down a formula for \( C \) where \( C \) pence is the total cost of the oranges.

I have a piece of string which is \( l \) cm long. I cut off a piece which is \( d \) cm long. Write down a formula for \( L \) if the length of string which is left is \( L \) cm.

A rectangle is \( 2b \) m long and \( b \) m wide. Write down a formula for \( P \) where \( P \) m is the perimeter of the rectangle.

Write down a formula for \( A \) where \( A \) m\(^2\) is the area of the rectangle described in question 11.

A lorry weighs \( T \) tonnes when empty. Steel girders weighing a total of \( S \) tonnes are loaded on to the lorry. Write down a formula for \( W \) where \( W \) tonnes is the weight of the loaded lorry.

A train travels \( p \) km in one direction, and then it comes back \( q \) km in the opposite direction. If it is then \( r \) km from its starting point, write down a formula for \( r \). (Assume that \( p > q \).)

Two points have the same \( y \)-coordinate. The \( x \)-coordinate of one point is \( a \) and the \( x \)-coordinate of the other point is \( b \). If \( d \) is the distance between the two points, write down a formula for \( d \) given that \( a \) is less than \( b \). Draw a sketch to illustrate this problem.
16 These are the coordinates of some points on a straight line.

<table>
<thead>
<tr>
<th>x-coordinate</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y-coordinate</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

Find the formula for \( y \) in terms of \( x \).

17 In the sequence

\[ 3, 5, 7, 9, 11, 13, \ldots \]

the first term is 3, the second term is 5, and so on.

If the \( n \)th term is \( u_n \), (e.g. the third term is \( u_3 \)), we can place these terms in a table, i.e.

<table>
<thead>
<tr>
<th>( n )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u_n )</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>9</td>
<td>11</td>
</tr>
</tbody>
</table>

Find a formula for \( u_n \).

18 A letter costs \( x \) pence to post. The cost of posting 20 such letters is \( £q \).

Write down a formula for \( q \). (Be careful: Look at the units given.)

19 One grapefruit costs \( y \) pence. The cost of \( n \) such grapefruit is \( £L \).

Write down a formula for \( L \).

20 A rectangle is \( l \) m long and \( b \) cm wide. The area is \( A \) cm\(^2\).

Write down a formula for \( A \).

21 On my way to work this morning the train I was travelling on broke down. I spent \( t \) hours on the train and \( s \) minutes walking.

Write down a formula for \( T \), if the total time that my journey took was \( T \) hours.

---

**Using brackets**

In this rectangle, the expression for length has two terms, and we use brackets to keep them together.

The perimeter of the rectangle is twice the length added to twice the width, i.e. if \( P \) cm is the perimeter then

\[ P = 2 \times (3x - 2) + 2 \times x \]

Now \( 2 \times x \) can be written as \( 2x \) and, in the same way, we can leave out the multiplication sign in \( 2 \times (3x - 2) \) and write \( 2(3x - 2) \).

Therefore

\[ P = 2(3x - 2) + 2x \]

Now \( 2(3x - 2) \) means ‘twice everything in the bracket’, i.e.

\[ 2(3x - 2) = 2 \times 3x - 2 \times 2 \]
\[ = 6x - 4 \]

**Note:** This process is called ‘multiplying out the bracket’.

Therefore the formula becomes

\[ P = 6x - 4 + 2x \]

i.e. \( P = 8x - 4 \)
Exercise 10b

Worked example

⇒ Multiply out $3(4x + 2)$
$3(4x + 2) = 12x + 6$

$Note: 3 \times 4x = 3 \times 4 \times x = 12x$

⇒ Multiply out $2(x - 1)$
$2(x - 1) = 2x - 2$

Multiply out the brackets.

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2(x + 1)$</td>
<td>4</td>
<td>$4(3x - 3)$</td>
<td>7</td>
<td>$8(3 - 2x)$</td>
</tr>
<tr>
<td>2</td>
<td>$3(3x - 2)$</td>
<td>5</td>
<td>$2(4 + 5x)$</td>
<td>8</td>
<td>$4(4x - 3)$</td>
</tr>
<tr>
<td>3</td>
<td>$5(x + 6)$</td>
<td>6</td>
<td>$2(6 + 5a)$</td>
<td>9</td>
<td>$3(6 - 4x)$</td>
</tr>
<tr>
<td>10</td>
<td>$5(x - 1)$</td>
<td>11</td>
<td>$7(2 - x)$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Worked example

To simplify an expression containing brackets, we first multiply out the brackets and then collect like terms.

⇒ Simplify $6x + 3(x - 2)$
$6x + 3(x - 2) = 6x + 3x - 6$
$= 9x - 6$

First multiply out the bracket.
Collect like terms.

⇒ Simplify $2 + (3x - 7)$
$2 + (3x - 7) = 2 + 3x - 7$
$= 3x - 5$

This means $2 + 1 \times (3x - 7)$.

Simplify the following expressions.

13 $2x + 4(x + 1)$
14 $3 + 5(2x + 3)$
15 $2(x + 4) + 3(x + 5)$
16 $6(2x - 3) + 2x$
17 $4 + (3x - 1)$
18 $3x + 3(x - 5)$
19 $3(x + 1) + 4$
20 $6(2x - 3) + 5(x - 1)$
21 $3x + (2x + 5)$
22 $7 + 2(2x + 5)$

Worked example

⇒ Simplify $4x - 2(x + 3)$
$4x - 2(x + 3)$ means $4x$ take away $2$ $x$s and $2$ threes
$4x - 2(x + 3) = 4x - 2x - 6$
$= 2x - 6$

⇒ Simplify $5 - (x + 7)$
$5 - (x + 7) = 5 - x - 7$
$= -2 - x$
Simplify the following expressions.

23  $3x - 2(3x + 4)$  
24  $5 - 4(5 + x)$  
25  $7c - (c + 2)$  
26  $5x - 4(2 + x)$  
27  $7a - (a + 6)$

28  $10 - 4(3x + 2)$  
29  $40 - 2(1 + 5w)$  
30  $6y - 3(3y + 4)$  
31  $8 - 3(2 + 5x)$

32 Roberto started the day with three unopened tubes of sweets. He ate all the sweets in one tube and 5 sweets from another tube. If $x$ is the number of sweets in each unopened tube, write down an expression for the number of sweets he has eaten and left.

33 The perimeter of this rectangle is $8x$ cm. Find the formula for $y$ in terms of $x$.

\[ (2x + 1) \text{ cm} \]
\[ y \text{ cm} \]

### Multiplication of directed numbers

From Book 7, we know that \[ \frac{+ (+a)}{(-a)} = +a \text{ and } \frac{+ (-a)}{(-a)} = -a \]

Consider the expression $6x - (x - 3)$.

Now $6x - (x - 3)$ means ‘$6x$ take away $x$ and take away $-3’$, i.e.

\[ 6x - (x - 3) = 6x - x - (-3) \]

We know that $-(-3) = +3$, so

\[ 6x - (x - 3) = 6x - x + 3 \]

Similarly $8x - 3(x - 2)$ means $8x - [3(x - 2)]$

Therefore

\[ 8x - 3(x - 2) = 8x - [3x - 6] \]

\[ = 8x - 3x + 6 \]

We can also interpret $8x - 3(x - 2)$ to mean $8x - 3 \times x - 3 \times 2$

Comparing

\[ 8x - 3 \times x - 3 \times 2 \]

With

\[ 8x - 3x + 6 \]

we see that $-3 \times x = -3x$, i.e. $(-3) \times (+x) = -3x$

and that $-3 \times -2 = +6$

In general,

\[ (+a) \times (+b) = +ab \text{ and } (-a) \times (+b) = -ab \]

\[ (-a) \times (-b) = +ab \text{ and } (+a) \times (-b) = -ab \]

i.e.

When two numbers are multiplied together, both signs the same give a positive answer, different signs give a negative answer.
### Worked example

Calculate $(+3) \times (-4)$

$(+3) \times (24) = -12$ Different signs give a negative answer.

Calculate $-3 \times 4$

$-3 \times 4 = -12$ Remember: If a number does not have a positive or negative sign, it is positive.

Calculate $(-5) \times (-2)$

$(-5) \times (-2) = 10$ The same signs gives a positive answer.

Calculate $-5(-4)$

$-5(-4) = 20$ $-5(-4)$ means $-5 \times (-4)$.

### Calculate

- $1 (3) \times (5)$
- $2 (+4) \times (-2)$
- $3 (-7) \times (-2)$
- $4 (+4) \times (+1)$
- $5 (+6) \times (-7)$
- $6 (-4) \times (3)$
- $7 (-6) \times (3)$
- $8 (-8) \times (-2)$
- $9 (+5) \times (-1)$
- $10 (-6) \times (-3)$
- $11 (-3) \times (-9)$
- $12 (-2) \times (+8)$
- $13 7 \times (-5)$
- $14 -6 (-4)$
- $15 -3 \times 5$
- $16 5 \times (-9)$
- $17 -6 (4)$
- $18 -2 (-4)$
- $19 - (-3)$
- $20 4 \times (-2)$
- $21 3 (-2)$
- $22 5 \times 3$
- $23 6 \times (-3)$
- $24 -5 (-4)$
- $25 6 \times \left(-\frac{1}{2}\right)$
- $26 -3 \left(+\frac{3}{4}\right)$
- $27 \left(+\frac{2}{3}\right) \times (+9)$
- $28 -\frac{3}{8} \times \frac{2}{3}$
- $29 \frac{3}{4} (-4)$
- $30 \left(-\frac{1}{4}\right) \times \left(-\frac{2}{3}\right)$

### Worked example

Division is the reverse of multiplication. Therefore the rules for dividing with directed numbers are the same as those for multiplying them, e.g.

$(-4) \div (-2) = +2$ and $6 \div (-3) = -2$
Worked example

Multiply out 

\(-4(3x - 4)\)

\(-4(3x - 4) = -12x + 16\)

Multiply out the following brackets.

1  \(-6(x - 5)\) 

2  \(-5(3c + 3)\) 

3  \(-2(5e - 3)\) 

4  \(-(3x - 4)\) 

5  \(-8(2 - 5x)\) 

6  \(-7(x + 4)\) 

7  \(-3(2d - 2)\) 

8  \(-2(4 + 2x)\) 

9  \(-7(2 - 3x)\) 

10 \(-(4 - 5x)\)

Worked example

Simplify 

\(4(x - 3) - 3(2 - 3x)\)

\(4(x - 3) - 3(2 - 3x) = 4x - 12 - 6 + 9x\)

\(= 13x - 18\)

Multiply out the brackets first.

Collect like terms.

Simplify

11  \(5x + 4(5x + 3)\) 

12  \(42 - 3(2c + 5)\) 

13  \(2m + 4(3m - 5)\) 

14  \(7 - 2(3x + 2)\) 

15  \(x + (5x - 4)\) 

16  \(9 - 2(4g - 2)\) 

17  \(4 - (6 - x)\) 

18  \(10f + 3(4 - 2f)\) 

19  \(7 - 2(5 - 2s)\) 

20  \(7x + 3(4x - 1)\) 

21  \(7(3x + 1) - 2(2x + 4)\) 

22  \(5(2x - 3) - (x + 3)\) 

23  \(2(4x + 3) + (x - 5)\) 

24  \(7(3 - x) - (6 - 2x)\) 

25  \(5 + 3(4x + 1)\) 

26  \(6x + 2(3x - 7)\) 

27  \(20x - 4(3 + 4x)\) 

28  \(4(x + 1) + 5(x + 3)\) 

29  \(3(2x - 3) - 5(x + 6)\) 

30  \(5(6x - 3) + (x + 4)\) 

31  \(3x + 2(4x + 2) + 3\) 

32  \(4(x - 1) - 5(2x + 3)\) 

33  \(7x + 8x - 2(5x + 1)\) 

34  \(4(x - 1) - 5(2x - 3)\) 

35  \(8(2x - 1) - (x + 1)\) 

36  \(4(x - 1) + 5(2x + 3)\) 

37  \(5 - 4(2x + 3) - 7x\) 

38  \(3(x + 6) - (x - 3)\) 

39  \(3(x + 6) - (x + 3)\) 

40  \(4(x - 1) + 5(2x - 3)\)
Worked example

The width of a rectangle is \(x\) cm and the length is 4 cm more than the width. 
The perimeter is \(P\) cm. Find a formula for \(P\).

The width is \(x\) cm, 
so the length is \((x + 4)\) cm.
\[
\therefore \quad P = 2x + 2(x + 4)
\]
\[
= 2x + 2x + 8
\]
i.e. \(P = 4x + 8\)

Simplify

41

The length of the base of an isosceles triangle is \(x\) cm and the two equal sides are 3 cm shorter. The perimeter of the triangle is \(P\) cm. Find a formula for \(P\).

42 I think of a number, \(x\), add 5 to it and treble the result. If the result is \(y\), find a formula for \(y\).

43 \(N\) sweets are divided among Amy, Madge and Amir. 
Amy has \(x\) sweets, Madge has 3 less than Amy and Amir has twice as many as Madge.

a Write down an expression for the number of sweets Madge has.

b Write down an expression for the number of sweets Amir has.

C Write down a formula for \(N\).

44 A group of \(n\) children are taken to the theatre. For each child, the cost of travel is \(x\) pence, and the theatre ticket costs three times as much as the travel. Each child is given an ice cream which costs 90 p less than the travel cost. The total cost is £\(P\). Find a formula for \(P\).

45 The terms in a sequence are formed from the rule ‘double the position number of a term and subtract 2’.
Hence the first term is \(2(1) - 2 = 0\), 
the second term is \(2(2) - 2 = 2\), and so on.

a Write down the first six terms of the sequence.

b If \(n\) is the position number of a term, find a formula for \(u_n\), where \(u_n\) is the \(n\)th term of the sequence.
Indices

Using index notation, we know that $2 \times 2$ can be written as $2^2$. We can also use index notation with letters that stand for unknown numbers, e.g.

- we can write $a \times a$ as $a^2$,
- and $b \times b \times b$ can be written as $b^3$

In the same way, $d^5$ means $d \times d \times d \times d \times d$ and $3b^4$ means $3 \times b \times b \times b \times b$

Note: The index applies only to the number it is attached to: $2x^2$ means that $x$ is squared, 2 is not. If we want to square $2x$, we write $(2x)^2$ or $2^2x^2$.

Exercise 10e

Worked example

$\rightarrow$ Simplify $3a \times 2a$

$3a \times 2a = 3 \times a \times 2 \times a$
$= 6 \times a \times a$
$= 6a^2$

Remember: Numbers can be multiplied in any order, i.e.

$3 \times a \times 2 \times a = 3 \times 2 \times a \times a$
$= 6 \times a^2$

Write in index notation.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$x \times x$</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>$p \times p \times p$</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>$s \times s \times s$</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>$t \times t \times t \times t$</td>
<td>8</td>
</tr>
</tbody>
</table>

Worked example

$\rightarrow$ Simplify $2a \times 3b \times a$

$2a \times 3b \times a = 2a \times a \times 3b$
$= 2a^2 \times 3b$
$= 6a^2b$

Simplify

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>$2b \times 3a \times 4b$</td>
<td>17</td>
</tr>
<tr>
<td>14</td>
<td>$4x \times 2x \times 3y$</td>
<td>18</td>
</tr>
<tr>
<td>15</td>
<td>$p \times 2q \times p$</td>
<td>19</td>
</tr>
<tr>
<td>16</td>
<td>$3s \times r \times s$</td>
<td>20</td>
</tr>
</tbody>
</table>
Worked example

- A rectangle is \(3a\) cm long and \(a\) cm wide. The area is \(A\) cm\(^2\). Find a formula for \(A\).

\[ A = 3a \times a \]
\[ A = 3a^2 \]

The area of a rectangle = length \(\times\) width

The area of each rectangle is \(A\) cm\(^2\). Find a formula for \(A\).

25

\[
\begin{array}{c}
\text{5a cm} \\
\text{2a cm}
\end{array}
\]

26

\[
\begin{array}{c}
\text{7cm} \\
\text{rcm}
\end{array}
\]

27

\[
\begin{array}{c}
\text{3l cm} \\
\text{4l cm}
\end{array}
\]

28

\[
\begin{array}{c}
\text{3bcm} \\
\text{3bcm}
\end{array}
\]

The volume of each cuboid is \(V\) cm\(^3\). Find a formula for \(V\).

29

\[
\begin{array}{c}
\text{2a cm} \\
\text{acm} \\
\text{acm}
\end{array}
\]

30

\[
\begin{array}{c}
\text{2w cm} \\
\text{5w cm}
\end{array}
\]

31

\[
\begin{array}{c}
\text{4a cm} \\
\text{b cm} \\
\text{acm}
\end{array}
\]

32

\[
\begin{array}{c}
\text{5s cm} \\
\text{t cm} \\
\text{scm}
\end{array}
\]
Substituting numbers into formulas

The diagram shows a pattern made with floor tiles, each of which is 3 times as long as it is wide, laid round a central square.

If \( l \) cm is the width of any one of these tiles, then its length is \( 3l \) cm and its area, \( A \) cm\(^2\), is given by the formula

\[
A = 3l^2
\]

If the width of the smallest of these rectangles is 5 cm, we can find its area directly from the formula by substituting 5 for \( l \), i.e.

when \( l = 5 \), \( A = 3 \times 5^2 \)
\[
= 3 \times 25 = 75
\]

Therefore the area of the smallest rectangle is 75 cm\(^2\).

We can use this formula to find the area of any rectangle in the pattern if we know its width.

---

Exercise 10f

Worked example

\( \Rightarrow \) If \( v = u + at \), find \( v \) when \( u = 2 \), \( a = \frac{1}{2} \) and \( t = 4 \).

\[
v = u + at
\]

When \( u = 2 \), \( a = \frac{1}{2} \) and \( t = 4 \),
\[
v = 2 + \frac{1}{2} \times 4
\]
\[
= 2 + 2 = 4
\]

Do multiplication and division before addition and subtraction.

1. If \( N = T + G \), find \( N \) when \( T = 4 \) and \( G = 6 \).
2. If \( T = np \), find \( T \) when \( n = 20 \) and \( p = 5 \).
3. If \( P = 2(l + b) \), find \( P \) when \( l = 6 \) and \( b = 9 \).
4. If \( L = x - y \), find \( L \) when \( x = 8 \) and \( y = 6 \).
5. If \( N = 4(I - s) \), find \( N \) when \( I = 7 \) and \( s = 2 \).
6. If \( S = n(a + b) \), find \( S \) when \( n = 20 \), \( a = 2 \) and \( b = 8 \).
7. If \( V = lbw \), find \( V \) when \( l = 4 \), \( b = 3 \) and \( w = 2 \).
8. If \( A = \frac{PRT}{100} \), find \( A \) when \( P = 100 \), \( R = 3 \) and \( T = 5 \).
9. If \( w = u(v - t) \), find \( w \) when \( u = 5 \), \( v = 7 \) and \( t = 2 \).
10. If \( s = \frac{1}{2}(a + b + c) \), find \( s \) when \( a = 5 \), \( b = 7 \) and \( c = 3 \).
11 If \( N = p + q \), find \( N \) when \( p = 4 \) and \( q = -5 \).

12 If \( C = RT \), find \( C \) when \( R = 4 \) and \( T = -3 \).

13 If \( z = w + x - y \), find \( z \) when \( w = 4 \), \( x = -3 \) and \( y = -4 \).

14 If \( r = u(v - w) \), find \( r \) when \( u = -3 \), \( v = -6 \) and \( w = 5 \).

15 Given that \( X = 5(T - R) \), find \( X \) when \( T = 4 \) and \( R = -6 \).

16 Given that \( P = d - rt \), find \( P \) when \( d = 3 \), \( r = -8 \) and \( t = 2 \).

17 Given that \( v = l(a + n) \), find \( v \) when \( l = -8 \), \( a = 4 \) and \( n = -6 \).

18 If \( D = \frac{a - b}{c} \), find \( D \) when \( a = -4 \), \( b = -8 \) and \( c = 2 \).

19 If \( Q = abc \), find \( Q \) when \( a = 3 \), \( b = -7 \) and \( c = -5 \).

20 If \( I = \frac{2}{3}(x + y - z) \), find \( I \) when \( x = 4 \), \( y = -5 \) and \( z = -6 \).

21 Oranges cost \( n \) pence each and a box of 50 of these oranges costs \( C \) pence.

   a Write down a formula for \( C \).

   b Use your formula to find the cost of a box of oranges if each
   orange costs 62 p.

22 Lemons cost \( n \) pence each and a box of 50 lemons costs \( £L \).

   a Write down a formula for \( L \) (be careful with the units).

   b Use your formula to find the cost of a box of these lemons when
   they cost 40 p each.

23 Given \( N = 2(n - m) \), find

   a \( N \) when \( n = 6 \) and \( m = 4 \)

   b \( N \) when \( n = 7 \) and \( m = -3 \)

   c \( n \) when \( N = 12 \) and \( m = -4 \)

24 If \( z = x - 3y \), find

   a \( z \) when \( x = 3\frac{1}{2} \) and \( y = \frac{3}{4} \)

   b \( z \) when \( x = \frac{3}{8} \) and \( y = -1\frac{1}{2} \)

   c \( x \) when \( z = \frac{1}{4} \) and \( y = \frac{7}{8} \)

25 If \( P = 10r - t \), find

   a \( P \) when \( r = 0.25 \) and \( t = 10 \)

   b \( P \) when \( r = 0.145 \) and \( t = 15.6 \)

   c \( t \) when \( P = 18.5 \) and \( r = 0.026 \)
26 A rectangular box is $3l$ cm long, $2l$ cm wide and $l$ cm deep.
The volume of the box is $V$ cm$^3$.
   a Write down a formula for $V$.
   b Use your formula to find the volume of a box measuring
      5 cm deep.

27 A rectangle is $a$ cm long and $(2a - 10)$ cm wide.
   a Write down a formula for $P$, if $P$ cm is the perimeter of the
      rectangle.
   b Use your formula to find the perimeter of a rectangle
      6.9 cm wide.
   c What is the smallest integer that $a$ can represent for the value
      of $P$ to make sense?

28 The length of a rectangle is twice its width. If the rectangle is
   $x$ cm wide, write down a formula for $P$ if its perimeter is $P$ cm.
   Use your formula to find the width of a rectangle that has a
   perimeter of 36 cm.

29 A roll of paper is $L$ m long. $N$ pieces each of length $r$ m are cut off
   the roll. If the length of paper left is $P$ m, write down a formula
   for $P$. A roll of paper 20 m long had 10 pieces, each of length 1.5 m
   cut from it. Use your formula to find the length of paper left.

Finding a formula for the $n$th term of a sequence

In the sequence 2, 6, 10, 14, 18, ..., the first term is 2, the second term is 6, the third term is
10 and so on.

If we use $n$ for the position number of a term and $u_n$ for that term, we can arrange the
terms in a table, i.e.

<table>
<thead>
<tr>
<th>$n$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_n$</td>
<td>2</td>
<td>6</td>
<td>10</td>
<td>14</td>
<td>18</td>
</tr>
</tbody>
</table>

We can see that each term is 4 more than the term before it. We can use this fact to find $u_n$
in terms of $n$.

Rewriting the table to give $u_n$ in terms of the first term, 2, and the difference between the
terms, 4, gives

<table>
<thead>
<tr>
<th>$n$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_n$</td>
<td>2</td>
<td>$2 + 1(4)$</td>
<td>$2 + 2(4)$</td>
<td>$2 + 3(4)$</td>
<td>$2 + 4(4)$</td>
</tr>
</tbody>
</table>

Comparing the value of $u_n$ with the value of $n$, we can see that $u_n$ is equal to 2 plus one
fewer number of 4s than the value of $n$, i.e.

$$u_n = 2 + (n - 1)4$$
$$= 2 + 4n - 4$$
$$= 4n - 2$$
Any sequence where the difference between the terms is always the same is called an arithmetic progression. This difference may be added or subtracted from each term to get the next term.

Some other sequences involve squaring the term number, for example, in the sequence

\[1, 4, 9, 16, 25, \ldots\]

the first term is 1, the second term is 4, the third term is 9, and so on.

If we use \(n\) for the position number of a term and \(u_n\) for that term, we can arrange the terms in a table, i.e.

<table>
<thead>
<tr>
<th>(n)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(u_n)</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>25</td>
</tr>
</tbody>
</table>

Now we can see that the \(n\)th term, \(u_n\), is equal to \(n^2\). Therefore the formula for the \(n\)th term of this sequence is \(u_n = n^2\)

**Exercise 10g**

In questions 1 to 4, find a formula for \(u_n\). You should be able to do this by sight (without calculation).

1

<table>
<thead>
<tr>
<th>(n)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(u_n)</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

3

<table>
<thead>
<tr>
<th>(n)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(u_n)</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
</tbody>
</table>

2

<table>
<thead>
<tr>
<th>(n)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(u_n)</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

4

<table>
<thead>
<tr>
<th>(n)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(u_n)</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

5 For each sequence given in question 1 to 4, write down, in words, the rule for generating the sequence.

6 The terms of a sequence are generated by starting with 2 and adding 3 each time.
   a Write the first five terms in a table like those given in questions 1 to 4.
   b Find a formula for \(u_n\) in terms of \(n\).

7 The terms of a sequence are generated by starting with 10 and subtracting 2 each time.
   a Write the first five terms in a table like those given in questions 1 to 4.
   b Find a formula for \(u_n\) in terms of \(n\).

8 The terms of a sequence are generated by starting with 5 and adding 5 each time.
   a Write the first five terms in a table like those given in questions 1 to 4.
   b Find a formula for \(u_n\) in terms of \(n\).
9 The terms of a sequence are generated by starting with 2 and subtracting $\frac{1}{2}$ each time.

a Write the first five terms in a table like those given in questions 1 to 4.

b Find a formula for $u_n$ in terms of $n$.

Questions 10 to 17 describe how the terms of a sequence are generated. For each question repeat parts a and b of question 6.

10 Double the position number and add one.

11 Subtract 5 from the position number.

12 Multiply the position number by itself and subtract one.

13 Multiply the position number by itself and add two.

14 Subtract the square of the position number from five.

15 Add twice the square of the position number to 3.

16 Multiply twice the square of the position number and subtract 3.

17 Add the position number to the next position number.

18 Look at this pattern of squares.

a Copy and continue this table.

<table>
<thead>
<tr>
<th>nth pattern</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of squares</td>
<td>1</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b If $N$ is the number of squares in the 10th pattern, find $N$.

19 Repeat question 18 for the number of sides in this pattern.
Consider again
When cooking a joint of beef in a microwave oven:
• Allow 6 minutes per 400 g.
Find a formula using letters and symbols for the time in minutes needed to cook a joint of beef in a microwave oven.

Investigation
This is the pattern from page 12. It is formed round a central square from rectangles each of which is 3 times as long as it is wide.

a  The width of the smallest rectangle is $a$ cm.
   Write down a formula for $A$, where $A \text{ cm}^2$ is its area.

b  Find a formula for $B$, where $B \text{ cm}^2$ is the area of each rectangle in the next ring of 4 rectangles.

c  Find a formula for $C$, where $C \text{ cm}^2$ is the area of each rectangle in the third, outer ring of the pattern.

d  The pattern is continued by adding further rings of rectangles.
   Find a formula for $N$, where $N \text{ cm}^2$ is the area of a rectangle in the $n$th ring.