5 Sets

Consider

Thirty students wrote down what kind of transport they used one day to get to school.

- 15 used a bus.
- 18 used a car.
- 6 used both a bus and a car.

How can you find the number that used neither a bus nor a car?

*You should be able to solve this problem after you have worked through this chapter.*

Class discussion

Thirty students were asked if they had a pencil with them. They were then asked if they had a pen with them.

- 25 students said they had a pencil.
- 21 students said they had a pen.

These two numbers add up to more than the number of students in the class. Why do you think this is the case?

Set notation

A set is a clearly defined collection of things that have something in common, for example, a set of drawing instruments, a set of books.

Things that belong to a set are called members or elements. These members are usually written down separated by commas and enclosed in curly brackets.

For example, the set of prime numbers between 0 and 10 can be written as \{2, 3, 5, 7\}.

We do not have to list all the members of a set when we can use words to describe them. For example, instead of \{1, 2, 3, 4, 5, ..., 19, 20\} we can write \{whole numbers from 1 to 20 inclusive\}.

Exercise 5a

Worked example

- Write \{the prime factors of 30\} as a list of members.

\[
30 = 2 \times 15 = 2 \times 3 \times 5
\]

Therefore \{the prime factors of 30\} = \{2, 3, 5\}
Write each of the following sets as a list of members.
1. {the days of the week}
2. {the even numbers between 1 and 11}
3. {the prime numbers between 10 and 20}
4. {the multiples of 3 between 1 and 20}
5. {the last four letters of the Roman alphabet}
6. {multiples of 5 between 20 and 50 inclusive}
7. {the prime factors of 70}
8. {the square numbers between 1 and 100 inclusive}
9. {the triangular numbers between 1 and 50 inclusive}
10. {the different factors of 12}

Finite and infinite sets
When we need to refer to a set several times, we can use a capital letter to label that set. For example

\[ A = \{\text{months of the year beginning with J}\} \]

or
\[ A = \{\text{January, June, July}\} \]

In many cases it is not possible to list all the members of a set. For example, if
\[ N = \{\text{positive whole numbers}\} \]
we can write
\[ N = \{1, 2, 3, 4, 5, \ldots\} \]

where the dots show that the list continues indefinitely.
\[ N \] is an example of an infinite set, whereas \( A \) is an example of a finite set.

Exercise 5b

State whether the following sets are finite or infinite.
1. \( A = \{\text{vowels of the Roman alphabet}\} \)
2. \( B = \{\text{the factors of 30}\} \)
3. \( C = \{\text{even numbers}\} \)
4. \( D = \{\text{even numbers between 3 and 9}\} \)
5. \( E = \{\text{students taking violin lessons}\} \)
6. \( F = \{\text{odd numbers}\} \)
7. \( G = \{\text{grains of sand on the Earth}\} \)
8. \( H = \{\text{multiples of 3}\} \)
9. \( I = \{\text{different shapes of triangle}\} \)
10. \( J = \{\text{different languages spoken in the world}\} \)
Universal sets
Consider the set \( X = \{ \text{whole numbers less than 16} \} \).
Now consider the sets \( A \) and \( B \) whose members are in \( X \) where
\[
A = \{ \text{prime numbers less than 16} \} = \{2, 3, 5, 7, 11, 13\}
\]
and
\[
B = \{ \text{multiples of 5 less than 16} \} = \{5, 10, 15\}
\]
The set \( X \) is called a **universal set** for the sets \( A \) and \( B \). It is a set that contains all the members of the sets \( A \) and \( B \), together with other members that are not in \( A \) or in \( B \). The universal set could also be any set of whole numbers that includes the members of \( A \) and \( B \), for example \( \{ \text{whole numbers less than 20} \} \). The universal set is usually labelled \( U \).

Exercise 5c

**Worked example**

Suggest a universal set for the sets
\[
A = \{ \text{pupils in my class having piano lessons} \}
\]
\[
B = \{ \text{pupils in my class having guitar lessons} \}
\]
\[
U = \{ \text{all the pupils in my class} \}
\]
(The universal set could be any other set that includes the pupils in \( A \) and \( B \).)

Suggest a universal set for
1 \( \{2, 4, 6, 8, 10\} \) and \( \{4, 8, 12\} \)
2 \( \{1, 3, 5, 7, 11\} \) and \( \{3, 6, 9, 12, 15\} \)
3 \{knives\} and \{forks\}
4 \{odd whole numbers\} and \{multiples of 3\}
5 \{students in my class who play football\} and \{students in my class who play tennis\}
6 \{cups and saucers\} and \{plates\}

Venn diagrams
Venn diagrams are a way of representing sets. A **Venn diagram** consists of a rectangle with circles inside, representing sets.
The rectangle represents a universal set. The members of a universal set are all the members of the given sets together with possible other members.
For example, if \( A = \{ \text{the odd numbers between 0 and 10} \} = \{1, 3, 5, 7, 9\} \) and \( B = \{ \text{multiples of 3 between 1 and 20} \} = \{3, 6, 9, 12, 15, 18\} \), the universal set could be \( \{ \text{all whole numbers between 1 and 20 inclusive} \} \).
Draw Venn diagrams to show the union of the following sets.

1. \( P = \{a, b, c, d, e\}, \quad Q = \{a, e, i, o, u\} \)
2. \( X = \{2, 4, 6, 8, 10\}, \quad Y = \{4, 8, 12, 16\} \)
3. \( A = \{\text{letters in the word ‘Bernado’}\}, \quad B = \{\text{letters in the word ‘Bashira’}\} \)
4. \( F = \{4, 8, 12, 16, 20\}, \quad G = \{3, 6, 9, 12, 15\} \)
5. \( P = \{2, 3, 5\}, \quad Q = \{2, 3, 7\} \)
6. \( X = \{\text{prime numbers less than 10}\}, \quad Y = \{\text{even numbers less than 10}\} \)
7. \( F = \{\text{square numbers less than 50}\}, \quad G = \{\text{triangular numbers less than 50}\} \)
8. \( M = \{\text{square numbers less than 100}\}, \quad N = \{\text{cube numbers less than 100}\} \)
9. \( A = \{\text{capital letters in the alphabet that can be written using only straight lines}\} \),
\[ B = \{\text{capital letters in the alphabet that cannot be written using only straight lines}\} \]

10. \( P = \{\text{letters in the word ALGEBRA}\} \),
\[ Q = \{\text{letters in the word GEOMETRY}\} \]

**Intersection of sets**

The **intersection of two sets** \( A \) and \( B \) is the set of members that is in both \( A \) and \( B \).

Consider again the sets \( A = \{1, 3, 5, 7, 9\} \) and \( B = \{3, 6, 9, 12, 15, 18\} \)

From the Venn diagram we can see that the members \{3, 9\} are in both \( A \) and \( B \).

The intersection of \( A \) and \( B \) is written as \( A \cap B \).

Therefore \( A \cap B = \{3, 9\} \)

**Exercise 5e**

Use the diagrams that you drew for Exercise 5D, questions 1 to 6 to write down the intersection of each pair of sets.

7. Draw a Venn diagram to illustrate the sets \( A = \{2, 4, 6, 8, 10, 12\} \) and \( B = \{3, 6, 9, 12\} \) with \( U = \{\text{whole numbers from 1 to 12 inclusive}\} \).
   Hence write down the set of numbers that are in \( U \) but not in either \( A \) or \( B \).

8. Draw a Venn diagram to illustrate the sets \( A = \{\text{factors of 6}\} \) and \( B = \{\text{factors of 10}\} \) with \( U = \{\text{whole numbers from 1 to 10 inclusive}\} \).
   Hence write down the set of numbers that are in \( U \) but not in either \( A \) or \( B \).

9. Given \( F = \{\text{square numbers less than 50}\} \), \( G = \{\text{triangular numbers less than 50}\} \) and \( U = \{\text{whole numbers from 1 to 50 inclusive}\} \)
   a) list the members of \( F \cap G \)
   b) find the number of members of the set that are not members of \( F \) or \( G \).

10. Given \( M = \{\text{square numbers less than 100}\} \), \( N = \{\text{cube numbers less than 100}\} \) and \( U = \{\text{whole numbers from 1 to 100 inclusive}\} \)
   a) list the members of \( M \cap N \)
   b) find the number of members of the set that are not members of \( M \) or \( N \).
Consider again

Thirty students wrote down what kind of transport they used one day to get to school.
- 15 used a bus. 18 used a car.
- 6 used both a bus and a car.

Now can you find the number that used neither a bus nor a car?

If you need some help, go to the STP website.

Investigation

Consider the following sets $X$ and $Y$.

$X = \{0, 2, 4, 6, 8, 10, \ldots \}$, $Y = \{1, 3, 5, 7, 9, 11, \ldots \}$

a. Describe $X$ and $Y$ in words.

b. Write down the next three numbers in $X$. Write down the next three numbers in $Y$.

c. Choose two numbers from $X$ and find their sum. Is this sum a member of $X$? Is this always true?

d. Choose two numbers from $Y$ and find their sum. Which set contains this sum? Is this always true?

e. What can you say about two numbers so that their sum is in $Y$?