Learning outcomes

- Construct simple algebraic expressions by using letters to represent numbers.
- Simplify linear expressions, e.g. collect like terms; multiply a constant over a bracket.
- Substitute positive integers into simple linear expressions.

Algebra

Mathematicians use letters to represent unknown numbers. Using letters to represent unknown numbers is called algebra.

Algebra is often called the language of mathematics. It is a language that is hundreds of years old and was first developed by the Babylonians.

Algebra is a language that is understood in every country around the world.

2.1 Using letters for unknown numbers

Solving problems is often much easier if you use letters to represent unknown numbers.

Consider the following problem:

A bag contains an unknown number of marbles.
Two more marbles are added to the bag.
How many marbles are there in the bag now?

Choose a letter to represent the unknown number of marbles.
Let \( n \) = the unknown number of marbles.

\( n \) is called a variable.

Two more marbles are added to the bag.
There are now \( n + 2 \) marbles in the bag.

\( n + 2 \) is called an expression.
Some rules of algebra:

- \(x + x\) means 2 lots of \(x\) \quad so \quad \(x + x = 2 \times x = 2x\)

- \(y \times x\) is the same as \(x \times y\) and is written as \(xy\)

- \(7 \times y \times x = 7xy\)

- \(x \div 2\) is written as \(\frac{x}{2}\)

**Worked example 1**

Write this sentence as an expression:

Start with the number \(n\), multiply by 2 and then add 5.

Build up the expression in stages:

Start: \(n\)

Multiply by 2: \(2n\)

Add 5: \(2n + 5\)

**Worked example 2**

A melon costs \(m\) cents and a peach costs \(p\) cents.

Write down an expression for the cost in cents of:

- **a** 7 melons
- **b** 5 peaches
- **c** 7 melons and 5 peaches.

- **a** Cost of 7 melons = \(7 \times m = 7m\)
- **b** Cost of 5 peaches = \(5 \times p = 5p\)
- **c** Cost of 7 melons and 5 peaches = \(7m + 5p\)

**Exercise 2.1**

1 Write these sentences as expressions.

- **a** Start with the number \(p\) and then multiply by 3.
- **b** Start with the number \(A\) and then add 4.
- **c** Start with the number \(d\) and then take away 8.
- **d** Start with the number \(b\) and then multiply by 5.
- **e** Start with the number \(x\), multiply by 4 and then add 3.
- **f** Start with the number \(y\), multiply by 2 and then take away 5.
- **g** Start with the number \(Q\), multiply by 7 and then add 1.
h. Start with the number $R$ and then take it away from 8.

i. Start with the number $c$, multiply by 5 and then add $d$.

j. Start with the number $f$, add $g$ and then add 2.

k. Start with the number $j$, take away $k$ and then add 2.

l. Start with the number $m$, multiply it by 5 and then take away from 2.

2. There are $x$ fish in a pond.
   Omar puts 5 more fish in the pond.
   How many fish are in the pond now?

3. A piece of string is $y$ cm long.
   Farrida cuts off 3 cm.
   How long is the piece of string now?

4. A brick has a mass of $m$ kg.
   Write an expression for the mass of 4 bricks.

5. Gregor was $h$ cm tall.
   His height increases by 2 cm.
   Write an expression for his new height.

6. Raju is $x$ years old.
   a. How old was Raju 4 years ago?
   b. How old will Raju be 6 years from now?

7. Robert, Anna and Helen have some marbles.
   Anna has 5 marbles more than Robert.
   Helen has 3 marbles fewer than Robert.
   If Anna has $x$ marbles:
   a. how many marbles does Robert have?
   b. how many marbles does Helen have?

8. A pencil costs $p$ cents and a ruler costs $r$ cents.
   Write an expression for the cost of:
   a. 5 pencils  
   b. 2 rulers  
   c. 5 pencils and 2 rulers.

9. A cup of coffee costs $c$ dollars and a cup of tea costs $t$ dollars.
   Write an expression for the cost of:
   a. 3 cups of coffee  
   b. 7 cups of tea  
   c. 3 cups of coffee and 7 cups of tea.

10. A CD costs $x$ dollars and a DVD costs $y$ dollars.
    Write an expression for the cost of 3 CDs and 5 DVDs.
11 Write an expression for the perimeter of each of these shapes.

a
\[
\begin{array}{ccc}
& x & \\
& x & \\
& x & \\
\end{array}
\]

b
\[
\begin{array}{ccc}
& y & \\
& y & \\
\end{array}
\]

c
\[
\begin{array}{ccc}
p & p & p \\
p & p & p \\
p & p & p \\
p & p & p \\
p & p & p \\
\end{array}
\]

12 Match each statement with the correct expression.

The first one has been done for you.

- Subtract 2 from \(x\) and then multiply by 5.
  \[5(x + 2)\]

- Add 2 to \(x\) and then multiply by 5.
  \[5 - 2x\]

- Multiply \(x\) by 2 and then subtract from 5.
  \[5x - 2\]

- Multiply \(x\) by 5 and then subtract 2.
  \[2x + 5\]

- Multiply \(x\) by 2 and then add 5.
  \[5(x - 2)\]

Note that \(5(x + 2)\) means \(5 \times (x + 2)\).

13 The cost of a theatre ticket for an adult is \(\$a\).

The cost of a theatre ticket for a child is \(\$c\).

Adult tickets are more expensive than child tickets.

a Match each statement with the correct expression.

- The total cost in dollars of 5 adult tickets and 4 child tickets
  \[4a + 5c\]

- The difference in price in dollars between an adult ticket and a child ticket
  \[100 - 4a\]

- The cost in dollars of 5 child tickets
  \[5c\]

- The cost in dollars of 5 child tickets
  \[5a + 4c\]

- The change in dollars from \(\$100\) when you buy 4 adult tickets
  \[a - c\]

\[100 - 9c\]

b Write a statement for each of the two expressions that are left.
2.2 Simplifying expressions

The expression $5x$ has one term.
The expression $2x + 5y - 3z + 7$ has four terms.
The expression $7x + 8y + 5x$ has two like terms that can be collected together.
The like terms are the $7x$ and the $5x$.

\[
\begin{align*}
7x + 8y + 5x &= 7x + 5x + 8y \\
&= 12x + 8y
\end{align*}
\]

Worked example

Simplify these expressions.

\begin{align*}
a & \quad 5x + 3x \\
b & \quad 7m - m \\
c & \quad 2x + 8y + 4x - 5y \\
d & \quad 8x + 3 - 4x - 10
\end{align*}

\begin{align*}
a & \quad 5x + 3x = 8x \\
b & \quad 7m - m = 7m - 1m = 6m \\
c & \quad 2x + 8y + 4x - 5y = 2x + 8y + 4x - 5y \\
d & \quad 8x + 3 - 4x - 10 = 8x + 3 - 4x - 10
\end{align*}

Exercise 2.2

1. Simplify these expressions.

\begin{align*}
a & \quad y + y + y \\
b & \quad 5x + 2x \\
e & \quad 5q - q \\
f & \quad 14m - 2m \\
i & \quad 5x + 2x + 4x \\
j & \quad 7y - 3y + 2y \\
m & \quad 7h - 2h - 8h \\
k & \quad 8p - 2p - 3p \\
l & \quad 5p + q - 4q \\
m & \quad 7p - 2p \\
o & \quad 8q - q - 7q \\
p & \quad 8a + a \\
q & \quad 5h - 6h \\
r & \quad 5p + q - 4q \\
s & \quad -8f + 2f + 2f
\end{align*}
2 This is an algebraic wall.

To find the expression in each block you add the expressions in the two blocks below it.

Copy and complete these algebraic walls.

![Algebraic wall a](image1)

![Algebraic wall b](image2)

![Algebraic wall c](image3)

3 To find the expression in each square you add the expressions in the two connecting circles.

Copy and complete these diagrams.

![Diagram a](image4)

![Diagram b](image5)

![Diagram c](image6)

4 Hassan has got his simplifying expressions homework wrong.

Copy out each question and correct his mistakes.

![Simplifying expressions](image7)

5 Simplify these expressions.

a $8y - 3x + 2y$

$= 8y - 2y + 3x$

$= 6y + 3x$

b $5 + 3x - 2 + 6x$

$= 5 - 2 + 3x + 6x$

$= 3 + 9x$

$= 12x$

c $4y + 8x + 2yx + 7x$

$= 15x + 4xy + 2yx$

6 Copy and complete.

a $5x + [ ] + 3x + 4y = 8x + 11y$

c $6x - 2y + [ ] - 4y = 9x - 6y$

e $9 + 3h + 4 - [ ] = 13 + 7h$

b $[ ] + 2x + 7 + 4x = 10 + 6x$

d $8p + 7q + [ ] - [ ] = 11p + 5q$

f $14 - 5x + 2y - [ ] + [ ] = 14 - 7x + 8y$
7 Copy and complete these algebraic walls.

\[
\begin{array}{c}
a & b & c \\
3x + 4y & 2x + 3y & 5x + 2y \\
11y + 20 & 3y + 7 & 3a + 1 \\
2y + 4 & 2a + 3 & 7a \\
\end{array}
\]

8 Write an expression for the perimeter of each of these shapes. Write your answers in their simplest form.

\[
\begin{array}{c}
a & b & c \\
3x & 2y & 3x \\
2y & x & 2y \\
3x & 2x & y \\
\end{array}
\]

9 Simplify these expressions.

\[
\begin{align*}
a) & \quad 5x + 3xy + 2x + 8xy \\
b) & \quad 7ab + 3ba + 5a \\
c) & \quad 5 + 3xy + 8yx \\
d) & \quad 6fg + 2g + 5gf \\
e) & \quad 7mn + 2 + 3nm \\
f) & \quad 8xy + 7 + 2x - 3yx \\
g) & \quad 8y - 9xy - 4y \\
h) & \quad 6fg - 4ad - 3gf - da \\
i) & \quad 3pq - 2qr + 4rq - 2qp \\
\end{align*}
\]

10 The diagram shows some expressions that are equivalent to \(5x + 2y\)

Copy the diagram and find four more expressions that are equivalent to \(5x + 2y\)

11 Copy and complete the algebraic wall.

12 Look at your diagrams for question 3.

Can you find a rule for finding the expressions in each of the circles when you know the expressions in the squares?

Write the rule for finding the expression in the orange circle using the letters \(a\), \(b\) and \(c\).
2.3 Expanding brackets

Some algebraic expressions contain **brackets**.

3(x + 2) means 3 lots of (x + 2)

So 3(x + 2) = x + 2 + x + 2 + x + 2 = 3x + 6

It is much quicker to multiply out the brackets.

When you multiply out the brackets you must multiply each term in the brackets by the term outside the brackets.

\[
3(x + 2) = 3 \times x + 3 \times 2 = 3x + 6 \\
7(2x + 3) = 7 \times 2x + 7 \times 3 = 14x + 21
\]

Multiplying out the brackets is also called **expanding brackets**.

**Worked example 1**

Multiply out the brackets.

\[
\begin{align*}
a & \quad 6(x + 7) \\
b & \quad 3(y - 2) \\
c & \quad 5(2 - 3x)
\end{align*}
\]

\[
\begin{align*}
a & \quad 6(x + 7) = 6 \times x + 6 \times 7 \\
& \quad = 6x + 42 \\
b & \quad 3(y - 2) = 3 \times y - 3 \times 2 \\
& \quad = 3y - 6 \\
c & \quad 5(2 - 3x) = 5 \times 2 - 5 \times 3x \\
& \quad = 10 - 15x
\end{align*}
\]

**Worked example 2**

Expand and simplify: 7(x + 5) + 6(2x - 3)

\[
\begin{align*}
7(x + 5) + 6(2x - 3) & = 7 \times x + 7 \times 5 + 6 \times 2x - 6 \times 3 \\
& = 7x + 35 + 12x - 18 \\
& = 19x + 17
\end{align*}
\]
Exercise 2.3

1 Nadia has got her expanding brackets homework wrong.

Copy out each question and correct her mistakes.

\[ a \quad 2(x + 1) \quad b \quad 5(2 + 3x) \quad c \quad 7(2x - 8) \]
\[ = 2x + 3 \quad = 5 \times 5x \quad = 14x - 8 \quad = 25x \]

2 Multiply out the brackets.

\[ a \quad 3(x + 4) \quad b \quad 5(y + 2) \quad c \quad 2(m + 7) \quad d \quad 4(p + 6) \]
\[ e \quad 8(3 + x) \quad f \quad 9(2 + g) \quad g \quad 6(9 + x) \quad h \quad 7(8 + k) \]
\[ i \quad 4(x - 3) \quad j \quad 5(y - 3) \quad k \quad 7(4 - y) \quad l \quad 6(15 - g) \]

3 Expand the brackets.

\[ a \quad 3(2x + 7) \quad b \quad 4(3y + 8) \quad c \quad 5(2m + 1) \quad d \quad 4(7p + 2) \]
\[ e \quad 6(3 + 7x) \quad f \quad 4(7 + 8g) \quad g \quad 3(9 + 5x) \quad h \quad 9(8 + 2k) \]
\[ i \quad 4(9x - 2) \quad j \quad 6(8p - 3) \quad k \quad 3(14 - 5y) \quad l \quad 7(25 - 4g) \]

4 Multiply out the brackets.

\[ a \quad 2(3x + 4y + 6) \quad b \quad 5(7x + 3y - 2) \quad c \quad 2(5m + 7n + 3) \]
\[ d \quad 4(8p - 6q + 5) \quad e \quad 8(8x - 3 + 2y) \quad f \quad 9(3f + 2g - 7h) \]
\[ g \quad 8(5x - 2y - 7) \quad h \quad 7(8p - 2q + 4r) \quad i \quad 4(9m + 3n - 4p) \]

5 Multiply out the brackets and simplify.

\[ a \quad 3(x + 4) + 2(x + 5) \quad b \quad 5(y + 3) + 4(y + 1) \quad c \quad 4(m + 6) + 3(m + 7) \]
\[ d \quad 4(p + 6) + 2(p + 5) \quad e \quad 7(2x + 3) + 4(3x + 5) \quad f \quad 7(2x + 9) + 4(5x + 2) \]
\[ g \quad 6(9y + 4) + 3(5 + 2y) \quad h \quad 4(8 + 7k) + 3(6k + 7) \quad i \quad 4(2x + 3) + 2(3x - 5) \]
\[ j \quad 5(4y - 3) + 4(2y + 7) \quad k \quad 9(4x - 2) + 7(3x + 5) \quad l \quad 6(3 - 2y) + 8(5y - 1) \]
\[ m \quad 4(5x - 8) + 3(5x - 4) \quad n \quad 2(3 - 4g) + 5(3g + 2) \quad o \quad 8(5 - 2x) + 2(7 - 3x) \]
\[ p \quad 4(2 + 5x) + 5(2 - 7x) \quad q \quad 3(7 + 8x) + 4(2 - 6x) \quad r \quad 4(7 - 3x) + 5(2x - 6) \]

6 Multiply out the brackets and simplify.

\[ a \quad 3(x + 5) + 2 \quad b \quad 5 + 7(y + 3) \quad c \quad 4(m + 6) + 8m \]
\[ d \quad 4 + 8(p + 6) + 2 \quad e \quad 7x + 3(2x + 4) + 9 \quad f \quad 7 + 8(2x - 9) + 4 \]
\[ g \quad 6y + 2(9y + 4) - 8 + 3y \quad h \quad 4(3x + 2y) + 5(2y - 9x) \quad i \quad 4(5x + 2y) + 4(2x - 5y) \]
Substitution into an expression

Substitution into an expression means replacing the letters in an expression by the given numbers.

Worked example

If \( a = 3 \), \( b = 4 \) and \( c = 5 \) find the value of these expressions.

\[ \begin{align*}
\text{a} & \quad 7a + c \\
\text{b} & \quad 4 + 2c \\
\text{c} & \quad ab - bc
\end{align*} \]

\[ \begin{align*}
\text{a} & \quad 7a + c = (7 \times 3) + 5 = 21 + 5 = 26 \\
\text{b} & \quad 4 + 2c = 4 + (2 \times 5) = 4 + 10 = 14 \\
\text{c} & \quad ab - bc = (3 \times 4) - (4 \times 5) = 12 - 20 = -8
\end{align*} \]
Exercise 2.4

1 If \( a = 6 \) and \( b = 7 \) find the value of:

\[
\begin{array}{llll}
\text{a} & a + 4 & \text{b} & a - b \\
\text{c} & a - 2 & \text{d} & 5a \\
\text{e} & 2a + 3b & \text{f} & 5b - 2a \\
\text{g} & 4 + 3b & \text{h} & 3ab \\
\text{i} & 4(a + b) & \text{j} & a(2 + b) \\
\text{k} & \frac{a}{3} & \text{l} & \frac{2ab}{a} \\
\text{m} & \frac{12}{a} + 3 & \text{n} & \frac{a + 2b}{2} \\
\text{o} & \frac{2b + 4}{a} & \text{p} & \frac{2ab}{a} \\
\end{array}
\]

2 If \( x = 4 \) and \( y = 5 \) find the value of:

\[
\begin{array}{llll}
\text{a} & x + y & \text{b} & 2y \\
\text{c} & x - y & \text{d} & y - 2x \\
\text{e} & \frac{12}{x} & \text{f} & 2 - \frac{x}{2} \\
\text{g} & \frac{2x + 2y}{3} & \text{h} & \frac{20}{x} + y \\
\end{array}
\]

3 If \( p = 3 \), \( q = 4 \) and \( r = 2 \) find the value of:

\[
\begin{array}{llll}
\text{a} & p + q + r & \text{b} & p + q - r \\
\text{c} & 2p + 3q + 4r & \text{d} & 3p - 4q - 2r \\
\text{e} & 5(p + q - 2) & \text{f} & pq + pr + qr \\
\text{g} & p(q + r) & \text{h} & 2qr + 5pq \\
\text{i} & \frac{q + r}{p} & \text{j} & \frac{pq}{r} \\
\text{k} & p + \frac{q}{r} & \text{l} & \frac{2p}{3} + \frac{3q}{2} + \frac{5r}{2} \\
\end{array}
\]

4 Find the odd one out when \( x = 8 \)

\[
\begin{array}{lll}
& x + 6 & 21 - x \\
& 2x - 2 &
\end{array}
\]

5 Find the value of \( x \) that makes each of these expressions equal.

\[
\begin{array}{lll}
& 2x + 1 & 22 - x \\
& 3x - 6 &
\end{array}
\]

6 Find the values of \( x \) and \( y \) that make each of these expressions equal.

\[
\begin{array}{lll}
& x + 2y + 16 & 2x + 2y + 10 \\
& x + 3y + 7 &
\end{array}
\]

Review

1 An apple costs \( a \) cents and a banana costs \( b \) cents.

Write an expression for the cost in cents of:

\[
\begin{array}{ll}
\text{a} & 8 \text{ apples} \\
\text{b} & 5 \text{ bananas} \\
\text{c} & 8 \text{ apples and } 5 \text{ bananas} \\
\text{d} & 15 \text{ apples and } 9 \text{ bananas.}
\end{array}
\]

2 Simplify these expressions.

\[
\begin{array}{lll}
\text{a} & 5h + 8h + 3h & \text{b} & 6x - 8x + 7x \\
\text{c} & 4b + 3c + 5b + 2c & \text{d} & 6f - 2g + 5f + g \\
\text{e} & 4 + 5x + 2y - 3x & \text{f} & 2xy - y + 5yx \\
\text{g} & 7x - 5y + 2x & \text{h} & 5ab + 2bc + 4ac - 2ba
\end{array}
\]
3 Copy and complete this algebraic wall.

4 Write an expression for the perimeter of each shape.

5 Multiply out the brackets.
   a $6(x + 3)$  b $2(x - 7)$  c $5(3 - x)$  d $3(x + y + 5)$
   e $8(2x + 3)$  f $3(4x - 1)$  g $9(2 - 7x)$  h $5(4a - 3b + 6c)$

6 Expand the brackets and simplify.
   a $3(5x + 4) + 2(6x - 3)$  b $5(5 - 2x) + 3(7 - 3x)$

7 If $x = 8$ and $y = 6$ find the value of:
   a $2x + 3y$  b $20 - 2x$  c $x(y - 3)$  d $2xy$
   e $9(2x - y)$  f $\frac{x}{4} + y$  g $\frac{xy}{12}$  h $\frac{3x}{y} + \frac{4y}{x}$

8 Find the value of $x$ that makes each of these expressions equal.