Nelson Mathematics
for Cambridge International A Level
Mechanics

L. Bostock • S. Chandler • D. A. Lee
# M1 Contents

## Introduction

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The Nelson Mathematics for Cambridge International A Level series has been specifically written for students of Cambridge’s 9709 syllabus by an experienced author team in collaboration with examiners who are very familiar with the syllabus and examinations. This means that, no matter which combination of modules you have chosen, the content of this series matches the content of the syllabus exactly and will give you firm guidelines on which to base your studies.

In this book, the content of the Mechanics 1 module is divided into ten chapters that give a sensible order for your studies. The chapters begin with a list of objectives that show you what is covered.

The following features help you to understand the concepts of the M1 module and to succeed in your exams.

- The introductions to concepts are accompanied by examples of questions together with their solutions. These show each step of working along with a commentary on the reasoning processes involved.
- There are numerous exercises for you to practise what you have learned and develop your skills.
- There are two Summary exercise sections with more detailed questions covering the content of the preceding chapters. These questions are similar to those found in exam papers and most are from real exam papers.
- Summaries of key information and formulae are at convenient points in the book to help you revise what you have covered in the last few chapters.
- Answers to all questions are provided at the back of the book for you to check your answers to exercises.
- Two sample exam papers have been created in the style of Cambridge’s International A Level M1 exams to give you the experience of working through a full exam paper.
DEFINING A VECTOR

A vector is a quantity for which direction is important as well as magnitude (i.e. size).

**Displacement** is a distance measured in a particular direction, e.g.

‘10 miles due north’ is a vector,

whereas ‘10 miles’ is a distance with no specified direction so is not a vector.

A quantity that has magnitude only is a **scalar** quantity.

Distance is the magnitude of displacement.

**Velocity** includes both speed and direction of motion, so

‘150 km h\(^{-1}\) on a bearing of 154°’ is a velocity, and is a vector,

whereas ‘a speed of 150 km h\(^{-1}\)’ is a scalar.

Speed is the magnitude of velocity.

**Acceleration** is the rate at which velocity is increasing so it follows that acceleration depends on both the speed and direction of motion, i.e. acceleration is a vector.

(Note that there is no different word for the magnitude of acceleration.)

**Force** is another quantity which plays an important part in the study of mechanics. Clearly if a force pushes an object we need to know both the size of the push and also which way it is acting, i.e. force is a vector.

Force is measured in newtons (N).

VECTOR REPRESENTATION

Any vector can be represented by a section of a line (called a **line segment**). The direction of the line gives the direction of the vector and the length of the line represents the magnitude of the vector.
RESULTANTS AND COMPONENTS

A man walks a distance of 30 m from \( A \) to \( B \) and then a distance of 40 m from \( B \) to \( C \). The man could have got to the same place by walking directly from \( A \) to \( C \).

Therefore the displacement from \( A \) to \( B \) followed by the displacement from \( B \) to \( C \) is equivalent to the displacement from \( A \) to \( C \). This is what is meant by adding vectors.

\[ AC = 50 \text{ m (Pythagoras' theorem)} \]

So \((30 \text{ m from } A \text{ to } B) + (40 \text{ m from } B \text{ to } C) = (50 \text{ m from } A \text{ to } C)\)

When adding vectors, ‘+’ means ‘together with’ or ‘followed by’
‘\( = \)’ means ‘is equivalent to’

Triangle \( ABC \) is called a triangle of vectors.

When two (or more) vectors are added, the single equivalent vector is called the resultant vector. The vectors that are combined are called components.

A heavy block is pulled along by two ropes. The unit in which the forces in the ropes are measured is the newton (N).

Although the ropes are pulling in different directions, the block moves in only one direction. This is the direction of the resultant of the tensions (i.e. the pulling forces) in the ropes. By drawing a triangle of vectors we can find both the magnitude and the direction of the resultant force. (We use a larger arrow for the resultant.)

Note that a triangle of vectors does not necessarily give the positions of the components or the resultant. In this case, for example, each of the components acts on the block so the equivalent resultant force also acts on the block.

The magnitude and direction of the resultant force can be found by calculation.
In \(\triangle ABC\): \(AC^2 = AB^2 + BC^2\) so \(AC\) represents 50 N

and \(\tan \hat{A} = \frac{3}{4} \left(\tan = \frac{\text{opp}}{\text{adj}}\right)\)

\[ \Rightarrow \hat{A} = 36.9^\circ \text{ (1 d.p.)} \]

Hence the resultant force is of magnitude 50 N acting at an angle of 37\(^\circ\) to \(AB\).

The calculation is easy because the components are at right angles.

This example can be extended to a general case where two components are perpendicular.

\[ \sqrt{p^2 + q^2} \]

If the magnitudes of the components are \(p\) and \(q\), the magnitude of the resultant vector \(\overrightarrow{AC}\) is given by \(\sqrt{p^2 + q^2}\).

**Example 1a**

Find, by calculation, the resultant of two velocities if one is 7 km h\(^{-1}\) south-west and the other is 12 km h\(^{-1}\) south-east.

A sketch is drawn starting with a line representing one velocity followed by a line representing the other one.

\(AB\) represents 7 km h\(^{-1}\) south-west.
\(BC\) represents 12 km h\(^{-1}\) south-east.
\(AC\) represents the resultant velocity.

In \(\triangle ABC\), \(AC^2 = AB^2 + BC^2 = 193\)

\[ \therefore AC = 13.9 \text{ (3 s.f.)} \]

\[ \tan \hat{A} = \frac{BC}{AB} = \frac{12}{7} = 1.714 \]

\[ \therefore \hat{A} = 59.7^\circ \text{ (1 d.p.)} \]

\[ \therefore \text{ the bearing of } AC \text{ is } 180^\circ - (59.7^\circ + 45^\circ) = 165.3^\circ \text{ (1 d.p.)} \]

The resultant velocity is 13.9 km h\(^{-1}\) on a bearing of 165\(^\circ\).
FINDING THE COMPONENTS OF A VECTOR

We have seen that two vectors can be combined into a single resultant vector. Now we will look at the reverse process, i.e. replacing one vector by an equivalent pair of vectors. This process is called resolving a vector into components.

For example a vector of magnitude 10 m east is to be replaced by two components, one of them due south and the other north-east.

In the diagram, $\overrightarrow{AC}$ represents the given vector. $\overrightarrow{AB}$ represents the component due south and $\overrightarrow{BC}$ represents the component north-east. The directions of these components are known but their magnitudes are not. However, the lengths of the lines $\overrightarrow{AB}$ and $\overrightarrow{BC}$, and hence the magnitudes of the components, can be found.

$\triangle ABC$ is an isosceles right-angled triangle, therefore $AB = AC = 10$.

Using Pythagoras’ theorem gives $BC^2 = AB^2 + AC^2 \Rightarrow BC = 14.1$ (3 s.f.)

Therefore the required components are 10 m due south and 14.1 m north-east.

Exercise 1a

Find the resultant of the given vectors.
Remember to give both the magnitude and the direction of the resultant.

1. A displacement of 12 km south followed by a displacement of 5 km east.

2. A displacement of 5 km east followed by a displacement of 12 km south. Is there any difference between your answer to this question and the answer to question 1?

3. A velocity of 24 m s$^{-1}$ north and a velocity of 7 m s$^{-1}$ east.


5. A force of 8N acting vertically downwards and a force of 6N acting horizontally.

6. A displacement of 7 m horizontally followed by a displacement of 3 m vertically upwards.

7. A velocity of 250 m s$^{-1}$ due east and a velocity of 300 m s$^{-1}$ due south.

8. ABCD is a square of side 10 m. A displacement from A to B followed by a displacement from B to the midpoint of BC.
Perpendicular components

A vector can be resolved into an infinite number of components in different directions but the most useful components, and the easiest to find, are a perpendicular pair.

Consider, for example, a velocity of 150 km h\(^{-1}\) at an angle of 30° above the horizontal.

![Diagram](image)

The horizontal and vertical components of the velocity can be found by using trigonometry, i.e.
- the horizontal component is given by \(PQ = 150 \cos 30° = 130\) (3 s.f.)
- the vertical component is given by \(QR = 150 \sin 30° = 75\)

Calculating the components of a vector plays a very important part in solving mechanics problems, so it is important that they can be written down immediately in the form above.

If you would first have written down \(\frac{PQ}{150} = \cos 30°\), practise going straight to the form \(PQ = 150 \cos 30°\); otherwise you will waste time.

### Examples 1b

1. A velocity has a magnitude of 5 m s\(^{-1}\) and a direction of 27° above the horizontal.
   
   (a) Find the horizontal and vertical components of the velocity.
   
   (b) What difference is there between the components found in part (a) and the horizontal and vertical components of a velocity of the same magnitude at 27° below the horizontal, as shown in the diagram?

1. (a)

   \[AB = 5 \cos 27°\]
   
   \[\therefore \text{ the horizontal component is } 4.46 \text{ m s}^{-1} \text{ (3 s.f.)}\]
   
   \[BC = 5 \sin 27°\]
   
   \[\therefore \text{ the vertical component is } 2.27 \text{ m s}^{-1} \text{ (3 s.f.)}\]
(b) Each velocity component has the same magnitude, but is in the opposite direction, from that in part (a).

This can be indicated by a minus sign, i.e.

the horizontal component is \(-4.46\, \text{m s}^{-1}\)

and the vertical component is \(-2.27\, \text{m s}^{-1}\)

2 A force of 98 N is pressing vertically downward on an inclined plane. If the angle of inclination of the plane is 40° find the components of the force parallel to, and perpendicular to, the plane.

The required components, although not horizontal and vertical in this case, are perpendicular and can again be found from a right-angled triangle.

\[
\begin{align*}
QR &= 98 \sin 40° = 62.99… \\
PQ &= 98 \cos 40° = 75.07…
\end{align*}
\]

Therefore, to three significant figures:

the component parallel to the plane is 63.0 N

the component perpendicular to the plane is 75.1 N.