3 Statistics

Probability

When you go into hospital for an operation many factors affect the chances of success. The reliability of the diagnosis, the risks and success rates for various treatment options, the surgeon’s experience, your general state of health, the cleanliness of the hospital,…

You will be expected to weigh up the various options and give your consent for a treatment plan.

What’s the point? Knowing how to estimate and combine probabilities can help you make better informed decisions about risky situations.

Check in

1. Simplify
   a \( \frac{6}{8} \)  
b \( \frac{85}{100} \)  
c \( \frac{18}{27} \)  
d \( \frac{21}{35} \)  
e \( \frac{51}{85} \)

2. Estimate these probabilities
   a obtaining an odd number on the roll of a fair die.
   b that you get heads tossing a biased coin if \( P(\text{tails}) = 0.45 \)
   c that you get a head and tail when you toss two fair coins
   d that two people in a class of 30 share a birthday.

3. Evaluate
   a \( \frac{3}{16} + \frac{4}{16} \)  
b \( \frac{5}{16} + \frac{3}{8} \)  
c \( \frac{3}{7} + \frac{5}{21} \)  
d \( \frac{1}{3} + \frac{1}{5} + \frac{1}{7} \)

4. A bag contains a large number of sweets. There are as many mints as jelly beans. Complete this table.

<table>
<thead>
<tr>
<th>Sweet</th>
<th>Fraction</th>
<th>Percentage</th>
<th>Decimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquorice</td>
<td>25%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chocolate button</td>
<td>( \frac{7}{20} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Toffee</td>
<td></td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>Mint</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jelly bean</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Keywords

- Equally likely outcomes
- Sample space

A restaurant offers a set meal. How many meal combinations could I choose?

Two fair dice are thrown.

- a) Show all possible outcomes in a table.
- b) Calculate the probability of getting a total score of 5.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,1</td>
<td>1,2</td>
<td>1,3</td>
<td>1,4</td>
<td>1,5</td>
</tr>
<tr>
<td>2</td>
<td>2,1</td>
<td>2,2</td>
<td>2,3</td>
<td>2,4</td>
<td>2,5</td>
</tr>
<tr>
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<td>3,1</td>
<td>3,2</td>
<td>3,3</td>
<td>3,4</td>
<td>3,5</td>
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<td>4</td>
<td>4,1</td>
<td>4,2</td>
<td>4,3</td>
<td>4,4</td>
<td>4,5</td>
</tr>
<tr>
<td>5</td>
<td>5,1</td>
<td>5,2</td>
<td>5,3</td>
<td>5,4</td>
<td>5,5</td>
</tr>
<tr>
<td>6</td>
<td>6,1</td>
<td>6,2</td>
<td>6,3</td>
<td>6,4</td>
<td>6,5</td>
</tr>
</tbody>
</table>

Sum 1 2 3 4 5 6

The sum of the two scores is shown in each of the 36 cells. Those with a total of 5 are shaded.

\[ P(\text{Sum} = 5) = \frac{4}{36} = \frac{1}{9} \]

You write \( P(A) \) for the probability that the event \( A \) occurs.

- Probability = \( \frac{\text{favourable outcomes}}{\text{all outcomes}} \)

Exercise 3a

1. Two fair coins are tossed.
   - a) List all possible outcomes.
   - b) Give the probabilities of obtaining
     i) 0 heads ii) 1 head iii) 2 heads.
   - c) How can you check your answers to part b?

2. A fair coin is tossed three times.
   - a) List all possible outcomes.
   - b) In how many of these are
     i) exactly 2 heads seen
     ii) at least 2 heads seen?
   - c) What is the probability of getting exactly 2 heads when 3 fair coins are tossed?

3. Anneka goes on a weekend break taking 1 skirt, 3 pairs of trousers and 4 tops.
   How many different combinations can she wear of a top with either a skirt or trousers?

4. Anil is buying an ice-cream. He has to choose between having it in a cone or a tub, whether to have vanilla or strawberry flavour and whether to have a flake, sprinkles or not to have either.
   Make a list of all the possible combinations he could choose.

5. Two fair dice are thrown.
   - a) Construct a sample space diagram which shows the higher score showing on the two dice.
   - b) What is the probability that the higher score showing is a 4?

The set menu in a restaurant has these options.

How many menu combinations are possible for someone
- a) who has no restrictions on what they will eat
- b) who does not like cheese
- c) who is a vegetarian?
Tree diagrams

- **Construct a simple tree diagram**

**Keywords**

Tree diagram

**A tree diagram** is used to show a sample space.

This representation is useful when you have more than two events or when the outcomes are not equally likely.

**Example**

Jacques wins a competition and gets to pick one of three identical envelopes which contain £100, £200 or £300. Before he opens his chosen envelope he also gets to decide whether to accept a 50:50 ‘double or nothing’ option.

a Show all the possible outcomes in a tree diagram.

b i What is the probability that he wins at least £300?

ii What is the probability that he wins nothing?

- **Exercise 3b**

1. A bag contains one red, one blue and one green ball which are identical except for their colours. A ball is taken out, its colour noted and it is put back in the bag, then a second ball is taken out.

   a. Draw a tree diagram to show all the possibilities for the colours of the two balls.

   b i What is the probability that red is seen at least once?

   ii What is the probability that green is not seen?

2. Repeat question 1 but this time assume that the first ball is not put back in the bag.

3. The journey Ms Atmar takes to school goes through two sets of traffic lights.

   a. Draw a tree diagram to show all the possibilities for having to ‘stop’ or ‘go’ at the two sets of lights.

   She notices that the first set of lights is on red \( \frac{1}{3} \) of the time and the second \( \frac{2}{5} \) of the time.

   b. If she makes 150 journeys to school in a year, add to the label on each branch the number of journeys that satisfy the conditions for that branch.

   c. Using your results from part b calculate the probabilities that she stops at

      i. neither set of lights

      ii. one set of lights

      iii. both sets of lights.

- **Challenge**

In the example, if Jacques could open the envelope before deciding whether to accept the ‘double or nothing’ option, what would you advise him to do?
1 A spinner with 3 equal sections coloured red, green and white is spun twice.
   a List all the possible outcomes.
   b In how many of these do you get a red and a green?
   c In how many of these do you not get a white?

2 Two fair dice are thrown.
   a Construct a sample space diagram which shows the product of the scores
      showing on the two dice.
   b What is the probability that the product is at least 20?

3 A lunch menu includes 3 starters, 4 main courses and 2 desserts.
   How many different menu combinations are there for someone who can eat
   anything on the menu?

4 A bag contains one black, one white and one purple ball which are identical
   except for their colours. A ball is taken out, its colour noted and then
   replaced before a second ball is taken out.
   a Draw a tree diagram to show the possibilities of the colours of the
      two balls.
   b i What is the probability that the two balls are a black and white?
      ii What is the probability that the two balls are the same colour?

5 A white and a black dice are thrown together and the events A to D are defined as
   A the sum of the scores is even
   B the white and the black dice show different scores
   C the total score is less than 3
   D the difference between the scores is not more than 1.
   Explain why these pairs of events are mutually exclusive or not
   i A and B
   ii B and D
   iii B and C

6 The faces of a regular tetrahedron are labelled 1–4 and those of a regular
   octahedron 1–8. They are both rolled and the number on the bottom face is
   counted.
   a List all possible outcomes.
   b Use your list to calculate the probabilities that
      i both show prime numbers
      ii only one shows a prime number.
   c Without looking at your list, what is the probability that neither
      shows a prime number?

7 Jorge is making stakes which should be about 1.3 m long.
   The lengths of a number of stakes he has made are listed below.
   1.27, 1.24, 1.27, 1.31, 1.30, 1.26, 1.25, 1.29, 1.32, 1.26, 1.28
   1.31, 1.25, 1.27, 1.26, 1.35, 1.26, 1.24, 1.27, 1.27, 1.25, 1.27
   a Estimate the probability that one of his stakes is longer than 1.3 m.
   b Explain how a better estimate of this probability could be made.

8 How could you estimate the following probabilities?
   a A vowel chosen at random in French is an `a`.
   b The National Lottery has a single jackpot winner in the next draw.
   c Seeing at least 1 six when 3 dice are thrown together.

9 A trainee in a bank is surprised at how often transactions he sees start
   with the digit 1. He does a quick tally of 100 transactions.
   
<table>
<thead>
<tr>
<th>first digit</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>frequency</td>
<td>33</td>
<td>19</td>
<td>14</td>
<td>10</td>
<td>6</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>
   
   Do you think 1 to 9 are equally likely to occur as the first digit of
   transactions?

10 An otherwise fair die is biased so that 5 and 6 are both three times as
    likely to occur as the digits 1–4.
    a What are the individual probabilities of obtaining the numbers 1–6?
    b Use a calculator to simulate rolling such a die three times. Write
      down the rules which you use and your results.
    c Repeat your simulation nine more times and use the results to
      estimate the average sum of the three scores.
    d How could you improve the accuracy of your estimate?
3 Summary

Assessment criteria
• Know the sum of probabilities of all mutually exclusive outcomes is 1 Level 6
• Systematically record mutually exclusive outcomes for single events and two successive events Level 6

1 Two spinners are numbered 1, 3, 5 and 2, 4, 6.
The pointers are spun at the same time.

a Draw a diagram to show all the possible outcomes.
b What is the probability that the sum of the spinners is 7?

Daisy’s answer

Daisy decides to draw a sample space diagram to show the outcomes.

The 3 outcomes when the sum is 7 are (6, 1), (4, 3) and (2, 5)

Daisy decides to write the probability as a fraction.

The 3 outcomes when the sum is 7 are (6, 1), (4, 3) and (2, 5)

In a bag, there are only red, white and yellow counters.
I am going to take a counter out of the bag at random.

The probability that it will be red is more than \( \frac{1}{4} \).
It is twice as likely to be white as red.

Give an example of how many counters of each colour there could be.
Write numbers in the sentence below.

There could be _____ red, _____ white and _____ yellow counters.

KS3 2008 4–6 Paper 2