Equations and graphs

Wind tunnels are used to investigate the aerodynamics (motion of air) around objects, for example aircraft or cars.

What’s the point? Models of the air movement using algebraic equations help engineers to improve speed and efficiency.

Check in

1. What numbers should be placed in these boxes to make the statements correct?
   a. $12 + □ = 21$
   b. $15 - □ = 9$
   c. $3 \times □ = 21$
   d. $\frac{□}{4} = 5$

2. The same number can be written in these pairs of boxes. What is the number?
   a. $6 \times □ = 42$ and $\frac{42}{6} = □$
   b. $7 \times □ = 56$ and $\frac{56}{7} = □$

3. I think of a number, add 6 to it, and get an answer of 14. What number am I thinking of?

4. a. Write the answer to $7 \times 8$.
   b. Write two division facts which are based on your answer to a.
Solving equations

Use inverse operations with simple equations

Here is a balance containing an unknown weight $x$.

It can be written as an equation. $x + 2 = 8$

To find $x$, you need to get it on its own on one side of the equation.

If you subtract 2 from the left, you must also subtract 2 from the right, to keep the balance.

The solution of the equation is $x = 6$

**Exercise 7a**

1. Find the value of $x$ in each of these balances.

2. Solve these equations by using inverse operations.

3. Solve these equations. They need various different operations.

Find the value of $x$ in each of these balances.

Keywords
- Balance
- Solution
- Equation
- Solve
- Inverse

**Challenge**
Solving some equations involves more than one step.

Example: Solve \(2x + 5 = 17\)

You can think of this equation as a balance.

\[
\begin{align*}
2x + 5 &= 17 \\
\text{Subtract 5 from both sides.} \\
x &= 12 \\
\text{Divide both sides by 2.} \\
x &= 6
\end{align*}
\]

This balance has the unknown in both scale pans.

To find the value of \(x\), subtract 3x from both sides because 3x is smaller than 4x.

The balance shows the equation \(4x + 6 = 3x + 14\).

\[
\begin{align*}
x + 6 &= 14 \\
\text{Subtract 6 from both sides.} \\
x &= 8
\end{align*}
\]

The solution \(x = 8\) can be checked by substitution:

\[
\begin{align*}
4x + 6 &= 3x + 14 \\
4 \times 8 + 6 &= 3 \times 8 + 14 \\
The check works. \\
Both sides are equal.
\end{align*}
\]

Example: Solve the equation \(3z + 4 = 9z - 8\)

Re-write the equation so that the most zs are on the LHS.

\[
\begin{align*}
9z - 8 &= 3z + 4 \\
\text{Subtract 3z from both sides.} \\
6z - 8 &= 4 \\
\text{Add 8 to both sides.} \\
6z &= 12 \\
\text{Divide both sides by 6.} \\
z &= 2
\end{align*}
\]

Example: Exercise 7b

1. Solve these equations. Each of them needs two steps.
   - a \(2x + 4 = 10\)
   - b \(2x + 3 = 13\)
   - c \(3x + 1 = 13\)
   - d \(3x + 5 = 11\)
   - e \(4x + 3 = 19\)
   - f \(10x + 7 = 57\)
   - g \(2x - 4 = 8\)
   - h \(2x - 1 = 8\)
   - i \(3x - 2 = 10\)
   - j \(3x - 5 = 4\)
   - k \(10x - 7 = 33\)
   - l \(2x + 3 = 10\)

2. Find the value of \(x\) in each of these balances.
   - a
   - b
   - c

3. Solve these equations.
   - a \(4x + 5 = 3x + 8\)
   - b \(8x + 4 = 7x + 6\)
   - c \(5x + 3 = 4x + 7\)
   - d \(3x + 1 = 2x + 10\)
   - e \(7x + 3 = 5x + 9\)
   - f \(10x + 4 = 8x + 8\)
   - g \(14x + 6 = 7x + 13\)
   - h \(9x + 4 = x + 24\)
   - i \(3 \frac{1}{2} + 1 = \frac{1}{2} + 7\)
   - j \(6x + 1 = 3x + 13\)
   - k \(12x + 7 = 2x + 27\)
   - l \(x + 7 = 3x + 1\)

4. Solve these equations.
   - a \(4x - 5 = 3x + 1\)
   - b \(6x - 2 = 5x + 4\)
   - c \(7x - 4 = 5x + 2\)
   - d \(9x - 1 = 7x + 7\)
   - e \(8x - 4 = 5x + 8\)
   - f \(6x - 7 = x + 3\)
   - g \(10x - 3 = 7x + 3\)
   - h \(5x - 2 = x + 10\)
   - i \(4 \frac{1}{2} - 8 = \frac{1}{2}\)
   - j \(5x - 3 = x + 5\)
   - k \(3x + 1 = 4x - 6\)
   - l \(6x = 8x - 4\)

5. Solve these equations. They need a mixture of methods.
   - a \(4x + 3 = 2x + 11\)
   - b \(4x - 3 = 2x + 11\)
   - c \(8x + 2 = x + 30\)
   - d \(6x - 7 = x + 3\)
   - e \(2x + 20 = 8x + 2\)
   - f \(3x + 1 = 5x - 13\)
   - g \(6x - 8 = 2x\)
   - h \(7x - 3 = 5x - 3\)
   - i \(2x = 9x - 28\)
   - j \(8x = 5x + 18\)
   - k \(2x = 9 - x\)
   - l \(3x = 21 - 4x\)

6. A joiner has seven boxes of screws and four extra screws. His workmate has five similar boxes and twenty eight extra screws. They have the same total number of screws. If there are \(n\) screws in each box:
   - a Form an equation involving \(n\)
   - b Find the value of \(n\).

   a Think of a number, multiply it by 5 and then subtract 3. If you double the same number and add 15, you get the same answer. Find the number.
   - b This mobile is made from different shapes. It can hang from the ceiling. If the square shape has a mass of 60 grams, find the masses of all the other shapes.
1 Find the value of \( x \) in each of these balances.
   \[ a \quad b \]

2 Solve these equations by using inverse operations.
   a \( x + 3 = 7 \)  b \( x - 3 = 7 \)  c \( 2x = 8 \)  d \( \frac{x}{2} = 5 \)  e \( 4 + x = 6 \)  f \( x - 4 = 1 \)  g \( 3x = 18 \)  h \( \frac{x}{4} = 2 \)

3 Solve these equations. Each of them needs two steps.
   a \( 2x + 4 = 14 \)  b \( 3x + 2 = 23 \)  c \( 2x - 1 = 11 \)  d \( 5x - 6 = 9 \)  e \( \frac{x}{2} + 1 = 6 \)  f \( \frac{x}{3} - 3 = 3 \)

4 Find the value of \( x \) in each of these balances.
   a \( 3x + 1 \)  b \( 2x + 5 \)  c \( x + 13 \)

5 Solve these equations. There are unknowns on both sides.
   a \( 4x + 2 = 3x + 7 \)  b \( 6x + 1 = 5x + 13 \)  c \( 3x + 6 = x + 10 \)  d \( 7x + 2 = 4x + 8 \)  e \( 6x + 9 = x + 24 \)  f \( 7x = 3x + 20 \)

6 Solve these equations. Take care with the negative signs.
   a \( 3x - 1 = 2x + 4 \)  b \( 7x - 2 = 5x + 6 \)  c \( 5x - 5 = 3x - 1 \)  d \( 8x - 11 = 5x - 2 \)  e \( 9x + 8 = 7x + 4 \)  f \( 6x + 14 = 3x + 5 \)

7 Sarah has six packets of Christmas cards and two loose cards. Her sister, Jane, has three similar packets of cards and seventeen loose cards. Each packet has \( x \) cards in it. When the sisters open all their boxes and count their cards, they find that they have the same total. Write an equation and find the value of \( x \).

8 Solve these equations.
   a \( 2(3x + 4) = 20 \)  b \( 3(2x - 1) = 21 \)  c \( 5(x - 2) = 20 \)  d \( 3(4x + 1) = 123 \)  e \( 4(2x + 1) = 6(x + 2) \)  f \( 5(3x + 2) = 4(3x + 4) \)

9 Solve these equations by expanding the brackets and collecting like terms.
   a \( 3(2x + 4) + 2(3x + 1) = 38 \)  b \( 5(2x + 1) + 2(x + 3) = 35 \)  c \( 4(x + 3) + 6(x + 1) = 28 \)  d \( 2(2x + 3) + 4(2x + 1) = 18 \)  e \( 3(5x + 1) + 2(1 - 6x) = 9 \)  f \( 5(x + 5) + 2(2x - 3) = 31 \)

10 Copy this mapping diagram three times.
   Complete your copies using these rules.
   a add five  b subtract two  c double then add five
   For each mapping diagram, copy and complete this table.

<table>
<thead>
<tr>
<th>x</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>y</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

11 Make two copies of this mapping diagram and the table of values.

   Complete your copies using these rules.
   a add five  b subtract two  c double then add five

12 a Copy and complete this table for the equation \( y = x + 5 \).
   Plot points on axes labelled as shown here.
   Draw the graph of the equation \( y = x + 5 \).
   b Repeat for the equations
   i \( y = x + 2 \)  ii \( y = 2x - 1 \)  iii \( x + y = 9 \)

13 Write the equations of these four straight lines A to D.

14 Here are the equations of some straight lines. Which lines are
   i parallel to the \( x \)-axis
   ii parallel to the \( y \)-axis
   iii sloping lines?
   a \( y = 2 \)  b \( x = 4 \)  c \( y = x + 3 \)  d \( y = 2x + 1 \)  e \( x = 5 \)  f \( y = -2 \)
### Assessment criteria
- Plot the graphs of simple linear functions Level 6
- Construct and solve linear equations Level 6
- Construct functions arising from real-life problems and plot their corresponding graphs Level 6

### 1. Solve the equation $5x + 4 = x + 12$
Show your working.

**Ollie’s answer**

Ollie decides to subtract 4 from both sides of the equation.

\[
\begin{align*}
5x + 4 &= x + 12 \\
5x &= x + 8 \\
4x &= 8 \\
x &= 2
\end{align*}
\]

He subtracts $x$ from both sides of the equation.

Ollie checks his answer $5 \times 2 + 4 = 2 + 12$ which is correct.

### 2. Each point on the straight line $x + y = 12$ has an $x$-coordinate and a $y$-coordinate that add together to make 12.
Copy this grid and draw the straight line $x + y = 12$