Complete Mathematics for Cambridge Secondary 1

Deborah Barton

Oxford excellence for Cambridge Secondary 1
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About this book
This book follows the Cambridge Secondary 1 Mathematics curriculum framework for Cambridge International Examinations in preparation for the Cambridge Checkpoint assessments. It has been written by a highly experienced teacher, examiner and author.

This book is part of a series of nine books. There are three student textbooks, each covering stages 7, 8 and 9 and three homework books written to closely match the textbooks, as well as a teacher book for each stage.

The books are carefully balanced between all the content areas in the curriculum framework: number, algebra geometry, measure, data handling and problem solving. Some of the questions in the exercises and the investigations within the book are underpinned by the final framework area: problem solving – providing a structure for the application of mathematical skills.

Features of the book:

- **Objectives** – taken from the Cambridge Secondary 1 curriculum framework.
- **What’s the point?** – providing rationale for inclusion of topics in a real-world setting.
- **Chapter Check in** – to assess whether the student has the required prior knowledge.
- **Notes and worked examples** – in a clear style using accessible English and culturally appropriate material.
- **Exercises** – carefully designed to gradually increase in difficulty, providing plenty of practice and techniques.
- **Considerable variation in question style** – encouraging deeper thinking and learning, including open questions.
- **Comprehensive practice** – plenty of initial practice questions followed by varied questions for stretch, challenge, cross-over between topics and links to the real world with questions set in context.
- **Extension questions** – providing stretch and challenge for students:
  - questions with a box e.g. 1 provide challenge for the average student
  - questions with a filled box e.g. 1 provide extra challenge for more able students.
- **Technology boxes** – direct students to websites for review material, fun games and challenges to enhance learning.
- **Investigation and Game boxes** – providing extra fun, challenge and interest.
- **Full colour with modern artwork** – to engage students and maintain their interest.
- **Consolidation examples and exercises** – providing review material on the chapter.
- **Summary and Check out** – providing a quick review of the chapter’s key points aiding revision and enabling you to assess progress.
- **Review exercises** – provided every six chapters with mixed questions covering all chapters.
- **Bonus chapter** – the work from Chapter 19 is not in the Cambridge Secondary 1 curriculum. It is in the Cambridge IGCSE® curriculum and is included to stretch and challenge more able students.

A note from the author:
If you don’t already love maths as much as I do, I hope that after working through this book you will enjoy it more. Maths is more than just learning concepts and applying them. It isn’t just about right and wrong answers. It is a wonderful subject full of challenges, puzzles and beautiful proofs. Studying maths develops your analysis and problem-solving skills and improves your logical thinking – all important skills in the workplace.

Be a responsible learner – if you don’t understand something, ask or look it up. Be determined and courageous. Keep trying without giving up when things go wrong. No-one needs to be ‘bad at maths’. Anyone can improve with hard work and practice in just the same way sports men and women improve their skills through training. If you are finding work too easy, say. Look for challenges, then maths will never be boring.

Most of all, enjoy the book. Do the ‘training’, enjoy the challenges and have fun!

Deborah Barton

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Fractions and indices

Objectives

- Consolidate writing a fraction in its simplest form by cancelling common factors.
- Add, subtract, multiply and divide fractions, interpreting division as a multiplicative inverse, and cancelling common factors before multiplying or dividing.
- Use the order of operations, including brackets and powers.
- Use positive, negative and zero indices and the index laws for multiplication and division of positive integer powers.

What’s the point?

Fractions are useful in many different areas of life. We can use fractions when decorating. We could mix a tin of white paint with a quarter of a tin of red paint to make pink, or we may use two thirds of a tin of paint to decorate a room. We can use fractions in farming. A farmer could use three quarters of his land for crops and one quarter for animals.

Before you start

You should know ...

1. Fractions can be simplified by dividing.
   For example:
   \[
   \frac{14}{35} = \frac{2}{5} = \frac{\div 7}{\div 7}
   \]

Check in

1. Simplify:
   \[
   a \quad \frac{14}{20} \quad b \quad \frac{16}{40} \quad c \quad \frac{32}{48}
   \]
   \[
   d \quad \frac{15}{75} \quad e \quad \frac{8}{96} \quad f \quad \frac{18}{162}
   \]
1 Fractions and indices

2 How to convert between mixed numbers and improper fractions.
   For example:
   \( \frac{8}{5} \) means \( 8 \div 5 = 1 \) remainder 3, so \( \frac{8}{5} = 1\frac{3}{5} \)
   \( 3\frac{1}{4} = \frac{(3 \times 4) + 1}{4} = \frac{13}{4} \)

3 The meaning of indices.
   For example:
   \( 5 \times 5 = 5^2 \) or five squared = 25
   \( 2 \times 2 \times 2 = 2^3 \) or two cubed = 8
   \( p \times p \times p \times p \times p = p^5 \) using index notation

1.1 Working with fractions

Before you start working with fractions you need to be comfortable cancelling fractions by dividing numerators and denominators by common factors. You also need to be able to convert between mixed numbers and improper fractions. These puzzles will help you practise these skills.

Adding and subtracting fractions

Fractions with common denominators are easily added or subtracted.

For example:
\[
\frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}
\]
\[
\frac{4}{5} - \frac{1}{5} = \frac{3}{5}
\]

Fractions without common denominators need to be changed to equivalent fractions with the same denominators.

EXAMPLE 1

Work out:
\[
a \quad \frac{3}{5} + \frac{1}{4} = \frac{12}{20} + \frac{5}{20} = \frac{17}{20}
\]
\[
b \quad \frac{3}{8} - \frac{2}{7} = \frac{21}{56} - \frac{16}{56} = \frac{5}{56}
\]

Addition and subtraction of mixed numbers is done in a similar manner.

Equivalent fraction puzzles

1 I am equivalent to \( \frac{21}{36} \) and my numerator is a prime number. What am I?
2 I am equivalent to \( \frac{64}{104} \) and my denominator is a prime number. What am I?
3 I am equivalent to \( \frac{45}{105} \) and my numerator and denominator are both prime numbers. What am I?
4 I am equivalent to \( \frac{3}{8} \) and the product of my numerator and denominator is 216. What am I?
5 I am equivalent to \( \frac{12}{7} \) and as an improper fraction the sum of my numerator and denominator is 40. What am I?
6 I am equivalent to \( \frac{60}{204} \) and my denominator is less than 20. What am I?
7 I am equivalent to \( \frac{12}{7} \) and as an improper fraction my numerator is 9 more than my denominator. What am I?
8 I am equivalent to \( \frac{5}{3} \) and my numerator is 72 less than my denominator. What am I?
9 I am equivalent to \( \frac{52}{65} \) and my numerator is smaller than 20 and a multiple of 3. What am I?
10 a When writing a fraction in its simplest form by dividing the numerator and denominator by a common factor, either the numerator or denominator or both will be prime. True or false?
   b What are the first eight prime numbers?
Exercise 1A

1 Work out:
   a \( \frac{1}{4} + \frac{1}{4} \)   
   b \( \frac{2}{5} + \frac{1}{5} \)   
   c \( \frac{5}{8} - \frac{3}{8} \)   
   d \( \frac{3}{4} + \frac{1}{4} \)   
   e \( \frac{4}{9} - \frac{2}{9} \)   
   f \( \frac{5}{9} - \frac{5}{9} \)

2 Calculate:
   a \( \frac{1}{2} + \frac{1}{4} \)   
   b \( \frac{1}{3} + \frac{1}{2} \)   
   c \( \frac{2}{3} + \frac{1}{5} \)   
   d \( \frac{2}{7} + \frac{3}{8} \)   
   e \( \frac{2}{9} + \frac{1}{3} \)   
   f \( \frac{2}{9} + \frac{1}{6} \)

3 Calculate:
   a \( \frac{3}{5} - \frac{1}{4} \)   
   b \( \frac{5}{8} - \frac{1}{4} \)   
   c \( \frac{3}{7} - \frac{2}{9} \)   
   d \( \frac{4}{7} - \frac{3}{8} \)   
   e \( \frac{3}{4} - \frac{1}{2} \)   
   f \( \frac{11}{12} - \frac{2}{3} \)

4 Work out:
   a \( \frac{3}{3} + \frac{2}{4} \)   
   b \( \frac{3}{2} - \frac{1}{4} \)   
   c \( \frac{7}{3} + \frac{1}{4} \)   
   d \( \frac{4}{5} - \frac{2}{7} \)

5 Write down three fractions, with different denominators, that add up to make \( 4\frac{5}{24} \).

6

Aakesh bought \( 3\frac{1}{4} \) kg of oranges. He gave his sister \( 1\frac{2}{3} \) kg.

What was the mass of the oranges he had left?

7 A plank of wood is 3 m in length. How long will it be if I cut \( \frac{5}{8} \) m of wood from it?

8 Karen spent \( \frac{4}{7} \) of her money on Monday.
   She spent \( \frac{1}{5} \) of her money on Tuesday. What fraction of money did she spend on Monday and Tuesday?
   a spend on Monday and Tuesday?
   b have left?

9 A container holds \( 4\frac{1}{2} \) litres of water.
   How much water is left in the container if Jerry drinks \( 2\frac{7}{8} \) litres?

10 Fill in the missing numbers:
   a \( \square + 3\frac{1}{3} - 2\frac{1}{3} = 5\frac{4}{13} \)
   b \( 4\frac{1}{2} - \square + 3\frac{1}{10} = 2\frac{9}{10} \)

11 Find the value of the letters in the following:
   a \( \frac{a}{5} + \frac{2}{10} = 4\frac{1}{10} \)
   b \( \frac{1}{3} + b^2_6 = 4\frac{1}{6} \)
   c \( 6\frac{c}{10} + 1\frac{3}{5} = 8\frac{1}{2} \)
   d \( 3\frac{7}{9} + \frac{7}{a} = 4\frac{1}{6} \)

12 Put these fractions into groups of three so that each group has a total of \( 4\frac{1}{2} \).
   \( \frac{2}{3}, \frac{1}{5}, \frac{2}{5} \quad \frac{2}{4}, \frac{2}{12}, \frac{7}{10} \quad \frac{1}{3}, \frac{1}{12}, \frac{11}{24} \quad \frac{1}{2}, \frac{3}{5} \)
A unit fraction is a fraction with a numerator of 1, for example \( \frac{1}{2} \), \( \frac{1}{3} \), or \( \frac{1}{7} \). The ancient Egyptians didn’t write fractions with a numerator greater than 1. Instead they would write them as a sum of two or more different unit fractions. For example they wouldn’t write \( \frac{2}{3} \); instead they would write \( \frac{2}{3} \) as \( \frac{1}{2} + \frac{1}{6} \). (They wouldn’t use \( \frac{1}{3} + \frac{1}{3} \) as this involves repeating a unit fraction.)

Investigate different non-unit fractions. Can all non-unit fractions be written as a sum of different unit fractions?

**Multiplying and dividing fractions**

To multiply fractions you just need to multiply the numerators and the denominators. For example:

\[
\frac{2}{3} \times \frac{2}{5} = \frac{2 \times 2}{3 \times 5} = \frac{4}{15}
\]

\[
1\frac{1}{2} \times \frac{3}{4} = \frac{3}{2} \times \frac{3}{4} = \frac{3 \times 3}{2 \times 4} = \frac{9}{8} = 1\frac{1}{8}
\]

You know that \( 3^2 \) means \( 3 \times 3 \); so we square fractions in the same way. Often you can simplify the fractions, by cancelling common factors, before doing the multiplication. For example:

\[
\frac{2}{21} \times \frac{3^3}{8} = \frac{2^4}{21} \times \frac{3^1}{8_4} = \frac{1 \times 1}{1 \times 4} = \frac{1}{4}
\]

\[
4\frac{2}{3} \times \frac{7}{33} = \frac{22}{5} \times \frac{7}{33} = \frac{22}{5} \times \frac{7}{33} = \frac{2 \times 7}{5 \times 3} = \frac{14}{15}
\]

You can also use the idea of cancelling when you are multiplying more than two fractions together.

**EXAMPLE 3**

Calculate:

a \( 1\frac{1}{15} \times \frac{3}{8} \times 10 \)

b \( \left( \frac{4}{5} \right)^2 \)

\[a \quad 1\frac{1}{15} \times \frac{3}{8} \times 10 = \frac{16}{15} \times \frac{3}{8} \times \frac{10}{1} \]

\[b \quad \left( \frac{4}{5} \right)^2 = \frac{16}{25} \times \frac{16}{25} \]

Division of fractions is a little more tricky. Remember:

\[
9 \div 3 = 9 \times \frac{1}{3} = 3
\]

\[
8 \div 2 = 8 \times \frac{1}{2} = 4
\]

Notice that the operation

\[
\div 3 \text{ is the same as } \times \frac{1}{3}
\]

\[
\div 2 \text{ is the same as } \times \frac{1}{2}
\]
In general, division by a number is the same as multiplying by the number’s inverse or reciprocal.

Under multiplication, the reciprocal of

\[ \frac{3}{3} = \frac{1}{3} \quad \left( 3 \times \frac{1}{3} = 1 \right) \]
\[ \frac{1}{2} = 2 \quad \left( \frac{1}{2} \times 2 = 1 \right) \]
\[ \frac{3}{4} = \frac{4}{3} \quad \left( \frac{3}{4} \times \frac{4}{3} = 1 \right) \]

Using this idea, all divisions of fractions can be turned into multiplications.

**EXAMPLE 4**

Calculate:

a. \[ \frac{3}{5} + \frac{2}{3} \]

\[ = \frac{3}{5} \times \frac{2}{3} = \frac{6}{15} = \frac{2}{5} \]

b. \[ 2\frac{1}{4} + 1\frac{3}{8} \]

\[ = \frac{9}{16} + \frac{33}{20} \]
\[ = \frac{9}{16} \times \frac{33}{20} = \frac{3}{4} \times \frac{33}{11} = \frac{15}{44} \]

The order of operations (BIDMAS) also applies to fractions. Remember in calculations we do:

**Brackets first** (note long dividing lines act like brackets)
**Then Indices (or powers)**
**Then Division and Multiplication**
**Then Addition and Subtraction**

**EXAMPLE 5**

Calculate:

\[ \left( \frac{3}{4} \right)^2 \]
\[ = \frac{3^2}{4^2} = \frac{9}{16} \]

\[ \frac{3}{3} - \frac{1}{4} \]

\[ = \frac{2}{3} - \frac{1}{4} \]
\[ = \frac{8}{12} - \frac{3}{12} = \frac{5}{12} \]

\[ \frac{3}{3} - \frac{1}{4} \]
\[ = \frac{2}{3} - \frac{1}{4} \]
\[ = \frac{8}{12} - \frac{3}{12} = \frac{5}{12} \]

**Exercise 1B**

1. Work out:

a. \[ 3 \times \frac{1}{2} \]

b. \[ \frac{1}{2} \times \frac{1}{2} \]

c. \[ \frac{3}{4} \times \frac{1}{2} \]

d. \[ \frac{3}{4} \times \frac{2}{3} \]

e. \[ \frac{7}{8} \times \frac{2}{5} \]

f. \[ \frac{3}{4} \times \frac{4}{7} \]

g. \[ \frac{6}{11} \times \frac{4}{9} \]

h. \[ \frac{5}{8} \times \frac{4}{5} \]

2. Calculate:

a. \[ 1\frac{1}{2} \times 3 \]

b. \[ 1\frac{1}{2} \times 1\frac{1}{2} \]

c. \[ 2\frac{1}{4} \times 1\frac{1}{4} \]

d. \[ 3\frac{3}{2} \times 2\frac{1}{2} \]

e. \[ 2\frac{1}{2} \times 2\frac{1}{2} \]

f. \[ 4\frac{1}{4} \times 1\frac{7}{8} \]

g. \[ 3\frac{1}{3} \times 4\frac{3}{4} \]

h. \[ 5\frac{2}{5} \times 3\frac{7}{11} \]
3 Calculate:
\[
\begin{align*}
    \text{a} & \quad \frac{1}{2} + \frac{1}{4} & \quad \text{b} & \quad \frac{1}{2} + \frac{1}{3} \\
    \text{c} & \quad \frac{3}{4} + \frac{1}{4} & \quad \text{d} & \quad \frac{7}{8} + \frac{3}{4} \\
    \text{e} & \quad \frac{7}{8} + \frac{2}{5} & \quad \text{f} & \quad \frac{3}{4} + \frac{4}{5} \\
    \text{g} & \quad \frac{6}{11} + \frac{4}{9} & \quad \text{h} & \quad \frac{3}{14} + \frac{4}{7}
\end{align*}
\]

4 Work out:
\[
\begin{align*}
    \text{a} & \quad 1\frac{1}{4} + 2\frac{1}{2} & \quad \text{b} & \quad 2\frac{1}{2} + 1\frac{1}{4} \\
    \text{c} & \quad 3\frac{1}{2} + 1\frac{1}{3} & \quad \text{d} & \quad 2\frac{2}{3} + 1\frac{3}{4} \\
    \text{e} & \quad 4\frac{2}{3} + 1\frac{3}{5} & \quad \text{f} & \quad 4\frac{3}{7} + 1\frac{6}{7} \\
    \text{g} & \quad 5\frac{4}{5} + 2\frac{7}{9} & \quad \text{h} & \quad 3\frac{4}{5} + 6\frac{2}{5}
\end{align*}
\]

5 Work out:
\[
\begin{align*}
    \text{a} & \quad \frac{7}{10} \text{ of } 8\frac{3}{5} \text{ m} & \quad \text{b} & \quad \frac{3}{5} \text{ of } 40\frac{1}{2} \text{ kg} \\
    \text{c} & \quad \frac{4}{5} \text{ of } 30\frac{2}{3} \text{ ml} & \quad \text{d} & \quad \frac{2}{3} \text{ of } 3\frac{1}{4} \text{ tonnes} \\
    \text{e} & \quad \frac{14}{15} \times \frac{3}{8} \times \frac{5}{7} & \quad \text{f} & \quad \frac{7}{8} \times \frac{4}{21} \times \frac{4}{5} \\
    \text{g} & \quad \left(\frac{7}{8}\right)^2 & \quad \text{h} & \quad 1\frac{1}{5} \times \left(\frac{2}{3}\right)^2 \\
    \text{i} & \quad 1\frac{1}{5} \times \frac{2^2}{3} & \quad \text{j} & \quad 2\frac{1}{5} \times 1\frac{1}{5} \times \frac{1}{3}
\end{align*}
\]

6 Work out:
\[
\begin{align*}
    \text{a} & \quad (1\frac{1}{2})^2 & \quad \text{b} & \quad 2\frac{1}{2} \times 1\frac{1}{2} \times \frac{1}{3} \\
    \text{c} & \quad 6 \times \left(1\frac{2}{3} + 1\frac{1}{2}\right) & \quad \text{d} & \quad \left(\frac{2^5}{6} + 3\frac{1}{4}\right) \div 4 \\
    \text{e} & \quad 3\frac{3}{4} - 1\frac{7}{8} + \frac{1}{2} & \quad \text{f} & \quad \frac{5}{8} - \frac{7}{9} + \frac{1}{3} \\
    \text{g} & \quad 1\frac{4}{5} + 2\frac{3}{4} \times 1\frac{1}{3} & \quad \text{h} & \quad 1\frac{1}{2} - 4\frac{1}{8} + 2\frac{3}{4} \\
    \text{i} & \quad 1\frac{1}{7} \times \left(2\frac{3}{4} + 1\frac{1}{3}\right) & \quad \text{j} & \quad \frac{2}{7} \times \left(1\frac{2}{3} + 1\frac{3}{5}\right) + \frac{1}{10}
\end{align*}
\]

7 Calculate:
\[
\begin{align*}
    \text{a} & \quad \frac{2}{7}\times \frac{17}{15} - 1\frac{1}{2} & \quad \text{b} & \quad \frac{3}{8} \text{ of } \left(1\frac{1}{2}\right)^2 \\
    \text{c} & \quad \frac{3}{8} \times \frac{4}{5} + 1\frac{1}{2} & \quad \text{d} & \quad 2\frac{2}{3} \times 2\frac{2}{5} \times 1\frac{1}{2} \\
    \text{e} & \quad \frac{3\frac{1}{2}}{1\frac{2}{5} + 1\frac{3}{4}} & \quad \text{f} & \quad \frac{2\frac{1}{2} \times 7\frac{1}{8}}{6 - \frac{3}{4}} \\
    \text{g} & \quad \frac{3\frac{7}{8} - 1\frac{5}{6}}{2\frac{3}{4} - 1\frac{2}{5}} & \quad \text{h} & \quad \frac{6\frac{2}{5} - 4\frac{3}{5}}{3\frac{1}{2} + 1\frac{3}{15}} \\
    \text{i} & \quad \frac{1\frac{7}{8} \times \frac{4}{5} + 3\frac{1}{2} \times 1\frac{3}{2} - 7\frac{1}{2}} & \quad \text{j} & \quad \left(1\frac{3}{4} + 2\frac{1}{5} \times \frac{5}{11} - 3 + 1\frac{1}{4}\right) \\
    \text{k} & \quad \left(1\frac{3}{5} - 1\frac{20}{29}\right) \times \left(1\frac{2}{3} + 1\frac{4}{5} - 2\frac{8}{15}\right)
\end{align*}
\]

8 A packet of biscuits has a mass of $1\frac{1}{3}$ kg. Jason eats $\frac{2}{7}$ of the packet. What is the mass of biscuits left?

9 A rectangle measures $4\frac{1}{7}$ cm by $2\frac{4}{5}$ cm. What is the area of the rectangle?

10 The area of a rectangular field is $130\frac{3}{5}$ m$^2$. What is the field’s length if its width is $10\frac{1}{8}$ m?
\[ \frac{3}{4} \text{-litre bottles of washing-up liquid are taken from a container holding } 164 \frac{1}{2} \text{ litres.} \]

a) How many bottles can be filled from the container?

b) When 75 bottles have been removed from the container, what fraction of the original amount of liquid remains in the container?

Work out:

a) \( -1 \frac{3}{8} \div \frac{2}{5} \)

b) \( -3 \frac{3}{5} \times \frac{5}{18} \)

c) \( -1 \frac{4}{4} \div 8 \)

d) \( \left( -1 \frac{4}{5} \right)^2 \)

What do the letters stand for in the following?

a) \( a \times 1 \frac{1}{4} = 3 \frac{1}{4} \)

b) \( b \div 3 \frac{1}{2} = \frac{9}{28} \)

c) \( 2 \frac{1}{3} \times 3 \frac{1}{4} = 7 \frac{7}{12} \)

d) \( 2 \frac{3}{7} \div d = \frac{3}{7} \)

e) \( (1 \frac{3}{7})^2 \times e = 1 \)

f) \( f^2 = 2 \frac{14}{25} \)

Fill in the missing numbers:

a) \( 1 \frac{1}{4} \times 40 = \square \div \frac{2}{5} \)

b) \( 20 \div 1 \frac{7}{8} = 5 \frac{1}{3} \times \square \)

c) \( \square \div \frac{1}{2} = \frac{1}{5} \times \square \)

d) There are many possible answers to part c. Find some more answer pairs and write down the connection between them.

INVESTIGATION

a) Find as many pairs of fractions as you can with a product of \( \frac{7}{9} \).

b) Find as many pairs of fractions as you can that when divided give \( \frac{1}{2} \).
1.2 Indices

A short way of writing
3 \times 3 \times 3 \times 3 = 3^4
3^4 is 3 raised to the power 4.

In the same way,
4^3 is 4 raised to the power 3.

4^3 = 4 \times 4 \times 4 = 64

The power is also called the index.

Notice that
4^3 \times 4^2 = 4 \times 4 \times 4 \times 4 \times 4 = 4^5
That is, \(4^3 \times 4^2 = 4^{3+2} = 4^5\)

- So long as the two numbers are powers of the same number you can multiply them by adding their indices. Using symbols,

\[ a^m \times a^n = a^{m+n} \]

**EXAMPLE 6**

Work out:

- a \(2^3 \times 2^4\)
- b \(2^2 \times 2^5 \times 2^7\)
- c \(7 \times 7^4\)

\[ a = 2^3 \times 2^4 = 2^{3+4} = 2^7 \]
\[ b = 2^2 \times 2^5 \times 2^7 = 2^{2+5+7} = 2^{14} \]
\[ c = 7 \times 7^4 = 7^5 \]

Notice also that

\[ 4^3 \div 4^2 = \frac{4 \times 4 \times 4}{4 \times 4} = 4^1 \]

That is \(4^3 \div 4^2 = 4^{3-2} = 4^1\)

- When two numbers are powers of the same number, you can divide them by subtracting their indices. Using symbols,

\[ a^m \div a^n = a^{m-n} \]

**EXAMPLE 7**

Work out:

- a \(5^7 \div 5^3\)
- b \(8^{12} \div 8^{10}\)

\[ a = 5^7 \div 5^3 = 5^{7-3} = 5^4 \]
\[ b = 8^{12} \div 8^{10} = 8^{12-10} = 8^2 \]

**Exercise 1C**

1 Copy and complete:
- a \(3^4 = 3 \times 3 \times \ldots\) = 81
- b \(5^4 = 5 \times 5 \times \ldots\) = □
- c \(2^7 = 2 \times 2 \times \ldots\) = □
- d \(25 = 5^2 = 5 \times \ldots\)
- e \(49 = 7^2 = 7 \times \ldots\)
- f \(8 = 2^3 = 2 \times \ldots\)
- g \(81 = 3^4 = 3 \times \ldots\)
- h \(10000 = 10 \times 10 \times \ldots = 10^\square\)
- i \(1000000 = 10 \times 10 \times \ldots = 10^\square\)

2 Copy and complete:
- a \(3^2 \times 3^4 = 3 \times 3 \times \ldots = 3^\square\)
- b \(2^3 \times 2^5 = 2 \times 2 \times \ldots = 2^\square\)
- c \(7^5 \times 7 = 7 \times 7 \times \ldots = 7^\square\)
- d \(5^6 \times 5^4 = 5 \times 5 \times \ldots = 5^\square\)

3 Simplify:
- a \(6^2 \times 6^3 	imes 6^5\)
- b \(7 \times 7^{10} \times 7^{12}\)
- c \(3^2 \times 3^{10} \times 3^5\)
- d \(10 \times 10 \times 10^{5}\)

4 Simplify these, if possible, leaving the answer in index form. If it is not possible, explain why.
- a \(2^6 \times 2^7\)
- b \(2^3 \times 3^2\)
- c \(3^2 \times 4^2 \times 5^2\)
- d \(4^3 \times 4^2 \times 4\)

5 Copy and complete:
- a \(\frac{3^4}{3^2} = \frac{3 \times 3 \times \ldots}{3 \times \ldots} = 3^\square\)
- b \(\frac{5^7}{5^3} = \frac{5 \times 5 \times \ldots}{5 \times \ldots} = 5^\square\)
- c \(\frac{7^{10}}{7^4} = \frac{7 \times 7 \times \ldots}{7 \times \ldots} = 7^\square\)

6 Simplify:
- a \(\frac{2^6}{2^3}\)
- b \(\frac{3^7}{3^4}\)
- c \(\frac{4^8}{4^3}\)
- d \(\frac{7^{10}}{7^5}\)
- e \(\frac{9^7}{9}\)
- f \(\frac{5^{12}}{5^8}\)

7 Simplify, leaving your answer in index form.
- a \(6^3 \div 6^2\)
- b \(5^7 \div 5^4\)
- c \(12^6 \div 12^3\)
- d \(7^5 \div 7\)
- e \(20^9 \div 20^4\)
- f \(q^2 \div q\)
- g \(b^6 \div b^3\)
- h \(y^4 \div y^m\)
- i \(8p^7 \div 2p^3\)
- j \(6x^6 \div 2x^2\)
- k \(10m^{10} \div 2m^3\)
Zero and negative indices

What does $5^0$ mean?

$5^2 \div 5^2 = 25 \div 25 = 1$, but

$5^2 \div 5^2 = 5^{2-2} = 5^0$

So $5^0 = 1$

Using symbols,

$a^0 = 1$

Indices can also be negative.

For example,

$6^3 \div 6^5 = \frac{6 \times 6 \times 6}{6 \times 6 \times 6 \times 6 \times 6} = \frac{1}{6 \times 6} = \frac{1}{6^2}$

but $6^3 \div 6^5 = 6^{3-5} = 6^{-2}$

so $6^{-2} = \frac{1}{6^2}$

Using symbols,

$a^{-n} = \frac{1}{a^n}$

Using the order of operations, including brackets and powers

The order of operations (BIDMAS) also applies to indices. Remember, in calculations we do:

Bracket first (note long dividing lines act like brackets)
Then Indices (or powers)
Then Division and Multiplication
Then Addition and Subtraction

**Example 8**

Simplify: $16 \times 2^{-3}$

$16 \times 2^{-3} = 16 \times \frac{1}{2^3} = 16 \times \frac{1}{2 \times 2 \times 2}$

$= 16 \times \frac{1}{8} = 2$

**Example 9**

Simplify: $\frac{(3^6 \times 3^4)^2}{3^2 \times 3^3}$

This long dividing line acts like brackets. Work out the numerator and denominator first.

Brackets on the numerator are worked out first.

Then the power of 2:

$(3^6)^2 = 3^{6 \times 2} = 3^{12}$

Then the division

$= \frac{3^{20}}{3^5}$

$= 3^{20-5} = 3^{15}$

**Exercise 1D**

1. Copy and complete:
   a. $2^3 \div 2^1 = 2^\square = 1$
   b. $3^5 \div 3^5 = 3^\square = 1$
   c. $7^2 \div 7^2 = 7^\square = 1$
   d. $9^{10} \div 9^{10} = 9^\square = 1$

2. Write down the value of:
   a. $5^0$
   b. $7^0$
   c. $(\text{-}2)^0$
   d. $8^0$
   e. $\left(\frac{1}{2}\right)^0$
   f. $m^0$

3. Copy and complete:
   a. $4^5 \div 4^6 = \frac{4 \times \ldots}{4 \times \ldots} = 1$
   b. $4^5 \div 4^6 = 4 \cdot \ldots = 4^{1}$
   c. $\frac{1}{\square} = 4^{-1}$
Exercise 1E

1. A piece of ribbon is 50 cm long. How many $10\frac{1}{2}$ cm lengths of ribbon can be cut from it? How much ribbon is left over?

2. I walk $2\frac{1}{3}$ km to school, then $2\frac{1}{2}$ km to the store, then $1\frac{1}{3}$ km back home. How far do I walk altogether?

3. On each bounce a ball rises to $\frac{2}{3}$ of its height at the start of the bounce. To what height will it rise after the fourth bounce, if it was originally dropped from a height of 81 cm?

4. A bottle of juice holds $2\frac{1}{4}$ litres. How many glasses of juice can be poured from the bottle if each glass holds $\frac{3}{8}$ litre?

5. A $\frac{1}{2}$-litre jug is filled with milk. It is used to fill two cups, one holding $\frac{1}{6}$ litre the other $\frac{2}{7}$ litre. How much milk remains in the jug?

6. Calculate the exact value of $\frac{3\frac{2}{3} - \frac{1}{7}}{\frac{2}{7}}$.

7. a. How many quarters in 16?
   b. How many thirds in 3?
   c. What is a tenth of 10?
   d. What is three tenths of 3?

8. a. Find a tenth of three quarters of $800$.
   b. Find three quarters of a tenth of $800$.

9. Leon has made a mistake in his homework. The question is: $2\frac{7}{35} \times 35$
   Leon has written:
   
   $$2\frac{7}{35} \times \frac{35}{1} = \frac{73}{5} \times 7$$
   $$= \frac{511}{5} \times \frac{7}{1}$$
   $$= \frac{1777}{5}$$
   $$= 355\frac{2}{5}$$

   What mistake has he made?
Consolidation

Example 1
Work out:
\( a \quad \frac{3}{4} + \frac{2}{5} \quad b \quad 4\frac{1}{2} \times 2\frac{2}{5} \quad c \quad 2\frac{1}{2} + 1\frac{3}{4} \)

\( a \quad \frac{3}{4} + \frac{2}{5} \)
\[ = 5 + \frac{1}{4} + \frac{2}{5} \]
\[ = 5 + \frac{5}{20} + \frac{8}{20} \]
\[ = 5 + \frac{13}{20} = 5\frac{13}{20} \]
\[ = 3 \times 4 = \frac{1}{1} = 12 \]

\( b \quad 4\frac{1}{2} \times 2\frac{2}{3} \)
\[ = \frac{9}{2} \times \frac{8}{3} \]
\[ = \frac{3}{2} \times \frac{4}{1} \]
\[ = 3 \]

\( c \quad 2\frac{1}{2} + 1\frac{3}{4} \)
\[ = \frac{5}{2} + \frac{7}{4} \]
\[ = \frac{5}{2} \times \frac{4}{7} \]
\[ = \frac{20}{14} = \frac{16}{14} = 1\frac{3}{7} \]

Example 2
Work out \( \frac{(1\frac{1}{2})^2}{2\frac{2}{5} + 1\frac{1}{3}} \)

\( \frac{(1\frac{1}{2})^2}{2\frac{2}{5} + 1\frac{1}{3}} = \frac{(\frac{3}{2})^2}{\frac{12}{5} + \frac{4}{3}} \)
\[ = \frac{\frac{16}{9}}{\frac{36}{15} + \frac{20}{15}} \]
\[ = \frac{16}{9} \div \frac{56}{15} \]
\[ = \frac{16}{9} \times \frac{15}{56} \]
\[ = \frac{26}{9} \times \frac{5}{56} = \frac{2}{3} \times \frac{5}{7} = \frac{10}{21} \]

Example 3
Simplify:
\( a \quad 2^3 \times 2^3 \quad b \quad \frac{4^3 \times 4^4}{4^7} \quad c \quad \frac{(2^3 \times 2^4)^2}{2^{16}} \)

\( a \quad 2^7 \times 2^3 = 2^{10} \)
\[ b \quad \frac{4^3 \times 4^4}{4^7} = \frac{4^7}{4^7} = 4^{7-7} = 4^0 = 1 \]

\( c \quad \frac{(2^3 \times 2^4)^2}{2^{16}} = \frac{(2^7)^2}{2^{16}} = \frac{2^7 \times 2^7}{2^{16}} = \frac{2^{14}}{2^{16}} = 2^{14-16} = 2^2 = \frac{1}{2^2} = \frac{1}{4} \)

Exercise 1

1 Work out:
\( a \quad \frac{1}{2} + \frac{2}{3} \quad b \quad \frac{3}{8} + \frac{4}{7} \quad c \quad 2\frac{3}{5} + 1\frac{2}{7} \quad d \quad \frac{6}{7} - \frac{2}{3} \quad e \quad 6\frac{1}{2} - 2\frac{5}{8} \)

2 Work out:
\( a \quad \frac{3}{5} \times \frac{2}{3} \quad b \quad 2\frac{1}{2} \times 3\frac{1}{3} \quad c \quad \frac{6}{7} \div \frac{2}{3} \quad d \quad \frac{4}{5} \times 1\frac{2}{3} \quad e \quad 4\frac{3}{4} \div 2\frac{2}{5} \)

3 Work out:
\( a \quad 1\frac{3}{4} + 1\frac{3}{2} \times 2\frac{1}{3} \quad b \quad \frac{2\frac{3}{4}}{2\frac{1}{4} + 1\frac{1}{3}} \quad c \quad 1\frac{1}{3} \times \left(4\frac{1}{4} - 3\frac{3}{8}\right) \div \frac{5}{6} \quad d \quad \frac{1\frac{1}{3} + \frac{2}{7}}{1\frac{1}{2} \times 2\frac{5}{6}} \quad e \quad \frac{(1\frac{1}{2})^2}{2\frac{2}{5} - 1\frac{1}{4}} \)

4 Simplify:
\( a \quad 2^4 \times 2^2 \quad b \quad 5^8 \div 5^2 \quad c \quad 4^8 \div 4^2 \quad d \quad 4^7 \times 4^3 \times 4^2 \quad e \quad 4^4 \times 4^3 \div 4^{10} \quad f \quad \frac{9^{20}}{(9 \times 9^9)^2} \quad g \quad \frac{(6^2 \times 6^3)^2}{6^9} \quad h \quad \frac{(8^4 \times 8^5)^2}{(8^{20} \div 8^2)} \)

5 A piece of cloth is 4\frac{1}{2} m long. Amandeep used \frac{2}{5} of it. What length did he use?

6 It takes 3\frac{3}{4} hours to plaster a wall. One third of this time is spent mixing the plaster. How long does the actual plastering take?

7 Janice bought 6\frac{1}{2} metres of cloth. She used 1\frac{1}{4} metres. What fraction did she use?
1 Fractions and indices

You should know ...

1 How to add and subtract fractions.
   For example:
   \[
   \frac{3}{4} + \frac{1}{3} = \frac{9}{12} + \frac{4}{12} = \frac{13}{12} = 1\frac{1}{12}
   \]

2 How to multiply and divide fractions.
   For example:
   \[
   \frac{3}{5} \times \frac{10}{12} = \frac{3}{5} \times \frac{10}{12} = \frac{2}{4} = \frac{1}{2}
   \]
   \[
   3^\frac{3}{4} \div \frac{5}{8} = \frac{15}{4} \div \frac{5}{8} = \frac{15}{4} \times \frac{8}{5} = \frac{3}{1} \times \frac{2}{1} = 6
   \]

3 How to multiply and divide with indices, and about the zero index.
   For example:
   \[
   3^2 \times 3^4 = 3^{2+4} = 3^6
   \]
   \[
   6^5 \div 6^2 = 6^{5-2} = 6^3
   \]
   \[
   4^0 = 1
   \]

8 7\frac{1}{2} kg of coffee are put into 1\frac{1}{4} kg bags. How many bags will be needed?

9 A tin holds 5\frac{1}{2} litres of oil. How many \frac{2}{3}-litre cans can be filled from it?

10 A piece of wood is 4\frac{1}{2} metres long. How many 1\frac{1}{2}-metre strips can be cut from it?

11 A man’s work day is 8\frac{1}{2} hours. He spends 3\frac{1}{2} hours dealing with customers and the remaining time working in his office.
   a What fraction of his work day does he spend with customers?
   b What fraction of the whole day is he in his office?
   (Hint: think about how many hours there are in a whole day.)

12 A man spends \frac{2}{7} of his weekly wage on food and \frac{2}{3} on rent. If he spends $200 on food, how much does he spend on rent?

13 What are the numbers \(p\), \(q\), \(r\) and \(s\) if
   a \(2^p = 4^q = 16^2 = 256\)
   b \(3^r = 9^s = 81^2 = 6561\)

14 Yafeu says that if he starts with a number and squares it he ends up with the same number he started with. What could his number be? Is there more than one answer?

### Summary

#### Check out

1 Work out:
   a \(\frac{1}{2} + \frac{1}{4}\)  b \(\frac{3}{5} + \frac{1}{3}\)
   c \(\frac{2}{3} - \frac{1}{4}\)  d \(\frac{7}{8} - \frac{3}{4}\)
   e \(\frac{4}{5} + \frac{3}{4}\)  f \(2\frac{2}{3} - 1\frac{3}{5}\)

2 Work out:
   a \(\frac{1}{2} \times \frac{3}{4}\)  b \(\frac{3}{4} \div \frac{1}{2}\)
   c \(\frac{3}{5} \times \frac{2}{3}\)  d \(\frac{1}{2} \div \frac{1}{4}\)
   e \(2\frac{1}{2} \times 2\frac{1}{2}\)  f \(2\frac{3}{4} + 1\frac{1}{4}\)
   g \(\left(\frac{2}{3}\right)^2\)
   h \(\frac{3}{4} \times \left(\frac{1}{8} + \frac{1}{2}\right)\)

3 Work these out. Write your answers in index form.
   a \(2^2 \times 2^3\)  b \(5^4 \times 5^7\)
   c \(7^8 \times 7^3\)  d \(9^5 \times 9^2\)
   e \(4^5 \div 4^2\)  f \(3^3 \div 3^3\)
   g \(2^8 + 2^4\)  h \(7^0\)
Expressions and formulae

Objectives

- Know the origins of the word algebra and its links to the work of the Arab mathematician Al’Khwarizmi.
- Simplify or transform algebraic expressions by taking out single-term common factors.
- Use index notation for positive integer powers; apply the index laws for multiplication and division to simple algebraic expressions.
- Construct algebraic expressions.
- Add and subtract simple algebraic fractions.
- Expand the product of two linear expressions of the form $x \pm n$ and simplify the corresponding quadratic expression.
- Substitute positive and negative numbers into expressions and formulae.
- Derive formulae and, in simple cases, change the subject; use formulae from mathematics and other subjects.

What’s the point?

Relationships between two variables are commonly expressed in algebraic form, as equations. For example, the equation $D = v \times t$ relates distance travelled, $D$, by a car to its average speed, $v$, and the time taken, $t$.

Before you start

You should know ...

1 How to add and subtract negative numbers.
   
   For example:
   
   $8 + -3 = 8 - 3 = 5$
   $8 - -3 = 8 + 3 = 11$

Check in

1 Work out:
   
   a $6 + -2$
   b $-2 - 3$
   c $-3 - -6$
   d $-6 + 5$
   e $-3 + -6 - -2$
2 Expressions and formulae

2 How to multiply and divide negative numbers.

For example:
\[
\begin{align*}
6 \times -3 &= -18 \\
-6 \times -3 &= 18 \\
6 \div -3 &= -2 \\
-6 \div -3 &= 2
\end{align*}
\]

3 How to substitute numbers for letters.

For example:
if \( x = 10 \) and \( y = 6 \)
then \( 2x + 3y = 2 \times 10 + 3 \times 6 = 20 + 18 = 38 \)

4 How to work with indices.

For example:
\[
\begin{align*}
7^2 \times 7^3 &= 7^2 + 3 = 7^5 \\
8^5 \div 8^2 &= 8^{5-2} = 8^3
\end{align*}
\]

5 The lowest common multiple (LCM) is the smallest multiple common to two or more numbers.

For example:
The LCM of 3 and 4 is 12.

2.1 The origins of algebra and algebraic expressions

Muhammad ibn Musa Al’Khwarizmi was a mathematician who lived more than a thousand years ago. He worked in Baghdad at the ‘House of Wisdom’, where philosophical articles were written and translated. He worked with Hindu-Arabic numbers and was among the first to use zero as a placeholder. He wrote the first book about algebra, called Al-Kitab al-mukhtasar fi hisab al-jabr wa-l-muqabala or ‘The Compendious Book on Calculation by Completion and Balancing’. The word ‘algebra’ comes from the al-jabr of the book’s title, and Al’Khwarizmi is known as the ‘father of algebra’.

Algebra involves using letters to stand for numbers, allowing us to manipulate letters in a similar way to numbers.

\[
\begin{align*}
3 + 3 + 3 + 3 &= 4 \times 3 = 12 \\
a + a + a + a &= 4 \times a = 4a \\
3 \times 3 \times 3 \times 3 &= 3^4 = 81 \\
a \times a \times a \times a &= a^4
\end{align*}
\]

4a and \( a^4 \) are algebraic expressions. This just means they contain letters to stand for numbers.

EXAMPLE 1

Construct an expression for the perimeter of this rectangle.

\[
\text{Perimeter} = 3d + d + 3d + d = 8d
\]

If \( d = 2 \) cm, then

\[
\text{perimeter} = 8 \times 2 \text{cm} = 16 \text{cm}
\]