Motion in a straight line

This chapter will show you how to
- distinguish between a vector quantity and a scalar quantity
- calculate average speed and velocity
- draw and interpret graphs of displacement, velocity and acceleration against time
- use the constant acceleration equations, including the motion of a particle moving vertically under gravity.

Before you start

You should know how to:

1. Find the gradient of a straight line on a graph.
2. Find the area of rectangles, triangles and trapeziums.
3. Substitute values into an algebraic formula.
4. Solve a linear equation.
5. Solve a quadratic equation using factorisation or the quadratic formula.
6. Convert between metric units.

Check in

1. Find the gradient of the line through the points (1, 4) and (3, 10).
2. A trapezium $ABCD$ has vertices $A(0, 0)$, $B(4, 9)$, $C(12, 9)$ and $D(15, 0)$. Find the area of $ABCD$.
3. In the equation $s = ut + \frac{1}{2}at^2$, find the value of $s$ when $u = 10$, $t = 4$ and $a = 6$.
4. Solve the equations
   a. $5x - 3 = 3x + 8$
   b. $\frac{1}{2}(x + 2) = \frac{1}{3}x + 2$
5. Solve the equations
   a. $x^2 - 5x + 6 = 0$
   b. $x^2 - 3x - 1 = 0$
6. Convert
   a. 350 g to kg
   b. 108 km h$^{-1}$ to m s$^{-1}$
This line, marked in metres (m), shows an origin, O, and two points, P and Q.

Suppose an object travels from P to Q and then to O. It moves from a position of 4 m at P to a position of −2 m at Q and then a position of 0 m at O.

The move from P to Q is a displacement of −6 m.
The move from Q to O is a displacement of 2 m.
The two moves give a combined displacement of −4 m.

**Displacement** is the change of position.

How far an object travels, ignoring direction, is called **distance**. This is not the same as displacement. The object in the diagram travels a total distance of 8 m in going from P to Q to O, but its resulting displacement from its starting point is −4 m.

Adding the magnitude of the displacements gives the distance travelled.

A girl stands at the top of a 30 metre high cliff. She throws a ball vertically up so that it rises 20 metres then falls to the bottom of the cliff.

Find

a the total displacement

b the total distance travelled.

Height of the girl’s hand above the ground can be ignored.

a Take the starting point of the ball as the origin, and upwards as the positive direction:

Total displacement = 20 + (−50) = −30 m

The ball finishes 30 m below the origin.

b Total distance = 20 + 50 = 70 m
Velocity and speed
Suppose the same object is moving at a constant rate, taking 3 seconds to go from \( P \) to \( Q \) and 1 second from \( Q \) to \( O \).

\[
\begin{array}{c}
\text{\(-6\) m in 3 s} \\
\text{2 m in 1 s} \\
\hline
-5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\end{array}
\]

From \( P \) to \( Q \) its displacement changes by \(-2\) m every second. Its velocity is \(-2\) metres per second (m s\(^{-1}\)).
From \( Q \) to \( O \) its displacement changes by 2 m every second. Its velocity is 2 m s\(^{-1}\).

Velocity is rate of change of displacement.

The speed of the object is 2 m s\(^{-1}\) throughout the whole journey.

Speed is the magnitude of the velocity. Speed is rate of change of distance.

Consider a ball hitting a wall.

George rolls a ball at a wall 30 m away. It hits the wall after 3 s and rebounds towards George, who stops it 20 m from the wall after a further 5 s.

First stage: 30 m

Second stage: 20 m

Take the initial direction of travel as positive:
First stage
Velocity = \(30 \div 3 = 10\) m s\(^{-1}\)
Speed = \(30 \div 3 = 10\) m s\(^{-1}\)
Second stage
Velocity = \((-20) \div 5 = -4\) m s\(^{-1}\)
Speed = \(20 \div 5 = 4\) m s\(^{-1}\)

You have assumed that the speed and velocity of the ball is constant during each stage.
If the motion of the ball varied during a stage, these results would give the average speed and average velocity, i.e. the constant speed and velocity needed to achieve the same distance and displacement in the same time.

Constant speed and velocity are also called uniform speed and velocity.
In reality objects rarely travel at a constant speed for any length of time, but the idea is a useful modelling assumption.
2 Motion in a straight line

Average speed = \( \frac{\text{total distance travelled}}{\text{total time taken}} \)

Average velocity = \( \frac{\text{total displacement}}{\text{total time taken}} \)

= \( \frac{\text{final position} - \text{initial position}}{\text{total time taken}} \)

You can find the average speed and velocity over several stages of a journey. Consider again the example of the ball hitting the wall:

The whole journey took 8 s.
The total distance travelled = 30 + 20 = 50 m
Average speed = \( \frac{50}{8} = 6.25 \text{ m s}^{-1} \)
The total displacement = 30 + (−20) = 10 m
Average velocity = \( \frac{10}{8} = 1.25 \text{ m s}^{-1} \)

A car starts from a point \( A \) and drives 300 m along a straight road to a point \( B \). It then immediately reverses 120 m to a point \( C \). It takes 15 s to go from \( A \) to \( B \) and 20 s from \( B \) to \( C \).

a Calculate the average speed
   i from \( A \) to \( B \)  
   ii from \( B \) to \( C \)  
   iii for the whole journey.

b Calculate the average velocity
   i from \( A \) to \( B \)  
   ii from \( B \) to \( C \)  
   iii for the whole journey.

c Express the result for a iii in km h\(^{-1} \).

a i Average speed = \( \frac{300}{15} = 20 \text{ m s}^{-1} \)

ii Average speed = \( \frac{120}{20} = 6 \text{ m s}^{-1} \)

iii Average speed = \( \frac{420}{35} = 12 \text{ m s}^{-1} \)

b Define the direction from \( A \) to \( B \) to be positive.

i Average velocity = \( \frac{300}{15} = 20 \text{ m s}^{-1} \)

ii Average velocity = \( -\frac{120}{20} = -6 \text{ m s}^{-1} \)

iii The total displacement = 300 − 120 = 180 m, so

Average velocity = \( \frac{180}{35} = 5.14 \text{ m s}^{-1} \)

c 12 m s\(^{-1} \) = 12 × 60 × 60 m h\(^{-1} \) = 43 200 m h\(^{-1} \) = 43.2 km h\(^{-1} \)

Even if the road had not been straight you could have modelled it as a straight line for the purpose of this calculation.

It is usual to model the car as a particle. You don’t need to allow for the length of the car in your calculations.

The average speed of a journey of several stages cannot be found by calculating the mean of the average speeds for the individual stages. In this example 12 m s\(^{-1} \) is not the mean of 20 m s\(^{-1} \) and 6 m s\(^{-1} \). The average speed for a journey must be found using the total distance and total time.
**Acceleration**

**Acceleration** measures the amount by which velocity changes in each second. The unit of acceleration is metres per second per second, or metres per second squared (m s\(^{-2}\)).

**Acceleration** is the rate of change of velocity.

You can calculate the constant acceleration of an object:

An object travelling at 1 m s\(^{-1}\) undergoes constant acceleration, and 3 s later is travelling at 7 m s\(^{-1}\).

Velocity changes by 6 m s\(^{-1}\) in 3 s.

Acceleration = \(\frac{6}{3} = 2\) m s\(^{-2}\).

An object travelling at 1 m s\(^{-1}\) undergoes a constant acceleration of -2 m s\(^{-2}\) for 3 s.

Velocity changes by \((-2) \times 3 = -6\) m s\(^{-1}\)

Final velocity = 1 - 6 = -5 m s\(^{-1}\).

In the real world constant (uniform) acceleration rarely happens, but it is sometimes a useful modelling assumption.

If an object is not accelerating at a constant rate you can find the **average acceleration**.

\[
\text{Average acceleration} = \frac{\text{change in velocity}}{\text{time taken}} = \frac{\text{final velocity} - \text{initial velocity}}{\text{time taken}}
\]

The velocity of an object increases from 2 m s\(^{-1}\) to 14 m s\(^{-1}\) in 4 s.

Average acceleration = \(\frac{14 - 2}{4} = 3\) m s\(^{-2}\)

Take care. This does not mean that the acceleration is constant at 3 m s\(^{-2}\) over the 4 s.
Exercise 2.1

1. A sentry on guard duty marches 50 m east from his sentry box. He then goes 90 m in a westerly direction before finally returning to the sentry box. Taking the sentry box as the origin and east as the positive direction, find
   a. his position at the end of each stage
   b. the displacement he undergoes in each of the three stages
   c. the total distance for the whole journey
   d. the total displacement for the whole journey.

2. A car travels 150 km between 10.30 am and 12.20 pm. Find its average speed
   a. in km h\(^{-1}\)
   b. in m s\(^{-1}\).

3. In a charity walk, a group walked for 2 hours, covering a distance of 6 km. They then stopped for lunch, which took an hour, and afterwards walked the final 12 km in 3 hours. Find, in km h\(^{-1}\), their average speed for the whole journey.

4. A cyclist travels 12 km at a speed of 15 km h\(^{-1}\) and then continues for 36 minutes at a speed of \(26\frac{2}{3}\) km h\(^{-1}\). Find her average speed
   a. in km h\(^{-1}\)
   b. in m s\(^{-1}\).

5. A man jogs 8 km at a constant speed of 6 km h\(^{-1}\), then cycles for 2 hours at a constant speed of 16 km h\(^{-1}\). Find, in km h\(^{-1}\), his average speed
   a. for the first 12 km
   b. for the whole journey.
6 A car travels 2 km at an average speed of 20 m s\(^{-1}\), then 2 km at an average speed of 25 m s\(^{-1}\).
   a Find the average speed for the whole journey.
   b Explain why, having finished the first stage at 20 m s\(^{-1}\), the car could not achieve an average speed of 40 m s\(^{-1}\) for the whole journey.

7 A canoe race involves paddling 1800 m upstream and then 1200 m downstream to the finish. A competitor takes 10 minutes for the upstream section and returns at 5 m s\(^{-1}\).
   Find
   a the average speed for the upstream section
   b the time for the downstream section
   c the average speed for the whole race
   d the average velocity for the whole race.

8 A spaceship is travelling at 15 m s\(^{-1}\). The forward thrusters are activated, giving an acceleration of \(-2\) m s\(^{-2}\).
   a How long will it take for the spaceship to come to rest?
   b If the thrusters fire for a total of 20 s, find the final velocity and speed of the spaceship.

9 A ball bearing is rolled up a slope. Its initial speed is 4 m s\(^{-1}\). After 8 seconds it is rolling down the slope at 6 m s\(^{-1}\). Find its average acceleration during this time.

10 A lift, parked on the third floor, is called by someone on the seventh floor, who then gets in and travels to the ground floor. Each floor is 3.5 m and the whole process takes 50 s.
   a Find the overall average speed of the lift.
   b Find the overall average velocity of the lift.
   c The lift passed the third floor at 3 m s\(^{-1}\) on the way down. If it came to rest at the ground floor 12 seconds later, find its average acceleration for that part of the journey.

11 Sharon drove a distance of 18 km from her home to the motorway before joining it for the rest of her journey. Her average speed for the first part of the journey was 15 m s\(^{-1}\) and her overall average speed was \(22\frac{2}{9}\) m s\(^{-1}\). The whole journey took 72 minutes. Find
   a the total distance she travelled
   b her average speed for the second part of the journey.

12 Amanda’s car has a maximum speed of \(2V\) m s\(^{-1}\). She wishes to complete a certain journey at an average speed of \(V\) m s\(^{-1}\). What is the minimum average speed at which she could complete the first half of the journey and still meet her target for the whole journey?
You can illustrate graphically the motion of an object in several ways. The simplest option is to plot its displacement (from the origin) against time.

A tiger pacing along the front of her enclosure moves in one direction at 2 m s\(^{-1}\) for 6 seconds, then turns and goes the other way at 3 m s\(^{-1}\) for 5 seconds. Represent this on a displacement–time graph.

Take the first direction as positive and the tiger’s starting point as the origin.

First-stage displacement = 6 × 2 = 12 m

Second-stage displacement = 5 × (−3) = −15 m

The tiger moves from a position of 0 m to a position of 12 m and then to a position of −3 m.

Use this information to draw the graph. Plot time on the horizontal axis and displacement on the vertical axis.

The symbol \(s\) is commonly used for displacement. The graph can be called an \(s-t\) graph (or sometimes a \(t-s\) graph).

The velocities here are uniform (constant), giving straight lines on the graph. Non-uniform velocity would give a curved graph.

You could also draw a distance–time graph. Both sections would then slope upwards, as the distance travelled is always increasing.

**Velocity from a displacement–time graph**

You know that \(\frac{\text{change of displacement}}{\text{time taken}} = \text{velocity}\),

but on the displacement–time graph

\(\frac{\text{change of displacement}}{\text{change of time}} = \text{gradient}\).
It follows that

The gradient of a displacement–time graph gives the velocity of the object.

E.g. The gradients of the two sections of the graph in Example 1 are 2 and −3. The velocities were 2 m s\(^{-1}\) and −3 m s\(^{-1}\).

**EXAMPLE 2**

If the graph is not a straight line, the gradient of the tangent to the curve at any point is the velocity of the object at that instant.

The gradient of a distance–time graph is the speed.

Find  

\( \text{a} \) the average speed  

\( \text{b} \) the average velocity of the tiger in Example 1.

\( a \) Average speed = \( \frac{\text{total distance travelled}}{\text{time taken}} \)

The distance travelled is

\[
(2 \times 6) + (3 \times 5) = 27 \text{ m}
\]

so average speed = \( \frac{27}{11} = 2.45 \text{ m s}^{-1} \)

\( b \)

![Displacement-time graph]

Average velocity = \( \frac{\text{total displacement}}{\text{time taken}} \)

The tiger’s overall displacement is −3 m, so

average velocity = \( \frac{-3}{11} = -0.27 \text{ m s}^{-1} \)
Exercise 2.2

1. The graph shows the displacement (in km) of a cyclist from a town, plotted against time (in hours).
   a) Describe the journey, giving the cyclist’s velocity during each stage.
   b) State three assumptions that have been made in drawing this graph.

2. A ball is rolled at a constant velocity of 4 m s\(^{-1}\). After travelling 6 m it strikes a wall at right angles and rolls back along the same line at a constant 2.5 m s\(^{-1}\). The ball is stopped 3 s after hitting the wall.
   a) Draw a displacement–time graph to illustrate the ball’s motion.
   b) Calculate the ball’s average speed.
   c) Calculate the ball’s average velocity.

3. A woman, walking a dog along a straight path, stops and releases the dog at a point A. She then continues forward at a constant speed of 1.4 m s\(^{-1}\). The dog runs 100 m forward in 10 s, stops and sniffs for 10 s, then runs forward a further 50 m in 20 s. It then spots another dog 100 m the other side of A and runs back to join it at a constant speed of 5 m s\(^{-1}\).
   a) Draw a graph to show the displacement against time of both the woman and the dog from A.
   b) From your graph estimate where and when the dog passes the woman.
   c) Find the average speed of the dog during the period described.
   d) Find the average velocity of the dog during the period described.

4. Harold lives at the bottom of a hill. The village shop is at the top of the hill, a distance of 1.2 km. Harold cycles to the shop in 7.5 minutes, spends 3.5 minutes in the shop, then free-wheels home at a speed of 4 m s\(^{-1}\).
   a) Draw a displacement–time graph to illustrate Harold’s shopping trip, stating any assumptions that you make.
   b) Calculate Harold’s average speed for the whole journey.
   c) What is Harold’s average velocity for the whole journey?
5  A cyclist starts from town A at 11 am to ride to town B 60 km away. He completes the journey in three stages of 20 km, each taking an hour, with 15 minute breaks between stages. A second cyclist starts from B at the same time as the first cyclist leaves A, and travels to A non-stop at 16 km h⁻¹.

a  On the same axes, draw graphs showing the displacements against time of the two cyclists from A.

b  At what time, and where, do the two cyclists pass each other?

6  At 10.00 am Adam joined a motorway at Junction 3 and travelled without stopping at 80 km h⁻¹ to Junction 36, 380 km north.

At the same time Marlon joined the motorway at Junction 5, 30 km north of Junction 3. He drove north at 90 km h⁻¹. After 150 km he took a 50 minute break, then continued his journey north, leaving the motorway at Junction 36 at 2.10 pm.

a  On the same axes draw distance–time graphs for Adam’s and Marlon’s journeys.

b  Between what times was Adam ahead of Marlon?

c  What was Marlon’s average speed after he took his break?

7  A particle starts from rest and travels with non-uniform velocity. Its displacements at intervals of 1 s were noted, as shown in the table.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement (m)</td>
<td>0</td>
<td>2.9</td>
<td>6.9</td>
<td>12.0</td>
<td>18.3</td>
<td>25.7</td>
<td>34.0</td>
<td>44.0</td>
<td>54.9</td>
<td>66.9</td>
<td>80.0</td>
</tr>
</tbody>
</table>

a  Draw a displacement–time graph to represent this data.

b  Use your graph to estimate the speed of the particle

i  after 1 s

ii  after 7 s.

c  Find the average speed of the particle over the first 7 s.
You can also draw a graph of velocity against time.
Plot time on the horizontal axis and velocity on the vertical axis.

### Example 1
An object starts from rest and accelerates uniformly to a velocity of $6 \text{ m s}^{-1}$ in 15 seconds. It then undergoes a constant negative acceleration so that after a further 20 seconds its velocity is $-4 \text{ m s}^{-1}$. Draw a graph of velocity against time.

The acceleration is uniform during each stage, so the graph consists of two straight lines.

**Acceleration from a velocity–time graph**
You know that

\[
\frac{\text{change of velocity}}{\text{time taken}} = \text{acceleration},
\]

but on the velocity–time graph

\[
\frac{\text{change of velocity}}{\text{change of time}} = \text{gradient}.
\]

It follows that

The gradient of a velocity–time graph gives the acceleration of the object.

This can be seen in Example 1. During the first stage the velocity increased by $6 \text{ m s}^{-1}$ in 15 s. This is an acceleration of $0.4 \text{ m s}^{-2}$, and the gradient of the line on the graph is 0.4.

During the second stage the velocity changed by $-10 \text{ m s}^{-1}$ in 20 s. This is an acceleration of $-0.5 \text{ m s}^{-2}$, and the gradient of the line on the graph is $-0.5$. 

The symbol $v$ is commonly used for velocity. The graph can be called a $v$–$t$ graph (or sometimes $a$–$t$, $v$ graph).

Straight lines on the graph correspond to uniform (constant) acceleration. Non-uniform acceleration would give a curved graph.
**Speed-time graphs**
You could plot speed rather than velocity.

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**Example 2**

Draw a speed–time graph for the situation described in Example 1.

As speed is a scalar quantity, its value is always positive, leading to the graph shown.

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**Displacement from a velocity–time graph**
You can relate displacement to a velocity–time graph, as shown by this example.

A cyclist travelling at a constant velocity of 8 km h\(^{-1}\) for 3 h achieves a displacement of 24 km.
The velocity–time graph is shown.
The shaded area between the graph and the time axis is
\[3 \times 8 = 24.\]
This area corresponds to the displacement.
If the cyclist had been travelling in the negative direction, the velocity would have been \(-8\) km h\(^{-1}\), as shown, and the displacement would have been \(-24\) km.
The shaded area is now \(3 \times (-8) = -24\).
The area again corresponds to the displacement.
This relationship also holds true for non-uniform velocity.

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The area between a velocity–time graph and the time axis gives the displacement of the object.

You may know from your study of calculus that you obtain negative results when finding areas below the horizontal axis by integration.

The area under a speed–time graph corresponds to the distance travelled.
For the situation described in Example 1, calculate

a  the overall displacement and
b  the total distance travelled by the object.

a  When time = 15 s, velocity = 6 m s\(^{-1}\).
   Acceleration from then on is \(-0.5\) m s\(^{-2}\),
   so \(v = 0\) after a further 12 s.
   Hence velocity = 0 m s\(^{-1}\) when time = 27 s.

   The triangular region above the axis has base 27
   and height 6.
   Its area = 81, so displacement = 81 m.

   The triangular region below the axis has base 8
   and height \(-4\).
   Its area = \(-16\), so displacement = \(-16\) m.

   Overall displacement = 81 m + (\(-16\)) m = 65 m.

b  The total distance travelled = 81 m + 16 m = 97 m.

**Acceleration–time graphs**

It is sometimes useful to draw an acceleration–time graph.

The acceleration–time graph corresponding to the velocity–time graph in Example 1 would look like this:

The area between the acceleration–time graph and the time axis gives the change of velocity. You should check that this is true for this graph.

You can summarise the features of the three types of motion graphs in a table.

<table>
<thead>
<tr>
<th></th>
<th>Displacement–time graph</th>
<th>Velocity–time graph</th>
<th>Acceleration–time graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gradient gives:</td>
<td>Velocity</td>
<td>Acceleration</td>
<td>–</td>
</tr>
<tr>
<td>Area gives:</td>
<td>–</td>
<td>Displacement</td>
<td>Change of velocity</td>
</tr>
</tbody>
</table>
Exercise 2.3

1. The displacement–time graph shows the progress of a hiker out on a day’s trek.

   ![Displacement–time graph](image)

   a. Describe the motion during each stage of the journey.
   b. Draw the corresponding velocity–time graph.

2. A car, travelling at 15 m s\(^{-1}\), accelerates uniformly to a velocity of 45 m s\(^{-1}\) in 12 s.
   a. Sketch the velocity–time graph.
   b. Calculate the car's acceleration.
   c. Calculate the distance the car travels while accelerating.

3. A lorry, travelling at 36 m s\(^{-1}\), is brought uniformly to rest with acceleration -1.2 m s\(^{-2}\).
   a. Sketch the velocity–time graph.
   b. Calculate the distance travelled by the lorry before it stops.

4. A lift starts from rest, accelerates upwards for 2 seconds at 1.5 m s\(^{-2}\), travels for 3 seconds at constant speed, and then decelerates to rest in 1.2 seconds.
   a. Sketch the velocity–time graph, stating any assumptions you have made.
   b. Sketch the corresponding acceleration–time graph.
   c. Calculate the displacement of the lift between stops.
5 The graph shows the acceleration of an object during a period of 7 seconds. At the start of the period the velocity of the object is 1 m s\(^{-1}\).

\[\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
\text{acceleration (m s}^{-2}\text{)} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\hline
\text{time (s)} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\hline
\end{array}\]

a Sketch the velocity–time graph.

b Calculate the overall displacement of the object.

c Calculate the distance travelled by the object.

6 A dragonfly, at rest on a bullrush, decides to fly to a second bullrush 18 m away. It accelerates uniformly to a speed of 5 m s\(^{-1}\), then immediately decelerates uniformly to rest on the second bullrush. Sketch the velocity–time graph and find how long the journey takes.

7 A boat starts from rest at a point A and accelerates uniformly at a rate of 0.5 m s\(^{-2}\) for 12 seconds. The boat then decelerates uniformly to rest in 15 seconds. The boat then accelerates as it reverses back towards A. It reaches a speed of 4 m s\(^{-1}\) in 20 seconds and then continues to reverse at that speed.

a Sketch the velocity–time graph.

b What was the boat’s greatest forward displacement from A?

c What was the total time between the boat’s leaving A and its return to A?
8 A car starts from rest at a point A and drives up a slope. It accelerates to a speed of 18 m s\(^{-1}\) in 6 s and maintains this speed for 4 s. The gears are then disengaged and the car coasts to rest with acceleration \(-2\) m s\(^{-2}\). Unfortunately, the driver forgets to put the handbrake on, so the car then rolls back down the slope with acceleration \(-1\) m s\(^{-2}\).

a Sketch a velocity–time graph of the motion.

b Calculate the acceleration on the first stage.

c Find how far the car was from A when it came instantaneously to rest.

d When will it return to A?

9 A car, P, is at rest at a point O when a second car, Q, passes at a constant speed of 20 m s\(^{-1}\). At the moment that Q passes, P moves off in pursuit. It accelerates uniformly at 2 m s\(^{-2}\) until it reaches a speed of 30 m s\(^{-1}\), then continues at this speed.

a Sketch the velocity–time graph for both cars on the same axes.

b How far is P behind Q at the moment when it reaches full speed?

c How much longer does it take for P to draw level with Q?

10 Chulchit drives to work along a straight road of length 8 km. His car accelerates and decelerates at 2.5 m s\(^{-2}\), and his preferred cruising speed is 90 km h\(^{-1}\).

a Assuming he has a clear run with no hold-ups, sketch a velocity–time graph of his journey and hence calculate his journey time.

b Chulchit hears on the radio that there are 2 km of road works, with a speed limit of 36 km h\(^{-1}\), somewhere along the route. Investigate the effect of the positioning of these road works on his best journey time, and find the maximum and minimum values of this.
Problems concerning the motion of objects involve some or all of the following quantities.

\[
\begin{align*}
    s &= \text{displacement} \\
    u &= \text{initial velocity} \\
    v &= \text{final velocity} \\
    a &= \text{acceleration} \\
    t &= \text{time}
\end{align*}
\]

If you can assume that acceleration is constant, these quantities are connected together by five simple formulae.

You can derive two of these formulae directly from the velocity–time graph.

Acceleration = gradient of velocity–time graph.

This gives \( a = \frac{v - u}{t} \)

\( \Rightarrow \quad v = u + at \) \[1\]

Displacement = area under velocity–time graph.

Use the formula for the area of a trapezium:

\( s = \frac{1}{2}(u + v)t \) \[2\]

You can combine equations [1] and [2] to give three more formulae.

Substitute for \( v \) from [1] into [2]:

\[ s = \frac{1}{2}(u + u + at)t \]

\( \Rightarrow \quad s = ut + \frac{1}{2}at^2 \) \[3\]
A car, travelling at a speed of 15 m s\(^{-1}\), suddenly accelerates at 3 m s\(^{-2}\). What is its speed after 5 seconds?

You need to find \(v\).

You know the values \(u = 15\) m s\(^{-1}\), \(a = 3\) m s\(^{-2}\) and \(t = 5\) s.

The formula containing these variables is \(v = u + at\).

Substitute the known values:

\[v = 15 + 3 \times 5 = 30\]

So the car’s speed after 5 seconds is 30 m s\(^{-1}\).
A hot air balloon is drifting at a constant velocity of 3 m s\(^{-1}\). A change in wind causes it to undergo an acceleration of \(-0.5\) m s\(^{-2}\) for a period of 16 s. Calculate

**a** the displacement of the balloon

**b** the distance travelled by the balloon during this period.

**a** You need to find \(s\).

You know the values \(u = 3\) m s\(^{-1}\), \(a = -0.5\) m s\(^{-2}\) and \(t = 16\) s.

The formula containing these variables is \(s = ut + \frac{1}{2}at^2\).

Substitute the known values:

\[
s = 3 \times 16 + \frac{1}{2} \times (-0.5) \times 16^2 = -16
\]

so the overall displacement is \(-16\) m.

**b** To find the distance travelled you need to know how far forward the balloon went before it began to move backwards. This is the point at which the balloon came instantaneously to rest.

You need to find \(s\).

You know the values \(u = 3\) m s\(^{-1}\), \(v = 0\) m s\(^{-1}\) and \(a = -0.5\) m s\(^{-2}\).

The formula containing these variables is \(v^2 = u^2 + 2as\).

Substitute the known values: \(0 = 3^2 + 2 \times (-0.5)s\)

\[
\Rightarrow s = 9
\]

So the balloon moved 9 m forward. It then drifted backwards 25 m to achieve an overall displacement of \(-16\) m.

Total distance travelled = 9 + 25 = 34 m.
A car, travelling at a speed of 10 m s\(^{-1}\), accelerates at 4 m s\(^{-2}\) until its speed has increased to 18 m s\(^{-1}\). How far does it travel while accelerating?

You need to find \(s\).
You know the values \(u = 10\) m s\(^{-1}\), \(v = 18\) m s\(^{-1}\) and \(a = 4\) m s\(^{-2}\).
The formula containing these variables is \(v^2 = u^2 + 2as\).

Substitute the known values: 
\[324 = 100 + 8s\]
\[\Rightarrow s = 28\]

So the distance travelled during the acceleration is 28 m.

A lorry is travelling at 30 m s\(^{-1}\). The driver applies the brakes and the lorry comes to rest after 5 seconds. How far did it travel during this time?

You need to find \(s\).
You know the values \(u = 30\) m s\(^{-1}\), \(v = 0\) m s\(^{-1}\) and \(t = 5\) s.
The formula containing these variables is \(s = \frac{1}{2}(u + v)t\).

Substitute the known values:
\[s = \frac{1}{2}(30 + 0) \times 5\]
\[= 75\]

So the distance travelled is 75 m.

A particle, accelerating at 10 m s\(^{-2}\), strikes a wall at 42 m s\(^{-1}\). How far did it travel in the final second of its motion?

You need to find \(s\).
You know the values \(v = 42\) m s\(^{-1}\), \(a = 10\) m s\(^{-2}\) and \(t = 1\) s.
The formula containing these variables is \(s = vt - \frac{1}{2}at^2\).

Substitute the known values:
\[s = 42 \times 1 - \frac{1}{2} \times 10 \times 1^2\]
\[= 37\]

So the distance it travelled in the final second is 37 m.
Some problems involve more than one object.

**Example 6**

A car, $P$, is accelerating at $2 \text{ m s}^{-2}$. At the point where its velocity is $10 \text{ m s}^{-1}$ it is overtaken by another car, $Q$, travelling at $16 \text{ m s}^{-1}$ and accelerating at $1 \text{ m s}^{-2}$. How long is it before car $P$ catches up with car $Q$?

You need to find the value of $t$ for which the displacements of the two cars are equal.

For car $P$ you have $u = 10 \text{ m s}^{-1}$ and $a = 2 \text{ m s}^{-2}$.

At time $t$ its displacement is

$$s_P = 10 \times t + \frac{1}{2} \times 2 \times t^2 = 10t + t^2$$

For car $Q$ you have $u = 16 \text{ m s}^{-1}$ and $a = 1 \text{ m s}^{-2}$.

At time $t$ its displacement is

$$s_Q = 16 \times t + \frac{1}{2} \times 1 \times t^2 = 16t + \frac{1}{2}t^2$$

The cars are level when $s_P = s_Q$

$$10t + t^2 = 16t + \frac{1}{2}t^2$$

$$t^2 - 12t = 0$$

$$t(t - 12) = 0 \quad \Rightarrow \quad t = 0 \text{ or } t = 12$$

$t = 0$ is the time when $Q$ passed $P$, so $P$ catches up with $Q$ 12 seconds later.

**Exercise 2.4**

1. A lorry starts from rest and accelerates uniformly at a rate of $3 \text{ m s}^{-2}$ for $30 \text{ s}$.
   a. How far does it travel?
   b. How fast is it travelling at the end of the period?

2. A stone is dropped from rest at the top of a tower. It takes 5 seconds to reach the ground, by which time it is travelling at $50 \text{ m s}^{-1}$.
   a. What is its acceleration?
   b. How high is the tower?

3. A body starts from rest with uniform acceleration and in $10 \text{ s}$ moves a distance of $150 \text{ m}$.
   a. What is its acceleration?
   b. How fast is it moving at the end of this period?
4 A stunt motorcyclist has 50 m in which to accelerate from rest to the 90 km h\(^{-1}\) needed at the ramp. How long does this ‘run up’ take?

5 A train leaves station \(A\) from rest with constant acceleration 0.2 m s\(^{-2}\). It reaches maximum speed after 2 minutes, maintains this speed for 4 minutes, then slows down to stop at station \(B\) with acceleration \(-1.5\) m s\(^{-2}\). Calculate the distance \(AB\).

6 A train accelerates uniformly from rest for 1 minute, at the end of which time its velocity is 30 km h\(^{-1}\). It maintains this speed until it is 500 m from the next station. It then decelerates uniformly and stops at the station. Calculate the train’s acceleration during the first and last phases of this journey.

7 A car crosses a speed hump with a velocity of 4 m s\(^{-1}\). It then accelerates at a rate of 2.5 m s\(^{-2}\) to a speed of 9 m s\(^{-1}\) when the driver applies the brakes, causing an acceleration of \(-3\) m s\(^{-2}\), reducing the speed of the car to 4 m s\(^{-1}\) to cross the next hump.
   a How far apart are the humps?
   b How long does the car take to travel from one hump to the next?
   c The question implies that the car is being modelled as a particle. In what way does this assumption affect your results?

8 A moon landing craft is 1 km above the lunar surface and descending at a speed of 80 m s\(^{-1}\). The rockets are then fired, giving it an upward acceleration \(a\) m s\(^{-2}\). Find the value of \(a\) if the craft is to make a perfect soft landing.

9 A car starts from rest at the bottom of a slope. It accelerates up the slope for 8 seconds at 1.5 m s\(^{-2}\), then disengages the engine and coasts. If its acceleration is now \(-1\) m s\(^{-2}\), find the time between the car’s leaving the bottom of the slope and returning to it.

10 A lift ascends from rest with an acceleration of 0.5 m s\(^{-2}\) before slowing with an acceleration of \(-0.75\) m s\(^{-2}\) for the next stop. If the total journey time is 10 s, what is the distance between the two stops?
11 A boat is travelling at a speed of $4 \text{ m s}^{-1}$. Its propeller is then put into reverse, giving it an acceleration of $-0.4 \text{ m s}^{-2}$ for a period of 25 seconds.
   a Find the overall displacement of the boat during this period.
   b Find the distance travelled by the boat during this period.

12 An object travels 10 m during one second and 15 m during the next second.
   a Find the acceleration of the object, assuming it to be constant.
   b How fast is the object going at the end of the two seconds?

13 An object moving with constant acceleration travels 10 m in 2 seconds. The next 10 m takes it 4 seconds.
   a Find the acceleration of the object.
   b For how much more time will the object travel before coming to rest?
   c How much further will the object travel before coming to rest?

14 Theresa is handing the baton on to Magda in a relay race. Theresa is running at a constant speed and when she is 4.5 m away Magda starts running with acceleration $1 \text{ m s}^{-2}$. Theresa continues at a constant speed and just manages to catch up and hand over the baton. How fast was Theresa running?

15 Trains A and B are travelling in the same direction on two parallel straight tracks. At the moment that A passes B, A is travelling at a constant $20 \text{ m s}^{-1}$ and B is travelling at $5 \text{ m s}^{-1}$ and accelerating at $0.5 \text{ m s}^{-2}$.
   a How much time passes before the trains are again level?
   b How far do they travel in this time?

16 Clare is driving along a road in her car, with Henry following 40 m behind in his car. They are both travelling at a speed of $25 \text{ m s}^{-1}$. Clare spots a problem ahead and brakes to a halt with an acceleration of $-5 \text{ m s}^{-2}$. Henry takes 0.2 seconds to react, then brakes, but his brakes are poor and only give an acceleration of $-4 \text{ m s}^{-2}$. Investigate what happens.
17. P is a point 10 m from the bottom of a slope. A ball is rolled up the slope from P with an initial velocity of 8 m s\(^{-1}\). It undergoes a constant acceleration of \(-4\) m s\(^{-2}\).
   
   a. i. How long does it take to reach the bottom of the slope?
      ii. How far up the slope does it travel?

b. One second after the first ball is rolled, a second ball is rolled up from the bottom of the slope. It has an initial velocity of 14 m s\(^{-1}\). It also has an acceleration of \(-4\) m s\(^{-2}\). Find when and where the two balls meet.

18. A particle travelling in a straight line with acceleration \(a\) passes a point A whilst moving with velocity \(u\). When it reaches a point B it has velocity \(v\), and at this point its acceleration changes to \(-a\). Show that when it again passes through A its speed is \(\sqrt{2v^2 - u^2}\).

19. A car, C, travelling at constant speed \(u\), passes a stationary police car, P, which immediately sets off in pursuit with acceleration \(a\). Find, in terms of \(u\) and \(a\), the greatest distance separating the two cars during the subsequent chase.

20. Two cars are travelling at a constant speed \(v\), with one car a distance \(d\) ahead of the other. As each car passes a marker post it accelerates with acceleration \(a\) to a new constant speed \(V\). Show that the new distance \(D\) between the cars is \(D = \frac{Vd}{v}\).

21. Two trains, A and B, are travelling on parallel tracks and in opposite directions. They start simultaneously from rest at stations a distance \(d\) m apart and head towards each other, with constant accelerations 0.4 m s\(^{-2}\) and 0.2 m s\(^{-2}\) respectively. After 50 s the fronts of the trains are level. Calculate
   
   a. the speed of each train when they meet
   b. the value of \(d\).
2.5 Free fall under gravity

You can assume that an object moving vertically under the effect of gravity has constant acceleration provided
- the object is so small and the speed is so low that air resistance can be neglected
- the distance it moves is small relative to the size of the Earth, so that the Earth's gravitational field can be assumed to be constant.

The acceleration due to gravity is denoted by $g$.

Near the Earth's surface the acceleration due to gravity is approximately

$$g \approx 9.8 \text{ m s}^{-2}$$

A stone is dropped from the top of a 20 m high tower.

a. How long does it take to reach the ground?

b. With what velocity does it hit the ground?

Take the origin as the top of the tower and take downwards as the positive direction.
You know $u = 0 \text{ m s}^{-1}$, $a = g = 9.8 \text{ m s}^{-2}$ and $s = 20 \text{ m}$.

a. Use $s = ut + \frac{1}{2}at^2$ to find $t$:

$$20 = \frac{1}{2} \times 9.8 \times t^2$$

$\Rightarrow \quad t = 2.02 \text{ or } -2.02$

You can obviously ignore the negative value, so $t = 2.02 \text{ s}$

b. Use $v^2 = u^2 + 2as$ to find $v$:

$$v^2 = 2 \times 9.8 \times 20$$

$$= 392$$

$\Rightarrow \quad v = 19.8 \text{ or } -19.8$

Again you can ignore the negative value, so $v = 19.8 \text{ m s}^{-1}$.
A ball is thrown vertically upwards from ground level at 20 m s\(^{-1}\). A boy, leaning out of a window 8 m above the point of projection, catches the ball on its way down.

**a** What is the time of flight of the ball?

**b** How fast is it travelling when the boy catches it?

Take the origin as ground level and take upwards as the positive direction.

You know \( u = 20 \text{ m s}^{-1}, a = g = -9.8 \text{ m s}^{-2} \).

**a** When the ball is caught, \( s = 8 \text{ m} \).

Use \( s = ut + \frac{1}{2}at^2 \) to find \( t \):

\[
8 = 20t + \frac{1}{2} \times (-9.8) \times t^2
\]

\[
\Rightarrow 4.9t^2 - 20t + 8 = 0
\]

\[
\Rightarrow t = 3.63 \text{ or } 0.450
\]

The ball is level with the boy at \( t = 0.450 \) going up, and at \( t = 3.63 \) going down.

So, the time of flight is 3.63 s.

**b** Use \( v = u + at \) to find \( v \):

\[
v = 20 + (-9.8) \times 3.63
\]

\[
= -15.6
\]

So, the ball is travelling at 15.6 m s\(^{-1}\) when it is caught. The negative sign shows that it is travelling downwards.

---

**Exercise 2.5**

1. A stone is dropped from the top of a 50 m high cliff.
   
   **a** How long does it take to reach the beach below?
   
   **b** With what velocity does it hit the beach?

2. A ball is thrown vertically upwards with speed 15 m s\(^{-1}\).
   
   **a** Find the greatest height it reaches.
   
   **b** How long does it take to reach this maximum height?
   
   **c** The ball returns to its starting position. What is the whole time of flight?
2 Motion in a straight line

3 A ball thrown vertically upwards rises 20 m before descending again.
   a What was its initial speed?
   b What is the whole time of flight?
   c With what speed is the ball travelling when it arrives back at its starting point?

4 A stone, thrown vertically upwards at 5 m s\(^{-1}\) from the edge of a 60 m high cliff, falls to the beach below.
   a With what speed does it hit the beach?
   b What is the time of flight?

5 A stone is thrown vertically upwards with speed 10 m s\(^{-1}\). One second later another stone is thrown vertically upwards from the same point and with the same speed.
   a How high are the stones when they meet?
   b How long after the first stone is thrown do the stones meet?

6 A boy drops a stone from the top of a building of height \(h\). Simultaneously his friend throws another stone vertically upwards from the ground below, with a speed of 30 m s\(^{-1}\). The stones meet 1.5 seconds later. Find \(h\).

7 A body falls from rest from the top of a tower. During the last second of its motion it falls \(\frac{7}{16}\) of the whole distance.
   a Show that the time of descent is independent of the value of \(g\).
   b Find the height of the tower in terms of \(g\).

8 A rocket ascends vertically from rest at ground level with acceleration 10 m s\(^{-2}\). Calculate
   a the height of the rocket after 4 s
   b the speed of the rocket after 4 s.
   After the rocket has been moving for 4 s a component breaks off and falls to the ground.
   c Calculate the further time which elapses before the component hits the ground.
9 A ball is projected vertically upwards from ground level at a speed of 30 m s\(^{-1}\).

Calculate

a the height to which the ball rises

b the time for which the ball is in the air

c the length of time for which the ball is over 30 m above the ground.

10 A stone falls past a window of height 2.5 m in 0.5 s. Taking \(g = 10 \text{ m s}^{-2}\), find the height from which the stone fell.

11 An object is thrown vertically downwards with speed \(V\). During the sixth second of its motion it travels a distance \(h\). Find \(V\) in terms of \(h\) and \(g\).

12 An object is projected vertically upwards with a velocity \(u\) m s\(^{-1}\). \(T\) seconds later another object is projected vertically upwards from the same point and with the same speed. Find, in terms of \(u\), \(T\) and \(g\), the further time which elapses before the objects collide.

13 a A particle is projected vertically upwards with initial speed \(u\) m s\(^{-1}\). After \(T\) s it reaches its greatest height, \(H\) m. Find, in terms of \(u\) and \(g\), an expression for

i \(T\) 

ii \(H\)

b At a certain time the particle is travelling upwards through a point \(A\), at a height of 19.6 m, and 2 s later it is at a point \(B\), at a height of 29.4 m.

i Show that it is on its way down through \(B\) at that time.

ii Find the value of \(u\).

14 A particle \(A\) is thrown vertically upwards from the bottom of a tower. At the same instant a second particle, \(B\), is dropped from the top of the tower. Given that when the particles collide they are travelling at the same speed, find the ratio between the distances they have travelled.
1. A train travels from rest at station A. It moves for 1 minute with a constant acceleration of 0.5 m s\(^{-2}\), continues at uniform speed for 3 minutes, and then slows to rest at station B in a further 30 s.
   a. Sketch a velocity–time graph to show the train’s journey.
   b. Calculate the maximum speed of the train during the journey.
   c. Calculate the distance between the two stations.
   d. Calculate the train’s average speed.

2. A runner moves from rest at a constant acceleration of 0.6 m s\(^{-2}\) for 10 s, then continues at a uniform velocity. A cyclist starts from rest at the same point and time and accelerates uniformly for 8 s before immediately slowing uniformly to rest after a further 22 s. At the moment that the cyclist stops they have both travelled the same distance.
   a. Sketch a velocity–time graph for the runner.
   b. Sketch a velocity–time graph for the cyclist.
   c. Calculate the distance travelled by the cyclist.
   d. Calculate the greatest speed reached by the cyclist.

3. The graph shows the velocity of a lift travelling upwards between two floors of a building.
   a. State any assumptions which have been made in drawing this graph.
   b. Calculate the acceleration of the lift during each phase of the motion.
   c. Calculate the distance between the two floors.
4 The graph shows the motion of a cat along the top of a straight fence.

a Describe the motion of the cat during this period of 20 s.

b Calculate the velocity of the cat during each phase of the motion.

c Calculate the average speed of the cat during this period of 20 s.

d Calculate the average velocity of the cat during this period of 20 s.

5 A car moves in a straight line for 6 s at a constant acceleration of 2.5 m s$^{-2}$. The brakes are then applied, bringing it to rest in 4 s. It is immediately put into reverse, accelerating uniformly so that it passes its original starting point travelling at a speed of 6 m s$^{-1}$.

a Sketch a velocity–time graph for the car’s journey.

b Calculate the greatest positive displacement of the car from its starting point.

c Calculate the total time for the motion described.

d Calculate the car’s acceleration when travelling in reverse.

e Sketch an acceleration–time graph for the motion described.
Motion in a straight line

6 A particle is sliding over a rough plane. Its initial speed is 12 m s\(^{-1}\) and its acceleration is \(-2\) m s\(^{-2}\).

a Calculate how long it takes to travel 20 m.

b Calculate how far it travels before coming to rest.

7 A tobogganer puts her feet down to try to avoid hitting a snowdrift that she sees 30 m ahead. She achieves a retardation of 0.2 m s\(^{-2}\), but this is not enough to stop her before she reaches the snowdrift. She runs into the snowdrift with a speed of 2 m s\(^{-1}\).

a For how long does she have her feet down?

b How fast was she travelling when she first put her feet down?

8 A competitor makes a dive from a high springboard into a diving pool. She leaves the springboard vertically with a speed of 4 m s\(^{-1}\) upwards. When she leaves the springboard, she is 5 m above the surface of the pool. The diver is modelled as a particle moving vertically under gravity alone and it is assumed that she does not hit the springboard as she descends. Find

a her speed when she reaches the surface of the pool

b the time taken to reach the surface of the pool.

c State two physical factors which have been ignored in the model.

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9 A small ball is projected vertically upwards from a point A. The greatest height reached by the ball is 30 m above A.

Calculate

a the speed of projection

b the time between the instant that the ball is projected and the instant it returns to A.
10 A car moves with constant acceleration along a straight horizontal road. The car passes the point A with speed 5 m s\(^{-1}\) and 4 s later it passes the point B, where AB = 50 m.

a Find the acceleration of the car.

When the car passes the point C, it has speed 30 m s\(^{-1}\).

b Find the distance AC.

11 A ball is dropped from the top of a cliff and hits the beach 4 seconds later.

a Assuming that there is no air resistance, calculate the height of the cliff.

b If air resistance in fact has a significant effect, would the actual height of the cliff be greater or less than that calculated in part a? Explain your answer.

12 A car starts from rest at a point S on a straight racetrack. The car moves with constant acceleration for 20 s, reaching a speed of 25 m s\(^{-1}\). The car then travels at a constant speed of 25 m s\(^{-1}\) for 120 s. Finally it moves with constant deceleration, coming to rest at a point F.

a Sketch a speed–time graph to illustrate the motion of the car.

The distance between S and F is 4 km.

b Calculate the total time the car takes to travel from S to F.

A motorcycle starts at S, 10 s after the car has left S. The motorcycle moves with constant acceleration from rest and passes the car at a point P which is 1.5 km from S. When the motorcycle passes the car, the motorcycle is still accelerating and the car is moving at a constant speed. Calculate

c the time the motorcycle takes to travel from S to P

d the speed of the motorcycle at P.
13 The stopping distance of a car consists of thinking distance plus braking distance. Harold’s car can brake with a deceleration of 5 m s$^{-2}$. There is a 1 s delay between Harold’s decision to stop and his foot pressing the brake.

a  Find an expression for Harold’s stopping distance when he is driving at a speed of $u$ m s$^{-1}$.

b  If Harold sees an obstacle in the road 60 m ahead, for what range of values of $u$ will he be able to stop in time?

14 Two cars, travelling in the same direction, are level as they pass a point $A$. At this stage the first car is travelling at 12 m s$^{-1}$, while the second car has a speed of 6 m s$^{-1}$. They both accelerate uniformly, the first car at 0.4 m s$^{-2}$ and the second at 0.6 m s$^{-2}$. The cars are again level when they pass a point $B$.

Calculate

a  the time they take to travel from $A$ to $B$

b  the distance $AB$

c  the speeds of the two cars at $B$.

15 Cars $A$ and $B$ are approaching the end of a race. $A$ is 1.5 km from the finish, is travelling at a speed of 30 m s$^{-1}$ and is accelerating uniformly at 0.7 m s$^{-2}$. $B$ is 210 m behind $A$, is travelling at a speed of 40 m s$^{-1}$ and is accelerating at 0.5 m s$^{-2}$.

a  Show that $B$ overtakes $A$ 285 m before the finish.

b  Calculate the difference in time between the arrivals of the two cars at the finish.

16 Arnold and Bruno are running for a bus, which pulls away from the bus-stop when they are 10 m short of reaching it. The bus has a uniform acceleration of 0.8 m s$^{-2}$.

a  Arnold runs at a constant speed of 5 m s$^{-1}$. Find how far the bus has moved before he catches it.

b  Bruno also runs at a constant speed but is slower than Arnold and only just catches the bus (that is, the bus has the same speed as Bruno at the moment that he draws level). Find Bruno’s running speed.
17 A parachutist drops from a helicopter $H$ and falls vertically from rest towards the ground. Her parachute opens 2 s after she leaves $H$ and her speed then reduces to $4 \text{ m s}^{-1}$. For the first 2 s her motion is modelled as that of a particle falling freely under gravity. For the next 5 s the model is motion with constant deceleration, so that her speed is $4 \text{ m s}^{-1}$ at the end of this period. For the rest of the time before she reaches the ground, the model is motion with constant speed of $4 \text{ m s}^{-1}$.

a Sketch a speed–time graph to illustrate her motion from $H$ to the ground.

b Find her speed when the parachute opens.

A safety rule states that the helicopter must be high enough to allow for the parachute to open and for the speed of a parachutist to reduce to $4 \text{ m s}^{-1}$ before reaching the ground.

Using the assumptions made in the above model,

c find the minimum height of $H$ for which the woman can make a drop without breaking this safety rule.

Given that $H$ is 125 m above the ground when the woman starts her drop

d find the total time taken for her to reach the ground.

e State one way in which the model could be refined to make it more realistic.

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Summary

- Quantities used to measure motion are vectors or scalars.
  - Displacement, velocity and acceleration are vectors.
  - Distance and speed are scalars.
- You can find average values for speed, velocity and acceleration.  
- You can draw graphs of displacement and distance against time.
  - The gradient of a displacement–time graph gives velocity.
  - The gradient of a distance–time graph gives speed.
- You can draw graphs of velocity, speed and acceleration against time.
  - The gradient of a velocity–time graph gives acceleration.
  - The area under a velocity–time graph gives displacement.
  - The area under an acceleration–time graph gives change of velocity.
- When acceleration can be assumed uniform (constant), the displacement ($s$), initial velocity ($u$), final velocity ($v$), acceleration ($a$) and time ($t$) are connected by these five equations:
  
  \[
  \begin{align*}
  v &= u + at \\
  s &= ut + \frac{1}{2}at^2 \\
  v^2 &= u^2 + 2as \\
  s &= \frac{1}{2}(u + v)t \\
  s &= vt - \frac{1}{2}at^2
  \end{align*}
  \]
  You need to memorise these.
- Objects moving vertically and freely under gravity have uniform acceleration of $g \approx 9.8 \text{ m s}^{-2}$.

Links

Railway operators need to calculate train accelerations and decelerations in order to plan their timetables, to position signals allowing for adequate stopping distances and, most importantly, to ensure the safe and efficient operation of railways.

Trains must be powerful enough to accelerate to the line speed limit within a short space of time as well as having reliable brakes to enable them to come to a standstill at a station or a signal as and when is needed.