Complete Mathematics for Cambridge IGCSE®
Teacher Resource Pack
Fifth edition
Extended
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For the updated syllabus
INTRODUCTION

This guide is designed to provide a structure for the teaching of the Cambridge IGCSE Mathematics syllabus at Extended level. There are a number of possible approaches suggested within the lessons but it does not give a definitive programme of study.

Students following the Extended syllabus are expected to have a significant amount of prior knowledge for many of the topics. As a result of this, many of the simpler topics and exercises are referred to only in passing.

Each lesson (or series of lessons) usually covers one or two key aspects of the syllabus and refers to the relevant pages of the Extended student textbook and associated exercises. It is expected that most of the main teaching points will be covered in the body of a single lesson, but that further lessons may be needed to consolidate and refresh the learning. Hence, each lesson is a guide to the time it might take to teach the topic rather than a prescriptive scheme of work.

Many of the starter activities are designed to get the students engaging with mathematics and are often only loosely connected with the body of the lesson. Often they are activities which are designed to refresh a fundamental skill required to achieve success in the topic(s) covered (for example, calculation skills). They are suggestions and can be modified or interchanged according to the needs of the specific group of students.

The main lesson commentary is designed to guide you through the topic(s), suggesting a possible order of teaching and picking up on certain key points which should be noted. These notes are not prescriptive and may often suggest activities that are not appropriate to either the teaching style of the teacher or the learning styles of the students. Flexibility of approach is to be expected. Suggested consolidation activities and/or extension activities are also noted where relevant.

The exercise commentary guides you through the various exercises associated with the topic. It suggests the types of question students of differing abilities should be attempting. Key misconceptions are also addressed. The nature and extent of the textbook exercises allow flexibility when students are consolidating, and a differentiated approach is encouraged.

The plenary activities are designed to summarise learning, but again these are not prescriptive. Card-matching activities, pairs and group work may not be appropriate, and a simple question and answer session or short test can be used instead. The aim is to suggest possible activities that might engage the learners in a different way.

There are many references to ‘mass response’ tools such as mini-whiteboards and response cards. Many schools do not have access to this type of equipment and therefore alternative methods have also been suggested, such as the students writing the answers in exercise books and checking them at the end.

There are references to the use of overhead projectors, interactive whiteboards, and other computer equipment. Not all schools will have access to these facilities, so all of the activities which suggest them can be easily modified to use pencil or prepared worksheets. Suggestions on resources:

- Response cards such as ‘True’ or ‘False’ can easily be made in class by the students by taking a piece of stiff card (A5 size or equivalent) and putting a ‘T’ on one side and an ‘F’ on the other. They work even better if the ‘T’ is one colour and the ‘F’ a different colour.
- Card-matching activities can be prepared beforehand and laminated (if facilities exist). They can be placed into a ‘bank’ of resources for use by other teachers and in future years.
- Mini-whiteboards can also be made by laminating stiff card, but dry-wipe pens and erasers will still be needed. Mini-chalkboards make an excellent alternative if the school has access to them.
- Scientific calculators should be available for all students following this course but if they are not, expected approximations to π are given and trigonometrical tables should be made available at the appropriate points.

The online components are designed to supplement and support the teaching of this course. There are supplementary worksheets for use at certain points during the course, practice papers, sample worked solutions and a glossary of key terms.

Access your support website at www.oxfordsecondary.com/9780198428077
Candidates can be entered for one of two different curriculums. The table below indicates the method of assessment for both the core and extended curriculum:

### Syllabus 0580

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<th>Extended curriculum (grades available A*–E):</th>
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</thead>
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<tr>
<td><strong>Paper 1, Short-answer questions</strong></td>
<td><strong>Paper 2, Short-answer questions</strong></td>
</tr>
<tr>
<td>1 hour, 35% of total marks</td>
<td>1 hour 30 minutes, 35% of total marks</td>
</tr>
<tr>
<td><strong>Paper 3, Structured questions</strong></td>
<td><strong>Paper 4, Structured questions</strong></td>
</tr>
<tr>
<td>2 hours, 65% of total marks</td>
<td>2 hours 30 minutes, 65% of total marks</td>
</tr>
</tbody>
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**Notes**

- Candidates should have an electronic calculator for all papers. Algebraic or graphical calculators are not permitted. Three significant figures will be required in answers except where otherwise stated.
- Candidates should use the value of \( \pi \) from their calculators if their calculator provides this. Otherwise, they should use the value of 3.142 given on the front page of the question paper only.
- Tracing paper may be used as an additional material for each of the written papers.
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CHAPTER 1
NUMBER

Lesson 1 – Decimals

Textbook pages 2–3

Expected prior knowledge Students should, at this level, have a working knowledge of the rules of arithmetic when applied to whole numbers and decimals fractions. This lesson provides a good opportunity to revise these methods.

Objectives
E1.8: Use the four rules for calculations with decimals, including correct ordering of operations and use of brackets.

Starter
Give students a range of questions where they are asked to multiply and divide by powers of 10. This could be done as a question and answer session or they could write the answers in their books for checking at the end.

Lesson commentary
• Students should be familiar with the basic rules of arithmetic and the idea of decimal place value. Use this as a starting point to demonstrate the methods for decimal calculations. Use a range of examples.

• Include examples of decimal addition and subtraction as well as multiplication and division, emphasising all the while the need to check the size of the answer relative to the numbers in the question. Incorrectly positioning the decimal point in the final answer is a common problem which can be overcome by checking the size of the answer. Students should be advised to first estimate the answer by approximating the numbers before comparing this to their actual answer. Simple arithmetical mistakes cannot be identified in this way but the checking of answers on a calculator could be encouraged.

• As an extension exercise, students could be encouraged to set their own decimal arithmetic questions for a partner to solve (ensuring that they can solve them first).

• If necessary, a second lesson can be used to provide students with additional time to consolidate this work.

Plenary
Give the students a starting number and then a series of operations involving decimal numbers. Students then work out each individual step before recording the final answer. An example could be:

4.2 × 3 – 1.3 + 6.7 ÷ 1.5 (= 12).

Exercise commentary
Exercise 1 provides routine practice in applying the rules of arithmetic to decimals. Questions 32, 33 and 35 to 40 require the correct use of the rules of BIDMAS so ensure students take this into account (students should have a working knowledge of the rules of BIDMAS from previous work but if they do not, these questions could be left out).

Exercise 2 is a problem-solving exercise which can be done at this point for interest or set as extension work or homework. Alternatively, a further lesson can be used to enable students to work on these applied problems as a class.

The support worksheet for Chapter 1 may be useful to support lessons 2, 7, 8 and 9.

The challenge worksheet for Chapter 1 may be useful to support lesson 4.
Lesson 2 – Fractions

Textbook pages 4–7

Expected prior knowledge Students should be familiar with simple equivalences and the methods for carrying out arithmetic with fractions.

Objectives

E1.5: Use the language and notation of simple vulgar and decimal fractions and percentages in appropriate contexts. Recognise equivalence and convert between these forms.

E1.6: Order quantities by magnitude.

E1.8: Use the four rules for calculations with decimals and fractions (including mixed numbers and improper fractions), including correct ordering of operations and use of brackets.

Starter

Simple fraction, decimal and percentage conversions: Ask students to write down the decimal and percentage equivalents of a half, a quarter, three quarters, a fifth and a tenth. The difficulty could be increased to include a number of hundredths or fractions such as one third and one eighth. They could write these down on mini-whiteboards or in their books.

Lesson commentary

● Assuming students are familiar with arithmetic involving fractions, ask them to write down a description of the methods for adding, subtracting, multiplying and dividing fractions as if they were writing a list of instructions for a classmate. Ask them to provide a worked example of each.

● Students should also be familiar with simple fraction and decimal equivalents (see starter activity) but should then be reminded of the more general methods for converting between forms.

● Demonstrate each of the techniques using a range of suitable examples taken from the textbook or other source.

● Decimal to fraction conversions can be done by writing the decimal over the appropriate power of ten and then simplifying. Provide a number of examples of this and then allow students time to consolidate if necessary.

● When converting from fractions to decimals, emphasise that the numerator goes inside the division and the denominator outside. Add extra zeros as appropriate and retain the position of the decimal point. Students should then treat the division the same as any other (short) division problem.

● Allow time for the students to practise converting between the forms. A second lesson can be used to ensure all students have fully consolidated both fraction arithmetic and fraction/decimal conversions.

● Model converting from recurring decimals to fractions using the examples in the textbook or elsewhere. Emphasise the importance of multiplying by the right power of 10 to shift the digits in order to get cancelling out on subtraction.

Plenary

A decimal/fraction card-matching activity could be given to students to complete in pairs. One pair could then check agreement with another pair and whole-class feedback can be given at the end. An alternative approach would be to give the students a prepared worksheet with lists of decimals and fractions that they have to match.

Exercise commentary

Questions 1 to 15 in exercise 3 are basic practice questions while questions 16 to 21 introduce mixed numbers. Ensure methods are consistent here. Questions 22 to 25 require the correct application of the rules of BIDMAS.

Questions 26 and 27 require students to use the idea of equivalent fractions to first order fractions and then find mid-way fractions. Questions 28 to 30 are problem-solving questions which could be given as extension work.

Exercise 4 looks at converting between decimals and fractions. The first 40 examples are basic practice while questions 41 to 48 ask students to add, subtract, multiply or divide when the numbers are in different forms. Encourage students to work in whichever form they prefer rather than insisting on one particular approach. Questions 49 to 52 require students to order numbers using the idea of equivalence.

The exercise finishes with questions where students convert from recurring decimals to fractions. Ensure they are multiplying by the correct power of 10.
Lesson 3 – Number facts

Textbook pages 7–10

Expected prior knowledge Students should be familiar with finding factors and multiples of small numbers (see starter activity).

Objectives
E1.1: Identify and use natural numbers, integers, prime numbers, common factors and common multiples, rational and irrational numbers.

Starter
Multiple and factor identification: Ask students to write down seven or eight numbers less than 50. Generate random numbers between 1 and 50 and tell students to cross their chosen numbers off if they are either a factor or a multiple of the random number. The first student to cross all their numbers off wins the exercise.

Lesson commentary
- Prime numbers under 100 can be revised using Eratosthenes’ sieve on a 100 number square. An interactive version on a 400 number square can be found at http://www.hbmeyer.de/eratosiv.htm (link correct at time of publication). Printed number squares can be given to the students to help them.
- Discuss the generalised method for testing for ‘primeness’ (divide by every prime less than the square root of the given number) and ask students to check some examples using standard divisibility rules and a calculator (for bigger numbers).
- In the second part of the lesson (or in the next lesson, if necessary) explain the different types of number which there could be and provide examples (natural numbers, integers, rational numbers). Discuss the meaning of irrational numbers (numbers that cannot be written as a ratio of two integers). Provide examples and ask students to classify a list of numbers provided. (This could be projected onto a screen or given as a worksheet. Examples could also be taken directly from the textbook.)
- Students might be familiar with writing numbers as the product of prime factors but it is worth working through an example and linking this to finding HCFs and LCMs. Students can then practise these skills with additional examples.

Exercise commentary
Exercise 5 looks at classifying primes, listing multiples and factors and finding simple common multiples, as well as prime factorisation and finding HCFs and LCMs. Question 7 could be used as an extension activity.
Exercise 6 asks students to first classify and then work with rational and irrational numbers. Emphasise the need to give reasons for their answers throughout.

Plenary
Ask students to give examples of numbers with certain features. Examples could include ‘a rational number with a denominator of 3’, ‘an irrational number between 5 and 7’ or ‘a number with exactly three factors’.
Lesson 4 – Sequences

Textbook pages 10–14

Expected prior knowledge Students should have met sequences beforehand so the emphasis should be on generating $n$th term formulae.

Objectives
E2.7: Continue a given number sequence. Recognise patterns in sequences including the term-to-term rule and relationships between different sequences. Find the $n$th term of sequences.

Starter
Simple addition/multiplication exercises – brain training: Pick a start number. Get students to successively add, subtract and multiply the start number. Try 10 operations and ask students to record the final answer.

Lesson commentary
- Students should be able to work out the next few terms of simple sequences already and a quick question and answer session will enable them to practise this. Start with simple examples such as 1, 3, 5, 7, ... and then extend this to include multiplicative sequences (1, 3, 9, ..., for example). Increase the complexity so that they are dealing with quadratic sequences such as 2, 5, 10, 17, ... and if appropriate, Fibonacci-style sequences such as 2, 3, 5, 8, ....
- Ask students to generate their own sequence and challenge a partner to continue it. Emphasise the need for a constant rule between terms. Students could then work in pairs to challenge another pair to spot the rule of a more complicated sequence. For example, one generated by multiplying successive terms by 2 and adding 3 (e.g. 1, 5, 13, 29, ...).
- The main part of the lesson should be about finding and identifying the $n$th term formulae for sequences. Using a series of examples, encourage students to think of the common difference as the multiplier for $n$ and then adjust accordingly. For example, 3, 7, 11, ... will be of the form $4n + ...$, what is the ...? (in this case, ‘–1’).
- Students can be given further examples and asked to practise generating the $n$th term formulae for a range of linear sequences.
- Sequences which arise out of ‘matchstick’ patterns or other geometrical arrangements can be introduced. Ask students to identify the pattern and continue it before tabulating and working out the $n$th term formulae. At this point, they could be encouraged to generate their own patterns and deduce the formulae for them.
- Non-linear patterns can be introduced as appropriate. Students should be encouraged to solve these by comparison to $n^2$ or $n^3$ by using a method of subtraction as shown in the example. Exponential sequences should be identified and solved by inspection.

Plenary
Give students a set of sequences and a set of rules. Ask them to match the sequences with the rules and then continue the sequences for two more terms. These could be provided using prepared worksheets, matching cards or projected onto a screen.

Exercise commentary
Exercise 7 asks students to simply write down the next terms for a variety of sequences and to state the term-to-term rule for some of them. It could be done as a whole-class question and answer exercise.

Exercise 8 looks at the idea of an $n$th term formula. In questions 1 and 2, students match sequences with given formulae while in questions 3 and 4 they are asked to generate sequences from given formulae.

In Exercise 9, students are expected to work out the $n$th term formulae for themselves.

Questions 1 to 10 in exercise 10 all relate directly to $n^2$. Questions 11 to 13 relate to $n^3$ and there is a hint to guide students. Likewise, a hint is given for questions 14 and 15 as these sequences are exponential in nature.
Lesson 5 – Rounding and estimation

Textbook pages 14, 18–19

Expected prior knowledge It is expected that students will be familiar with applying standard rounding conventions (see starter activity) but further revision of these methods may be necessary.

Objectives
E1.9: Make estimates of numbers, quantities and lengths, give approximations to specified number of significant figures and decimal places and round off answers to reasonable accuracy in the context of a given problem.

Starter
Give the students a quick question and answer quiz where given numbers are rounded to the nearest 10, one decimal place, etc.

Lesson commentary
● Students should be familiar with the concept of rounding numbers to the nearest ..., a given number of decimal places and a given number of significant figures. Revise these ideas as necessary, either using the exercise in the textbook or through a more extended question and answer session building on from the starter activity.

● Key problems might include truncating large numbers when rounding to a certain number of significant figures (23 450 rounds to 23 000 (2 s.f.) rather than just ‘23’), and confusion about the place of zeros in both rounding conventions. All zeros are counted when rounding to a number of decimal places, whereas only the zeros which come after the first non-zero digit are counted when rounding to a number of significant figures.

● One key area where rounding is important is in estimating the size of the answers to calculations. Students should be able to make sensible estimates for the value of certain calculations. Provide them with some examples of this type and ask them to round each number to one significant figure before working out the estimate. They can then check the accuracy of their calculations by working out the actual answer on a calculator. As an extension, ask them to also work out the percentage error between their answer and the actual answer.

Exercise commentary
Exercise 11 is a rounding exercise and could be done as a whole-class question and answer session if appropriate.

Exercise 14 is an exercise in estimation. Students should be advised that rounding to one significant figure is the accepted convention when estimating. After completing their estimates, students could check the answers using a calculator.

Plenary
A calculation quiz could be given where students have to estimate the answers to a number of questions, for example 3.13 × 4.98.
Lesson 6 – Measurements and bounds

Textbook pages 15–18

Objectives
E1.10: Give appropriate upper and lower bounds for data given to a specified accuracy. Obtain appropriate upper and lower bounds to solutions of simple problems given data to a specified accuracy.

Starter
Ask students to give examples of numbers which would be rounded to a given degree of accuracy. An example could be ‘give numbers which are rounded to 4.3 (to one decimal place)’ and expect responses such as 4.27 and 4.32.

Lesson commentary
● Following on from the previous lesson, a second aspect of this work on approximation and estimation is the idea of bounds for measurements. Emphasise that measurements are necessarily rounded to the nearest ... and that the true value could lie within half a unit either side (due to the conventions of rounding). Students should have no problem accepting the lower bound as 0.5 below but they may take some convincing that the upper bound is 0.5 above (‘but sir, that value rounds up!’). Discuss the idea that 0.4 can be ‘beaten’, as can 0.49, as can 0.499, etc. and therefore it makes sense to use 0.5 as the upper bound with the proviso that the actual value is strictly less than.
● Provide students with examples of calculations involving bounds and discuss the methods for getting the upper and lower bounds for the resulting sum. Emphasise that for subtraction and division, the lower bound is found by ‘lower bound (operation) upper bound’ and the upper bound is found by ‘upper bound (operation) lower bound’.

Exercise commentary
Exercises 12 and 13 test the students’ understanding of bounds in measurement and the questions in exercise 13 deal with calculations involving bounds.

Plenary
In order to effectively assess students’ progress, ask them to work out further simple sums involving bounds and write the answers in their exercise books for checking at the end.
Lessons 7 and 8 – Standard form

Textbook pages 19–21

Objectives
E1.7: Use the standard form $A \times 10^n$ where $n$ is a positive or negative integer, and $1 \leq A < 10$.

Starter
Ask the students to write down the answers to, or provide via question and answer, a series of ‘powers of 10’ questions. If you allow the use of calculators, you could include negative powers as well.

Exercise commentary
Exercise 15 provides routine examples where students put numbers into standard form. Questions 19 to 24 put the use of standard form into context and question 25 leads into the next exercise by giving students a problem involving division. The first 12 questions in exercise 16 are all calculation questions (use of calculators at your discretion). Question 16 links to the idea of bounds from the previous lesson while questions 17 and 18 provide applied calculations in standard form. Question 19 on googols could be reserved for extending able students.

Lesson commentary
• Explain that very large numbers or very small numbers which have lots of zeros after or before are quite difficult to manipulate. Provide examples where these numbers might be encountered (space measurements, atomic masses/sizes, etc.) and explain that there is a way that these numbers can be represented to make them more manageable.

• Introduce the idea of ‘standard form’ through some simple examples, both large and small, and refer back to the starter activity for powers of 10. Ensure students are comfortable with the value of $A$ being between 1 and 10 (could be decimal but not equal to 10). Explain how the power is determined (move the digits until there is single digit in front of the decimal point).

• Give them a series of examples of numbers written in standard form and ask them to convert them to normal numbers. Likewise, get them to work the other way. Address any problems with this conversion process as you go along and then explain how you use a calculator to work with numbers given in standard form (usually using the ‘EXP’ key).

• Students can then have a go at some more problem-solving type questions which use numbers given in standard form before introducing calculations involving numbers in standard form. Model multiplication and division examples and discuss the use of a calculator in this context.

• The second lesson can be used for further consolidation and/or problem-solving with numbers given in standard form.

Plenary
In order to check the students’ understanding of this topic, ask them to do further conversions and/or calculations using standard form. This could take the form of a quiz or short test.
Lesson 9 – Ratio

Textbook pages 21–23

Expected prior knowledge At this level, students should be familiar with using simple ratios so this lesson provides a good opportunity to revise some of the key applications of ratio.

Objectives

E1.11: Demonstrate an understanding of ratio and proportion.
Increase and decrease a quantity by a given ratio.

Starter

Give the students questions such as ‘divide 100 by 10 and multiply the answer by 3’, ‘divide 360 by 12 and multiply the answer by 7’, etc. Increase the complexity of the questions as appropriate.

Lesson commentary

- Students should be familiar with using simple ratios to divide quantities up but applying them to more complicated problems may need careful revision. Provide the class with some simple examples involving mixing paint, cordial and water mixes, etc. and ask them to respond by writing the answers in their exercise books or by whole class question and answer. How much red paint would be needed to make 15 litres of pink paint if red and white are mixed in the ratio 2 : 3, for example?

- Provide further examples where one of the sub-quantities are given and students are expected to work out the original amount that was divided up. For example, if John receives $70 and the money was divided in the ratio 7 : 3, how much money was originally divided up?

- Students should also be given examples where ratios are to be simplified into the form 1 : n (or n : 1). Problems of this type can be presented as a simple exercise in division and may require the use of a calculator.

- Allow students time to consolidate this work through the use of textbook exercises or other similar sets of questions.

Exercise commentary

Exercise 17 tests students on basic ratio techniques such as dividing into and writing ratios in the form 1 : n. Questions 26 to 30 are more difficult and could be used as extension questions.

Plenary

To assess the students at the end of the lesson, provide them with a summary question and answer session where they solve a variety of ratio problems quickly. Allow the use of calculators if appropriate.
Lesson 10 – Direct proportion

Textbook pages 23–24

**Objectives**
E1.11: Demonstrate an understanding of proportion.

**Starter**
What is one of ...? Give the students a scenario, for example 8 apples cost 96 cents, and ask them to work out what one apple would cost. Repeat for different situations and include questions which require the use of a calculator.

**Lesson commentary**
- The use of the ‘unitary’ method is a standard approach for direct proportion. Ensure students are happy with the principle used in the starter and then discuss the unitary method more fully. What would happen if you were given the 8-apple scenario but then asked for the price of five rather than one? Going via one apple is clearly the easy way to proceed.
- Provide students with further examples of direct proportion where the unitary method can be used. Students can work together on problems of this type, if appropriate.
- Ask students to think of examples of direct proportion that they may encounter in the real-world. Purchasing items from a store such as fruit and vegetables is a common example but examples could also include buying gas and currency exchanges.

**Exercise commentary**
Exercise 18 contains a mixture of questions on both direct and inverse proportion. Questions 1, 2, 4, 5, 6, 7, 9, 13, 14 and 15 are good examples of direct proportion. Exercises 19 and 20 look at the application of proportion to exchange rates and map scales. These could be introduced at this point or set aside for future consolidation work.

**Plenary**
Give the students a situation such as ‘9 pencils cost $1.08’. Ask them to quickly write down the answers to a number of follow-up questions such as ‘How much would 6 cost?’, ‘How much would 10 cost?’ and ‘How much would 100 cost?’
**Lessons 11 and 12 – Inverse proportion**

Textbook pages 23–27

**Objectives**

E1.11: Demonstrate an understanding of proportion.

E1.15: Calculate using money and convert from one currency to another.

**Starter**

Start with a number, say 100, expressed as 100 × 1. Ask students to write down the multiplication sum that would make 100 if the ‘100’ part of the multiplication was successively divided by two. Challenge the students to take the sequence as far as they can. The first few terms would be 50 × 2, 25 × 4, 12.5 × 8, etc.

**Lesson commentary**

- Students often find problems of inverse proportion difficult to understand. A common problem is with the recognition of when two quantities are inversely related so introduce the topic through a range of examples including men digging holes and the relationship between speed and time. Encourage active discussion of the problems as students work on them.

- Ask students if they can think of situations in the real-world that show an inversely proportional relationship. Further examples could include the relationship between density and volume or decorators painting a house.

- Allow sufficient time for students to consolidate this work and, if appropriate, begin to mix up the types of question to include examples of direct proportion as well. This will encourage students to think carefully about the problems rather than just assume they are of a single type.

- The second lesson can be used for consolidating this work on proportion further and for looking at exchange rate applications and map scales (see exercise commentary).

**Exercise commentary**

Exercise 18 contains a mixture of questions on both direct and inverse proportion. Questions 3, 8, 10, 11, 12 and 18 are good examples of inverse proportion.

Exercises 19 and 20 look at the application of proportion to exchange rates and map scales. These can be taught separately or incorporated within the lessons on this topic.

Exercise 21 extends the idea of map scales to look at area conversions and could be reserved as an extension activity for more able students.

**Plenary**

In order to test the students’ understanding of the work on proportion, a mixed question and answer activity can be carried out. Provide the students with a number of relatively straightforward examples of both direct and inverse proportion and ask them to either write the answers down in their exercise books or respond orally.
Lessons 13, 14 and 15 – Percentages

Textbook pages 28–32

Expected prior knowledge Students should be familiar with the equivalence between decimals and percentages (see starter activity) and they should also be able to calculate simple percentages of amounts. This series of lessons provides an opportunity to revise these ideas before developing them further.

Objectives
E1.12: Calculate a given percentage of a quantity. Express one quantity as a percentage of another. Calculate percentage increase or decrease. Carry out calculations involving reverse percentages.

Starter
Fraction, decimal and percentage conversions: Give the students a series of questions asking them to convert between commonly used fractions, decimals and percentages.

Lesson commentary
- Discuss the possible calculator and non-calculator methods for working out the percentage of an amount. These could include breaking the percentage down into 10%, 5%, 1%, etc. or via the unitary method of dividing by 100. Discuss the idea of a multiplier (the decimal equivalent of the percentage: 20% = 0.2, for example).
- Discuss the link to percentage increase and decrease. ‘If I can work out a percentage of an amount, then I can just add it or subtract it to work out a percentage increase or decrease.’
- Using a simple example, demonstrate the technique for finding the percentage increase (actual increase divided by original amount, multiplied by 100). Students will have to do this without a calculator for simple examples so encourage them to efficiently manipulate the fractions. Students will use a calculator for harder examples.
- Ask the students how they might modify the method to calculate percentage decrease. Provide them with a few examples to try (both with and without a calculator) and then they can do further practice as necessary. Link to profit and word-based problems as appropriate.

Exercise commentary
The questions in exercise 22 could be used as a starter activity or done via whole-class question and answer. Exercises 23 and 24 deal with the full range of percentage calculations and suitable questions can be selected from either to consolidate the techniques at different points throughout this series of lessons. The key ideas behind each one are the same and students should be encouraged to think carefully about each question rather than making assumptions that a block of questions are all of the same type. The later questions in both exercises could be used to extend stronger students.
• Ask students to come up with real-life examples where percentage increase and decrease are used (in-store sales, tax, service charges, etc.) They could set their own problems and challenge a partner to work out the new values.

• Link all of this into the idea of the multiplier and percentage change. Establish the basic rule ‘original value \times percentage multiplier = new value’. Model examples where the new value is to be found and also where the original value is to be found (reverse percentage change). A common mistake when finding the original value is to find the percentage of the new value and subtract it. For example, ‘x is increased by 15% and you get y, therefore to get back to x, find 15% of y and subtract it’. Emphasise that x is always ‘100%’ and if students continue to get these types of problems wrong, encourage them to check their solutions afterwards.

• There are a number of key percentage calculations that students are expected to be able to carry out. These include calculating the percentage of an amount, increasing and decreasing by a percentage, calculating percentage increase and decrease and calculating using reverse percentage change. This series of lessons is designed to cover all of these and provide students with the opportunity to practise them. It is at the discretion of the teacher the order in which they are taught but the lesson commentary provides a suggested order. Additional starter and plenary activities can be incorporated as appropriate.

**Plenary**

Provide students with questions and worked solutions which contain mistakes and ask them to study the solutions and identify the mistakes, correcting them as appropriate. This could be done as a paired activity, and be used to address some of the key misconceptions noted above.
Lesson 16 – Interest and tax

Textbook pages 32–35

Objectives
E1.16: Use given data to solve problems on personal and household finance involving earnings, simple interest and compound interest.

Starter
Ask students to use a calculator and record their answers on a mini-whiteboard or in their exercise book, provide them quick ‘multiplier’ questions. Examples could include 25 × 1.05 and 124 × 1.12.

Lesson commentary
- Discuss where students might encounter the concept of interest (bank loans, savings, mortgages, etc.) and set the problem ‘would you rather have a rate of 5% on your savings and get the money earned back every year, or a rate of 4% which is automatically reinvested?’ ‘How much would you have in each case after 1 year, 2 years, etc?’
- Discuss the link between percentage multipliers and the method for working out interest. Ask students to use a table to generate the growth of money in an account under both interest schemes. At what point does the 4% compound interest overtake the 5% simple interest?
- Students could then be given further examples where they have to work with both simple and compound interest.
- At Extended level, students should be able to work with both simple and compound interest in the context of a single lesson but if necessary, the lesson could be divided so that each type of interest is dealt with separately.
- Students should be familiar with using the compound interest formula and they need to learn it. Encourage them to use it when possible and get into the habit of inputting the calculation into their calculators in one go.
- The short exercise on tax which follows the work on interest could be set as homework for the students. They should follow the example and then try to solve the problems in the exercise. Alternatively, this work can be introduced as part of the second lesson on interest.

Plenary
Give the students three minutes to work out whether they would rather have 10% simple interest or 7% compound interest if they invested $100 over 5 years (10% simple interest is $150, 7% compound interest is $140.26 so simple interest is better). As an extension task, you could ask students to work out after how many years they would have more under the compound interest scheme.
Lesson 17 – Speed, distance and time, other rates

Textbook pages 36–39

Objectives
E1.11: Calculate average speed. Use common measures of rate.

Starter
Give students a quick question and answer activity testing simple division. Examples could include ‘What is 120 ÷ 3?’ and ‘What is 240 ÷ 6?’

Lesson commentary
• Students should be familiar with idea of average speed but may not have dealt with it formally. Discuss how you might interpret speed/distance/time information when given it for a scenario involving travelling to a nearby city. For example, ‘If I start at home and travel a distance of 120 kilometres, how long would it take me at an average speed of 50 kilometres per hour?’ Discuss the idea of using division to solve such a problem.
• What if I know the speed and the time? How could I use this information to work out the total distance of my journey?
• Develop the idea of the speed/distance/time ‘triangle’ and explain the ‘cover up’ method for working out the unknown quantity.
• Provide students with examples to practise (encourage collaborative working) and then consider an example where the information is given in metres and seconds and discuss how such a speed might be calculated in kilometres per hour. Is it easier to convert the units first or convert the final answer? Allow students flexibility to choose their own method.
• Explain that there are other rates that students might encounter such as litres per minute when filling a bath with water. Focus on the word per as indicating division of the two quantities: amount of water divided by the time gives the rate.

Exercise commentary
The questions in exercise 28 fall into two categories: finding speed, distance or time from the information given (using the ‘triangle’) and converting between units of speed. Questions 5, 6 and 7 require students to interpret multi-stage problems and work out overall average speed. Questions 8 onwards could be set aside for extending more able students as they are much more involved.

In question 1 in exercise 29 students are finding the rate, whereas in question 2 they have to find the time taken so a rearrangement of the ‘formula’ will be required. Questions 3 to 6 are of a problem-solving nature. Encourage students to write down all the information they know before proceeding.

Plenary
Ask students to write down approximately how far they live from school and the approximate time it takes them to get to school in the morning. Discuss modes of transport and get them to work out an estimate of the speed at which they travel to school.

Exercises 30, 31 and 32 on pages 40–42 are mixed exercises testing students’ practical application of number skills and could be set as homework or used in a consolidation lesson.
Lesson 18 – Using a calculator

Textbook pages 42–47

Objectives
E1.13: Use a calculator efficiently. Apply appropriate checks of accuracy.

Starter
Give the students a start number and then give them a series of operations to perform on a calculator. Make sure that they press the ‘equals’ key after each one. Ask for a volunteer to give their final answer and check that this matches the final answer of the rest of the class.

Lesson commentary

- Since there are many different makes and models of calculator on the market, it is difficult to teach calculator skills but there are several useful hints and tips that you can give to students to help them use their calculators efficiently.

- Make sure that they know where the key buttons are such as ‘\(x^2\)’ and ‘\(\sqrt{}\)’. Also make sure they can find the ‘power’ button (usually a ‘\(^\)’ or ‘\(\times x^y\)’).

- Make sure that they are confident in their understanding of the rules of BIDMAS and that they take these into account when keying in sums to the calculator. As a general rule, ‘if in doubt put some brackets in’ and ‘always bracket numerators and denominators when dealing with fractions’.

- Make sure they are aware of the function of the ‘ANS’ key when doing multi-stage calculations (the ‘memory’ functions are not as useful as they once were since the introduction of DAL (Direct Algebraic Logic) calculators and the ‘invention’ of the ‘ANS’ key).

- Always put negative numbers in brackets (and use the ‘(–)’ key rather than ‘take away’).

- Finally, encourage them to experiment and have the answers available to help them to check that what they are keying in correctly gives the answer.

Exercise commentary

Exercises provide plenty of calculator practice. Ensure students take into account all of the advice given and it is also worth emphasising the need to have an idea of the size of the answer beforehand.

Exercise 35 emphasises the need to check calculations for sense as well as the usefulness of estimating the answers beforehand.

Plenary

Further calculations with the answers written in exercise books could be used or students could challenge each other to come up with an ‘interesting’ sum that has a particular given answer.

The revision and examination-style exercises can be used for further practice as appropriate.
CHAPTER 2
ALGEBRA 1

Lesson 1 – Directed numbers

Textbook pages 56–59

Expected prior knowledge Students should be familiar with the arithmetic of negative numbers so this lesson provides a good opportunity to revise these ideas.

Objectives
E1.4: Use directed numbers in practical situations.
E2.2: Manipulate directed numbers.

Starter
Number line trail: Give the students a starting number, for example 3, and then ask them to work out the finishing number when a series of addition and subtraction sums are applied successively to it. For example: + 3 – 7 + 5 – 6 + 2 – 5 + 1 (= −7). Repeat the process for a different starting number and a different string of calculations. Include multiplication and division if appropriate.

Lesson commentary
● Students may need reminding of the key rules when adding/subtracting or multiplying/dividing negatives. Ensure they do not get the rules mixed up (adding two negative numbers does not give a positive answer, for example). Demonstrate examples of each type of calculation if necessary.
● Put negative numbers in context by discussing real-life situations such as temperature, depths of the ocean, etc. and ask students to try answering some examples.
● Thoroughly revise calculating with directed numbers through appropriate examples and question practice. There are lots of questions in the exercises for this lesson but able students will not need to do all of them. Use an appropriate approach such as ‘do just the odd numbers’ or ‘just do the questions in the first column’ to ensure students answer enough questions to consolidate the topic effectively.

Plenary
Quick negative number arithmetic: This could be done either using mini-whiteboards or as a short test to assess understanding.
Lessons 2 and 3 – Formulae

Textbook pages 59–63

Objectives
E2.1: Use letters to express generalised numbers and express basic arithmetic processes algebraically. Substitute numbers for words and letters in complicated formulae.

Starter
Provide students with a simple algebraic expression (\(2a + b\), for example) and then generate values for the unknowns (using different polyhedral dice, for example). Ask students to apply the substitutions and write the answers in their exercise books.

Lesson commentary
- Explain that students will be substituting numbers (including negatives) into a succession of formulae, some of which are simply abstract algebraic expressions and some of which are real-life formulae in context. Provide them with some examples (harder than the starter expression) and ask them to work together to check that they are correctly substituting the numbers and arriving at the correct answer.
- Increase the complexity of the expressions, introduce negative numbers and possibly numbers given in standard form (see exercise 5 question 6).
- Ask students to produce their own expressions (sensible ones) and challenge a partner to substitute a series of numbers into them.
- Students could be given a further activity where a series of expressions are given along with the answers when a certain set of numbers are substituted in. They should attempt to match the expressions with the values.
- The second lesson will enable students to have sufficient time to consolidate this work fully. There are a lot of questions in the exercises for this section so encourage students to attempt a variety of questions at different levels of complexity, rather than just doing 20 or 30 of the same type.

Exercise commentary
Exercise 5 requires students to substitute numbers into formulae in context and could be included in the main body of the lesson.
Questions 8 to 11 develop the idea of writing simple formulae from information given.
Exercises 6, 7 and 8 provide many examples of abstract expressions and negative substitutions and these questions can be used as appropriate. The examples in exercise 6 are standard linear expressions and most students should be able to attempt these. The examples in exercise 7 incorporate powers and brackets and some students may need guidance on these. Emphasise that a negative number squared is positive.
Exercise 8 can be used as a challenge exercise for more able students as the expressions are significantly more complex than in the previous two exercises.

Plenary
Order the expressions. Provide students with four expressions and then ask them to order them by size (increasing) upon substituting a given set of numbers. Repeat for different sets of numbers.
Lesson 4 – Brackets and simplifying

Textbook pages 64–65

Objectives
E2.2: Expand products of algebraic expressions.

Starter
Multiplication of algebraic terms: Ask the students a series of questions such as ‘What is $3 \times 2x$?’, ‘What is $4 \times 5y$?’ and ‘What is $7 \times 2z$?’

Lesson commentary
● Most students will be familiar with expanding simple brackets but they will certainly benefit from further practice. Demonstrate a number of examples, emphasising the need to multiply both of the terms inside the bracket by the term outside. Encourage them to draw arrows over the top of the bracket to indicate these two actions. Provide further examples for practice and encourage them to check their answers with a partner. Provide examples with negative terms outside and also examples with more than just a single bracket.

● Move on to collecting like terms and introduce brackets in this context as well. Students should be able to combine the two skills and include squares and negative terms.

● Students can then be asked to complete a series of questions from the exercise in order to consolidate this work fully.

Exercise commentary
Exercise 9 provides lots of examples of algebraic simplification. The first 24 questions are on simplification only while questions 25 to 48 introduce brackets in this context. More able students could begin on these directly.

Plenary
Provide students with a card-matching activity where they are given a series of expressions and their simplified equivalents. Ask them to work in pairs to match them up. Alternatively, ask the students further questions on brackets and simplifying and instruct them to write the answers in their exercise books or respond orally.
Lessons 5 and 6 – Two brackets

Textbook pages 65–67

Objectives
E2.2: Expand products of algebraic expressions.

Starter
‘Right or wrong?’ Provide students with a number of pairs of expressions and ask them to say whether they are equivalent or not. For example, ‘Is $3(x + y)$ equivalent to $3x + 3y$?’ (yes) and ‘Is $–2(2x + 3y)$ equivalent to $–4x + 6y$?’ (no).

Lesson commentary
- There are many methods available to students when expanding two brackets. The method used in the textbook is one alternative but there is also the grid method and FOIL (first, outside, inside, last). Demonstrate some examples using the different methods and ask students to use the one that they prefer for further examples. Students could then be grouped according to this choice and asked to work together to solve further expansion problems.
- Discuss the idea of a perfect square (a repeated bracket) and ensure that students write these types of problem as a product of two brackets before removing them. The common mistake when expanding perfect squares is to simply square the first term and square the second rather than considering them as the product of two separate brackets.
- Standard pairs of brackets and perfect squares could be dealt with in two separate lessons if appropriate. Alternatively, both types could be introduced in a single lesson and the second lesson used to provide time for consolidating this work on expanding pairs of brackets.

Exercise commentary
Exercise 10 provides routine practice in expanding pairs of brackets. From question 21 onwards, there is a third component in front of the two brackets which needs to be taken into account at the end. Discourage ‘short-cutting’ of steps.
Perfect squares are introduced in exercise 11. Questions 15 onwards and all of exercise 12 require two expansions followed by simplification of terms. These questions could be set aside as extension problems for more able students.

Plenary
Provide students with a number of worked solutions to expansion problems which contain one or more errors. Ask them to work in pairs to identify the mistakes and provide a correct solution to the problem. Alternatively, give the students further expansion examples and ask them to write the answers in their exercise books or respond orally.
Lessons 7, 8 and 9 – Linear equations

Textbook pages 67–72

Expected prior knowledge Students should be familiar with solving simple linear equations. These lessons can consolidate and develop this to include equations with a more complex structure.

Objectives
E2.5: Derive and solve simple linear equations in one unknown.

Starter
‘Think of a number’: Tell the students that you are thinking of a number and that when you, say: ‘add 2 and double it’, you get, for example, 10. What number did you think of? Responses can be via mini-whiteboards, written into exercise books or given orally by a member of the class.

Lesson commentary
- Students should be able to solve simple equations of the form $ax + b = c$. An introductory activity can be used to establish this by giving a sequence of examples and asking students to work out the values of the unknowns and write down their responses. Students may feel less confident with ‘take away’ and ‘divide’ and negative or fractional answers can also confuse some students who think the answer must always be a positive integer. Increase the complexity of the examples as appropriate to test this.

- Move on to examples where the unknown is on both sides. Ask students to test the validity of certain answers using ‘true’ or ‘false’. For example, ‘Is $x = 4$ a solution to $3x + 2 = 2x + 6$?’ Discuss the need to ‘balance’ the equation by always doing the same thing to both sides and encourage the collection of the unknowns on the side which has the most. Provide examples as appropriate and then allow students time to consolidate. Again, negatives are likely to cause confusion so encourage students to remove negatives where possible by adding terms to both sides.

- In addition to equations where $x$ appears once or on both sides of an equation, other examples involving brackets, pairs of brackets and equations where $x$ is in the denominator need to be covered. A worked example approach is generally the best way to ensure students develop the methods correctly. Encourage students to develop individual strategies for each type of equation such as removing negatives and fractions at an early stage and simplifying where possible.

Plenary
Provide worked examples of equation-solving which contain errors. Get the students to work in pairs to decipher the solution, find any mistakes and provide corrected solutions. Differentiate the questions as appropriate.

Exercise commentary
Questions 1 to 31 in exercise 13 have a single unknown while the remaining questions are a mixture of two unknowns, fractions, decimals and negatives. More able students should be able to work quickly through some simple examples before moving on to the later questions, while less able students will benefit from carefully working through the first part of the exercise.

Exercise 14 introduces brackets and the questions increase in complexity as they go on. Again, encourage more able students to move on to the later questions.

Pairs of brackets (and perfect squares) are the focus of exercise 15 and students should be encouraged to simplify whenever possible.

In exercise 16, the unknown is often in the denominator and all of the questions involve fractions. Encourage students to remove fractions at an early stage and collect like terms. More able students could be instructed to tackle the later problems in order to provide appropriate challenge.
**Lessons 10 and 11 – Problem-solving using linear equations**

Textbook pages 72–77

**Objectives**
E2.5: Derive and solve simple linear equations in one unknown.

**Starter**
Ask students to write down five numbers between 1 and 20. Roll a die and then perform a simple operation on the number, say, double it or add 7. If the students get the answer, they can cross it off their list.

**Lesson commentary**
- The emphasis here is to interpret the information from a word problem and create an equation which can then be solved by standard methods. Start by providing simple word problems along the lines of ‘think of a number’ and get the students to write down the equation that results. Students may not need to formally solve the equation but they should be encouraged to check that their intuitive solution and their equation do give the same answer.
- Provide students, possibly working in pairs or small groups, with practical problems involving the area and perimeter of rectangles, angle problems, consecutive number sums, etc. These could be provided on prepared worksheets or taken from the exercises. Ask them to write down the equations needed to solve the problems and then to find the value of the unknown. The task can be made different by providing an appropriate selection of questions for each student, pair, or group. Students could then be asked to make up their own problems of this type and challenge others. Emphasise the need to work out their own solutions first to make sure that they are consistent.

**Exercise commentary**
Exercise 17 is a series of typical examples which increase in complexity as they go on. Some students may have difficulty in identifying the ‘unknown’ and collaborative working on questions which provide appropriate challenge should be encouraged.

Exercise 18 is more difficult and these questions could be reserved for more able students working to provide model solutions.

**Plenary**
Provide students with further word problems (read aloud to encourage listening skills) and challenge them to solve the problems in the quickest time.
Lesson 12 – Solving simultaneous equations by substitution

Textbook pages 77–78

Objectives
E2.5: Derive and solve simultaneous linear equations in two unknowns.

Starter
Algebraic rearrangement: Give the students an example such as
\( x + 2y = 4 \) and ask them to make \( x \) the subject. Then ask them to then make \( y \) the subject. Change the example and repeat. This activity will enable students to get some initial practice in rearranging formulae.

Lesson commentary
● The topic of simultaneous equations could be introduced using a classic example of apples and bananas (or equivalent). Suggest the problem that you want to find the cost of one apple and one banana but that when you went into the fruit and vegetable shop, they sold you two apples and three bananas for 64 cents and the next time you went in they sold you four apples and 2 bananas for 80 cents. How much is one apple and one banana? (Apple: 14 cents, banana: 12 cents.)

● One method of solving simultaneous equations is the substitution method where one of the equations is given in (or rearranged into) the form ‘\( y = ... \)’ or ‘\( x = ... \)’. Students need to be comfortable with algebraic manipulation to solve simultaneous equations in this way and the starter activity is designed to check this understanding.
If students struggle with the starter activity, further practice in rearranging formulae of the type should be given.

● Demonstrate a solution using the method of substitution and make sure that the students can follow the algebraic steps. Ask them to complete an example of their own with guidance (possibly working together, if appropriate) and then they can practise further examples.

● This approach could be applied to examples from exercise 21 where simultaneous equations are required to solve a range of problems. Students could be directed to these if they quickly understand the necessary skills.

Exercise commentary
Exercise 19 contains several examples which students need to solve by substitution and these can be used as appropriate for consolidation and further practice.

Plenary
Give the students a further example of a pair of simultaneous equations to be solved by substitution. Allow them two minutes to work out the solution before writing it down. This can be repeated with different examples as time allows.
Lesson 13 – Solving simultaneous equations by elimination

Textbook pages 79–81

Objectives

E2.5: Derive and solve simultaneous linear equations in two unknowns.

Starter

Provide students with an algebraic expression (2x + y or similar) and then provide numbers for the unknowns and ask students to quickly substitute these in to work out the value of the expression. Change the expression and extend to include negative numbers or squared terms as appropriate.

Lesson commentary

- The elimination method is the most common method for solving simultaneous equations. Use the example of apples and bananas from the previous lesson or another suitable example to demonstrate the solution method including the multiplication of the equations to make the (second) term the same. Use the reminder ‘Same Sign Subtract (SSS)’ to indicate the next step in the solution and then solve fully.

- Some students may need additional examples which do not require the multiplication of equations in order to develop their understanding of the basic method. These can be provided as appropriate.

- Students can then work through further examples together (with guidance if appropriate) and then practise solving simultaneous equations by elimination with more questions.

- Problem-solving using simultaneous equations can be worked into the lesson as appropriate or looked at separately through further examples.

Exercise commentary

Exercise 20 is inserted to provide practice in manipulating negative numbers before students proceed to the examples in exercise 21 which require the use of the elimination method of solution. Questions 19 to 21 require initial manipulation and could therefore be omitted or set aside for extension work. Questions 25 to 30 involve fractions and decimals and could likewise be set aside for more able students.

Plenary

Give the students a further example of a pair of simultaneous equations to be solved by elimination. Allow them two minutes to work out the solution before writing it down. This can be repeated with different examples as time allows.
Lesson 14 – Problem-solving using simultaneous equations

Textbook pages 81–84

Objectives
E2.5: Derive and solve simultaneous linear equations in two unknowns.

Starter
Sum and product: Ask students to work out the two numbers when you tell them that the sum is 5 and the product 6 (2 and 3). Repeat for different sums and products.

Lesson commentary
- The previous two lessons have introduced students to the two standard methods for solving simultaneous equations and a quick summary of these is likely to be useful. Some students may also have had time to work on some problem-solving type questions as well.
- By selecting a range of examples, either from the textbook or elsewhere, demonstrate the formulation of the simultaneous equations from the information contained in the question. Emphasise that it is important to get this first action right because then the solution of the pair of equations can be solved by using one of the two standard methods.
- Discuss further examples as appropriate and then allow the students time to practise forming and solving simultaneous equations by completing further examples of their own. Encourage students to check they have correct equations before solving.

Exercise commentary
Problem-solving using simultaneous equations is tested in exercise 22. Students may need help with initially modelling the problem; encourage them to use the method of their choice to find the solutions. Questions 27 to 30 have three unknowns; these could be given to more able students as extension problems.

Plenary
Give the students a further example and ask them to work out the solution using the method of their choice. They could also be encouraged to set their own problems and challenge a partner to solve them.
Lesson 15 – Factorising

Textbook pages 84–85

Objectives
E2.2: Use brackets and extract common factors. Expand products of algebraic expressions. Factorise where possible expressions of the form: \( ax + bx + kay + kby, a^2x^2 - b^2y^2, a^2 + 2ab + b^2, ax^2 + bx + c. \)

Starter
Provide students with a list of algebraic expressions and ask them to write down the highest common factor of certain pairs of expressions. A good set of expressions might be 3\( x, 6x, 3y, 2y, x^2, y^2, 2x^2 \) and 3\( y^2. \) Include pairs where ‘no common factor’ is the answer.

Lesson commentary
• Explain that factorisation is the reverse process of taking out brackets. Explain that we are looking for ‘how the expression started out’ before the brackets were removed. Encourage the identification of the highest common factor from two (and eventually three) terms before writing this outside of the brackets. The terms inside the brackets can now be found by dividing each term by this highest common factor. Provide examples before letting the students practise some for themselves. Encourage them to check their answers.

• The factorisation of four terms into two brackets often causes problems for all but the most able students and could be avoided, if appropriate. If this technique is introduced, encourage students to follow the examples carefully before attempting to write their own list of instructions for performing this kind of factorisation. Encourage careful thought about the process and clear communication in written instructions. Students may need to consolidate this work more thoroughly than other topics in this chapter and further time could be allowed if necessary.

Exercise commentary
The examples in exercise 23 are routine practice questions where pairs of algebraic terms are to be factorised. Questions 31 and 32 include three terms and could be used as extension questions. Four-term factorisation is covered in exercise 24 and students may need clear modelling of this before proceeding.

Plenary
Ask students to write down the answers to several more factorisation questions. Keep them relatively simple and encourage speed of response.
Lessons 16 and 17 – Quadratic expressions 1

Textbook pages 85–86

Objectives
E2.2: Use brackets and extract common factors. Expand products of algebraic expressions. Factorise where possible expressions of the form: \( ax + bx + kay + kby, a^2x^2 - b^2y^2, a^2 + 2ab + b^2, ax^2 + bx + c. \)

Starter
Provide students with pairs of brackets to expand. Encourage them to work quickly and write the answers in their exercise books for checking at the end.

Lesson commentary
- Use examples from the starter activity to introduce the idea of ‘reversing the process’ and factorising quadratic expressions. Discuss the significance of the two numbers ‘\( b \)’ and ‘\( c \)’ as the sum and the product of the two numbers in the brackets and explain that when we factorise expressions of this type, we try to identify the bracket numbers using factor-pairs of \( c \).
- Students could then be encouraged to work together to solve a range of examples with both numbers positive, both numbers negative and one positive, one negative.
- Discuss examples which have ‘no middle term’ \( (b = 0) \) as an introduction to the ‘difference of two squares’ examples in exercise 27.
- The second lesson will enable students to have sufficient time to fully consolidate this work before moving on to harder examples in the next section.

Exercise commentary
The questions in exercise 25 are routine practice examples of factorising quadratic expressions. Questions 25 to 27 are slightly harder as the value of \( c \) is large and these could be used as extension questions for more able students. Questions 28 to 30 lead into the work on ‘difference of two squares’ covered in a later lesson.

Plenary
Provide students with a series of examples where some are correct and some incorrect. Ask them to decide which are correct or incorrect either by expansion of the brackets or factorisation of the quadratic expression. Alternatively, provide students with a set of dominos containing expanded and factorised expressions and ask them to join them up correctly.
Lessons 18 and 19 – Quadratic expressions 2

Textbook page 86

Objectives
E2.2: Use brackets and extract common factors. Expand products of algebraic expressions. Factorise where possible expressions of the form: \( ax + bx + kay + kby, a^2x^2 - b^2y^2, a^2 + 2ab + b^2, ax^2 + bx + c \).

Starter
Provide students with pairs of brackets to expand which include say ‘2x’ or ‘3x’ in one of them. Encourage them to work quickly and display their answers on mini-whiteboards or in their exercise books.

Lesson commentary
- Students often find the method of factorising general quadratic expressions of the form \( ax^2 + bx + c \) difficult to follow and consolidate. The best approach is to demonstrate examples and encourage a standard layout for the solution. A step-by-step approach could also be used in this context. Comparing the bracketed form and the expanded form will enable students to ‘see’ where the numbers come from and how they relate to each other. Encourage them to work together when solving problems of their own and check their solutions by re-expanding the brackets.
- An important check is that their bracket in the ‘factorise in pairs’ step is the same for both pairs.
- The second lesson will enable students to have sufficient time to fully consolidate this work before moving on to the next section.

Exercise commentary
All of the questions in exercise 26 are routine practice examples of this technique. Questions 24 to 28 are harder because the value of ‘ac’ is larger and so it is more difficult to find factors. Questions 29 and 30 lead into the work on ‘difference of two squares’ covered in the next lesson.

Plenary
A card-matching activity is ideal here to test understanding. Students could work in pairs to match the cards by equating quadratic expressions and their bracketed equivalents. Alternatively, students can be provided with further examples and work out the factorisation in their exercise books before checking the answers at the end.
Lesson 20 – Difference of two squares

Textbook page 87

Objectives
E2.2: Use brackets and extract common factors. Expand products of algebraic expressions. Factorise where possible expressions of the form: \( ax + bx + kay + kby,\ a^2x^2 - b^2y^2,\ a^2 + 2ab + b^2,\ ax^2 + bx + c.\)

Starter
Ask students to write down the square root of various numbers and algebraic expressions such as \( x^2,\ 9y^2,\ 25,\ \frac{1}{4},\ 16a^2 \) and \( \frac{x^2}{4}.\)

Lesson commentary
• Discuss the special case of a quadratic expression which has ‘no middle term’. Reference can be made back to the examples in the previous lessons as an introduction; encourage students to recognise the general form of these expressions:
  \[ x^2 - y^2 = (x - y)(x + y). \]
• Provide students with several examples using ‘non-standard’ letters and more complicated expressions for ‘\( x \)’ and ‘\( y \)’. Link back to the starter activity when deducing the square roots of ‘\( x \)’ and ‘\( y \)’ and allow students time to practise further examples.
• If appropriate, challenge students through the use of examples which require an initial ‘simple’ factorisation before using the ‘difference of two squares’ method and then challenge them to solve seemingly difficult arithmetical calculations using this method (examples could be taken from exercise 26, question 29 onwards).

Exercise commentary
Exercise 27 provides students with some routine practice examples which also include fractions. From question 17 onwards, initial ‘simple’ factorisation is necessary to get at the ‘difference of two squares’ form and questions 29 to 40 use the idea of ‘difference of two squares’ to solve arithmetical problems.

Plenary
This is an ideal opportunity to test all of the techniques from this lesson and the previous ones on factorisation. A mixed exercise, or short test could be used for this purpose.
Lessons 21 and 22 – Solving quadratic equations by factorisation

Textbook pages 87–89

Objectives
E2.5: Derive and solve quadratic equations by factorisation.

Starter
Factor pairs: Ask students to write down all of the factor pairs of given numbers (10, 16, 12, etc.).

Lesson commentary
- Students could be invited to solve a given quadratic equation (with integer solutions) by trial and improvement. Get them to substitute various numbers into the equation and find one which gives the answer zero. They could then be invited to find a second solution to the equation. Explain that quadratic equations will normally have two solutions (reference to the graph could be made at this point) and that there are several methods available for finding the solutions.
- Refer to the methods for factorising quadratic expressions encountered in previous lessons and, using the introductory example(s), demonstrate that the factorised form of the expression provides the solutions to the equation when each of the brackets is set equal to zero.
- Further examples can be demonstrated as appropriate including general quadratics \((a \neq 1)\) and special cases such as the difference of two squares and ones with no \(c\) term.
- The second lesson will provide students with more time to consolidate this work on solving quadratic equations by factorisation.

Exercise commentary
Exercises 28 and 29 provide lots of basic practice in solving quadratic equations through factorisation. Be selective in the questions given to certain groups in order to provide appropriate challenge and differentiation. For example, weaker students may be instructed to concentrate on examples where \(a = 1\) while more able students could be directed to ‘odd’ cases or ones which require transposition.

Plenary
Provide students with a worked example which contains errors and ask them to correct them. Ask volunteers to provide feedback for the whole class. Further examples can be given as appropriate. Alternatively, students can be asked to solve a variety of further examples and write the answers to these in their exercise books.
**Lessons 23 and 24 – Solving quadratic equations by completing the square**

Textbook pages 92–93

**Objectives**

E2.5: Derive and solve quadratic equations by completing the square.

**Starter**

Invite students to expand the brackets of a number of perfect algebraic squares such as \((x + 2)^2\) and \((x - 3)^2\).

**Lesson commentary**

- Solving quadratic equations by completing the square is not a common technique but it leads naturally on to using the formula.

- Provide students with a number of equivalent expressions such as \(x^2 + 4x - 6\) and \((x + 2)^2 - 10\). Ask them to show the equivalence through expanding the perfect square and simplifying. Explain that completing the square is the reverse process of this expansion and provide a modelled example for students to follow.

- The use of a step-by-step list of instructions often helps students become confident at applying this technique since it requires a standard approach for each example. Students could be provided with further examples and asked to follow the method before writing their own list of instructions for another student to follow.

- Further consolidation and practice can be provided as appropriate. The provision of a second lesson on this topic will ensure that the students have sufficient time to complete this practice on solving equations by competing the square.

**Exercise commentary**

Questions 1 to 10 in exercise 32 require students to complete the square. From question 7 onwards, \(a \neq 1\) so initial steps will be required to form an expression suitable for completing the square. These could be used as extension questions for more able students. Question 11 uses the technique to solve quadratic equations.

**Plenary**

Students could be provided with three sets of cards (one with quadratic equations, one with completed square forms and one with solutions) and invited to match one card in each set to form groups of three cards. Alternatively, further examples can be given to the students who work the answers out in their exercise book before feeding back the answers to the class through discussion.
Lessons 25 and 26 – Solving quadratic equations using the formula

Textbook pages 90–91

Objectives
E2.5: Derive and solve quadratic equations by use of the formula.

Starter
Ask students to write down some square roots of square numbers. This will test instant recall of square number facts. Then allow the use of a calculator to find the square roots of non-square numbers. Ask students to write down these numbers to three decimal places.

Lesson commentary
● Depending on the ability of the group, the quadratic formula could be derived (following on from completing the square in the previous lesson) or simply given to the students to use immediately.
● If the formula is to be derived, students could be provided with a list of steps and asked to follow them or they could be provided with the derivation and asked to work through it. This will encourage them to think about the process more carefully.
● When it comes to using the formula to solve quadratic equations, explain the importance of following the rules of BIDMAS and encourage students to work ‘step-by-step’ rather than trying to do the calculation in one go. Ensure there is no premature rounding (work in exact form) and then encourage the final answers to be given to a sensible degree of accuracy. Three significant figures is sensible, unless otherwise quoted in the question (usually one decimal place).
● The second lesson can be used to ensure sufficient time to practise solving quadratic equations using the formula. Depending on the pace of the group and the time, further examples could be demonstrated which require transposition before a solvable quadratic is arrived at. Students could be invited to ‘have a go’ at solving equations of this type without too much guidance.

Exercise commentary
Exercise 30 provides plenty of routine practice in using the quadratic formula. The later examples involving fractional or decimal coefficients could be used for extension questions.
The equations in exercise 31 need to be rearranged before solving the resulting quadratic. Depending on the ability of the group, these could be used sparingly or given to more able students to provide challenge.

Plenary
Provide further examples and invite students to ‘race against the clock’ to write down the correct solutions. Points could be awarded for the first five correct solutions to each example.
Lesson 27 – Problems solved by quadratic equations

Textbook pages 93–96

Objectives
E2.5: Derive and solve quadratic equations.

Starter
Work through the examples on pages 93 and 94 to demonstrate how solving a problem can sometimes mean solving a quadratic equation. Focus on the algebra in example 2, as some students may find this difficult. Also discuss why one solution is sometimes meaningless and must be discarded.

Lesson commentary
• When looking at questions 1 and 2 of exercise 33, discuss with the class why it doesn’t matter which of the two numbers we call x. Perhaps get one half of the class to use x and x + 3 in question 1, while the other half uses x and x − 3. Compare their results.
• Use questions 4 to 6 as another opportunity to discuss the idea of a meaningless negative solution. The class may also need reminding about Pythagoras’ theorem.
• More able students might like to briefly consider what assumptions are being made in question 7, and whether it depends on where in the world Sang Jae is walking.
• There are several other areas of mathematics that students will need to remember in order to complete all the questions. These include Pythagoras’ theorem, reciprocals, speed and bearings. It might be a good idea to have page references ready, so that students can look at these topics if they need to.

Exercise commentary
The examples and questions in exercise 33 provide a range of contexts in which quadratic equations need to be formed and solved. Questions 1 to 2 deal purely with numbers. Questions 3 to 6 deal with rectangles, and involve discarding the meaningless negative solution.

For Question 7 students would benefit from drawing a diagram.

Questions 8 and 9 concern money. From question 10 onwards, students need to draw on a wider knowledge of mathematics to construct their equations.

Questions 10 and 11 involve reciprocals. Questions 13 to 15 are about calculating speed.

Questions 16 to 20 are of a more problem-solving nature, and are significantly more challenging.

Plenary
In 2015, a GCSE maths question made the national news. As a class, search for ‘Hannah’s sweets’ on the internet and look at the question. It is a simple probability question that results in a quadratic equation that has to be solved.
Lesson 28 – Non-linear simultaneous equations

Textbook pages 96–97

Objectives
E2.5: Derive and solve simultaneous equations, involving one linear and one quadratic.

Starter
Revise solving linear simultaneous equations using question 1 from exercise 19 on page 78. Together, solve this question using elimination, then solve it again using substitution. Discuss with the class which method they prefer and why.

Lesson commentary
- It is important that students understand that, while the elimination method for linear simultaneous equations is often the quickest method, often it will not work when one equation is linear and the other is not. Use example 1 on page 96 to demonstrate why it is not generally possible to eliminate a variable that is present in more than one power. Ask students to explain why this is the case.
- Using questions 1 to 3 from exercise 34 as examples, tell students that if both equations are of the form $y = \ldots$, with an expression in terms of $x$ on the other side, the first thing they should do in their solution is equate the two right-hand sides.
- Encourage the use of substitution throughout this exercise. If students want to use a form of elimination for some of the questions, discuss this as a class and make them aware of the dangers and limitations involved in doing this.

Exercise commentary
Questions 1 to 3 in exercise 34 consist of two equations with only $y$ on the left-hand side.
The remaining questions are not of this form.

Plenary
Give students an exam-style question based on this topic.

The revision and examination-style exercises can be used for further practice as appropriate.

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