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About this book

This book has been written to cover the Cambridge AS & A Level International Mathematics (9709) course, and is fully aligned to the syllabus.

In addition to the main curriculum content, you will find:

- ‘Maths in real-life’, showing how principles learned in this course are used in the real world.
- Chapter openers, which outline how each topic in the Cambridge 9709 syllabus is used in real-life.

The book contains the following features:

Throughout the book, you will encounter worked examples and a host of rigorous exercises. The examples show you the important techniques required to tackle questions. The exercises are carefully graded, starting from a basic level and going up to exam standard, allowing you plenty of opportunities to practise your skills. Together, the examples and exercises put maths in a real-world context, with a truly international focus.

At the start of each chapter, you will see a list of objectives that are covered in the chapter. These objectives are drawn from the Cambridge AS & A Level syllabus. Each chapter begins with a Before you start section and finishes with a Summary exercise and Chapter summary, ensuring that you fully understand each topic.

Each chapter contains key mathematical terms to improve understanding, highlighted in colour, with full definitions provided in the Glossary of terms at the end of the book.

The answers given at the back of the book are concise. However, when answering exam-style questions, you should show as many steps in your working as possible. All exam-style questions, as well as Paper A and Paper B, have been written by the authors.
About the authors

Jim Fensom has many years’ experience of teaching and examining mathematics. He has authored a number of books. He recently retired after a career teaching in the UK and Singapore.

Phil Crossley is a senior examiner as well as a teacher at Carre’s Grammar School in England. He has many years of experience in teaching and examining mathematics.

Dr Martin Burgess has over nine years’ experience in teaching mathematics at secondary level and has also been an expert examiner for an A Level examination board. His PhD is in the field of data mining, specialising in statistical techniques, and he works at Nexus International School in Singapore.

Special thanks to James Nicholson for ‘Maths in real-life’.

A note from the authors

The aim of this book is to help students prepare for the Mechanics unit of the Cambridge International AS & A Level Mathematics syllabus, though it may also be found to be useful in providing support material for other AS and A Level courses. The book contains a large number of practice questions, many of which are exam-style.

In writing the book we have drawn on our experiences of teaching mathematics over many years, as well as our experience as examiners.
**Student Book:** *Complete Mechanics for Cambridge International AS & A Level*

**Syllabus:** *Cambridge International AS & A Level Mathematics: Mechanics (9709)*

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### Syllabus overview for 9709, first examined in 2020.

**Mechanics (Paper 4)**

Questions set will be mainly numerical, and will aim to test mechanical principles without involving difficult algebra or trigonometry. However, candidates should be familiar in particular with the following trigonometrical results:

\[
\sin(90° - \theta) = \cos \theta; \quad \cos(90° - \theta) = \sin \theta; \quad \tan \theta \equiv \frac{\sin \theta}{\cos \theta}; \quad \sin^2 \theta + \cos^2 \theta \equiv 1.
\]

Knowledge of algebraic methods from the content for Paper 1: Pure Mathematics 1 is assumed.

This content list refers to the equilibrium or motion of a ‘particle’. Examination questions may involve extended bodies in a ‘realistic’ context, but these extended bodies should be treated as particles, so any force acting on them is modelled as acting at a single point.

Vector notation will not be used in the question papers.

---

### 1. Forces and equilibrium

- Identify the forces acting in a given situation
- Understand the vector nature of force, and find and use components and resultants
- Use the principle that, when a particle is in equilibrium, the vector sum of the forces acting is zero, or equivalently, that the sum of the components in any direction is zero
- Understand that a contact force between two surfaces can be represented by two components, the normal component and the frictional component
- Use the model of a ‘smooth’ contact, and understand the limitations of this model
- Understand the concepts of limiting friction and limiting equilibrium; recall the definition of coefficient of friction, and use the relationship \( F = \mu R \) or \( F \leq \mu R \), as appropriate
- Use Newton’s third law

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### 2. Kinematics of motion in a straight line

- Understand the concepts of distance and speed as scalar quantities, and of displacement, velocity and acceleration as vector quantities (in one dimension only)
- Sketch and interpret displacement–time graphs and velocity–time graphs, and in particular appreciate that:
  - the area under a velocity–time graph represents displacement
  - the gradient of a displacement–time graph represents velocity
  - the gradient of a velocity–time graph represents acceleration

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<tr>
<td>Page 4</td>
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<td>Page 8</td>
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</table>
### 1. Use differentiation and integration with respect to time to solve simple problems concerning displacement, velocity and acceleration (restricted to techniques from the content for Paper 1: Pure Mathematics 1)

Page 37

### 3. Momentum

- Use the definition of linear momentum and show understanding of its vector nature
- Use the conservation of linear momentum to solve problems that may be modelled as the direct impact of two bodies (including direct impact where the bodies coalesce on impact). Note: knowledge of impulse and the coefficient of restitution is not required

Pages 112–114

### 4. Newton’s laws of motion

- Apply Newton’s laws of motion to the linear motion of a particle of constant mass moving under the action of constant forces, which may include friction, tension in an inextensible string and thrust in a connecting rod
- Use the relationship between mass and weight
- Solve simple problems that may be modelled as the motion of a particle moving vertically or on an inclined plane with constant acceleration
- Solve simple problems that may be modelled as the motion of connected particles, e.g. connected by a light inextensible string that may pass over a fixed smooth peg or light pulley

Pages 57–72

### 5. Energy, work and power

- Understand the concept of the work done by a force, and calculate the work done by a constant force when its point of application undergoes a displacement not necessarily parallel to the force (use of the scalar product is not required)
- Understand the concepts of gravitational potential energy and kinetic energy, and use appropriate formulae
- Understand and use the relationship between the change in energy of a system and the work done by the external forces, and use in appropriate cases the principle of conservation of energy
- Use the definition of power as the rate at which a force does work, and use the relationship between power, force and velocity for a force acting in the direction of motion
- Solve problems involving, for example, the instantaneous acceleration of a car moving on a hill with resistance

Pages 86–91

Pages 91–92

Pages 92–101

Pages 102–106

Pages 106–111
The longest straight stretch of train track in the world is in Australia. It runs from Ooldea, in South Australia, to Loongana, in Western Australia, a distance of 478 km. This section of track is part of the Trans-Australian Railway on which the Indian Pacific line from Sydney, in the East of Australia, to Perth, in the West, runs. It runs though the Nullarbor Plain, an area of flat, almost treeless, arid or semi-arid country that occupies an area of 200 000 square kilometres. The length of the journey is 4352 km one-way, and takes 65 hours. The average speed of trains is 85 km/h and its maximum speed is 115 km/h.

### Objectives

Sketch and interpret displacement–time graphs and velocity–time graphs, and in particular appreciate that
- the area under a velocity–time graph represents displacement
- the gradient of a displacement–time graph represents velocity
- the gradient of a velocity–time graph represents acceleration.

### Before you start

You should know how to:

1. Calculate the area of rectangles, triangles and trapeziums.
   - **e.g.**
     - Area = $2.4 \times 5 = 12 \text{ cm}^2$
     - Area = $6.2 \times 2 = 12.4 \text{ cm}^2$
     - Area = $\frac{1}{2} (4 + 7) \times 3 = 16.5 \text{ cm}^2$

2. Calculate the gradient of a straight line.
   - Use gradient = $\frac{y_2 - y_1}{x_2 - x_1}$.
   - **e.g.** Find the gradient of the line joining $(2, 4)$ and $(7, -1)$.
   - gradient = $\frac{-1 - 4}{7 - 2} = \frac{-5}{5} = -1$

### Skills check:

1. Calculate the area of the shape created between the red line of this graph and the $x$-axis.

2. Calculate the gradients of these lines.
   - **a)**
   - **b)**
3. Calculate **displacement**, **velocity**, **acceleration** and **time** using appropriate units (m, m s\(^{-1}\), m s\(^{-2}\), s).

*When velocity is constant then the formulae that connect these quantities are as follows:*  

\[
\text{displacement} = \text{velocity} \times \text{time}  
\]

**e.g.** Find the displacement of a particle travelling at 3 m s\(^{-1}\) for 5 s.

\[
\text{displacement} = 3 \times 5 = 15 \text{ m}  
\]

\[
\text{velocity} = \frac{\text{displacement}}{\text{time}}  
\]

**e.g.** Find the velocity when a particle has a displacement of 2.4 m in 3 s.

\[
\text{velocity} = \frac{2.4}{3} = 0.8 \text{ m s}^{-1}  
\]

\[
\text{time} = \frac{\text{displacement}}{\text{velocity}}  
\]

**e.g.** Find the time taken for a particle to be displaced 15 m with a velocity of 0.6 m s\(^{-1}\).

\[
\text{time} = \frac{15}{0.6} = 25 \text{ s}  
\]

*When acceleration is constant then the formulae that connect these quantities are as follows:*  

\[
\text{velocity} = \text{acceleration} \times \text{time}  
\]

**e.g.** Find the change in velocity when a particle accelerates for 2 s at 24 m s\(^{-2}\).

\[
\text{change in velocity} = 24 \times 2 = 48 \text{ m s}^{-1}  
\]

\[
\text{acceleration} = \frac{\text{velocity}}{\text{time}}  
\]

**e.g.** Find the acceleration when the velocity of a particle changes from 2 m s\(^{-1}\) to 10 m s\(^{-1}\) in 12 s.

\[
\text{acceleration} = \frac{10 - 2}{12} = \frac{2}{3} \text{ m s}^{-2}  
\]

\[
\text{time} = \frac{\text{velocity}}{\text{acceleration}}, \text{ e.g.} \text{ Find the time taken for a particle to accelerate to a velocity of 8 m s}^{-1} \text{ from 3 m s}^{-1} \text{ when its acceleration is 0.1 m s}^{-2}.  
\]

\[
\text{time} = \frac{8}{0.1} = 50 \text{ s}  
\]

4. Find the displacement of a particle travelling with

**a)** a velocity of 12 m s\(^{-1}\) for 12 s  
**b)** a velocity of 0.4 m s\(^{-1}\) for 10 s  
**c)** a velocity of 30 m s\(^{-1}\) for 0.5 s.

5. Find the velocity of a particle that has

**a)** a displacement of 24 m in 8 s  
**b)** a displacement of 45 m in 30 s  
**c)** a displacement of 10 m in 50 s.

6. Find the change in velocity when a particle accelerates at

**a)** 10 m s\(^{-2}\) for 10 s  
**b)** 0.2 m s\(^{-2}\) for 30 s.

7. Find the acceleration when a particle’s velocity changes

**a)** from 20 m s\(^{-1}\) to 50 m s\(^{-1}\) in 10 s  
**b)** from 44 m s\(^{-1}\) to 32 m s\(^{-1}\) in 6 s.

8. Find the time taken for a particle to accelerate from 15 m s\(^{-1}\) to 60 m s\(^{-1}\) at 15 m s\(^{-2}\).
1.1 Displacement–time graphs

A displacement–time graph is used to show the motion of a particle, in one dimension, along a straight line. We first look at examples where motion follows one or more stages of constant velocity, with the particle moving forwards and backwards along the straight line. In displacement–time graphs, time \((t)\) is shown on the horizontal axis. Displacement is often denoted by \(s\).

We know that velocity = \(\frac{\text{displacement}}{\text{time}}\), which can be abbreviated as velocity = \(\frac{s}{t}\).

Velocity is the gradient of the displacement–time graph.

Note that the concept of kinematics, or straight-line motion, refers to the motion of a particle. A particle has dimensions so small compared with other lengths that its position in space can be represented by a single point. A body is an object made up of particles. However, in Example 1, a body (in this case a car) is modelled as a particle for the purpose of the question.

Example 1

A car moves forward on a straight road from a point \(O\), at constant velocity for 20 s, travelling a distance of 60 m. During the next 20 s the car is stationary, remaining 60 m away from \(O\). The car then returns to \(O\), which takes 10 s.

a) Sketch a displacement–time graph of the first 50 s of the car’s journey.

b) Use the displacement–time graph to find the velocity of the car during each stage of the journey.

\[
\begin{array}{c|c}
\text{Time (s)} & \text{Displacement (m)} \\
0 & 0 \\
10 & 20 \\
20 & 40 \\
30 & 60 \\
40 & 60 \\
50 & 40 \\
60 & 20 \\
\end{array}
\]

Note: In this example, displacement away from \(O\) is regarded as positive; hence, on the return part of the journey, both the displacement and the velocity are negative.

b) Since velocity = \(\frac{\text{displacement}}{\text{time}}\), the gradient of a displacement–time graph is the velocity.

The gradient of \(OA\) is \(\frac{60}{20} = 3\).

The velocity in the first 20 s is 3 m s\(^{-1}\).

The gradient of \(AB\) is 0.

The velocity in the second 20 s is 0 m s\(^{-1}\).

The gradient of \(BC\) is \(\frac{-60}{10} = -6\).

The velocity in the final 10 s is –6 m s\(^{-1}\).
**Exercise 1.1**

1. A particle travelling in a straight line, starting from a point $O$ at a velocity of $2\,\text{m s}^{-1}$ for $10\,\text{s}$, rests for $20\,\text{s}$ and then returns to $O$ in $5\,\text{s}$.
   
a) Sketch the displacement–time graph of the motion of the particle.
   
b) What is the velocity of the particle on the return?

2. A car travels along a straight road from a town $O$. It travels $200\,\text{m}$ at a constant velocity of $20\,\text{m s}^{-1}$. It then stops for $5\,\text{s}$ before returning to the starting point in $8\,\text{s}$.
   
a) Sketch a displacement–time graph for the motion of the car.
   
b) Calculate the velocity on the return section of the journey.

3. A food container in a sushi restaurant travels along a straight track at a velocity of $0.5\,\text{m s}^{-1}$ for $10\,\text{s}$. It stops for $10\,\text{s}$ and then continues on its journey at a velocity of $0.6\,\text{m s}^{-1}$, coming to a halt after a further $10\,\text{s}$.
   
a) Sketch the displacement–time graph for the food container.
   
b) Calculate the total distance travelled by the food container.

---

**Note:** In the graphs that follow, displacement ($s$) is given in metres, and time ($t$) is in seconds.

4. Describe the motion of the particle in the graph.
   What is the velocity of the particle
   
a) in the first $10\,\text{s}$
   
b) between $t = 10$ and $t = 40$
   
c) in the last $30\,\text{s}$?

5. Describe the motion of a train moving along a track, as shown in the accompanying graph.
   What is the velocity of the train
   
a) in the first $5\,\text{s}$
   
b) between $t = 5$ and $t = 20$
   
c) in the last $15\,\text{s}$?
6. Describe the motion of an elevator moving in an elevator shaft, as shown in the graph.

What is the velocity of the elevator
a) in the first 8 seconds
b) between $t = 8$ and $t = 16$

c) in the last 12 seconds?

7. In a factory, a piece of steel travels a distance $S$ from $O$ in 40 s on a straight conveyor belt and then returns to $O$ 20 s later as shown in the diagram below.

If the initial velocity of the piece of steel is 8 m s$^{-1}$, calculate
a) the value of $S$
b) the velocity of the piece of steel between $t = 40$ and $t = 60$.

8. A particle travels 20 m in a straight line from $O$ in $T$ s. It remains stationary for a further $T$ s, and then returns directly to $O$ in $T$ s as shown in the graph below.

If the initial velocity of the particle is 30 m s$^{-1}$, calculate
a) the value of $T$
b) the time taken to complete the whole journey.
9. The displacement of a particle from $O$ is $S$ m in a time $4T$ s. The particle then returns to $O$ as shown in the graph.

![Graph showing displacement over time](image)

a) If the initial velocity is $4$ m s$^{-1}$, find the velocity on the return.
b) If the total time taken is $27$ s, find the value of $S$.

10. The graph shows the displacement $s$ of a model train moving along a track in time $t$.

![Graph showing displacement over time](image)

The velocity of the train from $O$ to $A$ is $V$ m s$^{-1}$ and from $A$ to $B$ the velocity is $-V$ m s$^{-1}$. The train is at rest between $B$ and $C$, and between $C$ and $D$ the velocity is $1.5V$ m s$^{-1}$. Calculate

a) the value of $V$
b) the displacement from $A$ to $B$
c) the displacement from $C$ to $D$
d) the displacement from $D$ to $E$
e) the velocity between $D$ and $E$.

**Did you know?**
The examples in Section 1.1 are a simplification of what happens in real-life. In practice, although a particle (or body) can travel at a constant velocity, change in velocity is never instantaneous. It involves acceleration or deceleration.

Imagine sitting in a car where the velocity changed abruptly. What would happen to your body if it sped up, slowed down, stopped, or changed direction in no time at all?
1.2 Velocity–time graphs

A velocity–time graph is also used to show the motion of a particle in one dimension, along a straight line. In the next set of examples, motion follows one or more stages of constant velocity or constant acceleration, with the particle moving forwards and backwards along the straight line. In velocity–time graphs, time \( (t) \) is shown on the horizontal axis. Velocity is denoted by \( v \).

![Velocity-Time Graph]

We know that acceleration = \( \frac{\text{velocity}}{\text{time}} \), which can be abbreviated as acceleration = \( \frac{v}{t} \).

Acceleration is the gradient of the velocity–time graph.

When velocity is constant, displacement = velocity \( \times \) time, or more simply, \( v \times t \). Therefore,

For constant velocity, displacement is found by calculating the area of the rectangle on a velocity–time graph.

![Constant Velocity Diagram]

When acceleration is constant, the velocity graph will be a straight line. Consider the area under the graph to be made of a series of very narrow rectangles. The area of each of these rectangles is the displacement of the particle over a very short time. Combining these areas, we get an approximation for the area of the trapezium under the graph, which improves as the time period for each rectangle becomes less.

Displacement is the area under the velocity–time graph.
Example 2

During the first 10 s of a journey along a straight road, a car accelerates from rest to a velocity of 20 m s⁻¹. It then continues for a further 20 s at constant velocity.

a) Sketch the velocity–time graph of the journey.

b) Calculate the acceleration during the first 10 s of the journey.

c) Describe the motion between the first 10 s and 30 s of the journey.

d) Calculate the total distance travelled in the first 30 s of the journey.

\[
\begin{align*}
\text{Initial velocity} & = 0 \\
\text{Final velocity} & = 20 \text{ m s}^{-1} \\
\text{Time} & = 10 \text{ s} \\
\text{Acceleration} & = \frac{20}{10} = 2 \text{ m s}^{-2}.
\end{align*}
\]

The motion during the first 10 s is acceleration. Thereafter, the motion is constant velocity.

The total distance travelled in the first 30 s is 500 m.
In a velocity–time graph, velocity can be positive (above the time axis) or negative (below the time axis). Performing calculations from values in the graph can result in areas that are negative as well, indicating a negative displacement. Care needs to be taken when finding displacement if both positive and negative velocities are involved.

Consider the case of a cricket ball thrown up in the air. There is a positive displacement as the ball travels up, and a negative displacement as it travels back down. Since the ball returns to the point where it started, the overall displacement is zero. The distance travelled, however, is not equal to zero. The difference between displacement (a vector quantity) and distance (a scalar quantity) will be discussed in Chapter 2.

Example 3

A ball is projected up a smooth plane at a velocity of 15 m s\(^{-1}\) from a point \(O\). The ball decelerates at a constant rate for 6 s. The ball is instantaneously at rest at \(t = 3\) s.

a) Sketch the velocity–time graph for the ball’s journey.

\[
\begin{array}{c|c|c|c|c|c|c|c|c|}
\hline
\text{time (t)} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline
\text{velocity (v)} & 20 & 15 & 10 & 5 & 0 & -5 & -10 \\ \hline
\end{array}
\]

b) Calculate the value of the ball’s acceleration.

Gradient = \(\frac{-30}{6} = \frac{-15}{3} = -5\)

The acceleration is \(-5\) m s\(^{-2}\).

c) What is the maximum displacement of the ball?

The maximum displacement occurs when it is at rest, when \(t = 3\) s.

\[
\text{Area} = \frac{1}{2} \times 3 \times 15 = 22.5
\]

The maximum displacement is 22.5 m.

d) What is the total distance travelled by the ball?

Between \(t = 3\) and \(t = 6\), the displacement is \(-22.5\) m, and hence the total displacement of the ball is 22.5 + (-22.5) = 0 m. This simply means that the ball has returned to its starting position.

The total distance travelled is \(2 \times 22.5 = 45\) m.
Another difference between vector and scalar quantities is that between velocity and speed. Consider two cars that hit each other on a highway travelling at the same velocity, and two other cars that hit each other head-on at the same speed.

- **Result**: the first two cars have only minor damage, while the second two have significant damage.

What is a possible explanation for this difference in damage?

- **The reason**: two cars travelling at the same velocity are travelling in the same direction, and so they hit each other in a side-on collision, causing less damage. In straight-line motion, speed could be in opposite directions, and a head-on collision would cause much more damage.

### Example 4

A car accelerates smoothly from rest for 30 s to a velocity of 20 m s\(^{-1}\). It continues at a steady velocity for 20 s before decelerating to rest in 20 s.

a) Sketch the velocity–time graph of the motion of the car in the first 70 s of motion.

b) Calculate the acceleration in the first 30 s and the final 20 s.

c) Calculate the total displacement.

d) Calculate the average speed during the journey.

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<table>
<thead>
<tr>
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<th>velocity (v)</th>
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<tbody>
<tr>
<td>0</td>
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<tr>
<td>10</td>
<td>2</td>
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<td>60</td>
<td>12</td>
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<tr>
<td>70</td>
<td>14</td>
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</tbody>
</table>
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b) Since acceleration \(= \frac{\text{velocity}}{\text{time}}\), the gradient of a velocity–time graph is the acceleration.

The gradient in the first 30 s is \(\frac{20}{30} = \frac{2}{3} \approx 0.667\).

The acceleration in the first 30 s is 0.667 m s\(^{-2}\).

The gradient in the last 20 s is \(-\frac{20}{20} = -1\).

The acceleration in the last 20 s is \(-1\) m s\(^{-2}\).
c) Since displacement = velocity × time, the displacement is found by calculating the area under the graph between \(t = 0\) and \(t = 70\).

Total area = area of trapezium
\[
= \frac{1}{2}(70 + 20) \times 20
= 900
\]

The total displacement is 900 m.

d) Average speed = \[
\frac{\text{total displacement}}{\text{time taken}}
\]
\[
= \frac{900}{70} \approx 12.9 \text{ m s}^{-1}
\]

**Examination advice**

It is a common error not to use the correct formula when calculating average speed. Make sure that you learn this formula:

\[
\text{average speed} = \frac{\text{total displacement}}{\text{time taken}}
\]

**Note:** Average speed is *not* found by taking the average of two speeds.

---

**Exercise 1.2**

1. A car is travelling at 30 m s\(^{-1}\). It continues at a constant velocity for 20 s, and then slows to a halt after a further 10 s. Sketch a velocity–time graph to show the motion of the car.

2. A baseball is thrown vertically upwards from the ground with an initial velocity of 20 m s\(^{-1}\). The acceleration due to gravity is 10 m s\(^{-2}\) downwards. Sketch a velocity–time graph to show the motion of the baseball from the time it is thrown until it reaches the ground again.

3. A train leaves a station and accelerates uniformly for 10 s until it reaches a velocity of 24 m s\(^{-1}\). It then travels at a constant velocity for 60 s until it approaches the next station when it decelerates uniformly at a rate of 2 m s\(^{-2}\). Sketch a velocity–time graph to show the motion of the train.

*Note: In the graphs that follow, velocity (\(v\)) is given in metres per second and time (\(t\)) in seconds.*

4. The velocity–time graph shows the motion of boat moving along a straight canal. The boat moves forward for 30 s, accelerating from rest to a velocity of 5 m s\(^{-1}\), then moves for another 50 s at a constant velocity and finally decelerates for another 50 s until it is at rest again. Calculate

a) the acceleration when \(t = 10\) and \(t = 100\)

b) the total distance covered

c) the average speed for the whole journey.
5. The velocity–time graph shows the first 35 s of the motion of a car as it moves onto a highway. In the first 10 s it accelerates from rest to 16 m s\(^{-1}\) on the slip road. It then travels for 10 s on the slip road at a constant velocity before joining the highway and accelerating for another 15 s to reach a velocity of 40 m s\(^{-1}\). Find
   a) the acceleration when \(t = 5\) and \(t = 30\)
   b) the distance travelled by the car on the slip road
   c) the total distance travelled by the car during the first 35 s.

\[ \begin{array}{c|c|c|c|c|c|c} \hline t (s) & 0 & 5 & 10 & 15 & 20 & 35 \\ \hline v (m s^{-1}) & 0 & 16 & 16 & 16 & 40 & 40 \\ \hline \end{array} \]

6. A particle accelerates to a velocity \(V\) in 20 s and then decelerates back to rest in 40 s. Find
   a) \(V\) if the total distance travelled is 450 m
   b) the acceleration in the first 20 s
   c) the total distance travelled in the first 40 s.

\[ \begin{array}{c|c|c|c|c|c|c} \hline t (s) & 0 & 20 & 40 & 60 \\ \hline v (m s^{-1}) & 0 & V & 0 & 0 \\ \hline \end{array} \]

7. In a mineshaft, an elevator is bringing coal from a coal mine below ground. The motion of the elevator is modelled by three straight-line segments. The first is acceleration from rest to 6 m s\(^{-1}\), the second is motion at a uniform velocity and the third is deceleration back to rest. Find
   a) \(T\), the time that it takes to accelerate if the initial acceleration is 0.2 m s\(^{-2}\)
   b) the total distance travelled by the elevator
   c) the average speed of the elevator.

\[ \begin{array}{c|c|c|c|c|c|c} \hline t (s) & 0 & T & 100 & 180 \\ \hline v (m s^{-1}) & 0 & 6 & 6 & 0 \\ \hline \end{array} \]
The velocity–time graph shows the motion of an elevator travelling in a building. There are seven stages in its journey. It accelerates from rest to 5 m s\(^{-1}\), and then travels at a constant velocity before decelerating back to rest. It remains stationary before moving downwards, again accelerating, moving with constant velocity and decelerating.

The times taken for each of these stages are shown on the graph. Calculate

a) the acceleration for each stage of the journey

b) the distance travelled moving upwards

c) the distance travelled moving downwards

d) If each floor in the building measures 2.5 m and the elevator starts on the 8th floor, on which floor does it first stop, and on which floor is it when it stops after 19 seconds?

Use the graph. Given that the total displacement is 420 m, find

a) the value of \( V \)

b) the acceleration at time \( t = 5 \)

c) the acceleration at time \( t = 20 \)

d) the times at which the speed is 6 m s\(^{-1}\).

Use the graph. If the total displacement is 380 m, find

a) the value of \( V \)

b) the value of the deceleration when \( t = 8 \) and \( t = 30 \)

c) the average speed for the whole journey.
11. Use the graph. Given that the total distance travelled is 70 m and that the initial deceleration is 2.5 m s$^{-2}$, find
a) the possible values of $V$
b) the possible values of $T$.

12. Use the graph. If the total displacement is 138 m and the initial deceleration is 1 m s$^{-2}$, find
a) the value of $V$
b) the value of $T$.

13. A ball is thrown upwards at a speed of 20 m s$^{-1}$ from a tower that is 25 m high. It goes up into the air and then falls all the way to the ground without hitting the tower. When it reaches the ground, it bounces back up at half the speed it hit the ground with. It comes to rest when it hits the ground for the second time. Times are shown on the velocity–time graph. If the acceleration due to gravity is $-10$ m s$^{-2}$, find
a) the greatest height it reaches above the ground
b) the time it takes to reach the ground
c) its velocity $V_1$ when it first hits the ground
d) its velocity $V_2$ when it rebounds
e) the greatest height it reaches above the ground after it bounces
f) the total time $T$ taken for the ball to come to rest.
Summary exercise 1

1. In a game of pool, a ball is directly hit towards the cushion at a speed of 10 m s\(^{-1}\). When it hits the cushion, it changes direction, returning at a speed of 8 m s\(^{-1}\). It takes 0.04 s for the ball to reach the cushion.

   a) Find the distance that the ball travels between the starting point and the cushion.

   b) Sketch a displacement–time graph showing the motion of the ball until the time that it returns to its starting position.

Exam-style question

2. Two stations at \(A\) and \(B\) are on a section of straight track and are 800 m apart. A train passes through \(A\) at \(t = 0\) at a constant speed of 20 m s\(^{-1}\) in the direction of \(B\). A second train passes through \(B\) at \(t = 5\) at a speed of 20 m s\(^{-1}\) in the opposite direction towards \(A\). After a further 10 s, the second train increases its speed to 25 m s\(^{-1}\) in the same direction.

   a) Sketch, on the same diagram, a displacement–time graph to model the motion of the two trains.

   b) Where and at what time do the two trains pass each other?

   c) Which train passes through the station at the opposite end of the track first?

A roller coaster in an amusement park is known as ‘The Long Drop’. The ride takes just over half a minute from beginning to end. The car starts at ground level and is carried upwards from rest. It accelerates uniformly and then travels for 10 s at constant velocity before decelerating and coming to a stop at the top of the track. Here it waits for 2 s before dropping back to the ground. After dropping for 6 s it decelerates rapidly for a further 2 s, coming to a halt at ground level. It reaches a maximum speed of 30 m s\(^{-1}\) on its descent. Calculate

   a) the distance that the car falls on its descent

   b) the maximum velocity \(V\) that it reaches on its ascent

   c) the value of the deceleration in the final phase before the car comes to a halt

   d) the distance that the car drops before it begins to decelerate.

Exam-style question

3. \(v(m \text{ s}^{-1})\)

A 100 m sprinter accelerates to 8 m s\(^{-1}\) in 1.5 s. He then accelerates to 12 m s\(^{-1}\) in the next 3 s and runs at a constant speed for the remainder of the race. He completes the race in \(T\) s. After he passes the finish, he decelerates and stops further along the track. Find

   a) the time he takes to complete the race

   b) his deceleration at the end of the race if he runs a further 20 m before coming to a stop.
5. A particle decelerates from a speed of 40 m s\(^{-1}\) to a speed \(V\) in 10 s. It travels at speed \(V\) for a further 10 s and then decelerates to rest in 16 s. The initial deceleration is 2 m s\(^{-2}\).
   a) Sketch a velocity–time graph for the particle.
   b) Find the value of \(V\).
   c) Find the average velocity for the whole journey.

6. A particle decelerates from an initial velocity of 12 m s\(^{-1}\) to a velocity \(V\) in \(T\) s. It continues at speed \(V\) until 15 s after it started. It then accelerates again so that 25 s after the particle started its velocity is 20 m s\(^{-1}\). The final acceleration is 1.8 m s\(^{-2}\). The particle travels a total distance of 179 m.
   a) Sketch a velocity–time graph for the particle.
   b) Calculate the value of \(V\).
   c) Calculate the value of \(T\).

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**Chapter summary**

**Displacement–time graph**
- A displacement–time graph is used to show the motion of a particle, in one dimension, along a straight line.
- Velocity = gradient of displacement–time graph
- Displacement is often denoted by \(s\).

**Velocity–time graph**
- A velocity–time graph is also used to show the motion of a particle in one dimension, along a straight line.
- Acceleration = gradient of velocity–time graph
- Displacement = area under velocity–time graph
  - For constant velocity, displacement is found by calculating the area of the rectangle on a velocity–time graph.
- Average speed = \(\frac{\text{total displacement}}{\text{time taken}}\)

*Note: It is a common error to use the incorrect formula for average speed, so make sure to learn this formula.*
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