Complete Pure Mathematics
for Cambridge International
AS & A Level
Second Edition

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Introduction

About this book

This book has been written to cover the Cambridge International AS & A Level Mathematics (9709) syllabus, and is fully aligned to the syllabus.

In addition to the main curriculum content, you will find:

- ‘Maths in real-life’, showing how principles learned in this course are used in the real world.
- Chapter introductions, which outline how each topic in the Cambridge 9709 syllabus is used in real-life.

The book contains the following features:

Throughout the book, you will encounter worked examples and a host of rigorous exercises. The examples show you the important techniques required to tackle questions. The exercises are carefully graded, starting from a basic level and going up to exam standard, allowing you plenty of opportunities to practise your skills. Together, the examples and exercises put maths in a real-world context, with a truly international focus.

At the start of each chapter, you will see a list of objectives that are covered in the chapter. These objectives are drawn directly from the Cambridge AS & A Level syllabus. Each chapter then begins with a Before you start section, which you should complete to ensure you have all the requisite skills for the chapter, and finishes with a Summary exercise and Chapter summary, ensuring that you fully understand each topic.

Each chapter contains key mathematical terms to improve understanding, highlighted in colour, with full definitions provided in the Glossary of terms at the end of the book.

The answers given at the back of the book are concise. However, when answering exam-style questions, you should show as many steps in your working as possible. All exam-style questions, as well as Paper A and Paper B have been written by the authors.
About the authors

Brian Western has over 40 years of experience in teaching Mathematics up to A level and beyond, and is also a highly experienced examiner. He taught Mathematics and Further Mathematics, and was an assistant head teacher in a large state school. Brian has written and consulted on a number of Mathematics textbooks.

Jean Linsky has been a Mathematics teacher for over 30 years, as well as head of Mathematics in Watford, Hertfordshire, in the United Kingdom. She currently teaches A Level Mathematics in a large mixed state school. Jean has authored and consulted on numerous Mathematics textbooks, is a very experienced examiner and has delivered many training courses to Mathematics teachers.

James Nicholson is an experienced teacher of Mathematics at secondary level; he has taught for 12 years at Harrow School and spent 13 years as Head of Mathematics in a large Belfast grammar school. He is the author of two A-level statistics texts, and editor of the Concise Oxford Dictionary of Mathematics. He has also contributed to a number of other sets of curriculum and assessment materials, is an experienced examiner and has acted as a consultant for UK government agencies on accreditation of new specifications.

James ran school workshops for the Royal Statistical Society for many years, and has been a member of the Schools and Further Education Committee of the Institute of Mathematics and its Application since 2000, including six years as chair; and is currently a member of the Outer Circle group for the Advisory Committee on Mathematics Education. He has served as a Vice-President of the International Association for Statistics Education for four years, and is currently Chair of the Advisory Board to the International Statistical Literacy Project.

A note from the authors

The aim of this book is to help students prepare for the Pure 1 unit of the Cambridge International AS & A Level Mathematics syllabus, though it may also be found to be useful in providing support material for other AS and A Level courses. The book contains a large number of practice questions, many of which are exam-style.

In writing the book we have drawn on our experiences of teaching Mathematics and Further Mathematics to A Level over the past 40 years as well as on our experience as examiners.

We are grateful to Robert Linsky and Graham Carter for their support during the completion of this project and to the many colleagues and students we have worked with over the years.
**Student Book: Complete Pure Mathematics 1 for Cambridge International AS & A Level**

**Syllabus: Cambridge International AS & A Level Mathematics: Pure (9709)**

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### 3. Coordinate geometry

- Find the equation of a straight line given sufficient information
- Interpret and use any of the forms \( y = mx + c \), \( y - y_1 = m(x - x_1) \), \( ax + by + c = 0 \) in solving problems
- Understand that the equation \( (x - a)^2 + (y - b)^2 = r^2 \) represents the circle with centre \((a, b)\) and radius \(r\)
- Use algebraic methods to solve problems involving lines and circles
- Understand the relationship between a graph and its associated algebraic equation, and use the relationship between points of intersection of graphs and solutions of equations

### 4. Circular measure

- Understand the definition of a radian, and use the relationship between radians and degrees
- Use the formulae \( s = r\theta \) and \( A = \frac{1}{2}r^2\theta \) in solving problems concerning the arc length and sector area of a circle

### 5. Trigonometry

- Sketch and use graphs of the sine, cosine and tangent functions (for angles of any size, and using either degrees or radians)
- Use the exact values of the sine, cosine and tangent of \(30^\circ\), \(45^\circ\), \(60^\circ\), and related angles, e.g. \(\cos 150^\circ = -\frac{\sqrt{3}}{2}\)
- Use the notations \(\sin^{-1}x\), \(\cos^{-1}x\), \(\tan^{-1}x\) to denote the principal values of the inverse trigonometric relations
- Use the identities \(\frac{\sin\theta}{\cos\theta} = \tan\theta\) and \(\sin^2\theta + \cos^2\theta = 1\)
- Find all the solutions of simple trigonometrical equations lying in a specified interval (general forms of solution are not included)

### 6. Series

- Use the expansion of \((a + b)^n\), where \(n\) is a positive integer (knowledge of the greatest term and properties of the coefficients are not required, but the notations \(\binom{n}{r}\) and \(n!\) should be known)
- Recognise arithmetic and geometric progressions
- Use the formulae for the \(n\)th term and for the sum of the first \(n\) terms to solve problems involving arithmetic or geometric progressions
- Use the condition for the convergence of a geometric progression, and the formula for the sum to infinity of a convergent geometric progression
### 7. Differentiation

- Understand the gradient of a curve at a point as the limit of the gradients of a suitable sequence of chords, and use the notations \( f'(x), f''(x), \frac{dy}{dx}, \frac{d^2y}{dx^2} \) for first and second derivatives
- Use the derivative of \( x^n \) (for any rational \( n \)), together with constant multiples, sums, differences of functions, and of composite functions using the chain rule
- Apply differentiation to gradients, tangents and normals, increasing and decreasing functions and rates of change (including connected rates of change)
- Locate stationary points, and use information about stationary points in sketching graphs (the ability to distinguish between maximum points and minimum points is required, but identification of points of inflexion is not included)

### 8. Integration

- Understand integration as the reverse process of differentiation, and integrate \( (ax + b)^n \) (for any rational \( n \) except –1), together with constant multiples, sums and differences
- Solve problems involving the evaluation of a constant of integration, e.g. to find the equation of the curve through \((1, -2)\) for which \( \frac{dy}{dx} = 2x + 1 \)
- Evaluate definite integrals (including simple cases of ‘improper’ integrals, such as \( \int_0^1 x^{-\frac{1}{2}} \, dx \) and \( \int_0^- x^{-2} \, dx \))
- Use definite integration to find:
  - the area of a region bounded by a curve and lines parallel to the axes, or between two curves
  - a volume of revolution about one of the axes
The Quadracci Pavilion is part of the Milwaukee Art Museum; it opened in 2001 and contains a movable, wing-like structure. The building, designed by Santiago Calatrava, received an outstanding structure award in 2004. The ‘wings’ open for a wingspan of 66 metres during the day and fold over the arched structure at night or during bad weather. Designing and constructing this building required the use of quadratic curves and parabolas. These same techniques are used in modelling bridges and a huge number of other structures.

Objectives

- Carry out the process of completing the square for a quadratic polynomial $ax^2 + bx + c$, and use a completed square form.
- Find the discriminant of a quadratic polynomial $ax^2 + bx + c$ and use the discriminant.
- Solve quadratic equations and quadratic inequalities, in one unknown.
- Solve by substitution a pair of simultaneous equations of which one is linear and one is quadratic.
- Recognise and solve equations in $x$ which are quadratic in some function of $x$.

Before you start

You should know how to:

1. Square a number that has a square root sign, e.g. $\sqrt{3} \times \sqrt{3} = 3$.
   e.g. $2\sqrt{5} = 4 \times 5 = 20$.
2. Factorise quadratic expressions, e.g. $2x^2 - 13x + 20 = (2x - 5)(x - 4)$
   e.g. $16x^2 - 49 = (4x + 7)(4x - 7)$.
3. Solve linear inequalities, e.g. $5x - 2 < 8x + 4$
   $-6 < 3x$
   $x > -2$.
4. Solve simultaneous linear equations, e.g. $2x + 3y = 5$
   $5x + 2y = -4$
   $10x + 15y = 25$
   $10x + 4y = -8$
   Subtracting: $11y = 33$
   $y = 3$
   Substituting: $2x + 3(3) = 5$
   $2x = -4 \Rightarrow x = -2$.

Skills check:

1. Simplify
   a) $\sqrt{7} \times \sqrt{7}$
   b) $(\sqrt{13})^2$
   c) $3\sqrt{11} \times 2\sqrt{11}$
   d) $(10\sqrt{2})^2$.
2. Factorise
   a) $6x^2 - x - 1$
   b) $1 - 100x^2$
   c) $21 + 11x - 2x^2$
   d) $2x^2 - 18$.
3. Solve
   a) $2x + 3 \geq 7x - 1$
   b) $5(2x - 3) < 2(1 - x)$
   c) $6 - 7x > 3(2x + 5)$.
4. Solve these simultaneous equations
   a) $3x - 4y = -5$
   $7x + 20y = 3$
   b) $2x - y = 3$
   $6x + 5y = 49$
   c) $4x + 3y = -11$
   $9x - 7y = -11$
   d) $6x - 2y = 1$
   $8x - 5y = -8$. 

1 Quadratics
1.1 Solve quadratic equations by factorising

You should know how to solve a quadratic equation by factorising. Here are some examples.

**Example 1**

Solve

a) $6x^2 + 11x - 35 = 0$

b) $20x^2 + 80 = 82x$.

**Example 2**

A rectangle has length $(x + 3)$ metres and width $(2x + 1)$ metres. The area of the rectangle is $12 \text{ m}^2$.

Find the length of the rectangle.

Area $= (2x + 1)(x + 3) = 2x^2 + 7x + 3 = 12$

$2x^2 + 7x - 9 = 0$

$(2x + 9)(x - 1) = 0$

$x = \frac{-9}{2}$ or $x = 1$

But the solution cannot be $x = \frac{-9}{2}$ as this will lead to a negative length and width, so $x = 1$.

Length $= x + 3 = 1 + 3 = 4 \text{ metres}$.

**Exercise 1.1**

1. Solve each of these quadratic equations.

   a) $x^2 + 2x - 35 = 0$
   
   b) $x^2 - 7x + 10 = 0$
   
   c) $x^2 - x - 12 = 0$
   
   d) $x^2 + 8x - 9 = 0$
   
   e) $x^2 - 3x = 18$
   
   f) $x^2 = x + 6$
2. A rectangle has length \((x + 4)\) cm and width \((3x + 4)\) cm. The area of the rectangle is 11 cm\(^2\). Find \(x\).

3. A piece of card has a length of \((2x - 1)\) cm and a width of \((x + 2)\) cm. A square of side \(x\) cm is removed from the card. The area of the card that is left is 68 cm\(^2\). Find the area of the card that has been removed.

4. Two numbers differ by 4. Their product is 21. Write down a quadratic equation and solve it to find the two numbers.

5. Solve \(\frac{2x^2 + 5x + 3}{x^2 + 3x + 2} = 4\).

### 1.2 Solving quadratic inequalities

To solve a quadratic inequality it is useful to sketch the curve.

If the coefficient of \(x^2\) is positive, the curve is \(\bigcup\) shaped.

If the coefficient of \(x^2\) is negative, the curve is \(\bigcap\) shaped.

**Example 3**

Solve the inequality \(x^2 - 3x - 4 \geq 0\).

\[
x^2 - 3x - 4 \geq 0
\]

\[
(x + 1)(x - 4) \geq 0
\]

We want the values of \(x\) when the curve is above the \(x\)-axis \((\geq)\).

Looking at the sketch we can see that this is true for all values of \(x\) shown by the red arrows.

Answer: \(x \leq -1\) or \(x \geq 4\)

**Note:** Unlike the linear inequality, where the solution range has only one boundary, e.g. \(x > -2\), the solution for the unknown variable in a quadratic inequality is a range of values with two boundaries, e.g. \(x < -1\) or \(x > 3\).
Example 4

Solve the inequality $6 + x - x^2 > 0$.

$6 + x - x^2 > 0$

$(3 - x)(2 + x) > 0$

We want the values of $x$ when the curve is above the $x$-axis ($>$).

Looking at the sketch we can see that this is true for the values of $x$ shown by the red arrows.

Answer: $-2 < x < 3$

Or

$6 + x - x^2 > 0$ can be written as $x^2 - x - 6 < 0$

$(x - 3)(x + 2) < 0$

Answer: $-2 < x < 3$

Note: There is only one region so we write the answer as one inequality. We read $-2 < x < 3$ as 'x is between −2 and 3'.

Remember: We change the symbol when multiplying each side by −1.

Example 5

Solve the inequality $(x + 5)(2x + 1) \leq 0$.

$(x + 5)(2x + 1) \leq 0$

As we have $+2x^2$, the curve is $\cup$ shaped.

We want the values of $x$ when the curve is on or below the $x$-axis ($\leq$).

Looking at the sketch we can see that this is the region shown by the red arrows.

Answer: $-5 \leq x \leq -\frac{1}{2}$.

Exercise 1.2

Solve each of these inequalities.

Sketch the curve for each, showing the interval that satisfies each inequality.

1. a) $(x + 2)(x - 5) \leq 0$   b) $(x - 1)(x - 3) > 0$   c) $(3x + 2)(x + 4) > 0$
   d) $(x - 7)(x + 10) < 0$   e) $(6 - x)(3 - x) \leq 0$   f) $(5 + x)(1 - 2x) \geq 0$
2. a) \( x^2 + 7x + 12 \leq 0 \)
   b) \( x^2 - x - 30 < 0 \)
   c) \( x^2 + 2x - 48 > 0 \)
   d) \( x^2 - 5x + 6 \geq 0 \)
   e) \( 2x^2 - 11x + 12 > 0 \)
   f) \( 3x^2 + 14x - 5 \leq 0 \)
   g) \( 15 + 2x - x^2 \leq 0 \)
   h) \( 16 - 6x - x^2 < 0 \)
   i) \( 21 - x - 2x^2 > 0 \)
   j) \( 10x^2 + 43x + 28 \geq 0 \)
   k) \( x^2 - 9 \geq 0 \)
   l) \( 50 - 2x^2 < 0 \)

3. a) \( x^2 - 2x > 35 \)
   b) \( x^2 + 6 \leq 5x \)
   c) \( x^2 \leq x + 20 \)
   d) \( x(x + 3) \geq 10 \)
   e) \( x^2 < 9x + 10 \)
   f) \( x^2 - 4(x + 6) > 21 \)
   g) \( 24 > 11x - x^2 \)
   h) \( 7 + 2(4x^2 - 15x) \leq 0 \)
   i) \( \frac{x^2 + 12}{2} \geq 4x \)

1.3 The method of completing the square

We can express a quadratic polynomial of the form \( ax^2 + bx + c \) in the form \( a(x + p)^2 + q \), where \( p \) and \( q \) are constants, by the method of completing the square. There is a strict method for expressing a quadratic expression as a perfect square, which can be seen in the following examples.

**Example 6**
Express \( x^2 + 10x - 3 \) in the form \((x + p)^2 + q\), where \( p \) and \( q \) are constants.

\[
x^2 + 10x - 3 = (x + 5)^2 - 25 - 3 = (x + 5)^2 - 28
\]

- **Always halve the coefficient of** \( x \) **to give the value of** \( p \).
- **We subtract 25 because** \((x + 5)^2 = x^2 + 10x + 25\) **therefore, (x + 5)^2 - 25 = x^2 + 10x.**

**Example 7**
Express \( 2x^2 - 12x + 1 \) in the form \( a(x + p)^2 + q \), where \( a \), \( p \) and \( q \) are constants.

\[
2x^2 - 12x + 1 = 2[x^2 - 6x] + 1 = 2[(x - 3)^2 - 9] + 1 = 2(x - 3)^2 - 18 + 1 = 2(x - 3)^2 - 17
\]

- **Divide the 1st two terms by the coefficient of** \( x^2 \) **and use square brackets.**
- **Complete the square for** \( x^2 - 6x. \)
- **Multiply out the square brackets.**

**Note:** We cannot divide \( 2x^2 - 12x + 1 \) by 2 as it is not an equation. If we had \( 2x^2 - 12x + 1 = 0 \), we could write \( x^2 - 6x + \frac{1}{2} = 0. \)
Example 8

Express \(3 + 4x - x^2\) in the form \(q - (x + p)^2\), where \(p\) and \(q\) are constants.

\[
3 + 4x - x^2 = -[x^2 - 4x - 3]
= -(x - 2)^2 - 4 - 3
= -(x - 2)^2 + 7
= 7 - (x - 2)^2
\]

We want the coefficient of \(x^2\) to be +1. Complete the square for \(x^2 - 4x\). Multiply the terms in the square brackets by \(-1\).

A quadratic equation of the form \(ax^2 + bx + c = 0\), where \(a\), \(b\) and \(c\) are constants and \(c \neq 0\), can also be solved by the method of completing the square.

Example 9

Solve the equation \(x^2 - 6x + 2 = 0\), giving your answer in the form \(x = p \pm \sqrt{q}\).

\[
(x - 3)^2 - 9 + 2 = 0
\]

\[
(x - 3)^2 - 7 = 0
\]

\[
x - 3 = \pm\sqrt{7}
\]

\[
x = 3 \pm \sqrt{7}
\]

Do not forget \(\pm\) when taking the square root of each side.

Example 10

Solve the equation \(x^2 - 3x - 1 = 0\) by completing the square.

Give your answer in the form \(p + \frac{\sqrt{q}}{2}\).

\[
\left(x - \frac{3}{2}\right)^2 - \frac{9}{4} - 1 = 0
\]

\[
\left(x - \frac{3}{2}\right)^2 - \frac{13}{4} = 0
\]

\[
\left(x - \frac{3}{2}\right)^2 = \frac{13}{4}
\]

\[
x - \frac{3}{2} = \pm\sqrt{\frac{13}{4}}
\]

\[
x - \frac{3}{2} = \pm\frac{1}{2}\sqrt{13}
\]

\[
x = \frac{3}{2} \pm \frac{1}{2}\sqrt{13}
\]

\[
x = \frac{3 \pm \sqrt{13}}{2}
\]

Leave \(\frac{13}{4}\) as an improper fraction.

\[
\frac{13}{4} = \frac{\sqrt{13}}{2} = \frac{\sqrt{13}}{2} = \frac{1}{2}\sqrt{13}.
\]
Example 11

Solve the inequality \(3x^2 + 24x + 2 < 0\) by completing the square. Leave your answer in surd form.

First consider \(3x^2 + 24x + 2 = 0\)

\[
x^2 + 8x + \frac{2}{3} = 0
\]

\[
(x + 4)^2 - 16 + \frac{2}{3} = 0
\]

\[
(x + 4)^2 = \frac{46}{3}
\]

\[
x = -4 \pm \sqrt{\frac{46}{3}}
\]

We want the answer in surd form.

Now consider \(3x^2 + 24x + 2 < 0\)

Answer: \(-4 - \sqrt{\frac{46}{3}} < x < -4 + \sqrt{\frac{46}{3}}\)

---

Exercise 1.3

1. Write each of these expressions in the form \((x + p)^2 + q\) or \(q - (x + p)^2\), where \(p\) and \(q\) are constants.
   a) \(x^2 + 2x - 5\)   b) \(x^2 - 10x + 20\)   c) \(x^2 - 4x + 1\)   d) \(6 - 8x - x^2\)
   e) \(10 - 16x - x^2\)   f) \(x^2 + 20x - 45\)   g) \(x^2 + 12x + 16\)   h) \(3 - 6x - x^2\)

2. Solve each of these equations by completing the square. Leave your answers in surd form.
   a) \(x^2 + 6x + 1 = 0\)   b) \(x^2 - 4x - 8 = 0\)   c) \(4 - 2x - x^2 = 0\)   d) \(x^2 - 20x + 30 = 0\)
   e) \(3 + 8x - x^2 = 0\)   f) \(x^2 + 14x - 1 = 0\)   g) \(x^2 - 3x - 2 = 0\)   h) \(x^2 - x - 3 = 0\)

3. Write each of these expressions in the form \(a(x + b)^2 + c\) or \(c - a(x + b)^2\).
   a) \(6x^2 + 12x - 3\)   b) \(3x^2 - 6x - 15\)   c) \(3x^2 - 18x + 4\)   d) \(4x^2 + 24x - 9\)
   e) \(5 - 2x - 2x^2\)   f) \(5x^2 - 20x + 2\)   g) \(4 + 3x - 2x^2\)   h) \(3x^2 + 5x + 1\)

4. Solve each of these equations by completing the square. Leave your answers in surd form.
   a) \(3x^2 + 12x + 2 = 0\)   b) \(3x^2 + 6x - 5 = 0\)   c) \(5x^2 + 50x - 7 = 0\)   d) \(3 + 20x - 2x^2 = 0\)
   e) \(2x^2 - 8x + 1 = 0\)   f) \(4x^2 - 48x - 26 = 0\)   g) \(2x^2 - x - 12 = 0\)   h) \(9 + 2x - 3x^2 = 0\)

5. Solve each of these inequalities by completing the square. Leave your answers in surd form.
   a) \(x^2 + 4x + 2 < 0\)   b) \(x^2 - 6x - 3 \geq 0\)   c) \(x^2 - 2x - 1 > 0\)   d) \(x^2 + 10x + 7 \leq 0\)
   e) \(2x^2 - 12x - 7 \geq 0\)   f) \(3x^2 + 6x + 1 < 0\)   g) \(5x^2 + 10x - 2 \leq 0\)   h) \(2x^2 - 8x - 3 > 0\)
1.4 Solving quadratic equations using the formula

A quadratic equation can be expressed in the form \( ax^2 + bx + c = 0 \), where \( a \), \( b \) and \( c \) are constants and \( a \neq 0 \).

We can use the method of completing the square to find the general formula for solving a quadratic equation.

\[
ax^2 + bx + c = 0
\]
\[
x^2 + \frac{b}{a}x + \frac{c}{a} = 0
\]
\[
\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a} = 0
\]
\[
\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 = -\frac{c}{a}
\]
\[
\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}
\]
\[
x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}
\]
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

This leads us to a formula for solving a quadratic equation of the form \( ax^2 + bx + c = 0 \). This is often called the quadratic formula.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Example 12

Solve the equation \( 3x^2 + x - 5 = 0 \). Give your answer correct to 3 significant figures.

\[
a = 3, \ b = 1, \ c = -5
\]
\[
x = \frac{-1 \pm \sqrt{(1)^2 - 4(3)(-5)}}{2(3)}
\]
\[
= \frac{-1 \pm \sqrt{61}}{6}
\]
\[
= \frac{-1 + \sqrt{61}}{6} \text{ or } \frac{-1 - \sqrt{61}}{6}
\]
\[
= 1.135042... \text{ or } -1.468375...
\]
\[
= 1.14 \text{ or } -1.47
\]
Example 13
Solve the inequality $2 - 6x - x^2 \leq 0$. Leave your answer in surd form.

Consider $2 - 6x - x^2 = 0$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(-1)(2)}}{2(-1)}$$

$$= \frac{6 + \sqrt{44}}{-2} \text{ or } \frac{6 - \sqrt{44}}{-2}$$

$$= 3 + \sqrt{11} \text{ or } 3 - \sqrt{11}$$

Now $2 - 6x - x^2 \leq 0$

$$x \leq -3 - \sqrt{11} \text{ or } x \geq -3 + \sqrt{11}$$

Example 14
Solve the equation $2x^2 = 5x - 1$. Write your answer correct to 2 decimal places.

When asked to solve to 2 decimal places, it is likely that the quadratic formula will be appropriate.

We must first rearrange to $ax^2 + bx + c = 0$.

In examinations, unless otherwise indicated, answers to all questions on all topics should be rounded to 3 significant figures. However, do not round your answers to 3 significant figures until the final answer. A significant number of candidates lose marks in examinations by rounding earlier in the question which then leads to an inaccurate answer. You should therefore keep all numbers unrounded until the final answer.

Exercise 1.4

1. Solve each of these quadratic equations. Write your answers correct to 2 decimal places.
   
   a) $2x^2 - 3x - 4 = 0$
   
   b) $5x^2 - 11x + 4 = 0$
   
   c) $3x^2 + 12x + 5 = 0$
   
   d) $x^2 + 5x - 2 = 0$
   
   e) $4x^2 + 2x - 5 = 0$
   
   f) $x^3 + x - 4 = 0$
   
   g) $3x^2 = 6x - 2$
   
   h) $9x = 6x^2 + 2$
   
   i) $3 - 8x = 2x^2$
   
   j) $x^2 + 3x = 1$

2. Use the formula to solve each of these quadratic equations. Leave your answers in surd form.
   
   a) $2x^2 - x - 5 = 0$
   
   b) $3x^2 - 6x + 1 = 0$
   
   c) $5x^2 - 3x - 7 = 0$
   
   d) $6x^2 - x - 4 = 0$
   
   e) $x^2 + 8x - 3 = 0$
   
   f) $x^2 + 7x + 3 = 0$
   
   g) $x^2 + 10x - 5 = 0$
   
   h) $2x^2 + 5x - 4 = 0$
   
   i) $4x^2 = 5 - 10x$
   
   j) $4x - 1 - x^2 = 0$

3. Use the formula to solve the following inequalities. Leave your answers in surd form.
   
   a) $3x^2 + 10x + 5 < 0$
   
   b) $x^2 - 6x + 7 \geq 0$
   
   c) $4x^2 + 3x - 2 > 0$
   
   d) $2x^2 + x - 2 \leq 0$
   
   e) $5x^2 - 8x - 2 \geq 0$
   
   f) $6x^2 - 6x + 1 \geq 0$
   
   g) $5 + 3x - x^2 \leq 0$
   
   h) $2x^2 + 2x - 3 < 0$
   
   i) $2x + 1 - 2x^2 > 0$
   
   j) $x^2 < 1 - x$
1.5 Solve more complex quadratic equations

You can now adapt the techniques you have learnt to solve more complex quadratic equations.

Example 15
Solve \( x^4 - 5x^2 + 4 = 0 \).

\[ x^4 - 5x^2 + 4 = 0 \]

Let \( y = x^2 \)

Hence \( y^2 - 5y + 4 = 0 \)

\( (y - 4)(y - 1) = 0 \)

\( y = 4 \) or \( y = 1 \)

\( x^2 = 4 \) or \( x^2 = 1 \)

\( x = \pm 2 \) or \( x = \pm 1 \)

Or

Factorise straight away \( (x^2 - 4)(x^2 - 1) = 0 \)

\( x^2 = 4 \) or \( x^2 = 1 \)

\( x = \pm 2 \) or \( x = \pm 1 \)

Example 16
Solve the equation \( 5x^4 - 20x^2 - 1 = 0 \) by completing the square.

Give your answer correct to 3 significant figures.

\[ 5x^4 - 20x^2 - 1 = 0 \]

Let \( y = x^2 \)

Hence \( 5y^2 - 20y - 1 = 0 \)

\( y^2 - 4y - \frac{1}{5} = 0 \)

\( (y - 2)^2 - 4 - \frac{1}{5} = 0 \)

\( (y - 2)^2 = \frac{21}{5} \)

\( y - 2 = \pm \sqrt{\frac{21}{5}} \)

\( y = 2 \pm \sqrt{\frac{21}{5}} = 4.04939... \) or \(-0.04939... \)

\( x^2 = 4.04939... \) or \(-0.04939... \)

\( x = \pm \sqrt{4.04939...} \) or \( x = \pm 2.0123... = \pm 2.01 \) (3 s.f.)

Hint: Try to adapt the equation to quadratic form \((\ldots)^2 - 5(\ldots) + 4 = 0\) by appropriate substitution.
Example 17

Solve the equation $2x^6 - 3x^3 = 8$. Write your answer correct to 2 decimal places.

We must first rearrange to $ax^2 + bx + c = 0$.

Let $y = x^3$ Hence $2y^2 - 3y - 8 = 0$

$$y = \frac{(-3) \pm \sqrt{(-3)^2 - 4(2)(-8)}}{2(2)}$$

$$= \frac{3 + \sqrt{73}}{4} \text{ or } \frac{3 - \sqrt{73}}{4}$$

$$= 2.886... \text{ or } -1.386...$$

$x^3 = 2.886... \text{ or } -1.386...$

$x = \sqrt[3]{2.886...} \text{ or } -\sqrt[3]{1.386...}$

$$= 1.4237... \text{ or } -1.1149...$$

Exercise 1.5

1. Solve each of these quadratic equations. Give your answers as exact answers.
   a) $x^4 - 4x^2 - 21 = 0$
   b) $6x^4 - x^2 = 2$
   c) $x^6 + 7x^3 + 10 = 0$
   d) $4 + 11x^2 - 3x^4 = 0$
   e) $6x^8 + 6 = 13x^4$
   f) $x^6 + 3x^3 = 40$

2. Solve each of these equations by completing the square. Write your answers correct to 2 decimal places.
   a) $x^6 - 8x^3 + 1 = 0$
   b) $3 - 6x^2 - x^4 = 0$
   c) $x^4 + 2x^2 = 10$
   d) $x^6 + x^3 - 4 = 0$
   e) $2x^8 - 20x^4 - 7 = 0$
   f) $3x^4 + 1 = 12x^2$

3. Use the formula to solve each of these quadratic equations. Write your answers correct to 3 significant figures.
   a) $x^4 + 3x^2 - 5 = 0$
   b) $2x^6 - 4x^3 + 1 = 0$
   c) $3x^6 - 7x^3 = 2$
   d) $5x^4 + 10x^2 + 2 = 0$
   e) $4x^{10} + x^5 - 2 = 0$
   f) $x^8 - x^4 - 1 = 0$
1.6 The discriminant of a quadratic equation

We sometimes call the solutions of a quadratic equation the roots of the equation. This also tells us where the quadratic graph crosses the x-axis. When we have an equation of the form $ax^2 + bx + c = 0$, we can tell the nature of the roots by looking at the discriminant of the quadratic equation.

The discriminant is the value of $b^2 - 4ac$ in $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

If $b^2 - 4ac > 0$ there are two distinct real roots.
If $b^2 - 4ac = 0$ there are equal roots (one repeated root).
If $b^2 - 4ac < 0$ there are no real roots.

Example 18

Work out whether each of these quadratic equations has two distinct roots, equal roots or no real roots.

a) $x^2 - 3x + 5 = 0$

b) $3x^2 + x - 6 = 0$

c) $25x^2 + 20x + 4 = 0$

Note: We know from 1.3 that if $y = ax^2 + bx + c$ and $a > 0$, then the curve is \( \cup \) shaped.
If $b^2 - 4ac < 0$, then there are no real roots and the curve does not cross the x-axis, so $y$ is positive for all values of $x$.

Example 19

The equation $x^2 + px + q = 0$, where $p$ and $q$ are constants, has roots $-1$ and $4$.

a) Find the value of $p$ and $q$.

b) Using these values of $p$ and $q$, find the constant $r$ for which the equation $x^2 + px + q + r = 0$ has equal roots.
The roots are the values of $x$.
Comparing with $x^2 + px + q = 0$.
Substitute the values of $p$ and $q$ from part (a).

**Exercise 1.6**

1. Work out the discriminant for each equation and then determine whether each equation has two distinct roots, equal roots or no real roots.
   a) $2x^2 - 3x + 4 = 0$
   b) $5x^2 - 11x + 4 = 0$
   c) $x^2 + 12x + 36 = 0$
   d) $2x^2 + x - 2 = 0$
   e) $4x^2 + 5x + 2 = 0$
   f) $4x^2 - 12x + 9 = 0$
   g) $7x^2 - 2x + 1 = 0$
   h) $x^2 - 6x + 7 = 0$
   i) $2 - x - 4x^2 = 0$
   j) $6x^2 = 6x - 1$
   k) $16x^2 = 24x - 9$
   l) $(x + 2)^2 - 8x - 2 = 0$

2. Show that $x^2 + 8x + 16 \geq 0$ for all values of $x$.
3. Show that $1 + 100x^2 - 20x \geq 0$ for all values of $x$.
4. If $2x^2 - ax + 8 = 0$ has no real roots, find the range of possible values of $a$.
5. If $6 - 2x - kx^2 = 0$ has a repeated root, find the value of $k$.
6. The equation $x^2 + px + q = 0$, where $p$ and $q$ are constants, has roots $-3$ and $2$.
   Find the value of $p$ and $q$.
7. Use the discriminant to find the nature of the roots of the equation $3x + 4 = \frac{5}{x}$.
8. The quadratic equation $kx^2 + 5x + 2 = 0$ has two distinct real roots. Find the range of possible values of $k$.
9. Find the value of $p$ for which the quadratic equation $px^2 - 4px + 2 - p = 0$ has equal roots.
10. Prove that the quadratic equation $(q - 5)x^2 + 5x - q = 0$ has real roots for any value of $q$.
11. The quadratic equation $x + k + \frac{9}{x} = 0$ has equal roots. Find the two possible values of $k$.
12. The equation $px^2 + qx + r = 0$, where $p$, $q$ and $r$ are constants, has roots $\frac{-1}{2}$ and $\frac{3}{4}$.
   Find the smallest possible integer values of $p$, $q$ and $r$. 

The discriminant of a quadratic equation
1.7 Solving simultaneous equations

We already know how to solve simultaneous equations where both equations are linear. We are now going to solve simultaneous equations where one equation is linear and the other is quadratic.

We will either get two distinct roots, one repeated root, or no real roots.

**Example 20**

Solve simultaneously \( y = x^2 - 3x - 1 \) and \( y = 2x - 7 \).

\[
x^2 - 3x - 1 = 2x - 7
\]
\[
x^2 - 5x + 6 = 0
\]
\[
(x - 3)(x - 2) = 0
\]
\[
x = 3 \text{ or } x = 2
\]

When \( x = 3 \), \( y = 2(3) - 7 = -1 \)

When \( x = 2 \), \( y = 2(2) - 7 = -3 \)

Solutions are \( x = 3, y = -1 \) and \( x = 2, y = -3 \)

**Example 21**

Show that there are no real roots for the simultaneous equations \( y = x^2 - 2x - 1 \) and \( y = x - 5 \).

The curve \( y = x^2 - 2x - 1 \) meets the line \( y = x - 5 \) when \( x^2 - 2x - 1 = x - 5 \)

\[
x^2 - 3x + 4 = 0
\]

\( b^2 - 4ac = (-3)^2 - 4(1)(4) = -7 < 0 \)

There are no real roots to the equation \( x^2 - 3x + 4 = 0 \).

Thus, there are no real roots for the simultaneous equations.
Example 22

Solve simultaneously \( y^2 + xy + 4x = 7 \) and \( x - y = 3 \).

\[
\begin{align*}
x - y &= 3 \\
x &= 3 + y \\
y^2 + (3 + y)y + 4(3 + y) &= 7 \\
y^2 + 3y + y^2 + 12 + 4y - 7 &= 0 \\
2y^2 + 7y + 5 &= 0 \\
(2y + 5)(y + 1) &= 0 \\
y &= -\frac{5}{2} \text{ or } y = -1 \\
x &= 3 + \left(-\frac{5}{2}\right) \text{ or } x = 3 + 1 \\
x &= \frac{1}{2} \text{ or } x = 2 \\
\text{Solutions: } x &= \frac{1}{2}, \quad y = -\frac{5}{2} \text{ or } x = 2, \quad y = -1
\end{align*}
\]

We can use the same method to solve other pairs of simultaneous equations where we do not have one linear equation and one quadratic equation.

We could also substitute \( y = x - 3 \).

Substitute for \( x \) in the non-linear equation.

Substitute for \( y \) in \( x = 3 + y \).

Exercise 1.7

Solve these simultaneous equations.

1. \( y = x + 1 \)  
   \( y = x^2 - 1 \)

2. \( y = 4x + 7 \)  
   \( y = 2x^2 + 1 \)

3. \( y = x^2 - 6x + 5 \)  
   \( y = x - 1 \)

4. \( y = x^2 - x - 2 \)  
   \( x + y = 7 \)

5. \( y = 2x \)  
   \( y = x^2 - x + 2 \)

6. \( y = x^2 - 2x - 5 \)  
   \( y = x + 5 \)

7. \( y = x \)  
   \( y = 6 - x^2 \)

8. \( y = 4x + 5 \)  
   \( y = x^2 \)

9. \( y + 3 = 2x \)  
   \( y = x^2 - 2x + 1 \)

10. \( y = 2x + 5 \)  
    \( y = x^2 - 3x - 1 \)

11. \( x^2 + y = 3 \)  
    \( y + 3x = 5 \)

12. \( y = x - 4 \)  
    \( y = 2 - x^2 \)

13. \( x - 2y = 8 \)  
    \( xy = 24 \)

14. \( y = 6x + 5 \)  
    \( y = 12x^2 - 5x \)

15. \( y = \frac{1}{2} (1 - x^2) \)  
    \( y = x - 1 \)

16. \( x^2 + y^2 = 8 \)  
    \( 2x = y + 2 \)

17. \( 3x + 4y = 15 \)  
    \( 2xy = 9 \)

18. \( y = 1 + 2x \)  
    \( y^2 = 2x^2 + x \)

19. \( y = 2x + 1 \)  
    \( x^2 - 2xy + y^2 = 1 \)

20. \( 3x = 1 + 2y \)  
    \( 3x^2 - 2y^2 + 5 = 0 \)

21. Show that there are no real solutions to the simultaneous equations \( y = 1 + 2x - x^2 \) and \( y = \frac{1}{2} x + 5 \).
22. The diagram shows a rectangle.
   The area of the rectangle is 3.5 m².
   The perimeter of the rectangle is 7.8 m.
   Find the dimensions of the rectangle.

23. The solid cuboid has a volume of 54 cm³ and a total surface area of 96 cm² and \( x > y \).
   a) Use this information to write down two equations in \( x \) and \( y \).
   b) Solve the equations simultaneously to find the value of \( x \) and the value of \( y \).

24. The diagram shows an L-shape.
   All measurements are in centimetres.
   a) Write down two equations in \( x \) and \( y \).
   b) Solve the equations simultaneously and hence find the perimeter of the L-shape.

25. The diagram shows a trapezium.
   All measurements are in centimetres.
   The area of the trapezium is 60 cm².
   The perimeter of the trapezium is 36 cm.
   Find the value of \( x \) and the value of \( y \).

1.8 Graphs of quadratic functions

You can sketch a graph of the quadratic equation \( y = ax^2 + bx + c \),
where \( a, b \) and \( c \) are constants and where \( a \neq 0 \), by considering the following:

If \( a > 0 \), the curve is \( \cup \) shaped and has a **minimum** value.

If \( a < 0 \), the curve is \( \cap \) shaped and has a **maximum** value.
If \( b^2 - 4ac > 0 \), there are **two distinct real roots**.

Or

If \( b^2 - 4ac = 0 \), there are **equal roots** (one repeated root).

Or

If \( b^2 - 4ac < 0 \), there are **no real roots**.

Or

When \( ax^2 + bx + c = 0 \) is expressed in the form \((x + p)^2 + q\), then the **minimum value** is when \( x = -p \) and the **minimum value** is \( q \).

\((-p, q)\)

When \( ax^2 + bx + c = 0 \) is expressed in the form \(-(x + p)^2 + q\), then the **maximum value** is when \( x = -p \) and the **maximum value** is \( q \).

\((-p, q)\)

The coordinates of the **vertex** of the curve are \((-p, q)\).

We can find these values by the method of completing the square.

The equation of the **line of symmetry** is \[ x = \frac{-b}{2a} \]

A vertex is a defining point in geometrical situations. It can be where the sides of a polygon intersect, where the edges of a polyhedron intersect or, in this case, where the curve and its line of symmetry intersect.
Example 23
Express \( y = x^2 - 6x + 10 \) in the form \( y = (x + p)^2 + q \).
Sketch the curve, stating the coordinates of the vertex.

\[ y = (x - 3)^2 - 9 + 10 = (x - 3)^2 + 1 \]
Coordinates of the vertex are (3, 1).
The minimum value of \( y \) is 1.
\( a > 0 \) therefore the graph is \( \cup \) shaped.
The line of symmetry:
\[ x = \frac{-(-6)}{2(1)} \]
\[ x = 3 \]
\[ b^2 - 4ac = (-6)^2 - 4(1)(10) \]
\[ = -4 < 0 \]
therefore no real roots.

Example 24
Express \( y = 3 - 8x - 2x^2 \) in the form \( y = a(x + b)^2 + c \).
Sketch the curve, stating the coordinates of the vertex.

\[ y = -2[x^2 + 4x] + 3 \]
\[ = -2[(x + 2)^2 - 4] + 3 \]
\[ = -2(x + 2)^2 + 8 + 3 \]
\[ = -2(x + 2)^2 + 11 \]
Coordinates of the vertex are (-2, 11).
The maximum value of \( y \) is 11.
\( a < 0 \) therefore the graph is \( \cap \) shaped.
The line of symmetry:
\[ x = \frac{-(-8)}{2(-2)} \]
\[ x = -2 \]
\[ b^2 - 4ac = (-8)^2 - 4(-2)(3) \]
\[ = 88 > 0 \]
therefore two distinct roots.

Example 25
A sheep pen is in the shape of a rectangle.
One of the sides of the pen is a wall.
A farmer puts fencing on the other three sides of the pen.
The two sides that touch the wall are each \( x \) metres.
He uses 40 m of fencing.
Find the maximum area of the pen.

The third side of the pen = 40 - 2x
Area of the pen = \( x(40 - 2x) \)
\[ = 40x - 2x^2 \]
\[ = -2[x^2 - 20x] \]
\[ = -2[(x - 10)^2 - 100] \]
\[ = -2(x - 10)^2 + 200 \]
The maximum area is 200 m\(^2\) (when \( x = 10 \)).
**Exercise 1.8**

In questions 1 to 10, express each of the quadratic equations in the form $y = a(x + b)^2 + c$. Sketch the curve, stating the coordinates of the vertex and whether there is a maximum or minimum value of $y$.

1. $y = x^2 + 2x$
2. $y = x^2 - 6x + 10$
3. $y = 3 - 2x - 4x^2$
4. $y = x^2 + 4x - 3$
5. $y = 5 + 2x - x^2$
6. $y = 3 + 4x - 4x^2$
7. $y = x^2 - 4x - 8$
8. $y = x^2 + 6x + 7$
9. $y = 5x^2 - 4x - 2$
10. $y = 2x^2 - 5x - 3$

11. A right-angled triangle has a width of $x$ cm.
   The length of the hypotenuse is 10 cm.
   The perimeter of the triangle is 24 cm.
   Find the maximum area of the triangle.

12. A rectangle has a width of $x$ cm.
    The perimeter of the rectangle is 32 cm.
    Find the maximum area of the rectangle.

13. Find the maximum area of the triangle on the right.
    **State** the value of $x$ when this occurs.

14. Faisal is $x$ years old. Faisal has a brother called Omar.
    The sum of the two boys’ ages is 20 years.
    a) Express the product of their ages in the form $y = a(x - b)^2 + c$.
    b) How old must Faisal be to make the product of their ages a maximum?
Summary exercise 1

1. Solve \((6x - 5)(4 - 3x) = 0\).

2. a) Factorise \(10x^2 - 29x - 21\).
   b) Hence or otherwise solve the equation \(10x^2 - 29x = 21\).

3. Solve \(y^6 - 9y^3 + 18 = 0\).

4. By using the substitution \(y = \sqrt{x}\) or otherwise, solve the equation \(2x - 5\sqrt{x} + 2 = 0\).

5. Solve the equation \(2x^6 - 16x^3 - 3 = 0\) by completing the square. Write your answer correct to 3 significant figures.

6. Solve the equation \(3x^4 + 5x^2 = 7\) by using the formula. Write your answer correct to 2 decimal places.

Exam-style question

7. A square piece of card with sides \(x\) cm has squares of sides 3 cm cut from the corners. The card is then folded to make an open box with volume 48 cm\(^2\). Find the dimensions of the card.

Exam-style questions

8. Find the set of values for \(x\) for which \((x - 1)(x + 2) < 18\).

9. Find the set of values for \(x\) for which \(20 + 7x - 6x^2 \geq 0\).

10. Solve the equation \(2x^2 + 5x + 1 = 0\). Write your answer correct to 3 significant figures.

11. Use the formula to solve \(7x^2 - 3x - 5 \leq 0\). Leave your answer in surd form.

Exam-style questions

12. Solve \(x(x - 1) - 3(x - 3) + 3(x - 2) = x + 6\).

13. Find the relationship between \(p\) and \(q\), if the equation \(px^2 + 3qx + 9 = 0\) has equal roots.

14. Work out whether each of these quadratic equations has two distinct roots, equal roots or no real roots.
   a) \(x^2 + x + 1 = 0\)
   b) \(4x^2 + 12x + 9 = 0\)
   c) \(3x^2 + 4x + 1 = 0\)

Exam-style questions

15. The quadratic equation \(x^2 + kx + 36 = 0\) has two different real roots. Find the set of possible values of \(k\).

16. The equation \(x^2 + px + q = 0\), where \(p\) and \(q\) are constants, has roots \(-1\) and 4.
   a) Find the values of \(p\) and \(q\).
   b) Using these values of \(p\) and \(q\), find the value of \(r\), where \(r\) is a constant, and the equation \(x^2 + px + q + r = 0\) has equal roots.
EXAM-STYLE QUESTIONS

17. Solve the simultaneous equations $x + y = 1$ and $x^2 - y^2 = 5$.

18. Solve the simultaneous equations $x + y = 0$ and $2x^2 + y^2 = 6$.

19. Solve the simultaneous equations $x + y = 3$ and $x^2 + 2xy = 5$.

20. Solve the simultaneous equations $3x + y = 8$ and $3x^2 + y^2 = 28$.

21. a) Express $3x^2 + 12x + 5$ in the form $p(x + q)^2 + r$.

   b) Find the minimum value of $3x^2 + 12x + 5$.

22. A square garden of side $x$ metres is surrounded by a path of width 1 metre. The area of the garden is the same as the area of the path.

   Find the value of $x$.

   Leave your answer in surd form.

23. A curve has equation $y = 2x^2 + x + 11$.

   i) Find the set of values of $x$ for which $y \leq 17$.

24. Rectangle $A$ has a height of $(x + 1)$ metres and an area of $4 \text{ m}^2$.

   Rectangle $B$ has a height of $(2x - 1)$ metres and an area of $6 \text{ m}^2$.

   The sum of the widths of the two rectangles is less than $8$ metres.

   Find the set of values for $x$.

25. Find the set of values of $k$ for which the line $y + 4 = kx$ intersects the curve $y = x^2$ at two distinct points.

26. The equation $x^2 + 8x - k(x + 8) = 0$ has equal roots. Find the value of $k$.

27. Find the real roots of the equation $\frac{8}{x^2} + \frac{5}{x^2} = 3$. 
Chapter summary

To solve quadratic equations

- **Factorise** and solve.
- Or use the **method of completing the square**.
- Or use the quadratic formula \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \).

To solve quadratic inequalities

- Solve the corresponding equation \( ax^2 + bx + c = 0 \) to find the values of \( x \).
- Sketch the graph of \( y = ax^2 + bx + c \).
- Write down the solutions using inequality signs.

The discriminant of the quadratic equation \( ax^2 + bx + c = 0 \)

- Discriminant = \( b^2 - 4ac \)
  - If \( b^2 - 4ac > 0 \) there are two distinct real roots.
  - If \( b^2 - 4ac = 0 \) there are equal roots (one repeated root).
  - If \( b^2 - 4ac < 0 \) there are no real roots.

\[
\begin{align*}
\text{ax}^2 + \text{bx} + \text{c} &= 0, \text{ where } a, b \text{ and } c \text{ are constants, } a \neq 0. \\
\text{Discriminant} &= b^2 - 4ac \\
&\quad \text{If } b^2 - 4ac > 0 \quad \text{there are two distinct real roots.} \\
&\quad \text{If } b^2 - 4ac = 0 \quad \text{there are equal roots (one repeated root).} \\
&\quad \text{If } b^2 - 4ac < 0 \quad \text{there are no real roots.}
\end{align*}
\]

Graphs of quadratic equations \( y = ax^2 + bx + c \) where \( a, b \) and \( c \) are constants, \( a \neq 0 \)

- If \( a > 0 \), the curve is \( \cup \) shaped and has a **minimum** value.
- If \( a < 0 \), the curve is \( \cap \) shaped and has a **maximum** value.
- The **equation of the line of symmetry** is \( x = \frac{-b}{2a} \).

To solve simultaneous equations where one is linear and one is quadratic

- Rearrange the linear equation to write \( x \) in terms of \( y \) or \( y \) in terms of \( x \).
- Then substitute this into the quadratic equation.

Recognise and solve equations in \( x \) which are quadratic in some function of \( x \)

- e.g. To solve \( x^6 - 2x^3 + 1 = 0 \), let \( y = x^3 \) and solve \( y^2 - 2y + 1 = 0 \).
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