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### E1.17 Use exponential growth and decay in relation to population and finance.

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### E2: Algebra and graphs

| E2.1 | Use letters to express generalised numbers and express basic arithmetic processes algebraically. Substitute numbers for words and letters in formulae. Construct and transform complicated formulae and equations. | 59–63, 176–183 |
| E2.2 | Manipulate directed numbers. Use brackets and extract common factors. Expand products of algebraic expressions. Factorise where possible expressions of the form: \( ax + bx + kay + kby, a^2x^2 - b^2y^2, a^2 + 2ab + b^2, ax^2 + bx + c. \) | 57–59, 64–67, 84–87 |
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| E2.6 | Represent inequalities graphically and use this representation in the solution of simple linear programming problems. | 196–202 |
| E2.7 | Continue a given number sequence. Recognise patterns in sequences including the term-to-term rule and relationships between different sequences. Find the \( n \)th term of sequences. | 10–14 |
| E2.8 | Express direct and inverse proportion in algebraic terms and use this form of expression to find unknown quantities. | 183–189 |
| E2.9 | Use function notation, e.g. \( f(x) = 3x - 5, f : x \rightarrow 3x - 5, \) to describe simple functions. Find inverse functions \( f^{-1}(x) \). Form composite functions as defined by \( g(f(x)) = g(f(x)) \). | 246, 305–310 |
| E2.10 | Interpret and use graphs in practical situations including travel graphs and conversion graphs. Draw graphs from given data. Apply the idea of rate of change to easy kinematics involving distance–time and speed–time graphs, acceleration and deceleration. Calculate distance travelled as area under a linear speed–time graph. | 247, 249–255, 260–266 |
| E2.11 | Construct tables of values and draw graphs for functions of the form \( ax^n \) (and simple sums of these) and functions of the form \( ax^b + c \). Solve associated equations approximately, including finding and determining roots by graphical methods. Draw and interpret graphs representing exponential growth and decay problems. Recognise, sketch and interpret graphs of functions. | 250, 255–260 |
| E2.12 | Estimate gradients of curves by drawing tangents. | 247–250 |
| E2.13 | Understand the idea of a derived function. Use the derivatives of functions of the form \( ax^n \), and simple sums of not more than three of these. Apply differentiation to gradients and turning points (stationary points). Discriminate between maxima and minima by any method. | 266–271 |
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### E4: Geometry

<p>| E4.1 | Use and interpret the geometrical terms: point, line, parallel, bearing, right angle, acute, obtuse and reflex angles, perpendicular, similarity and congruence. Use and interpret vocabulary of triangles, quadrilaterals, circles, polygons and simple solid figures including nets. | 140, 144, 147, 153, 156, 158, 162, 167 |
| E4.2 | Measure and draw lines and angles. Construct a triangle given the three sides using ruler and pair of compasses only. | 166–167 |
| E4.3 | Read and make scale drawings. | 219–220 |
| E4.4 | Calculate lengths of similar figures. Use the relationships between areas of similar triangles, with corresponding results for similar figures and extension to volumes and surface areas of similar solids. | 147–156 |
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| E4.6 | Recognise rotational and line symmetry (including order of rotational symmetry) in two dimensions. Recognise symmetry properties of the prism (including cylinder) and the pyramid (including cone). Use the following symmetry properties of circles: • equal chords are equidistant from the centre • the perpendicular bisector of a chord passes through the centre • tangents from an external point are equal in length. | 144–147, 162–164 |
| E4.7 | Calculate unknown angles using the following geometrical properties: • angles at a point • angles at a point on a straight line and intersecting straight lines • angles formed within parallel lines • angle properties of triangles and quadrilaterals • angle properties of regular polygons • angle in a semicircle • angle between tangent and radius of a circle • angle properties of irregular polygons • angle at the centre of a circle is twice the angle at the circumference • angles in the same segment are equal • angles in opposite segments are supplementary; cyclic quadrilaterals. | 137–141, 158–165 |</p>
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<td>formula area of triangle = ( \frac{1}{2} ab \sin C ).</td>
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<td></td>
<td>E6.5 Solve simple trigonometrical</td>
<td>220–222</td>
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</table>

<table>
<thead>
<tr>
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<th></th>
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<tr>
<td>E7.1 Describe a translation by using a vector</td>
<td>Describe a translation by using a vector</td>
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</tr>
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<td>represented by e.g. ( \begin{pmatrix} x \ y \end{pmatrix} ), ( \overrightarrow{AB} ) or ( \mathbf{a} ). Add and subtract vectors. Multiply a vector by a scalar.</td>
<td>represented by e.g. ( \begin{pmatrix} x \ y \end{pmatrix} ), ( \overrightarrow{AB} ) or ( \mathbf{a} ). Add and subtract vectors. Multiply a vector by a scalar.</td>
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<tr>
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<td>E7.2 Reflect simple plane figures. Rotate</td>
<td>310–324</td>
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<td></td>
<td>Recognise and describe reflections,</td>
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<td>E7.3 Calculate the magnitude of a vector</td>
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<td>( \begin{pmatrix} x \ y \end{pmatrix} ) as ( \sqrt{x^2 + y^2} ). Represent vectors by directed line segments. Use the sum and difference of two vectors to express given vectors in terms of two coplanar vectors. Use position vectors.</td>
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</table>
### E8: Probability

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<th>Calculate the probability of a single event as either a fraction, decimal or percentage.</th>
<th>366–370</th>
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<td>E8.2</td>
<td>Understand and use the probability scale from 0 to 1.</td>
<td>366–370</td>
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<tr>
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<td>E8.5</td>
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<td>373–381</td>
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<td>381–383</td>
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### E9: Statistics

<table>
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<th>337–338, 358–359</th>
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<tr>
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<td>359–361</td>
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<tr>
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<td>332–343, 349–354</td>
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<tr>
<td>E9.4</td>
<td>Calculate the mean, median, mode and range for individual and discrete data and distinguish between the purposes for which they are used.</td>
<td>344–347</td>
</tr>
<tr>
<td>E9.5</td>
<td>Calculate an estimate of the mean for grouped and continuous data. Identify the modal class from a grouped frequency distribution.</td>
<td>348–349</td>
</tr>
<tr>
<td>E9.6</td>
<td>Construct and use cumulative frequency diagrams. Estimate and interpret the median, percentiles, quartiles and inter-quartile range. Construct and interpret box-and-whisker plots.</td>
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<tr>
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Introduction

About this book
This revised 5th edition is designed to provide the best preparation for your Cambridge IGCSE examination, and has been completely updated for the latest Mathematics 0580 and 0980 extended/core syllabus.

Finding your way around
To get the most out of this book when studying or revising, use the:
- **Contents list** to help you find the appropriate units.
- **Index** to find key words so you can turn to any concept straight away.

Exercises and exam-style questions
There are thousands of questions in this book, providing ample opportunities to practise the skills and techniques required in the exam.
- **Worked examples and comprehensive exercises** are one of the main features of the book. The examples show you the important skills and techniques required. The exercises are carefully graded, starting from the basics and going up to exam standard, allowing you to practise the skills and techniques.
- **Revision exercises** at the end of each unit allow you to bring together all your knowledge on a particular topic.
- **Examination-style exercises** at the end of each unit consist of questions from past Cambridge IGCSE papers.
- **Examination-style papers**: there are two papers, corresponding to the papers you will take at the end of your course: Paper 2 and Paper 4. They give you the opportunity to practise for the real thing.
- **Revision section**: Unit 12 contains multiple-choice questions to provide an extra opportunity to revise.
- **Answers to numerical problems** are at the end of the book so you can check your progress.

Investigations
Unit 11 provides many opportunities for you to explore the world of mathematical problem-solving through investigations, puzzles and games.

Links to curriculum content
At the start of each unit you will find a list of objectives that are covered in the unit. These objectives are drawn from the Extended section of the Cambridge IGCSE syllabus.

What’s on the website?
The support website contains a wealth of material to help solidify your understanding of the Cambridge IGCSE Mathematics course, and to aid revision for your examinations.
All this material can be found online, at www.oxfordsecondary.com/9780198425076
Karl Friedrich Gauss (1777–1855) thought by many to have been the greatest all-round mathematician of all time. He considered that his finest discovery was the method for constructing a regular seventeen-sided polygon. This was not of the slightest use outside the world of mathematics, but was a great achievement of the human mind. Gauss would not have understood the modern view held by many that mathematics must somehow be ‘useful’ to be worthy of study.

E1.1 Identify and use natural numbers, integers (positive, negative and zero), prime numbers, square and cube numbers, common factors and common multiples, rational and irrational numbers (e.g. $\pi$, $\sqrt{2}$), real numbers, reciprocals.

E1.3 Calculate with squares, square roots, cubes and cube roots and other powers and roots of numbers.

E1.5 Use the language and notation of simple vulgar and decimal fractions and percentages in appropriate contexts. Recognise equivalence and convert between these forms.

E1.7 Use the standard form $A \times 10^n$ where $n$ is a positive or negative integer, and $1 \leq A < 10$.

E1.8 Use the four rules for calculations with whole numbers, decimals and fractions (including mixed numbers and improper fractions), including correct ordering of operations and use of brackets.

E1.9 Make estimates of numbers, quantities and lengths, give approximations to specified numbers of significant figures and decimal places and round off answers to reasonable accuracy in the context of a given problem.

E1.10 Give appropriate upper and lower bounds for data given to a specified accuracy. Obtain appropriate upper and lower bounds to solutions of simple problems given data to a specified accuracy.

E1.11 Demonstrate an understanding of ratio and proportion. Increase and decrease a quantity by a given ratio. Calculate average speed. Use common measures of rate.

E1.12 Calculate a given percentage of a quantity. Express one quantity as a percentage of another. Calculate percentage increase or decrease. Carry out calculations involving reverse percentages.

E1.13 Use a calculator efficiently. Apply appropriate checks of accuracy.

E1.14 Calculate times in terms of the 24-hour and 12-hour clock. Read clocks, dials and timetables.

E1.15 Calculate using money and convert from one currency to another.

E1.16 Use given data to solve problems on personal and household finance involving earnings, simple interest and compound interest. Extract data from tables and charts.

E1.17 Use exponential growth and decay in relation to population and finance.

E2.7 Continue a given number sequence. Recognise patterns in sequences including the term-to-term rule and relationships between different sequences. Find the $n$th term of sequences.
1.1 Arithmetic

Decimals

Example

Evaluate:

d) \( 0.84 \div 0.2 \)  

\[
\begin{array}{c}
\text{Multiply both numbers by 10 so that we can divide by a whole number.}
\end{array}
\]

\[
\begin{array}{c}
d) 0.84 \div 0.2 = 8.4 \div 2
\end{array}
\]

\[
\begin{array}{c}
e) 3.6 \div 0.004 = 3600 \div 4 = 900
\end{array}
\]

Exercise 1

Evaluate the following without a calculator:

1. \( 7.6 + 0.31 \)
2. \( 3.84 - 2.62 \)
3. \( 7.004 \div 0.368 \)
4. \( 0.06 + 0.006 \)

5. \( 4.2 + 42 + 420 \)
6. \( 2.6 \times 0.6 \)
7. \( 11.4 \div 9.73 \)
8. \( 4.61 - 3 \)

9. \( 17 - 0.37 \)
10. \( 8.7 + 19.2 - 3.8 \)
11. \( 25 - 7.8 + 9.5 \)
12. \( 3.6 \div 8.74 + 9 \)

13. \( 20.4 - 20.399 \)
14. \( 0.72 \times 0.04 \)
15. \( 2.1 \times 3.6 \)
16. \( 27.2 \div 0.08 \)

17. \( 0.1 \times 0.2 \)
18. \( (0.01)^2 \)
19. \( 3.6 \div 0.2 \)
20. \( 2.31 \times 0.34 \)

21. \( 0.36 \times 1000 \)
22. \( 0.34 \times 10000 \)
23. \( 0.968 \div 0.11 \)
24. \( 0.592 \div 0.8 \)

25. \( 0.1404 \div 0.06 \)
26. \( 3.24 \div 0.002 \)
27. \( 1100 \div 5.5 \)
28. \( 600 \div 0.5 \)

29. \( 0.007 \times 4 \)
30. \( 2640 \div 200 \)
31. \( (11 + 2.4) \times 0.06 \)
32. \( (0.1)^4 + 0.01 \)

33. \( (0.4)^2 \times 0.2 \)
34. \( 77 \div 1000 \)
35. \( (0.3)^2 + 100 \)
36. \( 63 \times 600 \div 0.2 \)

37. \( 92 \times 4.6 \)
38. \( 180 \times 4 \)
39. \( 0.55 \times 0.81 \)
40. \( 360 \times 7 \)
**Exercise 2**

1. A maths teacher bought 40 calculators at $8.20 each and a number of other calculators costing $2.95 each. In all she spent $387. How many of the cheaper calculators did she buy?

2. At a temperature of 20 °C the common amoeba reproduces by splitting in half every 24 hours. If we start with a single amoeba how many will there be after (a) 8 days (b) 16 days?

3. Copy and complete.
   
   \[
   3^2 + 4^2 + 12^2 = 13^2 \\
   5^2 + 6^2 + 30^2 = 31^2 \\
   6^2 + 7^2 + x^2 + y^2 = \\
   \]

4. Find all the missing digits in these multiplications.
   
   a) \[5 \times 9 \times \_ \times 6 \]
   b) \[\_ \times 4 \times 6 \]
   c) \[5 \times \_ \times \_ \times 4 \]

5. Pages 6 and 27 are on the same (double) sheet of a newspaper. What are the page numbers on the opposite side of the sheet? How many pages are there in the newspaper altogether?

6. Use the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 once each and in their natural order to obtain an answer of 100. You may use only the operations +, −, ×, ÷.

7. The ruler below has eleven marks and can be used to measure lengths from one unit to twelve units.

   ![Ruler with marks](image)

   Design a ruler which can be used to measure all the lengths from one unit to twelve units but this time put the minimum possible number of marks on the ruler.

8. Each packet of washing powder carries a token and four tokens can be exchanged for a free packet. How many free packets will I receive if I buy 64 packets?

9. Put three different numbers in the circles so that when you add the numbers at the end of each line you always get a square number.

10. Put four different numbers in the circles so that when you add the numbers at the end of each line you always get a square number.

11. A group of friends share a bill for $13.69 equally between them. How many were in the group?
Fractions

Common fractions are added or subtracted from one another directly only when they have a common denominator. Find the lowest common multiple of the two denominators to find the lowest common denominator.

Example
Evaluate:

a) \( \frac{3}{4} + \frac{2}{5} \)
\[= \frac{15}{20} + \frac{8}{20} = \frac{23}{20} = 1\frac{3}{20} \]

b) \( 2\frac{3}{8} - 1\frac{5}{12} \)
\[= \frac{19}{8} - \frac{17}{12} = \frac{57}{24} - \frac{34}{24} = \frac{23}{24} \]

c) \( \frac{2}{5} \times \frac{6}{7} \)
\[= \frac{12}{35} \]

d) \( 2\frac{2}{5} + 6 \)
\[= \frac{12}{5} + \frac{6}{1} = \frac{18}{5} \times \frac{1}{5} = \frac{2}{5} \]

Exercise 3

Evaluate and simplify your answer.

1. \( \frac{3}{4} + \frac{1}{5} \)
2. \( \frac{1}{3} + \frac{1}{8} \)
3. \( \frac{5}{6} + \frac{6}{9} \)
4. \( \frac{3}{4} - \frac{1}{3} \)
5. \( \frac{3}{5} - \frac{1}{3} \)

6. \( 1\frac{2}{5} - 2 \)
7. \( \frac{2}{3} \times \frac{4}{5} \)
8. \( \frac{1}{7} \times \frac{5}{6} \)
9. \( \frac{5}{8} \times \frac{12}{13} \)
10. \( \frac{1}{3} + \frac{4}{5} \)

11. \( \frac{3}{4} + \frac{1}{6} \)
12. \( \frac{5}{6} + \frac{1}{2} \)
13. \( \frac{3}{8} + \frac{1}{5} \)
14. \( \frac{3}{8} + \frac{1}{5} \)
15. \( \frac{3}{8} \times \frac{1}{5} \)

16. \( 1\frac{3}{4} - \frac{2}{3} \)
17. \( 1\frac{3}{4} \times \frac{2}{3} \)
18. \( \frac{3}{4} + \frac{2}{3} \)
19. \( \frac{3}{2} + 2\frac{3}{5} \)
20. \( \frac{3}{2} \times 2\frac{3}{5} \)

21. \( 3\frac{1}{2} + 2\frac{3}{5} \)
22. \( \left( \frac{3}{4} - \frac{2}{3} \right) + \frac{3}{4} \)
23. \( \left( \frac{3}{5} + \frac{1}{3} \right) \times \frac{5}{7} \)
24. \( \frac{3}{7} \times \frac{1}{2} \)
25. \( \frac{3}{3} + \frac{1}{3} \)

26. Arrange the fractions in order of size:

a) \( \frac{7}{12}, \frac{1}{2}, \frac{2}{3} \)
b) \( \frac{3}{4}, \frac{2}{3}, \frac{5}{6} \)
c) \( \frac{1}{3}, \frac{17}{24}, \frac{5}{8}, \frac{3}{4} \)
d) \( \frac{5}{6}, \frac{8}{9}, \frac{11}{12} \)
27. Find the fraction which is mid-way between the two fractions given:
   a) \( \frac{2}{5}, \frac{3}{5} \)   b) \( \frac{5}{8}, \frac{7}{8} \)   c) \( \frac{2}{3}, \frac{3}{4} \)   d) \( \frac{1}{3}, \frac{4}{9} \)   e) \( \frac{4}{15}, \frac{1}{3} \)   f) \( \frac{3}{8}, \frac{11}{24} \)

28. In the equation below all the asterisks stand for the same number.
    What is the number?

\[
\begin{bmatrix}
* & * & * \\
\star & 6 & \star \\
\end{bmatrix}
\]

29. When it hatches from its egg, the shell of a certain crab is 1 cm across. When fully grown the shell is approximately 10 cm across. Each new shell is one-third bigger than the previous one. How many shells does a fully grown crab have during its life?

30. Glass A contains 100 ml of water and glass B contains 100 ml of juice.
    A 10 ml spoonful of juice is taken from glass B and mixed thoroughly with the water in glass A. A 10 ml spoonful of the mixture from A is returned to B. Is there now more juice in the water or more water in the juice?

## Fractions and decimals

A decimal is simply a fraction expressed in tenths, hundredths, etc.

### Example 1

Change

<table>
<thead>
<tr>
<th>a) 7/8 to a decimal</th>
<th>b) 0.35 to a fraction</th>
<th>c) 1/3 to a decimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{7}{8} )</td>
<td>( \frac{7}{10} = \frac{35}{100} = \frac{7}{20} )</td>
<td>( \frac{1}{3} = 0.\overline{3} )</td>
</tr>
<tr>
<td>( \frac{7}{8} \div 8 = 0.875 )</td>
<td>( \frac{7}{8} \div 7 = 0.875 )</td>
<td>( \frac{1}{3} = 0.\overline{3} )</td>
</tr>
<tr>
<td>( 0.875 )</td>
<td>( 0.875 )</td>
<td>( 0.3 )</td>
</tr>
<tr>
<td>( 7 \div 8 = 0.875 )</td>
<td>( 7 \div 8 = 0.875 )</td>
<td>( 0.3 )</td>
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<tr>
<td>( 0.875 )</td>
<td>( 0.875 )</td>
<td>( 0.3 )</td>
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<td>( \overline{0.875} )</td>
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<td>( \overline{0.3} )</td>
</tr>
</tbody>
</table>
Example 2

a) Convert 0.\overline{7} to a fraction.

This recurring decimal has one place recursion so we multiply it by 10 and set out our working as follows:

\[10 \times: \ 7.7777\ldots \ [1]\]
\[1 \times: \ 0.7777\ldots \ [2]\]
\[\ [1] - [2]: \ 9 \times: \ 7.00000\ldots\]
Therefore: \ \[\frac{7}{9}\]

b) Convert 0.\overline{23} to a fraction.

Here we have two place recursion so we multiply it by 100:

\[100 \times: \ 23.232323\ldots \ [1]\]
\[1 \times: \ 0.232323\ldots \ [2]\]
\[\ [1] - [2]: \ 99 \times: \ 23.00000\ldots\]
Therefore: \ \[\frac{23}{99}\]

Exercise 4

In questions 1 to 24, change the fractions to decimals.

1. \[\frac{1}{4}\]
2. \[\frac{2}{5}\]
3. \[\frac{4}{5}\]
4. \[\frac{3}{4}\]
5. \[\frac{1}{2}\]
6. \[\frac{3}{8}\]
7. \[\frac{9}{10}\]
8. \[\frac{5}{8}\]
9. \[\frac{5}{12}\]
10. \[\frac{1}{6}\]
11. \[\frac{2}{3}\]
12. \[\frac{5}{6}\]
13. \[\frac{2}{7}\]
14. \[\frac{3}{7}\]
15. \[\frac{4}{9}\]
16. \[\frac{5}{11}\]
17. \[\frac{1}{5}\]
18. \[2\frac{5}{8}\]
19. \[2\frac{1}{3}\]
20. \[\frac{7}{10}\]
21. \[\frac{3}{16}\]
22. \[\frac{2}{7}\]
23. \[\frac{6}{7}\]
24. \[3\frac{19}{100}\]

In questions 25 to 40, change the decimals to fractions and simplify.

25. 0.2
26. 0.7
27. 0.25
28. 0.45
29. 0.36
30. 0.52
31. 0.125
32. 0.625
33. 0.84
34. 2.35
35. 3.95
36. 1.05
37. 3.2
38. 0.27
39. 0.007
40. 0.000 11

Evaluate, giving the answer to 2 decimal places:

41. \[\frac{1}{4} + \frac{1}{3}\]
42. \[\frac{2}{3} + 0.75\]
43. \[\frac{8}{9} - 0.24\]
44. \[\frac{7}{8} + \frac{5}{9} + \frac{2}{11}\]
45. \[\frac{1}{3} \times 0.2\]
46. \[\frac{5}{8} \times \frac{1}{4}\]
47. \[\frac{8}{11} + 0.2\]
48. \[\left(\frac{4}{7} - \frac{1}{3}\right) \div 0.4\]
Arrange the numbers in order of size (smallest first).

49. \( \frac{1}{3}, 0.33, \frac{4}{15} \)  
50. \( \frac{2}{7}, 0.3, \frac{4}{9} \)  
51. \( 0.71, \frac{7}{11}, 0.705 \)  
52. \( \frac{4}{13}, 0.3, \frac{5}{18} \)

Convert the following recurring decimals to fractions.

53. \( 0.\overline{6} \)  
54. \( 0.\overline{4} \)  
55. \( 0.\overline{12} \)  
56. \( 0.\overline{43} \)  
57. \( 0.\overline{134} \)  
58. \( 0.\overline{731} \)  
59. \( 0.2\overline{5} \)  
60. \( 0.6\overline{17} \)

### 1.2 Number facts and sequences

#### Number facts

- An **integer** is a positive or negative whole number or zero, e.g. 2, –3, ...
- A **prime** number is divisible only by itself and by 1, e.g. 2, 3, 5, 7, 11, 13, …
- The **multiples** of 12 are 12, 24, 36, 48, …
- The **factors** of 12 are 1, 2, 3, 4, 6, 12.
- A **square number** is the result of multiplying a number by itself, e.g. \( 5 \times 5 = 25 \), so 25 is a square number.
- A **cube number** is the result of multiplying a number by itself twice, e.g. \( 5 \times 5 \times 5 = 125 \), so 125 is a cube number.
- **Indices** are used as a neat way of writing products.
  
  \[ 2^4 = 2 \times 2 \times 2 \times 2 = 16 \quad [\text{2 to the power 4}] \]
  \[ 3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243 \quad [\text{3 to the power 5}] \]
- The **reciprocal** of a number is the result of dividing 1 by that number.
  
  The reciprocal of 4 is \( \frac{1}{4} \). [In index form, this is written as \( 4^{-1} \)]
  
  In general, the reciprocal of \( n \) is \( \frac{1}{n} \). This can be written as \( n^{-1} \).

#### Example

Find the Highest Common Factor (HCF) and Lowest Common Multiple (LCM) of 80 and 50. First write both 80 and 50 as the product of prime factors.

\[
80 = 2 \times 40 = 2 \times 2 \times 20 = 2 \times 2 \times 2 \times 10 = 2 \times 2 \times 2 \times 2 \times 5 = 2^4 \times 5
\]

\[
50 = 2 \times 25 = 2 \times 5 \times 5 = 2 \times 5^2
\]

The HCF is found by looking at the prime factorisation and examining the elements that are in **both** numbers. 80 and 50 both have a 2 and a 5 in the prime factorisation so the HCF is \( 2 \times 5 = 10 \).

The LCM is found by taking the HCF and multiplying it by the bits in each prime factorisation that are **left over**. For 80, we do not use \( 2^3 \) for the HCF and for 50 we do not use a 5.

So the \( \text{LCM} = \text{HCF} \times 2^3 \times 5 = 10 \times 8 \times 5 = 400 \).
Exercise 5

1. Which of the following are prime numbers?
   3, 11, 15, 19, 21, 23, 27, 29, 31, 37, 39, 47, 51, 59, 61, 67, 72, 73, 87, 99

2. Write down the first five multiples of the following numbers:
   a) 4        b) 6        c) 10        d) 11        e) 20

3. Write down the first six multiples of 4 and of 6. What are the first two common multiples of 4 and 6? [i.e. multiples of both 4 and 6]

4. Write down the first six multiples of 3 and of 5. What is the lowest common multiple of 3 and 5?

5. Write down all the factors of the following:
   a) 6        b) 9        c) 10        d) 15        e) 24        f) 32

6. a) Is 263 a prime number?
   By how many numbers do you need to divide 263 so that you can find out?
   b) Is 527 a prime number?
   c) Suppose you used a computer to find out if 1147 was a prime number. Which numbers would you tell the computer to divide by?

7. Make six prime numbers using the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 once each.

8. Write the following numbers as the product of prime factors:
   a) 24                    b) 60                    c) 90
   d) 144                   e) 1000                  f) 880

9. Find the Highest Common Factor of
   a) 24 and 60             b) 90 and 144         c) 60 and 1000
   d) 24 and 880            e) 90 and 1000        f) 24, 60 and 144

10. Find the Lowest Common Multiple of
    a) 24 and 60             b) 60 and 90          c) 144 and 1000
    b) 24 and 880            d) 90 and 1000        f) 24 and 1000

11. Work out without a calculator:
    a) \(4^2\)              b) \(6^2\)              c) \(10^2\)              d) \(3^3\)              e) \(10^3\)

12. Use the \(x^2\) button on the calculator to work out:
    a) \(9^2\)              b) \(21^2\)             c) \(1.2^2\)             d) \(0.2^2\)             e) \(3.1^2\)
    f) \(100^2\)            g) \(25^2\)             h) \(8.7^2\)             i) \(0.9^2\)             j) \(81.4^2\)
13. Find the areas of these squares.

a) 

\[
2.1 \text{ cm} \\
2.1 \text{ cm}
\]

b) 

\[
0.6 \text{ cm} \\
0.6 \text{ cm}
\]

c) 

\[
14 \text{ m} \\
14 \text{ m}
\]

14. A scientist has a dish containing $10^9$ germs.

One day later there are 10 times as many germs.

How many germs are in the dish now?

15. A field has $2^8$ daisies growing on the grass.

A cow eats half of the daisies.

How many daisies are left?

In questions 16 to 20, work out the value of the number given, both as a fraction and as a decimal.

16. $2^{-1}$

17. $10^{-1}$

18. $5^{-1}$

19. $4^{-1}$

20. $8^{-1}$

### Rational and irrational numbers

- A rational number can always be written exactly in the form $\frac{a}{b}$, where $a$ and $b$ are whole numbers.

\[
\begin{align*}
\frac{3}{7} & = \frac{1}{2} = \frac{3}{2} \\
5.14 & = \frac{257}{50} \\
0.6 & = \frac{2}{3}
\end{align*}
\]

All these are rational numbers.

- An irrational number cannot be written in the form $\frac{a}{b}$.

$\sqrt{2}, \sqrt{5}, \pi, \sqrt[3]{2}$ are all irrational numbers.

- In general $\sqrt{n}$ is irrational unless $n$ is a square number.

In this triangle the length of the hypotenuse is exactly $\sqrt{5}$. On a calculator, $\sqrt{5} \approx 2.23607$. This value of $\sqrt{5}$ is not exact and is correct to only 6 decimal places.
Exercise 6

1. Which of the following numbers are rational?
   a) $\frac{\pi}{2}$
   b) $\sqrt{5}$
   c) $(\sqrt{17})^2$
   d) $\sqrt{3}$
   e) 3.14
   f) $\frac{\sqrt{12}}{\sqrt{3}}$
   g) $\pi^2$
   h) $3^{-1} + 3^{-2}$
   i) $\frac{7^{-1}}{2}$
   j) $\frac{22}{7}$
   k) $\sqrt{2} + 1$
   l) $\sqrt{2.25}$

2. a) Write down any rational number between 4 and 6.
   b) Write down any irrational number between 4 and 6.
   c) Find a rational number between $\sqrt{2}$ and $\sqrt{3}$.
   d) Write down any rational number between $\pi$ and $\sqrt{10}$.

3. a) For each shape state whether the perimeter is rational or irrational.
   b) For each shape state whether the area is rational or irrational.

4. The diagram shows a circle of radius 3 cm drawn inside a square. Write down the exact value of the following and state whether the answer is rational or not:
   a) the circumference of the circle
   b) the diameter of the circle
   c) the area of the square
   d) the area of the circle
   e) the shaded area.

5. Think of two irrational numbers $x$ and $y$ such that $\frac{x}{y}$ is a rational number.

6. Explain the difference between a rational number and an irrational number.

7. a) Is it possible to multiply a rational number and an irrational number to give an answer which is rational?
   b) Is it possible to multiply two irrational numbers together to give a rational answer?
   c) If either or both are possible, give an example.

Sequences, the $n$th term

Look at the sequence which starts 2, 5, 8, 11, …

Wherever we are in the sequence, to get the next term we add 3 to the current term. Therefore, we say that the term-to-term rule for this sequence is ‘add 3’.
**Exercise 7**

Write down each sequence and find the next two numbers.

For questions 1 to 4, also state the term-to-term rule.

1. 2, 6, 10, 14  
2. 2, 9, 16, 23  
3. 95, 87, 79, 71  
4. 13, 8, 3, −2  
5. 7, 9, 12, 16  
6. 20, 17, 13, 8  
7. 1, 2, 4, 7, 11  
8. 1, 2, 4, 8  
9. 55, 49, 42, 34  
10. 10, 8, 5, 1  
11. −18, −13, −9, −6  
12. 120, 60, 30, 15  
13. 27, 9, 3, 1  
14. 162, 54, 18, 6  
15. 2, 5, 11, 20  
16. 1, 4, 20, 120  
17. 2, 3, 1, 4, 0  
18. 720, 120, 24, 6

Look at the sequence which starts 5, 9, 13, 17, …

What is the 10th number in the sequence?

What is the \(n\)th number in the sequence? \([n\) stands for any whole number.\]

Is there a formula so that we can easily find the 50th or 100th number in the sequence?

The 1st term in the sequence is \((4 \times 1) + 1 = 5\)

The 2nd term in the sequence is \((4 \times 2) + 1 = 9\)

The 10th term in the sequence is \((4 \times 10) + 1 = 41\)

The 50th term in the sequence is \((4 \times 50) + 1 = 201\)

The 1000th term in the sequence is \((4 \times 1000) + 1 = 4001\)

The \(n\)th term in the sequence is \((4 \times n) + 1 = 4n + 1\)

**Exercise 8**

1. Write down each sequence and select the correct formula for the \(n\)th term from the list given.

<table>
<thead>
<tr>
<th></th>
<th>11n</th>
<th>10n</th>
<th>2n</th>
<th>(n^2)</th>
<th>10n</th>
<th>3n</th>
<th>100n</th>
<th>(n^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>2, 4, 6, 8, …</td>
<td>b)</td>
<td>10, 20, 30, 40, …</td>
<td>c)</td>
<td>3, 6, 9, 12, …</td>
<td>d)</td>
<td>11, 22, 33, 44, …</td>
<td>e)</td>
</tr>
<tr>
<td>g)</td>
<td>10, 100, 1000, 10 000, …</td>
<td>h)</td>
<td>1(^3), 2(^3), 3(^3), 4(^3), …</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Look at the sequence: 5, 8, 13, 20, …

Decide which of the following is the correct expression for the \(n\)th term of the sequence.

<table>
<thead>
<tr>
<th></th>
<th>4n + 1</th>
<th>3n + 2</th>
<th>(n^2 + 4)</th>
</tr>
</thead>
</table>

3. Write down the first five terms of the sequence whose \(n\)th term is \(2n + 7\).

4. Write down the first five terms of the sequence whose \(n\)th term is

<table>
<thead>
<tr>
<th></th>
<th>a)</th>
<th>b)</th>
<th>c)</th>
<th>d)</th>
<th>e)</th>
<th>f)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(n + 2)</td>
<td>(5n)</td>
<td>(10n - 1)</td>
<td>(100 - 3n)</td>
<td>(\frac{1}{n})</td>
<td>(n^2)</td>
</tr>
</tbody>
</table>
Finding the $n$th term

- In an arithmetic sequence the difference between successive terms is always the same number.

Here are some arithmetic sequences:

- A  5, 7, 9, 11, 13
- B  12, 32, 52, 72, 92
- C  20, 17, 14, 11, 8

- The expression for the $n$th term of an arithmetic sequence is always of the form $an + b$.

The difference between successive terms is equal to the number $a$.

The number $b$ can be found by looking at the terms.

Look at sequences A, B and C above.

For sequence A, the $n$th term $= 2n + b$ [the terms go up by 2]
For sequence B, the $n$th term $= 20n + b$ [the terms go up by 20]
For sequence C, the $n$th term $= -3n + b$ [the terms go up by $-3$]

Look at each sequence and find the value of $b$ in each case.

For example in sequence A: when $n = 1$, $2 \times 1 + b = 5$
so $b = 3$

The $n$th term in sequence A is $2n + 3$.

Exercise 9

In questions 1 to 18 find a formula for the $n$th term of the sequence.

1. 5, 9, 13, 17, …
2. 7, 10, 13, 16, …
3. 4, 9, 14, 19, …
4. 6, 10, 14, 18, …
5. 5, 8, 11, 14, …
6. 25, 22, 19, 16, …
7. 5, 10, 15, 20, …
8. 2, 4, 8, 16, 32, …
9. $(1 \times 3)$, $(2 \times 4)$, $(3 \times 5)$, …
10. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, …$
11. 7, 14, 21, 28, …
12. 1, 4, 9, 16, 25, …
13. $\frac{5}{1^2}, \frac{5}{2^2}, \frac{5}{3^2}, \frac{5}{4^2}, \ldots$
14. $\frac{3}{1}, \frac{4}{2}, \frac{5}{3}, \frac{6}{4}, \ldots$
15. 3, 7, 11, 15, …
16. 5, 7, 9, 11, …
17. 7, 5, 3, 1, …
18. $-5, -1, 3, 7, \ldots$

19. Write down each sequence and then find the $n$th term.

a) 8, 10, 12, 14, 16, …
b) 3, 7, 11, 15, …
c) 8, 13, 18, 23, …
20. Write down each sequence and write the $n$th term.
   a) 11, 19, 27, 35, …
   b) $\frac{11}{2}, \frac{19}{2}, \frac{27}{2}, \frac{35}{2}, \ldots$
   c) −7, −4, −1, 2, 5, …

21. Here is a sequence of shapes made from sticks

The number of sticks makes the sequence 4, 7, 10, 13, …
   a) Find an expression for the $n$th term in the sequence.
   b) How many sticks are there in shape number 1000?

Example
The sequence 1, 4, 9, 16, 25, … does not have a common difference between terms but is the sequence of square numbers. We can write $t = n^2$. This is a quadratic sequence.

Find the $n$th term of the following sequence: 3, 6, 11, 18, …
We see that each term is 2 more than $n^2$ so the $n$th term formula is $t = n^2 + 2$.

Find the $n$th term of the following sequence: 3, 8, 15, 24, …
The difference goes up by 2 each time so the sequence is related to $n^2$. Subtract $n^2$ from each term: 2, 4, 6, 8, …
This sequence has $n$th term $2n$ so the complete sequence has $n$th term $t = n^2 + 2n$.

Find the $n$th term of this sequence: 2, 4, 8, 16, …
Notice each term doubles the previous one. This type of sequence is called an exponential sequence.
The $n$th term is $t = 2^n$.

Exercise 10
Find an expression for the $n$th term of each sequence.

1. 4, 7, 12, 19, …
2. 2, 8, 18, 32, …
3. 0, 3, 8, 15, …
4. 0.5, 2, 4.5, 8, …
5. −6, −3, 2, 9, …
6. −1, −4, −9, −16, …
7. 0, −3, −8, −15, …
8. 5, 12, 21, 32, …
9. 1, 6, 15, 28, …
10. 3, 9, 17, 27, …
11. 2, 9, 28, 65, . . .
12. 2, 16, 54, 128, . . .
13. –1, 6, 25, 62, . . .
14. 1, 3, 7, 15, . . .
15. 3, 9, 27, 81, . . .

1.3 Approximations and estimation

Example

a) \(7.8126 = 8\) to the nearest whole number.
   ↑ This figure is ‘5 or more’.

b) \(7.8126 = 7.81\) to three significant figures.
   ↑ This figure is not ‘5 or more’.

c) \(7.8126 = 7.813\) to three decimal places.
   ↑ This figure is ‘5 or more’.

d) \(0.078126 = 0.0781\) to three significant figures.
   ↑ 7 is the first significant figure.

e) \(3596 = 3600\) to two significant figures.
   ↑ This figure is ‘5 or more’.

Exercise 11

Write the following numbers correct to:

<table>
<thead>
<tr>
<th></th>
<th>a) the nearest whole number</th>
<th>b) three significant figures</th>
<th>c) two decimal places</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.174</td>
<td>20.041</td>
<td>0.814 52</td>
</tr>
<tr>
<td>2</td>
<td>19.617</td>
<td>5.311.14</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.275</td>
<td>7.007 47</td>
<td>15.62</td>
</tr>
<tr>
<td>4</td>
<td>11.241</td>
<td>11.241</td>
<td>0.5151</td>
</tr>
<tr>
<td>5</td>
<td>5.454</td>
<td>20.961</td>
<td>0.0851</td>
</tr>
<tr>
<td>6</td>
<td>0.0614</td>
<td>20.961</td>
<td>0.0851</td>
</tr>
<tr>
<td>7</td>
<td>11.241</td>
<td>11.241</td>
<td>0.5151</td>
</tr>
<tr>
<td>8</td>
<td>5.454</td>
<td>20.961</td>
<td>0.0851</td>
</tr>
<tr>
<td>9</td>
<td>3.071</td>
<td>3.071</td>
<td></td>
</tr>
</tbody>
</table>

Write the following numbers correct to one decimal place.

<table>
<thead>
<tr>
<th></th>
<th>a) 5.71</th>
<th>b) 0.7614</th>
<th>c) 11.241</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>17.0614</td>
<td>18.11.241</td>
<td>19.0.0614</td>
</tr>
</tbody>
</table>
## Measurements and bounds

*Measurement is approximate*

### Example 1

A length is measured as 145 cm to the nearest cm.
The actual length could be anything from 144.5 cm to 145.49999 … cm using the normal convention which is to round up a figure of 5 or more. Clearly 145.49999 … is effectively 145.5 and we say the *upper bound* is 145.5.
The *lower bound* is 144.5.
As an inequality we can write $144.5 \leq \text{length} < 145.5$
The upper limit often causes confusion. We use 145.5 as the upper bound simply because it is *inconvenient* to work with 145.49999 …

### Example 2

When measuring the length of a page in a book, you might say the length is 246 mm to the nearest mm.
In this case the actual length could be anywhere from 245.5 mm to 246.5 mm. We write ‘length is between 245.5 mm and 246.5 mm’.

### Example 3

a) If you say your mass is 57 kg to the nearest kg, your mass could actually be anything from 56.5 kg to 57.5 kg.

b) If your brother’s mass was measured on more sensitive scales and the result was 57.2 kg, his actual mass could be from 57.15 kg to 57.25 kg.

c) The mass of a butterfly might be given as 0.032 g. The actual mass could be from 0.0315 g to 0.0325 g.

Here are some further examples:

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>The diameter of a CD is 12 cm to the nearest cm.</td>
<td>11.5 cm</td>
<td>12.5 cm</td>
</tr>
<tr>
<td>The mass of a coin is 6.2 g to the nearest 0.1 g.</td>
<td>6.15 g</td>
<td>6.25 g</td>
</tr>
<tr>
<td>The length of a fence is 330 m to the nearest 10 m.</td>
<td>325 m</td>
<td>335 m</td>
</tr>
</tbody>
</table>
Exercise 12

1. In a DIY store the height of a door is given as 195 cm to the nearest cm. Write down the upper bound for the height of the door.

2. A vet measures the mass of a goat at 37 kg to the nearest kg. What is the least possible mass of the goat?

3. A farmer’s scales measure mass to the nearest 0.1 kg. What is the upper bound for the mass of a chicken which the scales say has a mass of 3.2 kg?

4. A surveyor using a laser beam device can measure distances to the nearest 0.1m. What is the least possible length of a warehouse which he measures at 95.6 m?

5. In the county sports Stefan was timed at 28.6 s for the 200 m. What is the upper bound for the time she could have taken?

6. Copy and complete the table.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) temperature in a fridge = 2 °C to the nearest degree</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b) mass of an acorn = 2.3 g to 1 d.p.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c) length of telephone cable = 64 m to the nearest m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d) time taken to run 100 m = 13.6 s to the nearest 0.1 s</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. The length of a telephone is measured as 193 mm, to the nearest mm. The length lies between:

   A  B  C
   192 and 194 mm  192.5 and 193.5 mm  188 and 198 mm

8. The mass of a suitcase is 35 kg, to the nearest kg. The mass lies between:

   A  B  C
   30 and 40 kg  34 and 36 kg  34.5 and 35.5 kg

9. Adra and Leila each measure a different worm and they both say that their worm is 11 cm long to the nearest cm.

   a) Does this mean that both worms are the same length?
   b) If not, what is the maximum possible difference in the length of the two worms?
Approximations and estimation

10. To the nearest cm, the length $l$ of a stapler is 12 cm. As an inequality we can write $11.5 \leq l < 12.5$.

For parts (a) to (j) you are given a measurement. Write the possible values using an inequality as above.

- a) mass = 17 kg (2 s.f.)
- b) $d = 256$ km (3 s.f.)
- c) length = 2.4 m (1 d.p.)
- d) $m = 0.34$ grams (2 s.f.)
- e) $v = 2.04$ m/s (2 d.p.)
- f) $x = 12.0$ cm (1 d.p.)
- g) $T = 81.4$ °C (1 d.p.)
- h) $M = 0.3$ kg (1 s.f.)
- i) mass = 0.7 tonnes (1 s.f.)
- j) $n = 52 000$ (nearest thousand)

11. A card measuring 11.5 cm long (to the nearest 0.1 cm) is to be posted in an envelope which is 12 cm long (to the nearest cm). Can you guarantee that the card will fit inside the envelope? Explain your answer.

**Exercise 13**

1. The sides of the triangle are measured correct to the nearest cm.
   - a) Write down the upper bounds for the lengths of the three sides.
   - b) Work out the maximum possible perimeter of the triangle.

2. The dimensions of a photo are measured correct to the nearest cm. Work out the minimum possible area of the photo.

3. In this question the value of $a$ is either exactly 4 or 5, and the value of $b$ is either exactly 1 or 2. Work out:
   - a) the maximum value of $a + b$
   - b) the minimum value of $a + b$
   - c) the maximum value of $ab$
   - d) the maximum value of $a - b$
   - e) the minimum value of $a - b$
   - f) the maximum value of $\frac{a}{b}$
   - g) the minimum value of $\frac{a}{b}$
   - h) the maximum value of $a^2 - b^2$.

4. If $p = 7$ cm and $q = 5$ cm, both to the nearest cm, find:
   - a) the largest possible value of $p + q$
   - b) the smallest possible value of $p + q$
   - c) the largest possible value of $p - q$
   - d) the largest possible value of $\frac{p^2}{q}$. 
5. If $a = 3.1$ and $b = 7.3$, correct to one decimal place, find the largest possible value of:
   i) $a + b$  
   ii) $b - a$

6. If $x = 5$ and $y = 7$ to one significant figure, find the largest and smallest possible values of:
   i) $x + y$  
   ii) $y - x$  
   iii) $\frac{x}{y}$

7. In the diagram, ABCD and EFGH are rectangles with AB = 10 cm, BC = 7 cm, EF = 7 cm and FG = 4 cm, all figures accurate to the nearest cm.
   Find the largest possible value of the shaded area.

8. When a voltage $V$ is applied to a resistance $R$ the power consumed $P$ is given by $P = \frac{V^2}{R}$.

   If you measure $V$ as 12.2 and $R$ as 2.6, correct to 1 d.p., calculate the smallest possible value of $P$.

**Estimation**

You should check that the answer to a calculation is ‘about the right size’.

**Example**

Estimate the value of $\frac{57.2 \times 110}{2.146 \times 46.9}$, correct to one significant figure.

We have approximately, $\frac{60 \times 100}{2 \times 50} = 60$

On a calculator the value is 62.52 (to 4 significant figures).

**Exercise 14**

In this exercise there are 25 questions, each followed by three possible answers.

Decide (by estimating) which answer is correct.

1. $7.2 \times 9.8$  
   [52.16, 98.36, 70.56]
2. $2.03 \times 58.6$  
   [118.958, 87.848, 141.116]
3. $23.4 \times 19.3$  
   [213.32, 301.52, 451.62]
4. $313 \times 107.6$  
   [3642.8, 4281.8, 33 678.8]
5. $6.3 \times 0.098$  
   [0.6174, 0.0622, 5.98]
6. $1200 \times 0.89$  
   [722, 1068, 131]
7. $0.21 \times 93$  
   [41.23, 9.03, 19.53]
When dealing with either very large or very small numbers, it is not convenient to write them out in full in the normal way. It is better to use standard form. Most calculators represent large and small numbers in this way.

The number $a \times 10^n$ is in standard form when $1 \leq a < 10$ and $n$ is a positive or negative integer.

### Example

Write the following numbers in standard form:

a) $2000 = 2 \times 1000 = 2 \times 10^3$

b) $150 = 1.5 \times 100 = 1.5 \times 10^2$

c) $0.0004 = 4 \times \frac{1}{10000} = 4 \times 10^{-4}$
**Exercise 15**

Write the following numbers in standard form:

1. 4000  
2. 500  
3. 70 000  
4. 60  
5. 2400  
6. 380  
7. 46 000  
8. 46  
9. 900 000  
10. 2560  
11. 0.007  
12. 0.0004  
13. 0.0035  
14. 0.421  
15. 0.000 055  
16. 0.01  
17. 564 000  
18. 19 million  
19. The population of China is estimated at 1100 000 000. Write this in standard form.
20. The mass of a hydrogen atom is 0.000 000 000 000 000 000 000 001 67 grams. Write this mass in standard form.
21. The area of the surface of the Earth is about 510 000 000 km². Express this in standard form.
22. An atom is 0.000 000 25 cm in diameter. Write this in standard form.
23. Avogadro’s number is 602 300 000 000 000 000 000 000. Express this in standard form.
24. The speed of light is 300 000 km/s. Express this speed in cm/s in standard form.
25. A very rich man leaves his fortune of $3.6 \times 10^8$ to be divided equally between his 100 grandchildren. How much does each child receive? Give the answer in standard form.

**Example**

Work out $1500 \times 8 \ 000 \ 000$.

$$1500 \times 8 \ 000 \ 000 = \left(1.5 \times 10^3\right) \times \left(8 \times 10^6\right)$$

$$= 12 \times 10^9$$

$$= 1.2 \times 10^{10}$$

Notice that we multiply the numbers and the powers of 10 separately.

**Exercise 16**

In questions 1 to 12 give the answer in standard form.

1. $5000 \times 3000$  
2. $60 \ 000 \times 5000$  
3. $0.000 \ 07 \times 400$  
4. $0.0007 \times 0.000 \ 01$  
5. $8000 \div \ 0.004$  
6. $(0.002)^2$  
7. $150 \times 0.0006$  
8. $0.000 \ 033 \div 500$  
9. $0.007 \div 20 \ 000$  
10. $(0.0001)^4$  
11. $(2000)^3$  
12. $0.005 \ 92 \div 8000$  

13. If $a = 512 \times 10^2$, $b = 0.478 \times 10^6$, $c = 0.0049 \times 10^7$

arrange $a$, $b$ and $c$ in order of size (smallest first).
14. If the number $2.74 \times 10^{15}$ is written out in full, how many zeros follow the 4?

15. If the number $7.31 \times 10^{-17}$ is written out in full, how many zeros would there be between the decimal point and the first significant figure?

16. If $x = 2 \times 10^5$ and $y = 3 \times 10^{-3}$ correct to one significant figure, find the greatest and least possible values of:
   i) $xy$
   ii) $\frac{x}{y}$

17. Oil flows through a pipe at a rate of $40 \text{ m}^3/\text{s}$. How long will it take to fill a tank of volume $1.2 \times 10^5 \text{ m}^3$?

18. Given that $L = 2 \sqrt{\frac{a}{k}}$, find the value of $L$ in standard form when $a = 4.5 \times 10^{12}$ and $k = 5 \times 10^7$.

19. a) The number 10 to the power 100 is called a ‘Googol’. If it takes $\frac{1}{5}$ second to write a zero and $\frac{1}{10}$ second to write a ‘one’, how long would it take to write the number 100 ‘Googols’ in full?
   
   b) The number 10 to the power of a ‘Googol’ is called a ‘Googolplex’. Using the same speed of writing, how long in years would it take to write 1 ‘Googolplex’ in full?

## 1.5 Ratio and proportion

The word ‘ratio’ is used to describe a fraction. If the ratio of a boy’s height to his father’s height is 4 : 5, then he is $\frac{4}{5}$ as tall as his father.

**Example 1**

Change the ratio 2 : 5 into the form

a) $1:n$

b) $m:1$

\[
\begin{align*}
a) & \quad 2:5 = 1: \frac{5}{2} = 1:2.5 \\
b) & \quad 2:5 = \frac{2}{5}:1 = 0.4:1
\end{align*}
\]

**Example 2**

Divide $60$ between two people A and B in the ratio 5 : 7.

Consider $60$ as 12 equal parts (i.e. $5 + 7$). Then A receives 5 parts and B receives 7 parts.

\[
\begin{align*}
\therefore & \quad \text{A receives } \frac{5}{12} \text{ of } 60 = $25 \\
& \quad \text{B receives } \frac{7}{12} \text{ of } 60 = $35
\end{align*}
\]
Example 3
Divide 200 kg in the ratio 1:3:4.
The parts are $\frac{1}{8}$, $\frac{3}{8}$ and $\frac{4}{8}$ (of 200 kg). i.e. 25 kg, 75 kg and 100 kg.

Exercise 17
In questions 1 to 8 express the ratios in the form 1:n.
1. 2:6  2. 5:30  3. 2:100  4. 5:8
5. 4:3  6. 8:3  7. 22:550  8. 45:360
In questions 9 to 12 express the ratios in the form n:1.
9. 12:5  10. 5:2  11. 4:5  12. 2:100
In questions 13 to 18 divide the quantity in the ratio given.
13. $40; (3:5)  14. $120; (3:7)  15. 250 m; (14:11)
16. $117; (2:3:8)  17. 180 kg; (1:5:6)  18. 184 minutes; (2:3:3)
19. When $143 is divided in the ratio 2:4:5, what is the difference between the largest share and the smallest share?
22. If $\frac{5}{8}$ of the children in a school are boys, what is the ratio of boys to girls?
23. A man and a woman share a prize of $1000 between them in the ratio 1:4. The woman shares her part between herself, her mother and her daughter in the ratio 2:1:1. How much does her daughter receive?
24. A man and a woman share a sum of money in the ratio 3:2. If the sum of money is doubled, in what ratio should they divide it so that the man still receives the same amount?
25. In a herd of x cattle, the ratio of the number of bulls to cows is 1:6. Find the number of bulls in the herd in terms of x.
26. If $x:3 = 12:x$, calculate the positive value of x.
27. If $y:18 = 8:y$, calculate the positive value of y.
28. $400 is divided between Kas, Jaspar and Jae so that Kas has twice as much as Jaspar and Jaspar has three times as much as Jae. How much does Jaspar receive?
29. A cake of mass 550 g has three dry ingredients: flour, sugar and raisins. There is twice as much flour as sugar and one and a half times as much sugar as raisins. How much flour is there?
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