DISCOVERING MATHEMATICS 1B

Teacher Guide

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OXFORD
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Teaching for Mastery in Discovering Mathematics

Teaching for mastery in Discovering Mathematics is:

- for all Key Stage 3 maths students: all stages, all levels of attainment
- for all secondary maths teachers: experienced and beginning, STEM graduates and non-STEM graduates.

In this series we use ‘mastery’ in reference to students’ outcomes: the learning they develop that is the consequence of the teaching they receive.

Students who show mastery in a particular topic or aspect of mathematics are able to solve problems because they have knowledge THAT, HOW and WHY related to that topic.

They have:

- factual knowledge – knowledge THAT (e.g. that the angles in a triangle add up to 180°)
- procedural fluency – knowledge HOW (e.g. how to work out the angles in an isosceles triangle given one of its angles)
- conceptual understanding – knowledge WHY (e.g. why the sum of the angles in a triangle add up to 180°)

We have adapted Discovering Mathematics for the UK towards our vision of teaching for mastery for all.

Teaching for mastery prioritises depth over breadth: an insistence that a student truly grasps the fundamental concepts and connections within the mathematics before they move on to the next topic.

A lesson for mastery may be focused on one idea only, and that one idea is explored in depth: with examples and followed by consolidation.

Problem-solving is seen as the end goal of a mastery education: the ultimate test that a student has truly grasped the maths.

In this section, we’ll show how Discovering Mathematics supports the mastery approach.

Lesson Design

When designing a maths mastery lesson, decide first what you want students to think in the lesson, and then decide what the students will do. Both in the Notes on Teaching in this book and in the Student Book, Discovering Mathematics encourages this approach.

How it does this is described in more detail on the next pages.

1. Motivating Students’ Learning (see page 3): students are shown upfront how each topic is related to real-life, as well as being encouraged to reflect on their prior learning to develop their knowledge further.

2. Guided Discovery (see pages 4–5): students learn best when they discover new concepts for themselves, with guidance from you, the teacher. Discovering Mathematics is structured to support this journey of discovery. Not every lesson will look the same, but there are certain elements that are carefully included:

   - the class activities and discussions to sharpen the students’ reasoning, and to prompt them to make connections within the current topic and between others
   - the design and sequencing of the intelligent practice that students undertake: the questions, tasks and activities that embed and deepen their factual knowledge, procedural fluency and conceptual understanding.

3. Concrete-Pictorial-Abstract (see page 6): the series supports the use of concrete and pictorial representations of a new concept, so students can understand it in the abstract.

4. Mathematical Language (see page 7): precise language and vocabulary is introduced, used and reinforced, so that students can express themselves clearly and accurately.

5. Assessment (see page 8): activities are included to assess students’ learning in the current lesson, to inform design of the next lesson and to allow for rapid intervention to prevent gaps in the moment.

Support: Support is given so that all students make progress and develop secure and connected knowledge of, fluency with and understanding about the mathematics in each lesson.
How to Use Discovering Mathematics

Chapter 7 introduction video

Notes on Teaching: Chapter 7

Links directly to

Scheme of Work – Chapter 7 Percentages:

- Teacher Guide 1B, page 18
- Links to all resources
- Extra practice on MyMaths
- Guidance on how to teach the lesson, including key points and common misconceptions
- Links to full coverage of the National Curriculum

Consolidation is provided throughout the Student Book as well as in the Workbook exercises. Graded Question Bank exercises and the online assessments on Kerboodle. Fully-worked solutions for all exercises, to encourage full understanding, are provided in the Teacher Guide (for the Student Book) and on Kerboodle (for the Workbook and Graded Question Bank).

Motivating Students’ Learning

It is vital to show students why they learn mathematics, alongside how. Although not exclusive to mastery, this is essential to it.

Every Student Book chapter starts with an example of how the maths can be related to real life. This is supported by a film on the digital book on Kerboodle.

Flashbacks (and Recall boxes, see page 4) are a reminder of prior learning. Teaching for mastery requires putting in place strong foundations in students’ knowledge. Flashbacks at the start of every chapter are where students and teachers can check those foundations before moving on to the next topic.

Learning objectives at the beginning of every chapter state what is covered.

At the end of every chapter, In a Nutshell summarises the key learning points that have been covered. This can be used for revision.

Maths boxes give interesting facts to enrich learning.

Spot Check boxes contain short questions to help clarify any misunderstanding.

Student Book 1B, pages 63-64

Student Book 1B, page 158

Workbook and Graded Question Bank).
Guided Discovery

In Singapore, educators talk about ‘Discovery’ rather than ‘Mastery’. The aim is to guide students on a journey of discovery to uncover mathematical concepts for themselves. When a student grasps a new concept or operation for themselves, they retain it more reliably than when they are fed it.

In Discovering Mathematics, Class Activities in the Student Books allow students to work through the problems with the teacher in class and encourage discussion and group learning. The activities should be mostly done through pair work and small group work, harnessing peer support. Students start to reason deliberately before they have fully understood the technique or procedure they will go on to study.

The Class Activity primes and motivates students:
- They want to know WHY and so they are eager to know HOW – reasoning and explaining is central to mastery.
- The structure ensures that every student has to reason and engage with a problem-solving task.
- If the lessons were instead designed ‘fluency first, reasoning afterwards’, then many students would not reach the reasoning activities before the end of the lesson.

Technology also supports some of the learning experiences in the Class Activities and throughout the series.

Students need practice to embed factual knowledge, acquire procedural fluency and develop conceptual understanding that enables them to solve mathematical problems. Practice includes repetition and variation to achieve proficiency and flexibility. It may be in the form of games, simple recall of facts, application of concepts, but above all it should be fun and motivating.

In Discovering Mathematics, Examples with full solutions appear throughout the Student Books. The solutions are always written in full sentences to encourage language development. Additional notes are provided to support understanding and to support the application of the maths in other contexts. The aim is to develop students’ confidence and independence.

Examples are always followed by Try It! practice questions. They are designed to develop and secure progression in small steps. They allow continuous assessment and rapid intervention.

Fully-worked solutions for all Try It! questions are given in the Teacher Guide. Key Try It! questions also have narrated solution videos on Kerboodle. Both formats show how to present mathematical solutions using precise mathematical language and encourage development of reasoning skills.

Class activities usually appear in Teacher Guides in traditional Mathematics textbooks – in Discovering Mathematics they occur throughout the Student Books.

Objective is clearly defined

Discuss boxes encourage paired and group working

Recall boxes remind students of previous learning

Questioning: students are asked to think about and explain their reasoning

The reasoning catalysed by the Class Activity prepares students to develop the procedural fluency

Algebra discs are used as a concrete resource to develop deep understanding

Student Book 1B, pages 80-81

Student Book 1B, pages 82-83

Rapid intervention

Throughout a lesson, teachers for mastery take constant note of their students’ learning by selectively noticing their understanding, knowledge, misconceptions and progress. Dealing with misconceptions quickly is important – ideally there and then, but certainly that same day. This helps avoid errors becoming embedded.
Concrete-Pictorial-Abstract (CPA) Approach

It matters how students learn. Most students develop secure mathematical knowledge by:

- experiencing concrete examples, e.g. by adding five physical objects to another three objects
- moving to pictorial representations, e.g. by drawing five objects, and then three more, and grouping them together
- progressing to abstract statements using words, e.g. ‘five and three makes eight’.

In Discovering Mathematics, we believe that all students should reason with concrete manipulatives and pictorial representations before doing so with abstract words and symbols. These different representations of mathematical concepts are key to developing deep understanding. Just as teaching for mastery is for all students, so are manipulatives and practical resources.

Word of warning: you and your students will need to invest some time in grasping new pictorial and concrete models for representing mathematics. When teaching for mastery, it’s important to introduce new models sparingly, and then to use them consistently.

In Discovering Mathematics, we focus on two models: bar models and algebra discs.

Bar models

Bar models are used throughout the series in problem-solving. It’s a big leap for students to take a word-based problem and then formulate an abstract mathematical solution. Bar models are a valuable stepping stone to achieving that. You will need to take time with your students: when they first encounter a bar model, it might even take them longer to solve the problem than without a bar model. But with practice, they’ll find the bar model becomes a powerful tool, especially for problems requiring multiplicative reasoning.

Algebra discs

Algebra discs are presented pictorially in worked examples and class activities throughout the Student Books. A class set of algebra discs is also available as part of this series. The class set contains 20 student sets of algebra discs together with double-sided discs, colour-matched precisely with the concrete-Pictorial-Abstract (CPA) Approach.

Concerto: Positive Integers

For positive numbers, we can use algebra discs to make calculations such as adding five and three. When we add an algebra disc to another, we can move the disc to the other side of the bar model.

In the example shown here, both algebra discs are positive. We can move the disc to the other side of the bar model.

In the example shown here, operations with negative numbers are modelled using algebra discs. Students can manipulate the physical discs to carry out subtractions, and then practise drawing them for calculations such as $5 + (-2)$. Together these support all students to develop confident and secure understanding of why subtracting any negative quantity is equivalent to adding the corresponding positive quantity.

Precise Mathematical Language

Discovering Mathematics places great emphasis on the use of precise mathematical language. Teachers for mastery strive to give all their students the confidence, the vocabulary and the opportunity to explain themselves and justify their mathematical conclusions. This is as relevant in Early Years as it is in Key Stage 3, GCSE and A-level. Maintaining consistent language will support understanding.

Students are given in full sentences in the Student Books, and in the fully-worked solutions in the Teacher Guide and on Kerboodle.

Note that there is no mention of the ‘bus stop’ method for division – division has no connection with a bus stop!

The use of complete sentences encourages students to think clearly, it helps them to grasp the mathematical language, and it allows you, the teacher, to check they have truly understood a given concept.

Mathematical reasoning is encouraged in the approach to questioning.

Questions and discussion points ask students to ‘Explain’ or to ‘Justify’. This is an opportunity for students to engage in reasoning and demonstrate their depth of understanding of a concept. This encourages mastery through not just knowing how to do mathematics, but also developing an understanding of why it works.

Open questions, which have more than one possible answer, are flagged in the series. These questions encourage depth of understanding and allow students to see that there can be more than one correct answer to a mathematics question.
Assessment

Assessment is at the heart of Discovering Mathematics. The mastery methodology for assessment is, at its core, straightforward:

1. **Formative assessment**: check students’ understanding of a topic and identify weak spots.
2. **Gap-filling**: support students with any weak spots.
3. **Summative assessment**: test students’ mastery of the topic.

### Formative assessment

- **End-of-chapter section exercises**
  - Split into Levels 1–3 to allow for differentiation within class: Level 1 – fluency; Level 2 – application; Level 3 – word-based problems.

### Gap-filling

Gap-filling can be covered through the Examples and Try It! questions in the Student Books, Try It! videos on Kerboodle and In a Nutshell in the Student Books.

It can be very effective to share students’ solutions and explanations with their peers – exemplary ones but also those that reveal a misconception, or those that reach a solution in a novel or sub-optimal way.

### Summative assessment

- **Revision Exercises for every chapter**
  - Covers all content of the chapter
  - Questions ramp-up in difficulty
  - The teacher-facing fully-worked solutions in the Teacher Guide have *GCSE grades assigned to each question

- **Online end-of-chapter printable test**
  - (with associated spreadsheet for class monitoring)

- **Science and Finance contexts flagged to encourage cross-curricular working**

### Supplementary Assessment

Additional assessment opportunities can be found throughout the Middle Tier resources in Discovering Mathematics.

- **Student Book**:
  - Integrated Examples and Review Exercises appear after every four chapters to support students to answer questions on a range of topics
  - Problems in Real-world Contexts question the content of the whole book in real-life situations
  - Self-assessment booklet for each Student Book on Kerboodle allows students to reflect on their own strengths and weaknesses

- **Workbook**:
  - Each chapter is matched to the corresponding Student Book chapter
  - Write-in questions are split into Levels 1–3 as in the Student Book
  - Self-assessment checklist for students to track confidence levels

- **Graded Question Bank**:
  - Each chapter directly corresponds to the Student Book chapter for Middle and Higher Tier
  - Questions split into Levels 1–3 in the Student Book
  - Also available in editable format on Kerboodle for teachers to make up their own tests

*GCSE grades are an assessment of actual grade, not a projection to GCSE result.
Three sub-grades are allocated, e.g. 1-, 1, 1+, 2-, 2, 2+.

The teacher-facing fully-worked solutions in the Teacher Guide have *GCSE grades assigned to each question

**Multi-tiered Grading**

Questions ramp-up in difficulty

Science and Finance contexts flagged to encourage cross-curricular working

Teacher enters class marks into an assessment spreadsheet, which provides GCSE grade analysis

Mark scheme provided for teachers with *GCSE grades assigned to each question

**Auto-generated feedback**

Questions similar to those in the Revision Exercises in the Student Book

End marks reflect gaps to be filled

Split into Levels 1–3 to allow for differentiation within class: Level 1 – fluency; Level 2 – application; Level 3 – word-based problems

Questions taken from the Revision Exercises in the Student Book

Problem-solving questions clearly identified

Questions from the Student Book

Example of a question from the Revision Exercise in the Student Book

Student Book 1A, page 28

Student Book 1B, page 38

Student Book 1B, page 281
Metacognition

Throughout Discovering Mathematics, students are encouraged to reflect on new concepts they’ve learnt and to test their own understanding. This self-reflection is a proven way of accelerating learning.

A Write in Your Journal feature at the end of every chapter asks students to reflect on concepts covered in the chapter and to practise communicating mathematically. We recommend providing a separate exercise book for your students, which they can use exclusively as their mathematics journal.

Discuss boxes appear in each chapter. These provide opportunities for students to work in pairs or groups to think and communicate mathematically.

Problem Solving

Skill and confidence at problem-solving is seen as the ultimate goal of a mastery education. It equips students to use mathematics in the real world, through their careers and through their lives.

Every exercise in the series includes problem-solving questions, with an emphasis on real-life contexts. They are all clearly identified in the Student Books, Workbooks and Graded Question Banks.

Every Student Book also has two separate sections dedicated to problem-solving:

1. Problem-Solving Processes and Heuristics – help students to learn about the steps to take to solve a problem: understand the problem, devise a plan, carry out the plan, and then look back. Examples are given to work through the problem-solving strategies.

2. Problems in Real-world Contexts – help students to recognise where mathematics is useful in everyday life, consolidate their learning of problem-solving strategies, and consider what mathematics they have learnt that is relevant to each problem.

The Discovering Mathematics full series Schemes of Work can be found at www.oxfordsecondary.co.uk/discoveringmathematics
<table>
<thead>
<tr>
<th>Term / Week</th>
<th>Chapter/Chapter Section</th>
<th>Learning Objectives</th>
<th>KS3 Programme of Study Reference</th>
<th>Series Resources (in addition to Student Book 1B)</th>
<th>MyMaths Codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summer term / Week 10</td>
<td>Chapter 12 Collecting, Organising and Displaying Data</td>
<td>• Recognise different methods of collecting data</td>
<td>DF2, 7 RM5 SP1–4 N1.2</td>
<td>Workbook 1B Section 12.1</td>
<td>1248, 1249</td>
</tr>
<tr>
<td>Summer term / Week 11</td>
<td>12.1 Collection of Data</td>
<td>• Recognise different methods of collecting data</td>
<td>DF2, 7 RM5 SP1–4 N1.2</td>
<td>Workbook 1B 12.1</td>
<td>1385, 1235, 1193</td>
</tr>
<tr>
<td>Summer term / Week 12</td>
<td>12.2 Organisation of Data</td>
<td>• Organise data • Create frequency tables</td>
<td>DF2, 7 SP1–4 N2, 10, 12 S1, 2</td>
<td>Workbook 1B 12.2</td>
<td>1193, 1205</td>
</tr>
<tr>
<td></td>
<td>12.3 Pictograms, Vertical Line Charts and Bar Charts</td>
<td>• Construct, analyse and interpret pictograms, vertical line charts, bar charts and compound bar charts</td>
<td>DF2, 7 SP1–4 N10, 12 S1, 2</td>
<td>Workbook 1B 12.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Review and assessment: Integrated Examples and Review Exercise 3 Problems in Real-world Contexts</td>
<td>Int Ex &amp; Rev Ex 3</td>
<td>DF2, 7 RM5 SP1–4 N10, 12, 13, 15 R3, 8 G1, 2, 3, 5, 7, 10, 11, 12, 15 S1, 2</td>
<td>Workbook 1B Review 3</td>
<td></td>
</tr>
</tbody>
</table>
Chapter 3
Introduction to Algebra

It is important to make a smooth transition from arithmetic expressions to algebraic expressions as this can often be a stumbling block for students. The previous two chapters focussed on the structure of the arithmetic expressions, the use of bar models and algebra discs, and these will prove to be valuable tools when developing understanding of algebraic expressions. Students will generalise simple daily-life scenarios to understand the value of using variables. Constant comparison with the numerical work will build confidence and understanding. The summary table in Section 3.1 (page 68), comparing the words to the model and the algebraic expression for the four rules, is a very useful review of the key concepts and should be compared to the numerical work in the Flashback (page 64) to embed understanding.

3.1 Letters to Represent Numbers

Learning Objectives:
- Use letters to represent numbers
- Interpret simple algebraic notations

By exploring some daily-life scenarios in the initial examples, students should find that the use of letters is an easy and efficient way to express a generalised arithmetic expression. Use of age, mass, length and money are all very accessible and relevant to the students. Ask students if a variable can represent any unknown number. They need to understand that it can and that a variable is used as an unknown quantity. It is important not to confuse students by assigning a letter to an object, for example three apples and four bananas is not 3a + 4b. Instead state the amount of something, like the amount of apples in a bag, so that $ax$ is the total number of apples in three bags. Ask students if it matters which letter is used for the unknown; for example, do they need to use $x$ if $x$ kg for the mass of a baby?

Students need to understand the algebraic conventions for expressing addition, subtraction, multiplication and division including the use of positive integer indices. Ask, what is the difference between $3ax$ and $a^2x$. Ensure students understand and can explain why they are different.

Ask students to discuss in pairs or small groups the rules for writing expressions. As a class, create a poster of the rules for the classroom wall.

3.2 Substituting Numbers for Letters

Learning Objective:
- Substitute integers in simple expressions and formulae

(For example: write division as $\frac{a}{b}$ rather than $a \div b$; write variables in alphabetical order; variables do not have units; a number next to a letter means multiply.) Make sure that the commutative and associative laws are understood here in relation to multiplication so that students are convinced $ax \times 4 = x \times 4b$ is the same as $4ax \times b$ and can therefore be written as $4(axb)$.

Bar models are a pictorial way of helping students to form expressions, which will help to embed understanding before moving to the abstract, see Examples 1, 2 and 3.

Discuss with students the meaning of the brackets in Example 7 and ensure students can explain for themselves why $3(x + 1)$ is different from $3x + 1$. Bar models of the four operations will be very useful in embedding understanding.

In problems it is important that students understand that a variable does not have a unit and that only the whole expression does. Therefore, in problems involving units, brackets should be used, for example $(x + 3)cm$.

Exercise 3.1 develops reasoning from numerical expressions to algebraic expressions. This is done at each level as the questions become more challenging. Completing this whole exercise will be very useful in building understanding. Discuss with students when a numerical answer is possible in a question and when an expression is required.
This section begins by introducing substitution into expressions before moving on to define formulae. This means students have the opportunity to consolidate their understanding of substitution by substituting into both expressions and formulae. Deepen students’ understanding by discussing the meaning of the coefficient for real-life contexts.

Make explicit the definitions of expression (as on page 65) and formula (page 72). Discuss with students what ‘is the same’ and ‘what is different’ about an expression and a formula. Make sure they understand and can explain the difference.

Ensure students understand the meaning of the word substitution. Relate it to a sporting substitution to embed the idea of a variable being exchanged for a number. Ask students again what 3x means and if necessary suggest they refer to the poster they made in Section 3.1. Ask students to explain why if x = 5, 3x becomes 3 x 5 and not 35.

Students must consider the order of operations when evaluating. Encourage students to use a bracket to enclose a negative number, so the negative sign is separate from the operation (see Example 11). Encourage students to write their solutions vertically lining up equal signs rather than all on one line.

In Class Activity 1, a spreadsheet allows students to compare different expressions. This is an opportunity for students to use spreadsheet notation for formulae. It is also an opportunity to challenge any student’s misconceptions regarding expressions. For example, by looking at the spreadsheet they should notice that (3n)² is not the same as 3n², and they should be able to explain why. Exercise 3.2 begins with evaluating expressions, before moving on to evaluate formulae. This is another opportunity to discuss the difference between the two.

3.3 Writing Algebraic Expressions and Formulae

Learning Objective:
● Write simple expressions and formulae

Bar models offer visual representations of problems and provide a ‘stepping stone’ from the words to the abstract algebra. They are used to demonstrate when one quantity is more or less than another, and by how much. They are also used to demonstrate when one quantity is n times another quantity (where n could be a fraction). Ensure students have plenty of practice using the bar models to support their understanding.

Use Example 13 to discuss the use of typical mathematical language: ‘Express y in terms of x.’

It takes time for students to master the skill of formulating algebraic expressions from word problems so plenty of practice is advisable.

Fully completing Exercise 3.3 and Workbook 1B Exercise 3.3 will develop this skill.

3.4 Like Terms and Unlike Terms

Learning Objective:
● Simplify expressions by collecting like terms

Ask students to identify the terms, coefficients and constants in different expressions. Use this language as much as possible when discussing expressions. Discuss how the terms of an expression can be repositioned to keep the expression equivalent. Students should understand that the sign is attached to the coefficient (which could be an unwritten 1) rather than the variable. Class Activity 2 allows them to look at how terms can be combined when only one variable is being used. Ask students to create their own questions based on this activity.

Bar models can be particularly useful as a visual representation and will help deepen understanding. It is a good idea to use bars of different colours for the variable and constant so students can readily see the difference.

After completing Class Activity 2, ask students if (x and x²) and (a/b and ab) are two pairs of like or unlike terms. Ask students to write down other terms that would match each of the terms in the brackets. This could be done as a paired activity where one student writes a term and the other has to write a like term that matches it.

Class Activity 3 introduces the use of the algebra discs to help with collecting like terms. Students should be familiar with zero pairs from their work on negative integers; this is now extended to include x, –x, etc. Using the discs, students can see how expressions can be written in different ways, which is important for manipulation later. Time needs to be spent on this activity so that all students become confident. Again, students can be asked to make up their own expressions to simplify to embed this understanding.

Questions at the end of Class Activity 3 promote student thinking about application of the distributive law. Discuss Question 2 to ensure students understand that 3x + 4y cannot be written as 7xy.

Examples 16 and 17 use zero pairs and should be worked through with the algebra discs; the Try Its should then be done in the same way.

Question 9 onwards of Exercise 3.4 can be used to reinforce that the variables in a formula or expression do not include units of measure.

3.5 Addition and Subtraction of Linear Expressions

Learning Objective:
● Add and subtract linear expressions

Discuss when a term is linear and when it is not. Students often quickly see that x² is not linear but need more convincing about xy.

To build on previous work, each linear expression is initially written in brackets which are then removed and like terms collected.

Recall how flipping discs was used when subtracting negative integers in Chapter 2. The process of flipping algebra discs when dealing with the negative of an expression can help students to understand and remember (a + b – c) = –a – b + c which then leads to subtraction of a linear expression. Remind students at this point that negation is by definition the same as multiplying by –1 (see page 30 in this Teacher Guide).

This is explored in Class Activity 4 before looking at addition and subtraction of expressions. Again, through the whole activity the discs should be used to consolidate understanding. However, when tackling Exercise 3.5, encourage students to use the discs only if they need to.

Revision Exercise 3 allows students to practise the algebraic concepts introduced in this chapter. This is also an opportunity to use the questions to check students’ knowledge of language and key algebraic terms they have learnt. The worded questions at the end bring more meaning back to the abstract nature of Questions 1–9, and provide opportunities for students to generate formulae in a variety of contexts and use them to solve problems.

The terminology used in algebra needs to be embedded. Ask students to complete the Write in Your Journal task to write their own definitions for these words. This will help students distinguish between the words.
Throughout these problems, there are opportunities to extend the work and incorporate students’ own experiences by making links to other curriculum areas such as design and technology, geography and history.

**A Paper Sizes**

Emphasise to students that, where possible, working with fractions will give more accurate answers. Where solutions to the nearest millimetre are required, care should be taken to avoid premature rounding during working which may affect the accuracy of the final answer.

**B Laying Carpet**

Encourage students to use sketch diagrams throughout this problem. Compare and discuss the various ways of splitting the shape or using an enclosing rectangle for the area calculation. Students may need some clarification over why they need to have all carpet running in the same direction.

**C Brick Wall**

This question gives a rich opportunity for students to consider the crucial need for approximation in building. The ‘one-half running bond’ technical vocabulary will probably be unfamiliar to most students, although the brick pattern described will not. When students work on Question 1, point out that they should calculate using measurements for bricks in a row where full bricks are used. They could then show by calculation that the brick parts at the ends of the other rows in the diagram will not be exactly half bricks. Question 2 enables plenty of discussion between students about how a reasonable estimate can be reached; encourage them to consider whether mortar should be accounted for and how to deal with the fact that the 2 metre wall will not be an exact number of rows high.

**D Magazine Holder**

Dealing with the quarter circles for the perimeter of the side pieces is likely to cause the most challenge. Remind students that they need to take the radius of the cut out circle into account when finding the lengths of two of the straight edges. A clearly labelled sketch will be an effective support here.

**E Clearway Road Sign**

Students may find the explanation in Question 2 particularly challenging. Ask students to draw their own version of the diagram with concentric circles to help them understand that the quarters do not have a consistent radius. Discuss how an approximation for the area of the red cross can be used to get an estimate for the area in Question 3.

**F London Tower Bridge**

A great deal of written material is presented in this question. Use this as an opportunity to discuss relevant and redundant information. For Question 3, it is essential that students make the connection that the bascules $AB$ and $DC$ come down together to form the entire length $AD$. Once this length, given as $61\text{ m}$, is established as $2x$, the remainder of the problem is accessible. Remind students to take account of the $9\text{ m}$ between the bridge and the water level for part (b).

**G Population Pyramid for the UK**

This diagram is a very rich source of information and may be familiar to students from their geography lessons. Point out that the population scale is measured in thousands so the readings will be estimates. Encourage students to refer to their knowledge of events in history such as the Second World War to offer explanations for features identified in the diagram.
### Class Activity 1
**Objective:** To compare the algebraic expressions $3 + n$, $3n$, $(3n)^2$, $3n^2$ and $9n^2$.

**Tasks**

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n</td>
<td>n+n</td>
<td>3n</td>
<td>$(3n)^2$</td>
<td>$3n^2$</td>
<td>$9n^2$</td>
</tr>
</tbody>
</table>

1. In a spreadsheet, enter the headings (row 1 and row 3) and the first column as shown.
2. Generate the values of other columns using formulae in the spreadsheet. You may use the copy and paste buttons to copy the formulae from one row to the other rows.

**Note:** In a spreadsheet, * stands for multiplication and ^ stands for the index.

**Questions**

1. Look at the values under the columns $3 + n$ and $3n$. Can you say that $3 + n = 3n$? Explain your answer.

   The values are different. $3 + n$ and $3n$ are not equal, that is, $3 + n \neq 3n$.

2. Compare the columns of $(3n)^2$, $3n^2$ and $9n^2$. What are the relationships between these three expressions? Explain your answer.

   - The values of $(3n)^2$ are equal to the values of $9n^2$. You say that $(3n)^2 = 9n^2$.
   - The values of $(3n)^2$ are three times the values of $3n^2$. You say that $(3n)^2 = 3(3n^2)$.
   - The values of $9n^2$ are three times the values of $3n^2$. You say that $9n^2 = 3(3n^2)$.

### Class Activity 2
**Objective:** To learn to add or subtract two terms with identical variable parts.

**Tasks**

1. (a) Look at the expressions below. What is the same about them? What is different?

   There are pairs of expressions with the same variable part (refer to (b), (c), (d)).
   - The expressions $3x^2$, $3y^2$ and $3z^2$ have the same coefficient 3.
   - Expressions are different when their variable parts and coefficients are different.
(b) Classify the expressions into separate groups based on their variable parts

<table>
<thead>
<tr>
<th>Expressions in x</th>
</tr>
</thead>
<tbody>
<tr>
<td>2x</td>
</tr>
<tr>
<td>4x</td>
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<table>
<thead>
<tr>
<th>Expressions in y</th>
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<tbody>
<tr>
<td>3x²</td>
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<td>2x²</td>
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<table>
<thead>
<tr>
<th>Expressions in xy</th>
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<tbody>
<tr>
<td>3xy</td>
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<tr>
<td>xy</td>
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</table>

<table>
<thead>
<tr>
<th>Expressions in y</th>
</tr>
</thead>
<tbody>
<tr>
<td>7y</td>
</tr>
<tr>
<td>3y</td>
</tr>
</tbody>
</table>

2. There are some red bars each of length \( x \) cm and some blue bars each of length \( y \) cm.

Using the given models, express the results of the following as simply as possible.

(a) \( + \)

(i) \( AB \) is formed by two red bars. Find the length of \( AB \) in terms of \( x \).

(ii) \( CD \) is formed by three red bars. Find the length of \( CD \) in terms of \( x \).

(iii) If \( AB \) and \( CD \) are joined together, what is the total length in terms of \( x \)?

(iv) Find \( 2x + 3x \).

(b) \(-\)

(i) \( EF \) is formed by five blue bars. Find the length of \( EF \) in terms of \( y \).

(ii) \( GH \) is formed by two blue bars. Find the length of \( GH \) in terms of \( y \).

(iii) If a part of length \( GH \) is cut off from \( EF \), find the length of the remaining part in terms of \( y \).

(iv) Find \( 5y = 2y \).

(c) If \( AB \) and \( EF \) are joined together, what will be the total length in terms of \( x \) and \( y \)?

Note: The expression in terms of \( x \) and \( y \) obtained in (c) cannot be further simplified.

Class Activity 3

Objective: To apply the process of collecting like terms using algebra discs.

As well as number discs, you have \( x \)-discs and \( y \)-discs.

Each disc has two sides. You can flip a disc to show either the front or the back. Just like 1 and \(-1\), \( x \) and \(-x\) and \( y \) and \(-y\) form zero pairs. That is, \( 1 + (-1) = 0 \), \( x + (-x) = 0 \) and \( y + (-y) = 0 \). Examples of zero pairs are:

(a) \( x + 2x = 3x \) and \( 2x + x = 3x \)

(b) \(-y + 3y = 2y\) and \(-3y + y = -2y\)

Some algebraic expressions can be represented by a set of algebra discs. For example,

(a) \( 2x + 3 \)

(b) \( y - 4 \)

(c) \( -x + 3y = 2 \)

Also, these sets of algebra discs can also represent \( 3 \) \( 2x \), \( \pm x \), \( \pm 3y \), \( 3 \) \( y \), \( x \) and \( \pm 3 \) \( y \).

Terms involving \( x \) and \( y \) can be simplified by collecting like terms as illustrated below:

![Diagram showing the process of simplification]

Tasks

1. Represent these algebraic expressions using the algebra discs.

(a) \( 3x + 2 \)

(b) \( x + 4y \)

(c) \( 2x + 3y = 4 \)

2. Simplify these expressions. Use algebra discs to help you.

(a) \( 2x + 5x \)

(b) \( 2x = 5x \)
Collect the like terms.

(c) \( ax + bx \)

(d) \( ax = bx \)

(e) \( 3y + l + 4y + 3 \)

(f) \( 3y + l + 4y = 3 \)

(g) \( 3y = l + 4y + 3 \)

(h) \( -3y = l - 4y - 3 \)

(i) \( 4x = 5y + 2y + 1 \)
### Chapter 3 Introduction to Algebra

#### Class Activity 4

**Objective:** To perform the addition and subtraction of linear expressions using algebra discs.

#### Tasks

1. To obtain the negative of an expression, you flip the discs that represent the expression. For example, simplify $(3x - 2)$.

   ![Diagram](image)

   - This means flipping over all the discs inside the box.

   $$\therefore (3x - 2) = -3x + 2$$

   - This means changing the sign of all the terms inside the bracket. Thus, $3x$ becomes $-3x$ and $-2$ becomes $2$.

   Simplify these expressions. Use algebra discs to help you.

   **(a)** $2x + 4$

   ![Diagram](image)

   $$\therefore 2x + 4 = 2x + 4$$

2. Can you write $3x + 4y$ as $7xy$? You may substitute $x$ and $y$ with numbers to help you illustrate your explanation.

   No, $3x + 4y$ is $7xy$. $3x$ and $4y$ are unlike terms. Therefore, $3x + 4y$ cannot be simplified.

#### Questions

1. Explain how you would simplify $ax + bx$, where $a$ and $b$ are given integers. You may substitute $a$ and $b$ with different integers to help you illustrate your explanation.

   $$ax + bx = (a + b)x$$

2. Can you write $3x + 4y$ as $7xy$? You may substitute $x$ and $y$ with numbers to help you illustrate your explanation.

   $3x + 4y$ is not $7xy$. $3x$ and $4y$ are unlike terms. Therefore, $3x + 4y$ cannot be simplified.
2. To add two expressions, you remove the brackets and collect the like terms.

For example, simplify \((2x + 3y) \bullet (x + y)\).

\[
(2x + 3y) \bullet (x + y) \Rightarrow \begin{cases} 2x \cdot x \quad 2x \cdot y \\ 3y \cdot x \quad 3y \cdot y \end{cases}
\]

Collect the like terms.

\[
\begin{align*}
2x^2 + 2xy + 3xy + 3y^2 \\
&= 2x^2 + 5xy + 3y^2
\end{align*}
\]

The signs of all terms in the bracket remain unchanged.

Simplify these expressions. Use algebra discs to help you.

(a) \((3x + 1) \bullet (-x + 2)\)

\[
\begin{align*}
(3x + 1) \bullet (-x + 2) &\Rightarrow \begin{cases} 3x \cdot (-x) \quad 3x \cdot 2 \\ 1 \cdot (-x) \quad 1 \cdot 2 \end{cases} \\
&\Rightarrow \begin{cases} -3x^2 + 6x \\ -x + 2 \end{cases} \\
&\Rightarrow -3x^2 - x + 8
\end{align*}
\]

Zero pair

(b) \((-2x - 3) \bullet (2x - y)\)

\[
\begin{align*}
(-2x - 3) \bullet (2x - y) &\Rightarrow \begin{cases} -2x \cdot 2x \quad -2x \cdot (-y) \\ -3 \cdot 2x \quad -3 \cdot (-y) \end{cases} \\
&\Rightarrow \begin{cases} -4x^2 + 2xy \\ -6x + 3y \end{cases} \\
&\Rightarrow -4x^2 - 6x + 2xy + 3y
\end{align*}
\]

Zero pair

(c) \((4y - 3) \bullet (y - 1)\)

\[
(4y - 3) \bullet (y - 1) \Rightarrow \begin{cases} 4y \cdot y \quad 4y \cdot (-1) \\ -3 \cdot y \quad -3 \cdot (-1) \end{cases} \\
&\Rightarrow \begin{cases} 4y^2 - 4y \\ -3y + 3 \end{cases} \\
&\Rightarrow 4y^2 - 7y + 3
\]

Collect the like terms.

(d) \((-3x - y) \bullet (2x + 3)\)

\[
\begin{align*}
(-3x - y) \bullet (2x + 3) &\Rightarrow \begin{cases} -3x \cdot 2x \quad -3x \cdot 3 \\ -y \cdot 2x \quad -y \cdot 3 \end{cases} \\
&\Rightarrow \begin{cases} -6x^2 - 9x \\ -2xy - 3y \end{cases} \\
&\Rightarrow -6x^2 - 2xy - 9x - 3y
\end{align*}
\]

Zero pair

(e) \((2x - 3y) \bullet (-2x + 2y)\)

\[
(2x - 3y) \bullet (-2x + 2y) \Rightarrow \begin{cases} 2x \cdot (-2x) \quad 2x \cdot 2y \\ -3y \cdot (-2x) \quad -3y \cdot 2y \end{cases} \\
&\Rightarrow \begin{cases} -4x^2 + 4xy \\ 6xy - 6y^2 \end{cases} \\
&\Rightarrow -4x^2 + 10xy - 6y^2
\]

Collect the like terms.

\[
\begin{align*}
&\Rightarrow -4x^2 + 10xy - 6y^2 \\
&\Rightarrow 2x \cdot (3y - 4x) \\
&\Rightarrow 2x \cdot 2y - 2x \cdot 3 \Rightarrow 4xy - 6x
\end{align*}
\]

Zero pairs

\[
\begin{align*}
&\Rightarrow 4xy - 6x - 2y \cdot (2x - 3y) \\
&\Rightarrow 4xy - 6x - 4xy + 6y \Rightarrow -6x + 6y
\end{align*}
\]

Collect the like terms.

\[
\begin{align*}
&\Rightarrow -6x + 6y - 2y \cdot (2x - 3y) \\
&\Rightarrow -6x + 6y - 4xy + 6y \Rightarrow -6x + 12y
\end{align*}
\]

Zero pairs

\[
\begin{align*}
&\Rightarrow -6x + 12y - 2y \cdot (2x - 3y) \\
&\Rightarrow -6x + 12y - 4xy + 6y \Rightarrow -6x + 18y
\end{align*}
\]

Collect the like terms.

\[
\begin{align*}
&\Rightarrow -6x + 18y - 2y \cdot (2x - 3y) \\
&\Rightarrow -6x + 18y - 4xy + 6y \Rightarrow -6x + 24y
\end{align*}
\]

Zero pairs

\[
\begin{align*}
&\Rightarrow -6x + 24y - 2y \cdot (2x - 3y) \\
&\Rightarrow -6x + 24y - 4xy + 6y \Rightarrow -6x + 30y
\end{align*}
\]

Collect the like terms.
3. Subtracting an expression is the same as adding the negative of that expression. For example, simplify $(2x - 3y) - (x + y)$.

\[
(2x - 3y) - (x + y) = 2x - 3y - x - y = x - 4y
\]

Simplify these expressions. Use algebra discs to help you.

(a) \(3x - (\triangle x)\)

\[
\begin{align*}
3x &= (\triangle x) \\
3x &= (\triangle x)
\end{align*}
\]

(b) \((2x - 1) - (3x + 2)\)

\[
\begin{align*}
(2x - 1) - (3x + 2) &= 2x - 1 - 3x - 2 \\
&= -x - 3
\end{align*}
\]

(c) \((-2x + 3y) - (x - 2y)\)

\[
\begin{align*}
(-2x + 3y) - (x - 2y) &= -2x + 3y - x + 2y \\
&= -3x + 5y
\end{align*}
\]

(d) \((-3x - 3 \cdot 4) - (3x - y - 1)\)

\[
\begin{align*}
(-3x - 3 \cdot 4) - (3x - y - 1) &= -3x - 12 - 3x + y + 1 \\
&= -6x + y - 11
\end{align*}
\]

(e) \((2x - 5) \cdot 6) - (2y - 3)\)

\[
\begin{align*}
(2x - 5) - (2y - 3) &= 2x - 5 - 2y + 3 \\
&= 2x - 2y - 2
\end{align*}
\]
Section 3.1

Try It!

1. Paul’s pocket money is £8 less than Lucy’s pocket money. Find Paul’s pocket money if Lucy’s is (a) £20, (b) £73, (c) £p.

Solution
Paul’s pocket money = Lucy’s - £8

(a) When Lucy’s pocket money is £20, Paul’s = £20 - £8 = £12
(b) When Lucy’s pocket money is £73, Paul’s = £73 - £8 = £65
(c) When Lucy’s pocket money is £p, Paul’s = (£p - 8)

2. The time Henry takes to complete a quiz is twice the time Rose takes. Find the time Henry takes to complete the quiz if the time Rose takes is (a) 20 minutes, (b) 35 minutes, (c) t minutes.

Solution
Time Henry takes = twice the time Rose takes

(a) If Rose takes 20 minutes, the time Henry takes = 2 × 20 = 40 minutes
(b) If Rose takes 35 minutes, the time Henry takes = 2 × 35 = 70 minutes
(c) If Rose takes t minutes, the time Henry takes = 2 × t = 2t minutes

3. Erica takes half the time that Harry takes to finish a mathematics assignment. Find the time Erica takes if Harry takes (a) 50 minutes, (b) 2 hours, (c) T hours.

Solution
Time Erica takes = \( \frac{1}{2} \) × time Harry takes

(a) If Harry takes 50 minutes, the time Erica takes = \( \frac{1}{2} \) × 50 = 25 minutes
(b) If Harry takes 2 hours, the time Erica takes = \( \frac{1}{2} \) × 120 = 60 minutes
(c) If Harry takes T hours, the time Erica takes = \( \frac{1}{2} \) × T = \( \frac{T}{2} \) hours

4. The price for a chocolate bar is £1 and the price for a pizza is £5. Find the total price for (a) three chocolate bars and two pizzas, (b) m chocolate bars and n pizzas in terms of m and n.

Solution
(a) Price of three chocolate bars = 3 × £1 = £3
Price of two pizzas = 2 × £5 = £10
Total price = £3 + £10 = £13
(b) Price of m chocolate bars = m × £1 = £m
Price of n pizzas = n × £5 = £5n
Total price = £m + £5n

5. Write these statements as algebraic expressions.
   (a) Add y to 3x
   (b) Subtract 7p from 3p
   (c) Multiply 4m and n
   (d) Divide (4x + 3b) by 9

Solution
(a) 3x + y
(b) 3p - 7p
(c) 4m × n
(d) \( \frac{4x + 3b}{9} \)

6. Simplify these expressions.
   (a) \( c \times 3 \times d \times 2 \)
   (b) \( h \times 7 \times h \)
   (c) \( r \times r \times 5 \times r \)

Solution
(a) \( 3 \times 2 \times c \times d \)
(b) \( 7 \times h \times h \)
(c) \( r \times r \times 5 \times r \)

7. Determine whether 5y^2 and (3y)^2 are equal.

Solution
5y^2 = 5y × y
(3y)^2 = 3y × 3y
Both expressions are equal.

Section 3.2

8. When p = 6, find the value of (a) 4p, (b) 20 + 3p.

Solution
(a) 4p = 4 × 6 = 24
(b) 20 + 3p = 20 + 3 × 6 = 20 + 18 = 38

9. The time Henry takes to complete a quiz is twice the time Rose takes. Find the time Henry takes if Rose takes (a) 35 minutes, (b) t minutes.

Solution
(a) If Rose takes 35 minutes, the time Henry takes = 2 × 35 = 70 minutes
(b) If Rose takes t minutes, the time Henry takes = 2 × t = 2t minutes
9. The height of a candle is (21 + 4) cm after burning for 3 hours. Find the height of the candle after burning for three hours.

Solution

Heights: (21 + 4) cm
When x = 3, height = 21 + 4 x 3 = 39 cm

10. The total length of x red rods and y blue rods in a model kit is (8x + 15) cm. Find the total length of six red rods and three blue rods.

Solution

Total length = (8x + 15)y cm
When x = 6 and y = 3, total length = 8 x 6 + 15 x 3 = 48 + 45 = 93 cm

11. Given the formula S = x + 2y, find the value of S when (a) x = 9, (b) y = 2.

Solution

(a) S = 9 + 2 x 2 = 13
(b) S = x + 2y = 9 + 2 = 11

12. For a rectangle of length L cm and width W cm, its area = L x W is given by the formula A = L x W. Using the formula, find the area of a rectangle of length 9 cm and width 5 cm.

Solution

A = L x W
When L = 9 and W = 5, A = 9 x 5 = 45
The required area is 45 cm².

Section 3.3

13. In a shot put event, Ada’s putting distance was 3 meters longer than Janet’s.

(a) Let x metres be Janet’s putting distance

Express Ada’s putting distance in terms of x.

(b) Let y metres be Ada’s putting distance. Write a formula connecting x and y.

Solution

(a) Janet’s putting distance = x

(b) Formula connecting x and y is x = 3 + y

14. The capacity of a cup is one-fifth the capacity of a bowl.

(a) Let b ml be the capacity of the bowl. Express the capacity of the cup in terms of b.

(b) Let c ml be the capacity of the cup. Write a formula connecting b and c.

Solution

(a) Capacity of a cup = \( \frac{1}{5} \times \) capacity of a bowl

\( = \frac{1}{5} \times b \) ml

(b) The formula is

\( \frac{b}{5} \)

15. There are p bicycles and q tricycles in a shop. Let \( W \) be the total number of wheels.

(a) Write a formula connecting \( p \) and \( q \) and \( W \).

(b) If there are five bicycles and seven tricycles, find the total number of wheels.

Solution

(a) Each bicycle has 2 wheels.

Number of wheels on \( p \) bicycles = \( 2p \)

Each tricycle has 3 wheels.

Number of wheels on \( q \) tricycles = \( 3q \)

Total number of wheels, \( W = 2p + 3q \)

(b) When \( p = 5 \) and \( q = 7 \),

\( W = 2 \times 5 + 3 \times 7 \)

\( = 10 + 21 \)

\( = 31 \)

The total number of wheels is 31.

Section 3.4

16. Simplify these expressions.

(a) \( 5x + 6y = 7t \)

Solution

(a) \( 5x + 6y = 7t \)

(b) \( \frac{a}{c} = 2x + 3y + 5z \)

Solution

(a) \( 5x + 6y = 7t \)

(b) \( \frac{a}{c} = 2x + 3y + 5z \)

17. Simplify these expressions.

(a) \( 5x + 6y = 7t \)

Solution

(a) \( 5x + 6y = 7t \)

(b) \( \frac{a}{c} = 2x + 3y + 5z \)

Solution

(a) \( 5x + 6y = 7t \)

18. The volume of a can of soft drink is \( (7t + 3) cm^3 \).

(a) Simplify the expression \( 7t + 3 + 2x \).

(b) Find the volume of the soft drink if \( t = 6 \).

Solution

(a) \( 7t + 3 + 2x \)

(b) When \( t = 6 \), volume of the soft drink \( = 563 ) \times 15 \)

\( = 330 ml \)

19. The total value of four stacks of coins is \( (£10m + 5s + 7n + 5y) \).

(a) Simplify the expression \( 10m + 5s + 7n + 5y \).

(b) Find the total value if \( m = 1 \) and \( n = 2 \).

Solution

(a) \( 10m + 5s + 7n + 5y \)

(b) When \( m = 1 \) and \( n = 2 \), total value \( = (£171 + 9) \)

\( = £180 \)

Section 3.5

20. Simplify \( (4a + b) + (3a + b) \).

Solution

\( (4a + b) + (3a + b) \)

\( = 4a + b + 3a + b \)

\( = 7a + 2b \)

21. Find the sum of \( 5p = 4q + 7 \) and \( 2y = 9 \).

Solution

\( (5p + 4q = 7) \)

\( + (2y = 9) \)

\( = 5p + 4q + 2y = 7 + 9 \)

\( = 5p + 4q + 2y = 16 \)

Exercise 3.1

Level 1

1. Tom is 13 years old. Find his age after

(a) 4 years, (b) 7 years, (c) 13 years.

Solution

(a) Tom’s age after 4 years = 13 + 4 = 17 years

(b) Tom’s age after 7 years = 13 + 7 = 20 years

(c) Tom’s age after 13 years = (13 + 13) years

2. Anna is 6 cm shorter than Fred. Find Anna’s height if Fred’s height is

(a) 163 cm, (b) 180 cm, (c) 167 cm.

Solution

Anna’s height = Fred’s height = 6 cm

Anna’s height = 163 cm

Fred’s height = 157 cm
3. Every chicken has two legs. Find the total number of legs in
(a) three chickens, (b) eight chickens, (c) n chickens.

Solution
(a) Number of legs in three chickens = 2 × 3 = 6
(b) Number of legs in eight chickens = 2 × 8 = 16
(c) Number of legs in n chickens = 2 × n = 2n

4. The capacity of a small bottle is one-third that of a large bottle. Find the capacity of the small bottle if the capacity of the large bottle is
(a) 600 ml, (b) 750 ml, (c) p ml.

Solution
Capacity of a small bottle = \( \frac{1}{3} \) × capacity of a large bottle

Small bottle

Larger bottle

(a) Capacity of the small bottle = \( \frac{1}{3} \) × 600
= 200 ml
(b) Capacity of the small bottle = \( \frac{1}{3} \) × 750
= 250 ml
(c) Capacity of the small bottle = \( \frac{1}{3} \) × p
= \( \frac{p}{3} \) ml

5. Some students share 360 jelly beans equally. Find the number of jelly beans each student gets if there are
(a) nine students, (b) 15 students, (c) m students.

Solution
(a) Share of each student = \( \frac{360}{9} \) = 40 jelly beans
(b) Share of each student = \( \frac{360}{15} \) = 24 jelly beans
(c) Share of each student = \( \frac{360}{m} \) = \( \frac{3600}{m} \) jelly beans

6. Find the number of wheels in
(a) eight tricycles, (b) N tricycles.

Solution
(a) Number of wheels in 8 tricycles = 3 × 8 = 24
(b) Number of wheels in N tricycles = 3N

7. Simplify these expressions.
(a) \( a \times b \) (b) \( h \times c \times e \)
(c) \( d \times d \) (d) \( h \times 7b \)
(e) \( 2m \times 6n \) (f) \( 3p + 5q \)

Solution
(a) \( a \times b = ab \) (b) \( h \times c \times e = 4bc \)
(c) \( d \times d = d^2 \) (d) \( h \times 7b = 7hb \)
(e) \( 2m \times 6n = 12mn \) (f) \( 3p + 5q \)

8. Write these statements as algebraic expressions.
(a) Add 5 to 3a.
(b) Subtract 4 from \( 15 \).
(c) The product of 2d and 3d.
(d) Divide m by 12n.

Solution
(a) Expression: 5 + 3a
(b) Expression: 15 − 4c
(c) Expression: \( 2d \times 3d = 6d^2 \)
(d) Expression: \( \frac{m + 12n}{12n} \)

Level 2 GCSE Grade 2

9. Simplify these expressions.
(a) \( r \times 4r \times r \) (b) \( 2x \times 3xy \times x \)
(c) \( 4u \times r \times u \) (d) \( 3x \times 3y \times 9 \)

Solution
(a) \( r \times 4r \times r = 4r^3 \)
(b) \( 2x \times 3x \times y = 6x^2y \)
(c) \( 4u \times r \times u = 4ru^2 \)
(d) \( 3x \times 3y \times 9 = 27xy \)

10. 400-gram tins of baked beans are placed in a box with a mass of 250 grams. Find the total mass of the box which contains
(a) five tins of baked beans, (b) n tins of baked beans in terms of n.

Solution
(a) Total mass of the box with 5 tins of baked beans = 400 × 5
= 2000 grams
(b) Total mass of the box with n tins of baked beans = 400n grams

11. The price of a calculator is £10 and the price of a pen is £2. Find the total price of
(a) three calculators and four pens, (b) p calculators and q pens in terms of p and q.

Solution
(a) Total price = \( 3 \times £10 + 4 \times £2 \)
= £38
(b) Total price = \( p \times £10 + q \times £2 \)
= £(10p + 2q)

12. A rod of length 120 cm is cut into three pieces, A, B and C.
(a) If the length of A is 47 cm and the length of B is 52 cm, find the length of C.
(b) If the length of A is \( x \) cm and the length of B is \( y \) cm, find the length of C in terms of \( x \) and \( y \).

Hint: You may draw a bar model to help.

Solution
(a) Length of C = \( 120 - 47 - 52 \) cm
= 21 cm
(b) Length of C = \( 120 - (x + y) \) cm

13. Find an expression for the total value of
(a) six £1 coins and m £2 coins, (b) h £1 coins and k £2 coins.

Solution
(a) Total value = \( 6 \times £1 + m \times £2 \)
= £(6 + 2m)
(b) Total value = \( h \times £1 + k \times £2 \)
= £(h + 2k)

Level 3

14. A history book is 2 cm thick and a science book is 3 cm thick.
(a) If a stack consists of four history books and three science books, find
(i) the number of books in the stack, (ii) the height of the stack.
(b) If m history books and n science books are placed in a stack, find, in terms of \( m \) and \( n \), (i) the number of books in the stack, (ii) the height of the stack.

Solution
(i) Number of books = \( 4 + 3 = 7 \)
(ii) Height of the stack = \( 2 \times 4 + 3 \times 3 \)
= 17 cm
(ii) Number of books = \( m + n \)
(ii) Height of the stack = \( 2 \times m + 3 \times n \)
= \( 2m + 3n \) cm

15. There is one £5 note, \( x \) £10 notes and \( y \) £50 notes in a wallet. Express, in terms of \( x \) and \( y \),
(a) the number of bank notes in the wallet, (b) the total value of the bank notes in the wallet.

Solution
(a) Number of bank notes = \( 1 + x + y \)
(b) Total value of the bank notes = \( £(5 \times 1 + 10x + 50y) \)
= \( £(10x + 50y) \)

16. A boy is \( N \) years old. His father is four times as old as him. His mother is three years younger than his father. Express, in terms of \( N \), the age of
(a) his father, (b) his mother.

Hint: You may draw a bar model to help.

Solution
(a) Age of father = \( 4 \times N \) years
(b) Age of mother = \( (4N - 3) \) years
Exercises 3.2

Level 1

1. Find the value of \(2x + 1\) when
   (a) \(x = 7\),
   (b) \(x = -1\).

   Solution
   (a) When \(x = 7\),
   \[2x + 1 = 2(7) + 1 = 15\]
   (b) When \(x = -1\),
   \[2x + 1 = 2(-1) + 1 = -1\]

2. Find the value of \(9y + y\) when
   (a) \(y = 5\),
   (b) \(y = -4\).

   Solution
   (a) When \(y = 5\),
   \[9y + y = 9(5) + 5 = 44\]
   (b) When \(y = -4\),
   \[9y + y = 9(-4) + (-4) = -40\]

3. Find the value of \(\frac{z}{3}\) when
   (a) \(z = 2\),
   (b) \(z = -8\).

   Solution
   (a) When \(z = 2\),
   \[\frac{z}{3} = \frac{2}{3}\]
   (b) When \(z = -8\),
   \[\frac{z}{3} = \frac{-8}{3}\]

4. Find the value of \(3k + 2k\) when
   (a) \(k = 2\) and \(m = 5\),
   (b) \(k = 3\) and \(m = 4\).

   Solution
   (a) When \(k = 2\) and \(m = 5\),
   \[3k + 2k = 3(2) + 2(5) = 22\]
   (b) When \(k = 3\) and \(m = 4\),
   \[3k + 2k = 3(3) + 2(4) = 17\]

5. Find the value of \(8 - 2b + 3k\) when
   (a) \(b = 7\) and \(k = 5\),
   (b) \(b = -2\) and \(k = 4\).

   Solution
   (a) When \(b = 7\) and \(k = 5\),
   \[8 - 2b + 3k = 8 - 2(7) + 3(5) = 5\]
   (b) When \(b = -2\) and \(k = 4\),
   \[8 - 2b + 3k = 8 - 2(-2) + 3(4) = 24\]

6. Given that \(y = 3x - 1\), find the value of \(y\) when
   (a) \(x = 4\),
   (b) \(x = 0\).

   Solution
   (a) When \(x = 4\),
   \[y = 3(4) - 1 = 11\]
   (b) When \(x = 0\),
   \[y = 3(0) - 1 = -1\]

7. Given that \(P = 9 - 2Q\), find the value of \(P\) when
   (a) \(Q = 4\),
   (b) \(Q = -3\).

   Solution
   (a) When \(Q = 4\),
   \[P = 9 - 2(4) = 1\]
   (b) When \(Q = -3\),
   \[P = 9 - 2(-3) = 15\]

8. Given that \(s = \frac{b}{y}\), find the value of \(s\) when
   (a) \(b = 24\),
   (b) \(y = 6\).

   Solution
   (a) When \(b = 24\),
   \[s = \frac{24}{y}\]
   (b) When \(y = 6\),
   \[s = \frac{24}{6} = 4\]

9. Given that \(w = 2v + 3r = 7\), find the value of \(w\) when
   (a) \(v = 5\) and \(r = 4\),
   (b) \(v = -4\) and \(r = -1\).

   Solution
   (a) When \(v = 5\) and \(r = 4\),
   \[w = 2(5) + 3(4) = 22\]
   (b) When \(v = -4\) and \(r = -1\),
   \[w = 2(-4) + 3(-1) = -11\]

10. Given that \(A = bh\), find the value of \(A\) when
    (a) \(b = 4\) and \(h = 3\),
    (b) \(b = 7\) and \(h = 8\).

    Solution
    (a) When \(b = 4\) and \(h = 3\),
    \[A = 4 \times 3 = 12\]
    (b) When \(b = 7\) and \(h = 8\),
    \[A = 7 \times 8 = 56\]

Level 2

11. Find the value of \(3x - 24\) when
    (a) \(x = 0\),
    (b) \(x = 2\).

    Solution
    (a) When \(x = 0\),
    \[3x - 24 = 3(0) - 24 = -24\]
    (b) When \(x = 2\),
    \[3x - 24 = 3(2) - 24 = 0\]

12. Given the formula \(m = \frac{a + b}{c}\), find the value of \(m\) when
    (a) \(a = 4\) and \(b = 18\),
    (b) \(a = -3\) and \(b = 7\).

    Solution
    (a) When \(a = 4\) and \(b = 18\),
    \[m = \frac{4 + 18}{2} = 11\]
    (b) When \(a = -3\) and \(b = 7\),
    \[m = \frac{-3 + 7}{2} = 2\]

13. Given the formula \(y = l - 2x^2\), find the value of \(y\) when
    (a) \(x = 0\),
    (b) \(x = 3\).

    Solution
    (a) When \(x = 0\),
    \[y = l - 2(0)^2 = l\]
    (b) When \(x = 3\),
    \[y = l - 2(3)^2 = l - 18\]
14. Given the formula \( P = 30 \times (1 + r) \), find the value of \( P \) when
(a) \( r = 2 \) and \( n = 1 \),  
(b) \( r = -3 \) and \( n = -4 \).

**Solution**
(a) \( P = 30 \times (1 + 2) = 30 \times 3 = 90 \)
(b) \( P = 30 \times (1 - 3) = 30 \times (-2) = -60 \)

15. Erica is a part-time clerk. Her weekly wage is £(3 x 145), which is the number of hours she works in a week.
(a) If Erica works 30 hours in a week, find her weekly wage.
(b) What do you think the number 14 in the expression 14w stands for?

**Solution**
(a) Erica's weekly wage = £(14 x 30) = £420
(b) The number 14 stands for the hourly wage in £.

16. The total price for \( m \) burgers and \( n \) cups of coffee is £(3m + 2n).
(a) Find the total price for five burgers and four cups of coffee.
(b) What do you think the numbers 3 and 2 in the expression 3m + 2n stand for?

**Solution**
(a) The total price = £(3 x 5 + 2 x 4) = £23
(b) In the expression 3m + 2n, the number 3 is the price of a burger and 2 is the price of a cup of coffee in £.

17. The conversion of \( x \)°C (Celsius) to \( y \)°F (Fahrenheit) is given by the formula \( y = \frac{9}{5}x + 32 \).
(a) The freezing point of water is 0°C. Work out the freezing point in Fahrenheit.
(b) The boiling point of water is 100°C. Work out the boiling point in Fahrenheit.

**Solution**
(a) When \( x = 0 \), \( y = \frac{9}{5}(0) + 32 = 32 \)
(b) The freezing point of water is 32°F.

**Exercise 3.3**

**Level 1**

1. The mass of Jim is \( p \) kg and the mass of Lily is \( q \) kg.
(a) Express \( x \) in the bar model in terms of \( p \) and \( q \).
(b) What does \( x \) represent?

**Solution**
(a) \( x = \frac{p + q}{2} \)
(b) \( x \) represents the combined masses of Jim and Lily.

2. Tom's long jump distance is \( x \) metres. Shaun's long jump distance is \( y \) metres.
(a) Express \( z \) in the bar model in terms of \( x \) and \( y \).
(b) What does \( z \) represent?

**Solution**
(a) \( z = \frac{x - y}{2} \)
(b) \( z \) represents how much further Tom's long jump distance is than Shaun's.

3. Richard is \( m \) years old and Susan is \( n \) years old.
The bar model represents their ages.
(a) Express \( n \) in terms of \( m \).
(b) What can you say about the relationship between their ages?

**Solution**
(a) \( n = \frac{1}{2}m + 4 \)
(b) Susan's age is one third of Richard's age, or Richard is three times as old as Susan.

4. The length of a bed is 2 metres shorter than the length of a bedroom. If the length of the bedroom is \( L \) metres, express the length of the bed in terms of \( L \).

**Solution**
Length of bedroom = \( L \) metres
Length of bed = \( L - 2 \) metres.

5. Tom's age is four times the age of his son. If the age of his son is \( x \) years, express Tom's age in terms of \( x \).

**Solution**
Tom's age = \( 4x \) years.

6. Johnson spends one-quarter of his weekly salary on a pair of sports shoes. If his weekly salary is £w, express the price of the pair of sports shoes in terms of \( w \).

**Solution**
Price of shoes = \( \frac{1}{4}w \) £

If the weekly salary is £w, the price of the sports shoes is £(1/4w) or £(w/4).

**Level 2**

7. The initial temperature of a cup of water is \( x \)°C. The temperature drops 25°C after an hour and reaches \( y \)°C. Write down a formula connecting \( x \) and \( y \).

**Solution**
The formula is \( y = x - 25 \).

8. The thickness of a dictionary is twice the thickness of another book. Let \( d \) cm and \( h \) cm be the thicknesses of the dictionary and the other book respectively. Express \( d \) in terms of \( h \).

**Solution**
The formula is \( d = 2h \).

9. The height of a London Eye souvenir is one-thousandth of the actual height of the London Eye. Let \( h \) cm and \( H \) cm be the heights of the souvenir and the London Eye respectively. Express \( h \) in terms of \( H \).

**Solution**
The formula is \( h = \frac{H}{1000} \) or \( h = 0.001H \).
10. The length of a folk song is 4 minutes. The length of a pop song is 1 minute. The length of the pop song is half the length of the folk song.

(a) Express y in terms of x.
(b) If the length of the folk song is 4 minutes, find the length of the pop song.

Solution
(a) \( y = \frac{1}{2}x \)
(b) If \( x = 4 \), \( y = \frac{1}{2} \times 4 = 2 \) minutes.

The length of the pop song is 2 minutes.

11. The area of Paul’s flat is \( p \) m\(^2\) and the area of Queenie’s flat is \( q \) m\(^2\). Paul’s flat is 15 m\(^2\) more than Queenie’s flat.

(a) Express \( p \) in terms of \( q \).
(b) If Queenie’s flat is 90 m\(^2\), find the area of Paul’s flat.

Solution
(a) \( p = q + 15 \)
(b) If \( q = 90 \), \( p = 90 + 15 = 105 \) m\(^2\).

The area of Paul’s flat is 105 m\(^2\).

**Level 3**

12. Mrs Atkins gives a money box with £9 in it to her son. She asks her son to put £2 into the money box each week.

(a) Express the amount in the money box after \( n \) weeks in terms of \( n \).
(b) Find the amount in the money box after 10 weeks.

Solution
(a) \( \text{Amount} = 9 + 2n \) £
(b) \( \text{Amount} = 9 + 2 \times 10 = 29 \) £

The amount in the money box after 10 weeks is £29.

13. Frank has two part-time jobs. The weekly salary of the second job is three times the weekly salary of the first job. Let \( m \) be the weekly salary of the first job.

(a) Express the weekly salary of the second job in terms of \( m \).
(b) Let \( LT \) be the total weekly salary of Frank’s part-time jobs. Write a formula connecting \( m \) and \( LT \).

Solution
(a) \( \text{Weekly salary of second job} = 3m \) £
(b) \( LT = m + 3m = 4m \) £

**Exercise 3.4**

**Level 1**

1. State the number of terms and the constant term in each of these expressions.
(a) \( 2a + 3b + 1 \)
(b) \( 7x^2 + 2x + 6 \)
(c) \( 3p + 4q \)
(d) \( 2x^2 + 7y + 8x^3 + 4xy \)

Solution
(a) \( \text{Number of terms} = 3 \)
(b) \( \text{Number of terms} = 3 \)
(c) \( \text{Number of terms} = 2 \)
(d) \( \text{Number of terms} = 5 \)

2. Write down the coefficients of \( x \) and \( y \) in each of these expressions.
(a) \( 3x + 4y + 6 \)
(b) \( 2x + 7y - 8 \)
(c) \( x^2 + 4x^5 + 8 \)
(d) \( 3y + 5y^2 + x \)

Solution
(a) \( \text{Coefficient of } x = 3 \)
(b) \( \text{Coefficient of } y = 8 \)
(c) \( \text{Coefficient of } x = 1 \)
(d) \( \text{Coefficient of } y = 5 \)

3. Simplify these expressions.
(a) \( 7a + 2a \)
(b) \( 5b + 8b \)
(c) \( 4x + 6x \)
(d) \( 2a - 3y \)

Solution
(a) \( 9a \) £
(b) \( 13b \) £
(c) \( 10x \) £
(d) \( 2a - 3y \) £

4. Simplify these expressions.
(a) \( 3a + 10 + 4a - 11 \)
(b) \( 2x + 3x + 4e + 7 \)
(c) \( 8s + 7z + 9x + 7y \)
(d) \( 3p - 4r + 2x + 5m \)
(e) \( 7l + 11m + 4r \)
(f) \( -2x + 3.5y + 5 - 3x - 7y \)

Solution
(a) \( 7a + 10 + 4a - 11 = 10a - 1 \) £
(b) \( 2x + 3x + 4e + 7 = 5x + 4e + 7 \) £
(c) \( 8s + 7z + 9x + 7y = 8s + 7z + 9x + 7y \) £
(d) \( 3p - 4r + 2x + 5m = 3p - 4r + 2x + 5m \) £
(e) \( 7l + 11m + 4r = 7l + 11m + 4r \) £
(f) \( -2x + 3.5y + 5 - 3x - 7y = -5x + 3.5y + 5 \) £

5. (a) Simplify the expression \( 2a + 5x + 3y \).
(b) Find the value of the expression when \( x = 2 \).

Solution
(a) \( 2a + 5x + 3y \) £
(b) \( \text{Value when } x = 2 = 2a + 5(2) + 3y = 2a + 10 + 3y \) £
6. (a) Simplify the expression 7a + 2b + 5b – a = 3.
(b) Find the value of the expression when a = 4 and b = 2.

Solution
(a) 7a + 2b + 5b – a = 3
(b) When a = 4 and b = 2, 6a + 3b = 3

7. (a) Simplify the expression 3x – 2y + 1 = x.
(b) Find the value of the expression when x = –2 and y = 6.

Solution
(a) 3x – 2y + 1 = x
(b) When x = –2 and y = 6, 3x + 2y = 1

Level 3 GCSE Grade 3

8. The total number of atoms in the compound CH₄ is n + 2a + 2b. Find the total number of atoms if n = 10.

Solution
(a) n + 2a + 2b = 32

9. The total price of a student ticket, an ordinary ticket and a VIP ticket for a concert is given by 4p + 50p + 2p = 300.
(a) Simplify p + 50 + 2p = 30.
(b) Find the total price if p = 90.

Solution
(a) 4p + 50p + 2p = 300
(b) When p = 90, 4p + 20 + 50 + 20 = 300

10. The height of a stack of 10 books is given by (2x + 3y) cm.
(a) Simplify the expression 2x + 3y + x + y.
(b) Find the height of the stack if x = 2 and y = 3.

Solution
(a) 2x + 3y + x + y = 3x + 4y
(b) When x = 2 and y = 3, 6x + 5y = 24

11. The lengths of the sides of a triangle are 2cm, 4cm and 3cm. Express the perimeter of the triangle in terms of x and y.

Solution
Perimeter of the triangle = (2x + 4y + 3x) cm

12. A rectangle is 3cm long and 2cm wide.
(a) Express the perimeter of the rectangle in terms of x.
(b) When p = 12, find the perimeter of the rectangle.

Solution
(a) Perimeter of the rectangle = 2(3x) + 2(2p)
(b) When p = 12, perimeter of the rectangle = 10(12)

13. A woman works 2 hours each day from Monday to Friday. She works 2a + y hours on Saturdays. She does not work on Sundays.
(a) Express her working hours in a week in terms of x and y.
(b) How many hours does she work each week when x = 4 and y = 3?

Solution
(a) Working hours in a week = [2a + 5 + (2a – y)]
(b) When r = 4 and y = 3, working hours in a week = [12(4)]

Exercise 3.5

Level 1

1. Simplify these expressions.
(a) (2x + 1)
(b) (–7) + 6
(c) (x + 9)
(d) (7)
(e) (4x + 3)
(f) (8y + 7)

Solution
(a) 2x + 1
(b) –7 + 6 = 1
(c) x + 9
(d) 7
(e) 4x + 3
(f) 8y + 7

2. Simplify these expressions.
(a) (2x + 3) + (x – 5)
(b) (–7) + (30 – 1)
(c) (2x + 6) + (1 – 2x)
(d) (–2x + 2y) + (–x + 0)
(e) (8x + 7y) + (35m + 2n)
(f) (–2x + 3y) + (x + y)

Solution
(a) 2x + 6
(b) 30 – 1
(c) 0
(d) –x + 2y
(e) 35m + 2n
(f) –x + 4y

Level 2 GCSE Grade 3

4. Simplify these expressions.
(a) (2a + 3k) + (2b + 5k)
(b) (9x + 4y) + (6x + 5y)
(c) (a + 10b) + (6c + 3)
(d) (5y + 4a) + (2b + 7a)
(e) (7x + 2y) + (4x + 6) + (3y + 9)
(f) (2a + 3b) + (9 – 4d) + (2c + 7)

Solution
(a) 2a + 3k + 2b + 5k
(b) 9x + 4y + 6x + 5y
(c) a + 10b + 6c + 3
(d) 5y + 4a + 2b + 7a
(e) 7x + 6y + 3y + 9
(f) 2a + 3b + 9 – 4d + 2c + 7

Level 3 GCSE Grade 3

3. Simplify these expressions.
(a) (6p + 7) + (3q + q)
(b) (6x + 5y) + (3z + 1)
(c) (9w + 5t) + (3t + 2)
(d) (2x + 4y) + (2p + 4q)

Solution
(a) 6p + 7 + 3q + q
(b) 6x + 5y + 3z + 1
(c) 9w + 5t + 3t + 2
(d) 2x + 4y + 2p + 4q

Chapter 3 Introduction to Algebra
10. Subtract one from $t = 5$ and $w = 4.$

Solution:

When $p = 10$ and $q = 30,$

\[
\text{mass} = 8p + 17q + 16
\]

\[
= 8(10) + 17(30) + 16 = 506 \text{ grams}
\]

11. The perimeter of a triangle is $(7x + 3) \text{ cm}.$ The lengths of two sides of the triangle are $(2x + y) \text{ cm}$ and $(x + 2y) \text{ cm}.$

(a) Find the length of the third side in terms of $x$ and $y.$

(b) If $t = 1$ and $w = 1,$ find the length of the third side.

Solution:

(a) Length of the third side

\[
= (7x + 3 - (2x + y))
\]

\[
= x + 3 - y
\]

(b) When $x = 5$ and $y = 1,$

\[
l = 5(3) - 1 = 14 \text{ cm}
\]

12. Peter's mother is four times as old as Peter. Peter's sister is three years younger than Peter. Let $n$ years be the age of Peter. Express, in terms of $n$,

(a) the age of Peter's mother

(b) the age of Peter's sister.

(c) the sum of the ages of Peter, his mother and his sister.

Solution:

(a) Age of Peter's mother $= 4n$ years

(b) Age of Peter's sister $= (n - 3)$ years

(c) Sum of the ages $= n + 4n + (n - 3) = 6n - 3$ years

13. In an experiment, the original temperature of a beaker of pure water is $(40 \degree C)$ under standard pressure. It is heated up by $(3r + 15) \degree C$ and then cools down by $(2t + 10) \degree C$.

(a) Find the final temperature in terms of $t.$

(b) If $t = 11,$ find the final temperature.

(c) Is it possible that $t = 25?$ Explain briefly.

Solution:

(a) Final temperature

\[
= (3r + 15) + (2t + 10)
\]

\[
= 3r + 2t + 25
\]

When $t = 11,$

\[
= 3(25) + 2(11) + 15 = 110 \degree C
\]

14. Suppose nine dates in a certain month are enclosed by a rectangle as shown above.

(a) Explain a quick way to calculate the sum of the nine numbers.

(b) Let $n$ be the number at the top left-hand corner of the rectangle. Express the sum of the nine numbers in terms of $n$.

(c) Let $m$ be the middle number in the rectangle. Express the sum of the nine numbers in terms of $m$.

(d) If $n$ is a multiple of 9, on which day will the first day of the month be?

(e) Describe some other interesting properties about the numbers within the rectangle.

Solution:

(a) A quick way to calculate the sum of the nine numbers is to

(i) add the two numbers on the opposite ends of each diagonal, that is $(7 + 23) + (9 + 21),$

(ii) add the two numbers on the opposite side of the rectangle, that is $(9 + 22) + (14 + 16),$

(iii) add the middle number $15$ to (i) and (ii) Hence the sum of $(7 + 23 + 9 + 21 + 8 + 22 + 14 + 16) + 15 = 135$

(b) If the number at the top left-hand corner of the rectangle is $n,$ you have

\[
\begin{array}{ccc}
\text{Sun} & \text{Mon} & \text{Tue} \\
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9 \\
10 & 11 & 12 \\
13 & 14 & 15 \\
16 & 17 & 18 \\
19 & 20 & 21 \\
22 & 23 & 24 \\
25 & 26 & 27 \\
28 & 29 & 30 \\
22 & 21 & 20 \\
19 & 18 & 17 \\
16 & 15 & 14 \\
13 & 12 & 11 \\
10 & 9 & 8 \\
7 & 6 & 5 \\
6 & 5 & 4 \\
5 & 4 & 3 \\
4 & 3 & 2 \\
3 & 2 & 1 \\
2 & 1 & 0 \\
\end{array}
\]

Using the method in (a), the sum of the nine numbers

\[
= (n + 16 + 6) + (2r + 14 + 8)
\]

\[
= 40 \degree C
\]

\[
= 2n + 16 + 2r + 14 + 8
\]

\[
40 \degree C + 90 \degree C = 135 \degree C
\]

However $45 \degree C\text{ to }90 \degree C = 135 \degree C,$ which exceeds the boiling point, $100 \degree C,$ of pure water. Hence, it is not possible that $t = 25.$
3. Write these statements as algebraic expressions.
(a) Add \( c \) squared to \( 3d \).
(b) Subtract \( 2 \times k \) from \( k \).
(c) Multiply \( p \) and \( q \) and \( p \times 3 \).
(d) Divide \( s \) cubed by \( 7 \).

Solution
(a) \( d^2 + c^2 \)
(b) \( k - (2 \times k) \)
(c) \( p \times q + p \times q \)
(d) \( \frac{s^3}{7} \)

4. Find the values of these expressions when \( n = 4 \).
(a) \( 2n + 5 \)
(b) \( 5n + 2 \)
(c) \( 7(1 - 3n) \)

Solution
(a) \( 2 \times 4 + 5 = 13 \)
(b) \( 5 \times 4 + 2 = 22 \)
(c) \( 7 \times (1 - 3 \times 4) = 7 \times (-11) = -77 \)

5. Find the value of the expression \( a^2 - 5b \) when \( a = 3 \) and \( b = 2 \).

Solution
When \( a = 3 \) and \( b = 2 \),
\[ a^2 - 5b = (3)^2 - 5(2) = 9 - 10 = -1 \]

6. Given the formula \( F = mn \), find the value of \( F \) when \( m = 4 \) and \( n = 5 \).

Solution
When \( m = 4 \) and \( n = 5 \),
\[ F = mn = 4 \times 5 = 20 \]

7. Given the formula \( E = \frac{1}{2}mv^2 \), find the value of \( E \) when \( m = 3 \) and \( v = 2 \).

Solution
When \( m = 3 \) and \( v = 2 \),
\[ E = \frac{1}{2}mv^2 = \frac{1}{2} \times 3 \times 2^2 = 6 \]

8. Simplify these expressions.
(a) \( 3 + 4b - 2 + 5n \)
(b) \( 2a + 3b - 4a + 2n \)
(c) \( \frac{5}{3}p \times 2q = 10pq \)
(d) \( 3x - 7y + 2x + 2y \)

Solution
(a) \( = 3 + 4b + 5n - 2 \)
(b) \( = 2a + 3b - 4a + 2n \)
(c) \( = \frac{5}{3}pq \times 2 = \frac{10}{3}pq \)
(d) \( = 3x + 2x - 7y + 2y \)

9. Simplify these expressions.
(a) \( 3 \times (2a - 3b) \)
(b) \( (2c - 3d)^2 - (1c - 5d) \)
(c) \( (2m + 3n)(2m - 9n) \)
(d) \( (5x - 6y)(-2x + 4y) \)

Solution
(a) \( = 3 \times 2a - 3 \times 3b \)
(b) \( = 2c^2 - 7cd + 9ad - 3c + 5d \)
(c) \( = 4m^2 + 6mn + 3n^2 - 27mn \)
(d) \( = 10x - 12xy + 20y - 24y^2 \)

10. The price \( LP \) of planting a square flowerbed of side \( x \) metres is given by the formula:
\[ P = \frac{8x^2 + 20x}{3} \]

Find the price of planting a square flowerbed of side 3 metres.

Solution
\[ P = \frac{8 \times 3^2 + 20 \times 3}{3} = \frac{72 + 60}{3} = \frac{132}{3} = 44 \]

11. The temperature of an iron bar is 17°C. When it is heated, the temperature rises by 20°C every minute.
(a) Find the temperature of the iron bar after 3 minutes.
(b) If the temperature of the iron bar after \( t \) minutes is \( T \)°C, express \( T \) in terms of \( t \).
(c) Find the temperature of the iron bar after 9 minutes.

Solution
(a) \( T = 17 + 20 \times 3 = 77°C \)
(b) \( T = 17 + 20t \)
(c) \( T = 17 + 20 \times 9 = 197°C \)

12. There are three consecutive odd numbers. The smallest number is \( n \).
(a) Express the other two numbers in terms of \( n \).
(b) Let \( S \) be the sum of these three numbers. Write a formula connecting \( n \) and \( S \).
(c) Hence find the value of \( S \) when the smallest number is 13.

Solution
(a) The next two odd numbers are \( n + 2 \) and \( n + 4 \).
(b) \( S = n + (n + 2) + (n + 4) = 3n + 6 \)
(c) When \( n = 13 \), \( S = 3(13) + 6 = 45 \)

13. The price for a bag of pasta is £2 and the price for a pack of sausages is £3. Mrs Stocks buys \( m \) bags of pasta and \( n \) packs of sausages.
(a) Express the total price of the items in terms of \( m \) and \( n \).
(b) Mrs Stocks pays with a £50 note at the counter and she gets £C change. Write a formula connecting \( m \), \( n \) and \( C \).
(c) Find the change that Mrs Stocks receives if she buys four bags of pasta and two packs of sausages.

Solution
(a) Total price of the items \( = 2m + 3n \)
(b) \( C = 50 - (2m + 3n) \)
(c) \( C = 50 - 2m - 3n \)

14. The mass of three apples is 3 grams. The mass of four apples and a basket is (4x, 120) grams.
(a) Express the total mass of these seven apples and the basket in terms of \( x \).
(b) Find the total mass of these seven apples and the basket when \( x = 150 \).

Solution
(a) Total mass of the seven apples and the basket \( = 3x + 4(x + 120) \)
(4x + 120 + 7x + 120) grams
(b) When \( x = 150 \),
\[ \text{total mass} = 7(150) + 120 = 1170 \text{ grams} \]

15. In the Leeds Triathlon, an athlete’s swimming time is \( t \) minutes. His cycling time is twice the swimming time. His running time is 20 minutes more than the swimming time.
(a) Express his cycling time and running time in terms of \( t \).
(b) Let \( T \) minutes be the time taken by the athlete in the race. Write a formula connecting \( t \) and \( T \).
(c) If the athlete’s swimming time is 40 minutes, find his time taken, in hours, to complete the race.

Solution
(a) Cycling time \( = 2t \) minutes
Running time \( = t + 20 \) minutes
\[ T = t + 20 \]

Chapter 3 Introduction to Algebra
Class Activity 1

Objective: To explore how to solve an equation.

Tasks

In each question, you will explore two possible methods for finding the solution. In part A, you will use a bar model and in part B, you will use balancing scales. For each part, copy and complete the diagram and equation.

1. Solve the equation \(2x + 3 = 7\).

A. Using a Bar Model

1. Represent the equation using a bar model.

2. Take away 3. Copy and fill in the diagram and the equation.

3. Divide the bar into two equal parts. Each part is \(x\). Copy and fill in the diagram and the equation.

B. Using Balancing Scales

The left-hand side (LHS) and the right-hand side (RHS) of the scales must be equal in value to keep the supporting beam of the two sides in a level position.

1. Place the corresponding discs on the LHS and RHS of the scales.

2. To remove 3 from the LHS, you add \(\text{zero pairs}\) to both sides. Copy and complete the equation.