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Please refer to the text for a full list of acknowledgments.
This book has been written by an experienced author and reviewed by a team of maths consultants and practising teachers. It is based on the top-selling Singaporean maths series and has been carefully adapted to make it suitable for Year 7. The features and exercises have been designed to help you discover the underlying principles of mathematics and to set you on the road to mastery.

Every chapter starts with an example of how the maths you will learn can be related to real life. This is supported by a film on the digital book on Kerboodle.

Learning objectives at the beginning of the chapter tell you what you will learn.

At the end of every chapter: In a Nutshell summarises the key learning points that have been covered. This can be used for revision.

A graph on the digital book provides the links to the film and the student version of the film.

You will be told when you should or shouldn’t use a calculator. If there is no instruction, you can decide for yourself or ask your teacher.

At the end of every chapter: Write in Your Journal sections encourage you to reflect on your learning.
Most cars and lorries carry a spare tyre. Cars have 4 working tyres + 1 spare tyre, so 5 tyres in total. Some lorries have 8 working tyres as well as the spare tyre. All these vehicles can be said to carry ‘\(N + 1\) tyres’. What does \(N\) stand for in this expression?
1. Four Operations on Numbers

(a) Addition

<table>
<thead>
<tr>
<th>Statement</th>
<th>Expression and Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>add 4 to 3</td>
<td>3 + 4 = 7</td>
</tr>
<tr>
<td>sum of 5 and −6</td>
<td>5 + (−6) = 5 − 6 = −1</td>
</tr>
<tr>
<td>total of −9 and 2</td>
<td>(−9) + 2 = −9 + 2 = −7</td>
</tr>
<tr>
<td>−8 plus −1</td>
<td>(−8) + (−1) = −8 − 1 = −9</td>
</tr>
</tbody>
</table>

(b) Subtraction

<table>
<thead>
<tr>
<th>Statement</th>
<th>Expression and Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>subtract 9 from 4</td>
<td>4 − 9 = −5</td>
</tr>
<tr>
<td>take away 2 from −3</td>
<td>(−3) − 2 = −3 − 2 = −5</td>
</tr>
<tr>
<td>−7 minus −8</td>
<td>(−7) − (−8) = −7 + 8 = 1</td>
</tr>
</tbody>
</table>

(c) Multiplication

<table>
<thead>
<tr>
<th>Statement</th>
<th>Expression and Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>multiply 4 by 6</td>
<td>4 × 6 = 24</td>
</tr>
<tr>
<td>product of −5 and 3</td>
<td>(−5) × 3 = −15</td>
</tr>
<tr>
<td>twice −8 or double −8</td>
<td>2 × (−8) = −16</td>
</tr>
<tr>
<td>−9 times −7</td>
<td>(−9) × (−7) = 63</td>
</tr>
</tbody>
</table>

(d) Division

<table>
<thead>
<tr>
<th>Statement</th>
<th>Expression and Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>divide 8 by 3</td>
<td>8 ÷ 3 = (\frac{8}{3})</td>
</tr>
<tr>
<td>quotient of 12 divided by −6</td>
<td>12 ÷ (−6) = −2</td>
</tr>
<tr>
<td>half −8</td>
<td>(−8) ÷ 2 = −4</td>
</tr>
<tr>
<td>−15 divided by 5</td>
<td>(−15) ÷ 5 = −3</td>
</tr>
<tr>
<td>−25 over −28</td>
<td>(−25) ÷ (−28) = (\frac{25}{28})</td>
</tr>
</tbody>
</table>

2. Square and Cube of a Number

(a) A number multiplied by itself is called the square of the number.

  e.g. 3 × 3 = 3², called 3 squared.

(b) A number multiplied by itself and itself again is called the cube of the number.

  e.g. 7 × 7 × 7 = 7³, called 7 cubed.

3. Order of Operations

When an expression involves more than one operation, the order of the operations is:

1. Brackets
2. Indices, such as square and cube
3. Multiplication and division from left to right
4. Addition and subtraction from left to right

Example: 3 × 4 − 72 = (−9) = 12 − (−8) = 12 + 8 = 20

3.1 Letters to Represent Numbers

A bicycle has two wheels.

If there are three bicycles, the total number of wheels = 3 × 2 = 6.
If there are five bicycles, the total number of wheels = 5 × 2 = 10.
If there are n bicycles, the total number of wheels = n × 2 or 2 × n, usually written as 2n.

In algebra, you use letters to represent numbers. Here, you use the letter \(n\) to represent the number of bicycles. The letter \(n\) is called a variable as its value can be any positive number such as 3, 5 or 10. The expression 2n involves numbers and letters and is called an algebraic expression.

Example: Anne is 28 years older than Sophia. Find Anne’s age when Sophia is (a) 3 years old, (b) 15 years old, (c) s years old.

Solution: From the given information, Anne’s age = Sophia’s age + 28.

You can represent this using a bar model.

Anne = \(s + 28\) years old.

(a) When Sophia is 3 years old, Anne’s age = 3 + 28 = 31 years
The length of a nail is one-quarter the length of a screw. Find the length of the nail if the length of the screw is (a) 28 mm, (b) 52 mm, (c) \(L\) mm.

**Solution**

From the given information, the length of the nail = \(\frac{1}{4}\) × the length of the screw.

You can represent this using a bar model.

For (a) When the length of the screw is 28 mm, the length of the nail = \(\frac{1}{4}\) × 28 = 7 mm.

For (b) When the length of the screw is 52 mm, the length of the nail = \(\frac{1}{4}\) × 52 = 13 mm.

For (c) When the length of the screw is \(L\) mm, the length of the nail = \(\frac{1}{4}\) × \(L\) = \(\frac{L}{4}\) mm.

Note: \(\frac{L}{4}\) or \(\frac{L}{4}\) is written as \(\frac{L}{4}\), where the multiplication sign is omitted and the number is placed in front of the variable.

Erica takes half the time that Harry takes to finish a mathematics assignment. Find the time Erica takes if Harry takes (a) 50 minutes, (b) 2 hours, (c) \(T\) hours.

**Example 2**

Try It!

For a school play, the price of an adult ticket is £3 and the price of a child ticket is £1. Find the total price of (a) two adult tickets and four child tickets, (b) \(x\) adult tickets and \(y\) child tickets in terms of \(x\) and \(y\).

**Solution**

(a) Price of two adult tickets = £3 × 2
   Price of four child tickets = £1 × 4
   \[\therefore \text{ total price} = (3 \times 2 + 1 \times 4)\]
   \[= 10\text{ pounds}\]

(b) Price of \(x\) adult tickets = £3 \(x\)
   Price of \(y\) child tickets = £1 \(y\)
   \[\therefore \text{ total price} = (3 \times x + 1 \times y)\]
   \[= 3x + y\text{ pounds}\]
Note: • $3x + y$ is an algebraic expression in two variables $x$ and $y$.
• 'Find in terms of $x$ and $y$' means finding an expression using $x$ and $y$ for the answer. It is different from finding a value for the answer.
• $1 \times y$ or $y \times 1$ is written as $y$ NOT $1y$.
• $3 \times 2$ can be written as $(3)(2)$.

The price for a chocolate bar is £1 and the price for a pizza is £5. Find the total price for
(a) three chocolate bars and two pizzas,
(b) $m$ chocolate bars and $n$ pizzas in terms of $m$ and $n$.

The table summarises algebraic expressions for the basic operations. You can compare them with the arithmetic expressions in the Flashback.

<table>
<thead>
<tr>
<th>Operation</th>
<th>Statement</th>
<th>Algebraic Expression</th>
<th>Bar Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition</td>
<td>• add $b$ to $a$</td>
<td>$a + b$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• sum of $a$ and $b$</td>
<td>$a + b$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• total of $a$ and $b$</td>
<td>$a + b$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• $a$ plus $b$</td>
<td>$a + b$</td>
<td></td>
</tr>
<tr>
<td>Subtraction</td>
<td>• subtract $c$ from $d$</td>
<td>$d - c$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• take away $c$ from $d$</td>
<td>$d - c$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• $d$ minus $c$</td>
<td>$d - c$</td>
<td></td>
</tr>
<tr>
<td>Multiplication</td>
<td>• multiply $p$ by $q$</td>
<td>$pq$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• product of $p$ and $q$</td>
<td>$pq$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• $p$ times $q$</td>
<td>$pq$</td>
<td></td>
</tr>
<tr>
<td>Division</td>
<td>• Divide $x$ by $y$</td>
<td>$\frac{x}{y}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Quotient of $x$ divided by $y$</td>
<td>$\frac{x}{y}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• $x$ over $y$</td>
<td>$\frac{x}{y}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Assume $y \neq 0$.</td>
<td>$\frac{x}{y}$</td>
<td></td>
</tr>
</tbody>
</table>

Assume $a$ and $b$ are positive.

Assume $c$ and $d$ are positive.

Assume $p = 3$ and $q$ is positive.

Assume $x$ is positive and $y < 0$.

Example 5

Write these statements as algebraic expressions.
(a) Add 6$m$ to $n$.
(b) Subtract $24k$ from $5d$.
(c) Multiply $3a$ and $b$.
(d) Divide $(2x - 5y)$ by $7$.

Solution
(a) Add 6$m$ to $n$: $n + 6m$
(b) Subtract $24k$ from $5d$: $5d - 24k$
(c) Multiply $3a$ and $b$: $3a \times b$ = $3ab$
(d) Divide $(2x - 5y)$ by $7$: $(2x - 5y) \div 7$ = $\frac{2x - 5y}{7}$

Write these statements as algebraic expressions.
(a) Add $y$ to $3x$.
(b) Subtract $7p$ from $3q$.
(c) Multiply $4m$ and $n$.
(d) Divide $(4a + 3b)$ by $9$.

Try It! $\text{Example 5}$

In arithmetic, you write $7 \times 7$ as $7^2$ and $5 \times 5$ as $5^2$. In algebra, the notation is similar.

You read $a \times a = a^2$ as $a$ squared, and $b \times b \times b = b^3$ as $b$ cubed.

The number 2 is called the index of $a$ in $a^2$ and 3 is called the index of $b$ in $b^3$.

You should note that $a^2 = a \times a$ and $2a = a + a$.

So $a^2$ and $2a$ are different.

Similarly, $b^3 = b \times b \times b$ and $3b = b + b + b$.

So $b^3$ and $3b$ are different.

Example 6

Simplify these expressions.
(a) $4 \times a \times 5 \times b$
(b) $3 \times t \times t$
(c) $u \times 2 \times u \times u$

Solution
(a) To simplify an expression means to rewrite it as simply as possible.
$4 \times a \times 5 \times b = 4 \times 5 \times a \times b$
Rearrange and write the numbers first.
Write letters in alphabetical order.
$= 20 \times ab$
$b \times t \times t = t \times t \times t$
$= 3a^2$
(c) $u \times 2 \times u \times u = 2 \times u \times u \times u = u \times u \times u \times u = 2a^3$

Try It! $\text{Example 6}$

Simplify these expressions.
(a) $c \times 3 \times d \times 2$
(b) $b \times 7 \times b$
(c) $r \times r \times 5 \times r$
Chapter 3 Introduction to Algebra

Solution

Example 8

When \( m = 5 \), find the value of

(a) \( 3m \)
(b) \( 7 + 2m \).

Solution

(a) \( 3m = 3 \times 5 \)
(b) \( 7 + 2m = 7 + 2 \times 5 \)

\[ 15 \]

12. A rod of length 120 cm is cut into three pieces, A, B and C.
   (a) If the length of A is 47 cm and the length of B is 52 cm, find the length of C.
   (b) If the length of A is \( x \) cm and the length of B is \( y \) cm, find the length of C in terms of \( x \) and \( y \). 

HINT: You may draw a bar model to help.

13. Find an expression for the total value of
   (a) six £1 coins and five £2 coins,
   (b) \( h \) £1 coins and \( k \) £2 coins.

14. A history book is 2 cm thick and a science book is 3 cm thick.
   (a) If a stack consists of four history books and three science books, find
       (i) the number of books in the stack,
       (ii) the height of the stack.
   (b) If \( m \) history books and \( n \) science books are placed in a stack, find, in terms of \( m \) and \( n \),
       (i) the number of books in the stack,
       (ii) the height of the stack.

LEVEL 3

15. There is one £5 note, \( x \) £10 notes and \( y \) £50 notes in a wallet. Express, in terms of \( x \) and \( y \),
   (a) the number of bank notes in the wallet,
   (b) the total value of the bank notes in the wallet.

16. A boy is \( N \) years old. His father is four times as old as him. His mother is three years younger than his father. Express, in terms of \( N \), the age of
   (a) his father,
   (b) his mother.

HINT: You may draw a bar model to help.

17. A survey was conducted about the number of children in a family. The table below summarises the results.

<table>
<thead>
<tr>
<th>Number of children in a family</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
</table>
| Number of families            | 17 | \( p \) | \( q \)

Express, in terms of \( p \) and \( q \),
(a) the number of families in the survey,
(b) the total number of children in the families in the survey.

3.2 Substituting Numbers for Letters

A Evaluation of Algebraic Expressions

As mentioned in Section 3.1, if there are \( n \) bicycles,
   the total number of wheels on the bicycles is \( 2n \).

If there are 18 bicycles, you can calculate the total number of wheels by putting \( n = 18 \)
into the above expression.

Thus, 
the total number of wheels on the bicycles is \( 2 \times 18 \) = 36.

If there are 37 bicycles, then \( n = 37 \) and
the total number of wheels on the bicycles is \( 2 \times 37 \) = 74.

This process of replacing each variable with a number to calculate the value of an
algebraic expression is called substitution.

Example 8

When \( m = 5 \), find the value of

(a) \( 3m \)
(b) \( 7 + 2m \).

Solution

(a) \( 3m = 3 \times 5 \) \[ 15 \]

Substitute \( m \) by 5.
You have learnt that the total number of wheels on $n$ bicycles is given by the expression $2n$.

If you represent this total by the variable $T$, you have

$T = 2n$.

An equality connecting two or more variables is called a **formula**.

When the value of $n$ is known, you can find the value of $T$ in the formula by substitution. For example, when $n = 3$,

$T = 2 \times 3$

$= 6$.

That means the total number of wheels on three bicycles is 6.

**Formulas** are often used in mathematics and physics to show the relationship between different quantities.

You have learnt that the total number of wheels on $n$ bicycles is given by the expression $2n$.

If you represent this total by the variable $T$, you have

$T = 2n$.

An equality connecting two or more variables is called a **formula**.

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$T = 2 \times 3$

$= 6$.

That means the total number of wheels on three bicycles is 6.

**Formulas** are often used in mathematics and physics to show the relationship between different quantities.
For a rectangle of length \( L \) cm and width \( W \) cm, its perimeter \( P \) cm is given by the formula \( P = 2L + 2W \). Using the formula, find the perimeter of a rectangle of length 36 cm and width 25 cm.

\[ L \text{ cm} \quad W \text{ cm} \]

**Solution**

\[ P = 2L + 2W \]

When \( L = 36 \) and \( W = 25 \),

\[ P = 2(36) + 2(25) = 72 + 50 = 122 \text{ cm} \]

i.e. the required perimeter is 122 cm.

For a rectangle of length \( L \) cm and width \( W \) cm, its area \( A \) cm\(^2\) is given by the formula \( A = LW \). Using the formula, find the area of a rectangle of length 9 cm and width 5 cm.

**Exercise 3.2**

**Level 1**

1. Find the value of \( 2x + 1 \) when
   (a) \( x = 7 \),
   (b) \( x = -1 \).

2. Find the value of \( 9 - y \) when
   (a) \( y = 5 \),
   (b) \( y = -4 \).

3. Find the value of \( \frac{z}{6} \) when
   (a) \( z = 2 \),
   (b) \( z = -18 \).

4. Find the value of \( 3b + c \) when
   (a) \( b = 2 \) and \( c = 5 \),
   (b) \( b = -1 \) and \( c = -4 \).

5. Find the value of \( 8 - 2k + 3k \) when
   (a) \( k = 2 \) and \( k = 5 \),
   (b) \( k = -2 \) and \( k = 4 \).

6. Given that \( y = 3x - 1 \), find the value of \( y \) when
   (a) \( x = 4 \),
   (b) \( x = 0 \).

7. Given that \( P = 9 - 2Q \), find the value of \( P \) when
   (a) \( Q = 4 \),
   (b) \( Q = -3 \).

8. Given that \( x = \frac{24}{3} \), find the value of \( x \) when
   (a) \( t = 4 \),
   (b) \( t = -3 \).

9. Given that \( w = 2u - 3v - 7 \), find the value of \( w \) when
   (a) \( u = 5 \) and \( v = 4 \),
   (b) \( u = -4 \) and \( v = -1 \).

10. Given that \( A = bh \), find the value of \( A \) when
    (a) \( b = 4 \) and \( h = 5 \),
    (b) \( b = 7 \) and \( h = -8 \).

**Level 2**

11. Find the value of \( 3x - 24 \) when
    (a) \( x = 0 \),
    (b) \( x = -2 \).

12. Given the formula \( m = \frac{a+b}{2} \), find the value of \( m \) when
    (a) \( a = 4 \) and \( b = 18 \),
    (b) \( a = -3 \) and \( b = 7 \).

13. Given the formula \( y = 1 - 2x \), find the value of \( y \) when
    (a) \( x = 0 \),
    (b) \( x = 3 \).

14. Given the formula \( P = 30 \times (1 + rt) \), find the value of \( P \) when
    (a) \( r = 2 \) and \( t = 1 \),
    (b) \( r = -3 \) and \( t = -4 \).

**Level 3**

15. Erica is a part-time clerk. Her weekly wage is £14h, where \( h \) is the number of hours she works in a week.
    (a) If Erica works 20 hours in a week, find her weekly wage.
    (b) What do you think the number 14 in the expression 14h stands for?

**3.3 Writing Algebraic Expressions and Formulae**

Algebraic expressions and formulae can be used to express the relationships between two or more quantities in your daily life. Consider some examples.

**Example 13**

In a 100 m run, Charles was two seconds faster than Eric.
(a) Let \( x \) seconds be the time taken by Eric. Express the time taken by Charles in terms of \( x \).
(b) Let \( y \) seconds be the time taken by Charles. Write a formula connecting \( x \) and \( y \).

**Solution**

(a) Charles’ time = Eric’s time – 2 seconds
\[ x = (x - 2) \text{ seconds} \]
(b) From (a), you have the formula
\[ y = x - 2 \]

**Try It!**

In a shot put event, Ada’s putting distance was 3 metres longer than Janet’s.
(a) Let \( x \) metres be Janet’s putting distance. Express Ada’s putting distance in terms of \( x \).
(b) Let \( y \) metres be Ada’s putting distance. Write a formula connecting \( x \) and \( y \).
For a concert, the price of a child ticket is half the price of an adult ticket.
(a) Let £a be the price of an adult ticket. Express the price of a child ticket in terms of a.
(b) Let £c be the price of a child ticket. Write a formula connecting a and c.

(a) Price of a child ticket
= \( \frac{1}{2} \) x price of an adult ticket
= £ \( \frac{1}{2} \) a
(b) From (a), the formula is
\[ c = \frac{a}{2} \]

Example 9

There are \( m \) birds and \( n \) rabbits in a pet shop. Let \( T \) be the total number of legs.
(a) Write a formula connecting \( m \), \( n \) and \( T \).
(b) If there are 10 birds and six rabbits in the shop, find the total number of legs.

(a) Each bird has two legs.
Number of legs of \( m \) birds = \( 2m \).
Each rabbit has four legs.
Number of legs of \( n \) rabbits = \( 4n \).
Total number of legs of these animals = \( 2m + 4n \).
\[ T = 2m + 4n \]
(b) When \( m = 10 \) and \( n = 6 \),
\[ T = 2 \times 10 + 4 \times 6 \]
Substitute \( m = 10 \) and \( n = 6 \).
= 20 + 24
= 44
\[ \therefore \] the total number of legs is 44.

There are \( p \) bicycles and \( q \) tricycles in a shop. Let \( W \) be the total number of wheels.
(a) Write a formula connecting \( p \), \( q \) and \( W \).
(b) If there are five bicycles and seven tricycles, find the total number of wheels.

EXERCISE 3.3

LEVEL 1

1. The mass of Jim is \( plkg \) and the mass of Lily is \( qkg \).
(a) Express \( x \) in the bar model in terms of \( p \) and \( q \).
(b) What does \( x \) represent?

2. Tom’s long jump distance is \( x \) metres. Shaun’s long jump distance is \( y \) metres.
(a) Express \( z \) in the bar model in terms of \( x \) and \( y \).
(b) What does \( z \) represent?

3. Richard is \( m \) years old and Susan is \( n \) years old.
The bar model represents their ages.
(a) Express \( n \) in terms of \( m \).
(b) What can you say about the relationship between their ages?

4. The length of a bed is 2 metres shorter than the length of a bedroom. If the length of the bedroom is \( L \) metres, express the length of the bed in terms of \( L \).

5. Tom’s age is four times the age of his son. If the age of his son is \( x \) years, express Tom’s age in terms of \( x \).

LEVEL 2

6. Johnson spends one-quarter of his weekly salary on a pair of sports shoes. If his weekly salary is £w, express the price of the pair of sports shoes in terms of \( w \).

7. The initial temperature of a cup of water is \( x^\circ C \). The temperature drops 23\(^\circ C \) after an hour and reaches \( y^\circ C \). Write down a formula connecting \( x \) and \( y \).

8. The thickness of a dictionary is twice the thickness of another book. Let \( d \) cm and \( h \) cm be the thicknesses of the dictionary and the other book respectively. Express \( d \) in terms of \( b \).

9. The height of a London Eye souvenir is one-thousandth of the actual height of the London Eye. Let \( h \) cm and \( H \) cm be the heights of the souvenir and the London Eye respectively. Express \( b \) in terms of \( H \).

10. The length of a folk song is \( x \) minutes. The length of a pop song is \( y \) minutes. The length of the pop song is half the length of the folk song.
(a) Express \( y \) in terms of \( x \).
(b) If the length of the folk song is 4 minutes, find the length of the pop song.

11. The area of Paul’s flat is \( pm^2 \) and the area of Queenie’s flat is \( qm^2 \). Paul’s flat is 15 \( m^2 \) more than Queenie’s flat.
(a) Express \( p \) in terms of \( q \).
(b) If Queenie’s flat is 90 \( m^2 \), find the area of Paul’s flat.
12. Mrs Atkins gives a money box with £9 in it to her son. She asks her son to put £2 into the money box each week.
   (a) Express the amount in the money box after 10 weeks in terms of $n$.
   (b) Find the amount in the money box after 10 weeks.

13. Frank has two part-time jobs. The weekly salary of the second job is three times the weekly salary of the first job. Let £$m$ be the weekly salary of Frank's first part-time job. Write a formula connecting variables $m$ and $T$.
   (a) Express the weekly salary of the second job in terms of $m$.
   (b) Let £$T$ be the total weekly salary of Frank's part-time jobs. Write a formula connecting $m$ and $T$.
   (c) If the weekly salary of the first job is £$130$, find the total weekly salary of Frank's part-time jobs.

3.4 Like Terms and Unlike Terms

In an algebraic expression, the parts separated by plus or minus signs are called the terms of the expression. For example, there are three terms in the expression $2x - 3y + 8$, and they are $2x$, $-3y$ and $8$.

- **term**: A part of the expression separated by plus or minus signs.
- **constant term**: A term that does not contain a variable.
- **variable**: A symbol that represents a number.

The number part, including the sign, of a term is called the coefficient of its variable part. In the above example, $2$ is the coefficient of $x$ and $-3$ is the coefficient of $y$. The term $8$ has no variable, so it is called a constant term.

Consider another expression, $4p - 3q$. It consists of two terms which are $4p$ and $-3q$. Here, the coefficients of $p$ and $q$ are $4$ and $-3$ respectively.

**RECALL**

In algebra, a letter can be used to represent any number. You call this letter a variable.

14. Carbon and hydrogen form a compound. The number of hydrogen atoms in the compound is twice the number of carbon atoms.
   (a) If there are $n$ carbon atoms, express, in terms of $n$,
      (i) the number of hydrogen atoms,
      (ii) the total number of atoms in the compound.
   (b) If the compound has three carbon atoms, find the total number of atoms in the compound.

Note: Propene, whose molecular formula is $C_3H_6$, is an example of this type of compound.

15. Create a situation that forms a formula connecting variables $M$, $x$ and $y$ such that $M = 28$ when $x = 5$ and $y = 6$.

**Objective:** To learn to add or subtract two terms with identical variable parts.

**Tasks**

1. **(a)** Look at the expressions below. What is the same about them? What is different?
   **(b)** Classify the expressions into separate groups based on their variable parts.

2. There are some red bars each of length $x$ cm and some blue bars each of length $y$ cm.

Using the given models, express these results as simply as possible.

   **(a)**
   - $AB$ is formed by two red bars. Find the length of $AB$ in terms of $x$.
   - $CD$ is formed by three red bars. Find the length of $CD$ in terms of $x$.
   - If $AB$ and $CD$ are joined together, what is the total length in terms of $x$?
   - Find $2x + 3x$.

   **(b)**
   - $EF$ is formed by five blue bars. Find the length of $EF$ in terms of $y$.
   - $GH$ is formed by two blue bars. Find the length of $GH$ in terms of $y$.
   - If a part of length $GH$ is cut off from $EF$, find the length of the remaining part in terms of $y$.
   - Find $5y - 2y$.

   **(c)** If $AB$ and $EF$ are joined together, what is the total length in terms of $x$ and $y$?

   **Note:** The expression in terms of $x$ and $y$ obtained in (c) cannot be further simplified.
When two terms have identical variable parts, i.e. they consist of the same variable(s) having the same indices, they are called like terms.

Referring to Question 1 in Class Activity 2, one pair of like terms is $3xy$ and $-6xy$ as they have identical variable parts, $xy$. All constant terms are like terms. When two terms are not like terms, they are called unlike terms.

<table>
<thead>
<tr>
<th>Like terms</th>
<th>Unlike terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3x$, $7a$</td>
<td>$4a$, $8b$</td>
</tr>
<tr>
<td>$9x^2$, $-5x^4$</td>
<td>$2y$, $y^2$</td>
</tr>
<tr>
<td>$-p^2q^2$, $-6p^2q^2$</td>
<td>$-p^2q^2$, $-6p^2q^2$</td>
</tr>
<tr>
<td>$11$, $-2$</td>
<td>$15w$, $15$</td>
</tr>
</tbody>
</table>

You can simplify an algebraic expression by adding or subtracting like terms. For example, $2x + 3x = 5x$ and $5y - 2y = 3y$.

This process of simplification is called collecting like terms.

**Objective:** To apply the process of collecting like terms using algebra discs.

As well as number discs, you have $x$-discs and $y$-discs.

Each disc has two sides. You can flip a disc to show either the front or the back. Just like 1 and $-1$, $x$ and $-x$ and $y$ and $-y$ form zero pairs. That is, $1 + (-1) = 0$, $x + (-x) = 0$ and $y + (-y) = 0$. Examples of zero pairs are:

(a) $2x + (-2x) = 2x - 2x = 0$
(b) $(-3y) + 3y = -3y + 3y = 0$

each of these pairs form zero pairs.

Some algebraic expressions can be represented by a set of algebra discs. For example,

(a) $2x + 3$:  
(b) $y - 4$: 

This set of algebra discs can also represent $3 + 2x$.

You can write $y - 4$ as $y + (-4)$.

**RECALL**

$2 + (-2) = 2 - 2 = 0$
$(-3) + 3 = -3 + 3 = 0$

**Tasks**

1. Represent these algebraic expressions using algebra discs.
   (a) $3x + 2$  
   (b) $x + 4y$  
   (c) $-2x - 3y + 4$

2. Simplify these expressions. Use algebra discs to help you.
   (a) $2x + 5x$
   (b) $2x - 5x$
   (c) $-2x + 5x$
   (d) $2x - 5x$
   (e) $3y + 1 + 4y + 3$
   (f) $3y + 1 - 4y - 3$
   (g) $-3y + 1 + 4y + 3$
   (h) $-3y - 1 - 4y - 3$
   (i) $4x - 5y + x + 2y - 1$
   (j) $-3x + 2y - 4 - x + y + 1$

**Questions**

1. Explain how you would simplify $ax + bx$, where $a$ and $b$ are given integers. You may substitute $a$ and $b$ with different integers to help you illustrate your explanation.

2. Can you write $3x + 4y$ as $7xy$? You may substitute $x$ and $y$ with numbers to help you illustrate your explanation.
Example 16  

Simplify these expressions.  

(a) $7x - 3x + x$  

(b) $-5y + 4y - 6y$  

Solution  

(a) $7x - 3x + x = 4x + x = 5x$  

(b) $-5y + 4y - 6y = -y - 6y = -7y$  

Note: Alternatively, when the variable parts are the same, you can simplify like this:  

$7x - 3x + x = (7 - 3 + 1)x$  

$= 5x$  

Alternatively, $-5y + 4y - 6y = (-5 + 4 - 6)y = (-1 - 6)y = -7y$  

Try It!  

Simplify these expressions.  

(a) $5x + 6x - 7t$  

(b) $-4z - 3z + 5z$  

Example 17  

Simplify these expressions.  

(a) $3a + 4b - 2a + 5b$  

(b) $-3w + 8x - 1 + w - 5x - 2$  

Solution  

(a) $3a + 4b - 2a + 5b = 3a - 2a + 4b + 5b = (3 - 2)a + (4 + 5)b = a + 9b$  

(b) $-3w + 8x - 1 + w - 5x - 2 = (-3 + 1)w + (8 - 5)x - 3 = -2w + 3x - 3$  

Example 18  

The mass of a turkey is $(3p - 4 + 2p + 7)$ grams.  

(a) Simplify the expression $3p - 4 + 2p + 7$.  

(b) Find the mass of the turkey if $p = 960$.  

Solution  

(a) $3p - 4 + 2p + 7 = 3p + 2p - 4 + 7 = 5p + 3$  

(b) When $p = 960$,  

mass of turkey $= 5(960) + 3 = 4803$ grams  

Try It!  

The volume of a can of soft drink is $(7v + 9 - 2v + 6)$ ml.  

(a) Simplify the expression $7v + 9 - 2v + 6$.  

(b) Find the volume of the soft drink if $v = 63$.  

Example 19  

The total monthly salary of the employees at a small firm is £$(3x + 3y + 4x + y)$.  

(a) Simplify the expression $5x + 3y + 4x + y$.  

(b) Find their total monthly salary if $x = 3000$ and $y = 3500$.  

Solution  

(a) $5x + 3y + 4x + y = 5x + 4x + 3y + y = 9x + 4y$  

(b) When $x = 3000$ and $y = 3500$,  

total monthly salary $= £(9(3000) + 4(3500)] = £41000$
Try It!

(a) Simplify the expression $10m + 4n + 7m + 5n$.
(b) Find the total value if $m = 1$ and $n = 2$.

EXERCISE 3.4

LEVEL 1

1. State the number of terms and the constant term in each of these expressions.
   (a) $2a - 3b - 1$
   (b) $7x + 6 - 4y + 5z$
   (c) $3p - 4q$
   (d) $2 - x + 7y - 8x^2 + 4xy$

2. Write down the coefficients of $x$ and $y$ in each of these expressions.
   (a) $3x - 4y + 6$
   (b) $-2x + 7y - 8$
   (c) $x^2 + x + y + 8$
   (d) $9 - 4y - 5x + x$

3. Simplify these expressions.
   (a) $7a + 2a$
   (b) $5b - 6b$
   (c) $-4x + 6x$
   (d) $-2y - 3y$
   (e) $c + c + c$
   (f) $d + 2d - 9d$
   (g) $4y - 9y + 5y$
   (h) $-4m - 2m + 5m - m$
   (i) $-4p - 9p + 7p$
   (j) $-2x + 6x - 4x$

LEVEL 2

4. Simplify these expressions.
   (a) $3a + 10 - 4a - 11$
   (b) $-6 + 3k - 4k + 7$
   (c) $8x - 7y - 9x + 7y$
   (d) $3m + 4n - 2n + 5m$

(e) $7t + 4r - 5z - 4v$
(f) $-2x + 3y - 5x - 7y$

5. (a) Simplify the expression $-4 - 2x + 5 + x$.
   (b) Find the value of the expression when $x = 2$.

6. (a) Simplify the expression $7a - 2b + 5b - a - 3$.
   (b) Find the value of the expression when $a = -1$ and $b = 2$.

7. (a) Simplify the expression $3x - 2 - y + 1 - x$.
   (b) Find the value of the expression when $x = -2$ and $y = -6$.

LEVEL 3

8. The total number of atoms in the compound $C_{14}H_{22}O_{4}$ is $n + 2n$. (a) Simplify $n + 2n$.
   (b) Find the total number of atoms if $n = 10$.

9. The total price of a student ticket, an ordinary ticket and a VIP ticket for a concert is given by $(4p + 50 + p + 2p - 30)$. (a) Simplify $p + 50 + p + 2p - 30$.
   (b) Find the total price if $p = 90$.

10. The height of a stack of 10 books is given by $(2x + 3y + 4x + y)$ cm. (a) Simplify the expression $2x + 3y + 4x + y$.
    (b) Find the height of the stack if $x = 2$ and $y = 3$.

3.5 Addition and Subtraction of Linear Expressions

An algebraic expression, such as $3a + 1$ and $-2x + 5y - 4$, where each term has at most one variable to the index 1, is called a linear expression. The addition and subtraction of linear expressions can be performed by removing brackets and collecting like terms. You can explore the operations using algebra discs.

REMARK

The expressions $x^2 - 3$ and $6 - 5xy$ are not linear expressions. This is because the term $x^2$ involves $x$ to the index 2 and the term $-5xy$ involves two variables which are $x$ and $y$.

CLASS ACTIVITY 4

Objective: To perform the addition and subtraction of linear expressions using algebra discs.

Tasks

1. To obtain the negative of an expression, you flip the discs that represent the expression.
   For example, simplify $-(3x - 2)$.
   This means flipping the expression.

   

   $-(3x - 2) = -3x + 2$

   This means changing the sign of all the terms inside the bracket. Thus, $3x$ becomes $-3x$ and $-2$ becomes $2$.

   

The ‘$-$’ sign means flipping over all $8$ discs to get $8$ discs.

   

The ‘+$’ sign means flipping over all $8$ discs to get $0$ discs.
Collect the like terms.

2. To add two expressions, you remove the brackets and collect the like terms.

For example, simplify \((2x - 3y) + (-3x + y)\).

\[
(2x - 3y) + (-3x + y) \quad \text{Collect the like terms.} \quad -x - 2y
\]

3. Subtracting an expression is the same as adding the negative of that expression.

For example, simplify \((2x - 3y) - (-3x + y)\).

\[
(2x - 3y) - (-3x + y) \quad \text{Remove brackets.} \quad 5x - 4y
\]
There are two consecutive even integers. If the smaller integer is $2n$ find the sum of the two integers in terms of $n$.

Since the smaller integer = $6n + 1$, the larger integer = $(6n + 1) + 2$

Their sum = $(6n + 1) + (6n + 3)$

= $6n + 1 + 6n + 3$

= $6n + 6n + 1 + 3$

= $12n + 4$

There are two consecutive odd integers. If the smaller integer is $6n + 1$, find the sum of the two integers in terms of $n$.

Try It! Simplify $(7y - 2) - (4y - 9)$.

Example 2

Solution

The number of passengers on a bus is $5m + 1$. At a particular bus stop, $(3m - 2)$ passengers get off while $(2m + 4)$ passengers board the bus. Express, in terms of $m$, the number of passengers on the bus right after leaving the bus stop.

Number of passengers on the bus $= (5m + 1) - (3m - 2) + (2m + 4)$

$= 5m + 1 - 3m + 2 + 2m + 4$

$= 5m - 3m + 2m + 1 + 2 + 4$

$= 4m + 7$

Try It!

A lift goes up $(7x + 3)$ metres, then goes down $(4x - 6)$ metres, and finally goes up $(2x - 1)$ metres. How high is the lift above its starting point?

EXERCISE 3.5

LEVEL 1

1. Simplify these expressions.
   (a) $-(2x + 1)$
   (b) $-(-3x + 6)$
   (c) $-(4y - 9)$
   (d) $-(-2y + 7)$
   (e) $-(4x - 3y + 8)$
   (f) $-(-5x + 8y - 7)$

2. Simplify these expressions.
   (a) $(2a + 3) + (a - 4)$
   (b) $-(2b - 5) + (3b - 1)$
   (c) $2c + 6 - (5 - 2c)$
   (d) $(-4c + 2d) + (-3c + d)$
   (e) $(8m - 7n) + (-5m - 2n)$
   (f) $(-6x + 3y) + (x - 3y)$

3. Simplify these expressions.
   (a) $(6y + 7) - (3y + q)$
   (b) $-4 - (6x - 3)$
   (c) $(7 - 6x) - (-2 - 3)$
   (d) $-2x - 6 + (3x + 1)$
   (e) $-(4x + 5y) - (3x - 6y)$
   (f) $(2p - 4g) - (2p + 4g)$

LEVEL 2

4. Simplify these expressions.
   (a) $(2h - 3k + 6) + (8h - 5k - 2)$
   (b) $(9 - 3x - 4y) - (6 + 2x - 5y)$
   (c) $(-m - 8u + 1) + (-7m + 6u + 3)$
   (d) $(5 - 6p + 7q) - (2 - 3p - 7q)$
   (e) $(7x + 2y) + (4x - 6) - (-3 + 2y)$
   (f) $(2t - 3s) - (9 - 4c) + (2t + 1 - 5)$

5. Add $7x - 2y - 4z$ to $-2x + 3y - 5c$.

6. Find the sum of $5a - 3b, 7b - 3c$ and $9c - a$.

7. Subtract $a - 4b - 3c$ from $a + 2b - 6c$.

8. Subtract $t - 3v$ from the sum of $7t - 2u - 3v$ and $3t + 5u - 8v$.

LEVEL 3

9. There are three consecutive integers. If the smallest one is $n$, find the sum of these three integers in terms of $n$.

   Hint: For consecutive numbers, the difference between one number and the next number is 1.
10. The masses of three boxes of chocolates are \((3p + 4q + 2)\) grams, \((4p + 6q + 5)\) grams and \((p + 7q + 9)\) grams.
   (a) Find their total mass in terms of \(p\) and \(q\).
   (b) If \(p = 10\) and \(q = 30\), find the total mass.

11. The perimeter of a triangle is \((7x – 3y + 6)\) cm.
The lengths of two sides of the triangle are \((2x – y)\) cm and \((x + 2y)\) cm.
   (a) Find the length of the third side in terms of \(x\) and \(y\).
   (b) If \(x = 5\) and \(y = -1\), find the length of the third side.

12. Peter’s mother is three years younger than Peter. Peter’s sister is three years younger than Peter. Let \(n\), \(s\), and \(m\) represent
   (a) the age of Peter’s mother, 
   (b) the age of Peter’s sister,
   (c) the sum of the ages of Peter, his mother and his sister.

13. In an experiment, the original temperature of a beaker of pure water is \((t + 20)\)°C under standard pressure. It is heated up by \((3t + 15)\)°C and then cools down by \((2t – 10)\)°C.
   (a) Find the final temperature in terms of \(t\).
   (b) If \(t = 1\), find the final temperature.
   (c) Is it possible that \(t = 25\)? Explain briefly.

---

Suppose nine dates in a certain month are enclosed by a rectangle as shown above.
(a) Explain a quick way to calculate the sum of the nine numbers.
(b) Let \(n\) be the number at the top left-hand corner of the rectangle. Express the sum of the nine numbers in terms of \(n\).
(c) Let \(m\) be the middle number in the rectangle. Express the sum of the nine numbers in terms of \(m\).
(d) If ‘13’ falls on a Friday, on which day will the first of the month be?
(e) Describe some other interesting properties about the numbers within the rectangle.

Hint: Observe the sum of the three numbers on each diagonal.

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In a Nutshell

Notation in Algebra

In the algebra, letters are used to represent numbers.

The operation signs in algebra are the same as in arithmetic.

- add = \(x + y\)
- subtract = \(x - y\)
- product = \(xy\) or \(x \times y\)
- quotient = \(\frac{x}{y}\) or \(x \div y\)

(For division, \(y \neq 0\))

\(x \times 3 = 3x\)
\(x \times x = x^2\)
\(x \times x \times x = x^3\)

Algebraic Expressions

An algebraic expression involves numbers and letters connected by operations.

\(3a + 1\) and \(2b – 5\) are linear expressions.

\(3\) is called the coefficient of \(a\).

\(-5\) is a constant term.

Substitution

When the variables in an algebraic expression are substituted by numbers, the value of the expression can be found.

\(p \times q = \frac{p}{q}\)
\(p \times q = \frac{p}{q}\)
\(a^2 = 2\)

4. Find the values of these expressions when \(n = 4\).
   (a) \(2n + 5\)
   (b) \(5n - 2\)
   (c) \(7 \times 3\)

5. Find the values of the expression \(a^2 - 5b\) when \(a = 3\) and \(b = -2\).

6. Given the formula \(F = ma\), find the value of \(F\) when \(m = 4\) and \(a = 5\).

7. Given the formula \(E = \frac{1}{2}mv^2\), find the value of \(E\) when \(m = 3\) and \(v = 2\).
8. Simplify these expressions.
   (a) \(3 + 4e - 2 + 5e\)  \(\text{Simplify: } 3 + 4e - 2 + 5e\)
   (b) \(2a - 3b - 4a - b\)  \(\text{Simplify: } 2a - 3b - 4a - b\)
   (c) \(-3x + 5 + 2q - 6p\)  \(\text{Simplify: } -3x + 5 + 2q - 6p\)
   (d) \(3x - 7y + 8 - x - 2y\)  \(\text{Simplify: } 3x - 7y + 8 - x - 2y\)

9. Simplify these expressions.
   (a) \((1 + 5a) + (3 + 4a)\)  \(\text{Simplify: } (1 + 5a) + (3 + 4a)\)
   (b) \((2c - 7d) + (-d - 5c)\)  \(\text{Simplify: } (2c - 7d) + (-d - 5c)\)
   (c) \((-3m + n) - (2m + n)\)  \(\text{Simplify: } (-3m + n) - (2m + n)\)
   (d) \((5x - 6y) - (-7y + 2x)\)  \(\text{Simplify: } (5x - 6y) - (-7y + 2x)\)

10. The price \(LP\) of planting a square flowerbed of side \(x\) metres is given by the formula
    \[P = 8x + 20°\]
    Find the price of planting a square flowerbed of side 3 metres.

11. The temperature of an iron bar is 17°C. When it is heated, the temperature rises by 20°C every minute.
    (a) Find the temperature of the iron bar after 3 minutes.
    (b) If the temperature of the iron bar after \(t\) minutes is \(T°C\), express \(T\) in terms of \(t\).
    (c) Find the temperature of the iron bar after 9 minutes.

12. There are three consecutive odd numbers.
    The smallest number is \(n\).
    (a) Express the other two numbers in terms of \(n\).
    (b) Let \(S\) be the sum of these three numbers.
        Write a formula connecting \(n\) and \(S\).
    (c) Hence find the value of \(S\) when the smallest number is 13.

13. The price for a bag of pasta is £2 and the price for a pack of sausages is £3. Mrs Stocks buys \(m\) bags of pasta and \(n\) packs of sausages.
    (a) Express the total price of the items in terms of \(m\) and \(n\).
    (b) Mrs Stocks pays with a £50 note at the counter and she gets £C change.
        Write a formula connecting \(m\), \(n\) and \(C\).
    (c) Find the change that Mrs Stocks receives if she buys four bags of pasta and two packs of sausages.

14. The mass of three apples is 3x grams. The mass of four apples and a basket is (4x + 120) grams.
    (a) Express the total mass of these seven apples and the basket in terms of \(x\).
    (b) Find the total mass of these seven apples and the basket when \(x = 150\).

15. In the Leeds Triathlon, an athlete’s swimming time is \(t\) minutes. His cycling time is twice the swimming time. His running time is 30 minutes more than the swimming time.
    (a) Express his cycling time and running time in terms of \(t\).
    (b) Let \(T\) minutes be the time taken by the athlete in the race. Write a formula connecting \(t\) and \(T\).
    (c) If the athlete’s swimming time is 40 minutes, find his time taken, in hours, to complete the race.

16. A rectangular lawn is \((2x + 3y)\) metres long and \((6y - x)\) metres wide.
    (a) Let \(P\) metres be the perimeter of the lawn.
        Find a formula connecting \(P\), \(x\) and \(y\).
    (b) The cost of making the fence around the lawn is £3 per metre. Find the cost of making the fence when \(x = 5\) and \(y = 2\).