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Acknowledgements

The publishers would like to thank the following for permission to use copyright material:

1. Four Operations on Numbers
   
   (a) Addition
   
   Statement | Expression and Result |
   -----------|------------------------|
   add 4 to 3 | 3 + 4 = 7             |
   sum of 5 and −6 | 5 + (−6) = 5 − 6 = −1 |
   total of −9 and 2 | (−9) + 2 = −9 + 2 = −7 |
   −8 plus 1 | (−8) + (−1) = −8 − 1 = −9 |

   (b) Subtraction
   
   Statement | Expression and Result |
   -----------|------------------------|
   subtract 9 from 4 | 4 − 9 = −5             |
   take away 2 from −3 | (−3) − 2 = −3 − 2 = −5 |
   −7 minus −8 | (−7) − (−8) = −7 + 8 = 1 |

   (c) Multiplication
   
   Statement | Expression and Result |
   -----------|------------------------|
   multiply 4 by 6 | 4 × 6 = 24             |
   product of −5 and 3 | (−5) × 3 = −15         |
   twice −8 or double −8 | 2 × (−8) = −16        |
   −9 times −7 | (−9) × (−7) = 63       |

   (d) Division
   
   Statement | Expression and Result |
   -----------|------------------------|
   divide 8 by 3 | 8 ÷ 3 = 8/3            |
   quotient of 12 divided by −6 | 12 ÷ (−6) = −2      |
   half −8 | (−8) ÷ 2 = −4         |
   −15 divided by 5 | (−15) ÷ 5 = −3        |
   −25 over −28 | (−25) ÷ (−28) = 25/28 |

2. Square and Cube of a Number
   
   (a) A number multiplied by itself is called the square of the number.
      e.g. 3 × 3 = 9, called 3 squared.
   
   (b) A number multiplied by itself and itself again is called the cube of the number.
      e.g. 7 × 7 × 7 = 343, called 7 cubed.
   
   (c) A number multiplied by itself four times is called a number to the power 4.
      e.g. 5 × 5 × 5 × 5 = 625. 4 is called the index and 5 is called the base.

3. Order of Operations
   When an expression involves more than one operation, the order of the operations is:
   1. Brackets
   2. Indices, such as square and cube
   3. Multiplication and division from left to right
   4. Addition and subtraction from left to right
      e.g. 3 × 4 − 72 + (−9) = 12 − (−8) = 12 + 8 = 20
   
      −7 + 24 × 2 = −7 + 24 × 16 = −7 + 384 = 377
      14 − 2 + (16 − 23) = 14 − 2 + (−7) = 7 + (−7) = 7 − 7 = 0

   Most cars and lorries carry a spare tyre. Cars have 4 working tyres + 1 spare tyre, so 5 tyres in total. Some lorries have 8 working tyres as well as the spare tyre. All these vehicles can be said to carry ‘N + 1 tyres’. What does N stand for in this expression?

LET’S LEARN TO

- use letters to represent numbers
- interpret simple algebraic notations
- substitute integers into simple expressions and formulae
- write simple expressions and formulae
- simplify expressions by collecting like terms
- add and subtract linear expressions
- expand a single bracket

INTRODUCTION TO ALGEBRA

Most cars and lorries carry a spare tyre. Cars have 4 working tyres + 1 spare tyre, so 5 tyres in total. Some lorries have 8 working tyres as well as the spare tyre. All these vehicles can be said to carry ‘N + 1 tyres’. What does N stand for in this expression?
3.1 Letters to Represent Numbers

A butterfly has four wings.
If there are five butterflies, the total number of wings = $5 \times 4 = 20$.
If there are nine butterflies, the total number of wings = $9 \times 4 = 36$.
If there are $n$ butterflies, the total number of wings = $n \times 4$ or $4 \times n$, usually written as $4n$.

In algebra, you use letters to represent numbers. Here, you use the letter $n$ to represent the number of butterflies. The letter $n$ is called a variable as its value can be any positive number such as 5, 9 or 10. The expression $4n$ involves numbers and letters and is called an algebraic expression.

Example 1

Anne is 28 years older than Sophia. Find Anne’s age when Sophia is (a) 3 years old, (b) $s$ years old.

Solution

From the given information,
Anne’s age = Sophia’s age + 28.
You can represent this using a bar model.

(a) When Sophia is 3 years old,
Anne’s age = $3 + 28$
= 31 years

(b) When Sophia is $s$ years old,
Anne’s age = $(s + 28)$ years

Paul’s pocket money is £8 less than Lucy’s pocket money.
Find Paul’s pocket money if Lucy’s is (a) £20, (b) £$p$.

Try It!

(a) The mass of a dog is three times the mass of a cat. Find the mass of the dog if the mass of the cat is (a) 4 kg, (b) $m$ kg.

Solution

From the given information,
the mass of the dog = 3 times the mass of the cat.
You can represent this using a bar model.

(a) If the mass of the cat = 4 kg,
the mass of the dog = $3 \times 4$
= 12 kg

(b) If the mass of the cat = $m$ kg,
the mass of the dog = $3 \times m$
= 3$m$ kg

Note: $3 \times m$ or $m \times 3$ is written as $3m$, where the multiplication sign is omitted and the number is placed in front of the variable.

The time Henry takes to complete a quiz is twice the time Rose takes. Find the time Henry takes to complete the quiz if the time Rose takes is (a) 20 minutes, (b) $t$ minutes.

Try It!

(a) The length of a nail is one-quarter the length of a screw. Find the length of the nail if the length of the screw is (a) 52 mm, (b) $l$ mm.

Solution

From the given information,
the length of the nail = $\frac{1}{4} \times$ the length of the screw
You can represent this using a bar model.

(a) When the length of the screw is 28 mm, the length of the nail is \( \frac{1}{4} \times 28 = 7 \text{ mm} \)

(b) When the length of the screw is \( L \) mm, the length of the nail is \( \frac{1}{4} \times L \) mm

Note: \( \frac{L}{4} = L + 4 \). In algebra, you usually express \( L + 4 \) as \( \frac{L}{4} \).

Erica takes half the time that Harry takes to finish a mathematics assignment. Find the time Erica takes if Harry takes (a) 50 minutes, (b) \( 7 \) hours.

**Example 4**

For a school play, the price of an adult ticket is £3 and the price of a child ticket is £1. Find the total price of

(a) two adult tickets and four child tickets,
(b) \( x \) adult tickets and \( y \) child tickets in terms of \( x \) and \( y \).

**Solution**

(a) Price of two adult tickets = £3 \times 2
Price of four child tickets = £1 \times 4
∴ Total price = £(3 \times 2 + 1 \times 4)
= £10

(b) Price of \( x \) adult tickets = £3 \times x
Price of \( y \) child tickets = £1 \times y
∴ Total price = £(3 \times x + 1 \times y)
= £(3x + y)

Note: • 3x + y is an algebraic expression in two variables \( x \) and \( y \).
  • Find in terms of \( x \) and \( y \) means finding an expression using \( x \) and \( y \) for the answer. It is different from finding a value for the answer.
    • 1 \times y or \( y \times 1 \) is written as \( y \), NOT \( 1y \).
    • 3 \times 2 can be written as \( 3(2) \).

The price for a chocolate bar is £1 and the price for a pizza is £5. Find the total price for

(a) three chocolate bars and two pizzas,
(b) \( m \) chocolate bars and \( n \) pizzas in terms of \( m \) and \( n \).

The table summarises algebraic expressions for the basic operations. You can compare them with the arithmetic expressions in the Flashback.

<table>
<thead>
<tr>
<th>Operation</th>
<th>Statement</th>
<th>Algebraic Expression</th>
<th>Bar Model</th>
</tr>
</thead>
</table>
| Addition  | • add \( b \) to \( a \)  
  • sum of \( a \) and \( b \)  
  • total of \( a \) and \( b \)  
  • \( a \) plus \( b \) | \( a + b \) | \( a + b \) |
|           | (b) Note: | Assume \( a \) and \( b \) are positive. |
| Subtraction | • subtract \( c \) from \( d \)  
  • take away \( c \) from \( d \)  
  • \( d \) minus \( c \) | \( d - c \)  
  \( \# c - d \)  
  \( \text{(in general)} \) | \( d - c \)  
  \( \# c - d \)  
  \( \text{(in general)} \) |
|           | (b) Note: | Assume \( c \) and \( d \) are positive. |
| Multiplication | • multiply \( p \) by \( q \)  
  • product of \( p \) and \( q \)  
  • \( p \) times \( q \) | \( pq \) | \( pq \) |
| Division  | • divide \( x \) by \( y \)  
  • Quotient of \( x \) divided by \( y \)  
  • \( x \) over \( y \) Assume \( y \) \( \neq 0 \). | \( \frac{x}{y} \)  
  \( \# \frac{x}{y} \)  
  \( \text{(in general)} \) | \( \frac{x}{y} \)  
  \( \# \frac{x}{y} \)  
  \( \text{(in general)} \) |
|           | (b) Note: | Assume \( x \) is positive and \( y \) \( \neq 0 \). |

**Example 5** Write these statements as algebraic expressions.

(a) Add \( 6n \) to \( n \).
(b) Subtract \( 24k \) from \( 5d \).
(c) Multiply \( 3a \) and \( b \).
(d) Divide \( (2x - 5y) \) by \( 7 \).

**Solution**

(a) Add \( 6n \) to \( n \): \( n + 6n \)
(b) Subtract \( 24k \) from \( 5d \): \( 5d - 24k \)
(c) Multiply \( 3a \) and \( b \): \( 3a \times b \)
(d) Divide \( (2x - 5y) \) by \( 7 \): \( \frac{2x - 5y}{7} \)

\( \therefore \) you need the brackets for \( (2x - 5y) \). However, \( 2x - 5y = (2x - 5y) \), so you do not need the brackets for the answer.
In arithmetic, you write \(7 \times 7\) as \(7^2\) and \(5 \times 5\) as \(5^2\). In algebra, the notation is similar.
You read \(a \times a = a^2\) as \(a\) squared, and
\[
\begin{align*}
    b \times b \times b &= b^3, \\
    c \times c \times c \times c &= c^4, \\
    d \times d \times d \times d \times d &= d^5.
\end{align*}
\]
In general, \(a \times a \times a \times \ldots \times a = a^n\), where \(a\) is called the base and \(n\) is called the index.

\(a\) multiplied by itself \(n\) times.

You should note that:
\[a^2 = a \times a \quad \text{and} \quad 2a = a + a.
\]
So \(a^2\) and \(2a\) are different.

Similarly,
\[b^3 = b \times b \times b \quad \text{and} \quad 3b = b + b + b.
\]
So \(b^3\) and \(3b\) are different.

In general, \(a^2\) and \(na\) are not different.

\[\text{Example 6}\]
Simplify these expressions.
(a) \(4 \times a \times 5 \times b\)
(b) \(3 \times t \times t\)
(c) \(u \times 2 \times u \times u\)

\[\text{Solution}\]
(a) To ‘simplify’ an expression means to rewrite it as simply as possible.
\[
4 \times a \times 5 \times b = 4 \times 5 \times a \times b = 20 \times ab = 20ab
\]
Rearrange and write the numbers first. Write letters in alphabetical order.

(b) \(3 \times t \times t = 3 \times t^2\)
(c) \(u \times 2 \times u \times u \times u = 2 \times u \times u \times u = 2 \times u^3\)

\[\text{Example 7}\]
Determine whether \((3a)^2\) and \(3a^2\) are equal.

\[\text{Solution}\]
\[
\begin{align*}
(3a)^2 &= 3a \times 3a \\
&= 3 \times 3 \times a \times a \\
&= 9a^2 \\
3a^2 &= 3 \times a \times a \\
\therefore \quad (3a)^2 &\neq 3a^2
\end{align*}
\]

\[\text{Remark}\]
\[
(3 \times 2)^2 = 6^2 = 36 \\
3 \times 2^2 = 3 \times 4 = 12
\]

Note: The square of a multiplication expression in brackets means every letter or number within the brackets is squared. For example, \((7x)^2 = 7^2 \times x^2 = 49x^2\).
On the other hand, in the expression \(7x^2\), only \(x\) is squared, and so it means \(7\) times \(x\) squared.
In general, \((ax)^2 \neq ax^2\).

\[\text{Try it! 5}\]
Write these statements as algebraic expressions.
(a) Add \(y\) to \(3x\).
(b) Subtract \(7p\) from \(3q\).
(c) Multiply \(4m\) and \(n\).
(d) Divide \((4a + 3b)\) by \(9\).

\[\text{Example 8}\]
In general, \(a \times a \times a \times \ldots \times a = a^n\), where \(a\) is called the base and \(n\) is called the index.
\(a\) multiplied by itself \(n\) times.

You should note that:
\[a^2 = a \times a \quad \text{and} \quad 2a = a + a.
\]
So \(a^2\) and \(2a\) are different.

Similarly,
\[b^3 = b \times b \times b \quad \text{and} \quad 3b = b + b + b.
\]
So \(b^3\) and \(3b\) are different.

In general, \(a^2\) and \(na\) are not different.

\[\text{Example 6}\]
Simplify these expressions.
(a) \(4 \times a \times 5 \times b\)
(b) \(3 \times t \times t\)
(c) \(u \times 2 \times u \times u\)

\[\text{Solution}\]
(a) To ‘simplify’ an expression means to rewrite it as simply as possible.
\[
4 \times a \times 5 \times b = 4 \times 5 \times a \times b = 20 \times ab = 20ab
\]
Rearrange and write the numbers first. Write letters in alphabetical order.

(b) \(3 \times t \times t = 3 \times t^2\)
(c) \(u \times 2 \times u \times u \times u = 2 \times u \times u \times u = 2 \times u^3\)

\[\text{Example 7}\]
Determine whether \((3a)^2\) and \(3a^2\) are equal.

\[\text{Solution}\]
\[
\begin{align*}
(3a)^2 &= 3a \times 3a \\
&= 3 \times 3 \times a \times a \\
&= 9a^2 \\
3a^2 &= 3 \times a \times a \\
\therefore \quad (3a)^2 &\neq 3a^2
\end{align*}
\]

\[\text{Remark}\]
\[
(3 \times 2)^2 = 6^2 = 36 \\
3 \times 2^2 = 3 \times 4 = 12
\]

Note: The square of a multiplication expression in brackets means every letter or number within the brackets is squared. For example, \((7x)^2 = 7^2 \times x^2 = 49x^2\).
On the other hand, in the expression \(7x^2\), only \(x\) is squared, and so it means \(7\) times \(x\) squared.
In general, \((ax)^2 \neq ax^2\).

\[\text{Try it! 7}\]
Determine whether \(5y^2\) and \((5y)^2\) are equal.
13. A history book is 2 cm thick and a science book is 3 cm thick. If \( n \) history books and \( n \) science books are placed in a stack, find, in terms of \( m \) and \( n \),
(a) the number of books in the stack,
(b) the height of the stack.

14. There is one £5 note, \( a \) £10 notes and \( b \) £50 notes in a wallet. Express, in terms of \( a \) and \( b \),
(a) the number of bank notes in the wallet,
(b) the total value of the bank notes in the wallet.

15. A boy is \( N \) years old. His father is four times as old as him. His mother is three years younger than his father. Express, in terms of \( N \), the age of
(a) his father,
(b) his mother.

Hint: You may draw a bar model to help.

16. Alice got \( m \) marks in a test. Bob's score was 15 more than Alice's score. Carol's score was half of Bob's score. Express, in terms of \( m \),
(a) Bob's score,
(b) Carol's score.

17. Describe a real-life situation that could be represented by the expression \( 2x + 3y \).

### 3.2 Substituting Numbers for Letters

#### Evaluation of Algebraic Expressions

As mentioned in Section 3.1, if there are \( n \) butterflies,
the total number of wings = \( 4n \).

If there are 18 butterflies, you can calculate the total number of wings by putting \( n = 18 \)
into the above expression.

Thus, the total number of wings = \( 4 \times 18 = 72 \).

If there are 37 butterflies, then \( n = 37 \) and
the total number of wings = \( 4 \times 37 = 148 \).

This process of replacing each variable with a number to calculate the value of an algebraic expression is called **substitution**.

**Example 8**

When \( m = 5 \), find the value of
(a) \( 3m \),
(b) \( 7 + 2m \).

**Solution**

(a) \( 3m = 3 \times 5 \quad \text{Substitute } m \text{ by } 5 \)

\[ = 15 \]

(b) \( 7 + 2m = 7 + 2 \times 5 \quad \text{Substitute } m \text{ by } 5 \)

\[ = 17 \]

**Try It!**

When \( p = 6 \), find the value of
(a) \( 5p \),
(b) \( 20 - 3p \).

**Example 9**

The charge for a service by a plumber is \( £(40 + 30t) \), where \( t \) is the number of hours taken to complete the job. Find the charge for a two-hour job.

**Solution**

\[ \text{Charge} = £(40 + 30t) \]

When \( t = 2 \),

\[ \text{charge} = £(40 + 30 \times 2) = £(40 + 60) = £100 \]

**Try It!**

The height of a candle is \( (21 - 4t) \) cm after burning for \( t \) hours. Find the height of the candle after burning for three hours.

**Example 10**

When \( p = 2 \) and \( q = -3 \), work out the values of these expressions.
(a) \( 10pq \)
(b) \( 5p - 2q \)

**Solution**

(a) \( 10pq = 10 \times 2 \times (-3) \quad \text{Substitute } p = 2 \text{ and } q = -3 \)

\[ = 20 \times (-3) = -60 \]

(b) \( 5p - 2q = 5 	imes 2 - 2 \times (-3) \)

\[ = 10 + 6 = 16 \]

**Try It!**

When \( m = 3 \) and \( n = -2 \), work out the values of these expressions.
(a) \( 12mn \)
(b) \( 7m - 2n \)

**Example 11**

The total mass of \( m \) forks and \( n \) teaspoons is \( (65m + 30n) \) grams. Find the total mass of four forks and five teaspoons.

**Solution**

Total mass = \( (65m + 30n) \) grams

When \( m = 4 \) and \( n = 5 \),

\[ \text{total mass} = 65 \times 4 + 30 \times 5 \]

\[ = 260 + 150 = 410 \text{ grams} \]

**Try It!**

The total length of \( x \) red rods and \( y \) blue rods in a model kit is \((8x + 15y)\) cm. Find the total length of six red rods and three blue rods.
**Chapter 3 Introduction to Algebra**

**Example 12**

Given the formula $L = 2r + 7$, find the value of $L$ when

(a) $r = 4$,
(b) $r = -6$.

**Solution**

(a) $L = 2r + 7$

When $r = 4,

\[ L = 2 \times 4 + 7 \]

\[ = 8 + 7 \]

\[ = 15 \]

(b) When $r = -6$,

\[ L = 2 \times (-6) + 7 \]

\[ = -12 + 7 \]

\[ = -5 \]

**Try It! 12**

Given the formula $S = 8 + 3p$, find the value of $S$ when

(a) $p = 9$,
(b) $p = -4$.

**Example 13**

Given the formula $T = 2m^2 + 7$, find the value of $T$ when $m = 4$.

**Solution**

\[ T = 2m^2 + 7 \]

When $m = 4$,

\[ T = 2 \times 4^2 + 7 \]

\[ = 2 \times 64 + 7 \]

\[ = 128 + 7 \]

\[ = 135 \]

**Try It! 13**

Given the formula $w = 8 + 3t^2$, find the value of $w$ when $t = 2$.

**Example 14**

For a rectangle of length $L$ cm and width $W$ cm, its perimeter $P$ cm is given by the formula $P = 2L + 2W$. Using the formula, find the perimeter of a rectangle of length 36 cm and width 25 cm.

**Class Activity 1**

**Objective:** To compare the algebraic expressions $3 + n$, $3n$, $(3n)^2$, $3n^2$, $9n^2$, $(5n)^3$ and $5n^3$.

**Task**

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>Algebraic Expressions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
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<td>n</td>
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<td>$9n^2$</td>
<td>$(5n)^3$</td>
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<td>$27$</td>
<td>$81$</td>
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<td>$81$</td>
<td>$27$</td>
<td>$81$</td>
<td>$3375$</td>
</tr>
</tbody>
</table>

1. In a spreadsheet, enter the headings (row 1 and row 3) and the first column as shown.
2. Generate the values of other columns using formulae in the spreadsheet. You may use the copy and paste buttons to copy the formulae from one row to the other rows.

**Note:** In a spreadsheet, * stands for multiplication and ^ stands for the index.

**Questions**

1. Look at the values under the columns $3 + n$ and $3n$. Can you say that $3 + n = 3n$? Explain your answer.
2. Compare the columns of $(3n)^2$, $3n^2$ and $9n^2$. What are the relationships between these three expressions? Explain your answer.
3. Compare the columns of $(5n)^3$ and $5n^3$. What is the relationship between these two expressions? Explain your answer.
Chapter 3 Introduction to Algebra

3. Introduction to Algebra

3.1 Algebraic Expressions

3.2 Writing Algebraic Expressions and Formulae

**Exercise 3.2**

**Level 1**

1. Find the value of $2x + 1$ when
   \( a \) $x = 7$, \( b \) $x = -1$.

2. Find the value of $9 - y$ when
   \( a \) $y = 5$, \( b \) $y = -4$.

3. Find the value of $\frac{x}{6}$ when
   \( a \) $z = 2$, \( b \) $z = -18$.

4. Find the value of $3b + c$ when
   \( a \) $b = 2$ and $c = 5$, \( b \) $b = -1$ and $c = -4$.

5. Find the value of $8 - 2b + 3k$ when
   \( a \) $k = 7$ and $k = 5$, \( b \) $k = -2$ and $k = 4$.

6. Given $y = 1 - 3x$, find the value of $y$ when
   \( a \) $x = 4$, \( b \) $x = 0$.

7. Given that $x = \frac{5y}{4}$, find the value of $x$ when
   \( a \) $t = 24$, \( b \) $t = -8$.

8. Given that $w = 2u - 3v$, find the value of $w$ when
   \( a \) $u = 5$ and $v = 4$, \( b \) $u = -4$ and $v = -1$.

9. Given that $A = bh$, find the value of $A$ when
   \( a \) $b = 4$ and $h = 3$, \( b \) $b = 7$ and $h = -3$.

**Level 2**

10. Find the value of $3x - 24$ when
    \( a \) $x = 0$, \( b \) $x = -2$.

11. Given the formula $m = \frac{x + b}{2}$, find the value of $m$ when
    \( a \) $a = 4$ and $b = 18$, \( b \) $a = -3$ and $b = 7$.

12. Given the formula $y = 1 - 2x^2$, find the value of $y$ when
    \( a \) $x = 0$, \( b \) $x = 3$.

13. Given the formula $P = 30 \times (1 + rt)$, find the value of $P$ when
    \( a \) $r = 2$ and $t = 1$, \( b \) $r = -3$ and $t = -4$.

14. Given the formula $S = 6 + n^2$, find the value of $S$ when
    $n = 5$.

15. Given the formula $T = \frac{n^2}{4}$, find the value of $T$ when
    $q = 2$.

**Level 3**

16. Erica is a part-time clerk. Her weekly wage is £46h, where $h$ is the number of hours she works in a week.
    \( a \) If Erica works 30 hours in a week, find her weekly wage.
    \( b \) What do you think the number 14 in the expression $14h$ stands for?

17. The total price for $m$ burgers and $n$ cups of coffee is
    \( 5m + 2n \).
    \( a \) Find the total price for five burgers and four cups of coffee.
    \( b \) What do you think the numbers 3 and 2 in the expression $3m + 2n$ stand for?

18. The conversion of $x$°C (Celsius) to $y$°F (Fahrenheit) is given by the formula $y = \frac{9}{5}x + 32$.
    \( a \) The freezing point of water is 0°C. Work out the freezing point in Fahrenheit.
    \( b \) The boiling point of water is 100°C. Work out the boiling point in Fahrenheit.

19. The cost, $L$, of making a square glass frame of side length $x$ cm is given by the formula
    \( C = 12x + 30x^2$.
    How much more is the cost of making a square glass frame of side length 3 m than that of side length 2 m?

20. The mass, $M$, grams, of a glass cube of side length $x$ cm is given by the formula $M = \frac{51}{2}$. The mass, $N$, grams, of an iron pyrite cube of side length $ycm$ is given by the formula $N = 54$. Find the total mass of two glass cubes of side length 4 cm and seven iron pyrite cubes of side length 2 cm.

21. Create a formula connecting $x$ and $y$ such that $y = 5$ when $x = 3$.

22. Create a formula connecting $s$ and $t$ such that
    \( a \) $s = 64$ when $t = 2$.
    \( b \) $s = 61$ when $t = 2$.
    Hint: You may use indices.

**3.3 Writing Algebraic Expressions and Formulae**

Algebraic expressions and formulae can be used to express the relationships between two or more quantities in your daily life. Consider some examples.

**Example 15**

In a 100m run, Charles was two seconds faster than Eric. Find the time taken by each.

\( a \) Let $x$ seconds be the time taken by Eric. Express the time taken by Charles in terms of $x$.
\( b \) Let $y$ seconds be the time taken by Charles. Write a formula connecting $x$ and $y$.

**Solution**

\( a \) Charles’s time = Eric’s time – 2 seconds
\( b \) From (a), you have the formula
\( y = x - 2$.

**Try It!**

In a shot-put event, Ada’s putting distance was 3 metres longer than Janet’s.
\( a \) Let $x$ metres be Janet’s putting distance. Express Ada’s putting distance in terms of $x$.
\( b \) Let $y$ metres be Ada’s putting distance. Write a formula connecting $x$ and $y$.

**Remark**

The bar model for this situation is:

- Ada's putting distance $= x + 3$ metres
- Janet's putting distance $= x$ metres
Example 16

For a concert, the price of a child ticket is half the price of an adult ticket.
(a) Let \( £a \) be the price of an adult ticket. Express the price of a child ticket in terms of \( a \).
(b) Let \( £c \) be the price of a child ticket. Write a formula connecting \( a \) and \( c \).

Solution

(a) Price of a child ticket = \( \frac{1}{2} \times \) price of an adult ticket
= \( £ \left( \frac{1}{2} \times a \right) \)
= \( £ \frac{a}{2} \)

(b) From (a), the formula is
\[ c = \frac{a}{2} \]

The capacity of a cup is one-fifth the capacity of a bowl.
(a) Let \( b \) ml be the capacity of the bowl. Express the capacity of the cup in terms of \( b \).
(b) Let \( c \) ml be the capacity of the cup. Write a formula connecting \( b \) and \( c \).

Example 17

There are \( m \) birds and \( n \) rabbits in a pet shop. Let \( T \) be the total number of their legs.
(a) Write a formula connecting \( m \), \( n \) and \( T \).
(b) If there are 10 birds and six rabbits in the shop, find the total number of legs.

Solution

(a) Each bird has two legs.
Number of legs of \( m \) birds = \( 2m \).
Each rabbit has four legs.
Number of legs of \( n \) rabbits = \( 4n \).
Total number of legs of these animals = \( 2m + 4n \).
\[ T = 2m + 4n \]

(b) When \( m = 10 \) and \( n = 6 \),
\[ T = 2 \times 10 + 4 \times 6 = 44 \]
the total number of legs is 44.

There are \( p \) bicycles and \( q \) tricycles in a shop. Let \( W \) be the total number of their wheels.
(a) Write a formula connecting \( p \), \( q \) and \( W \).
(b) If there are five bicycles and seven tricycles, find the total number of wheels.

EXERCISE 3.3

LEVEL 1

1. The mass of Jim is \( p \) kg and the mass of Lily is \( q \) kg.
   (a) Express \( x \) in the bar model in terms of \( p \) and \( q \).
   (b) What does \( x \) represent?

2. Tom’s long jump distance is \( x \) metres. Shaun’s long jump distance is \( y \) metres.
   (a) Express \( z \) in the bar model in terms of \( x \) and \( y \).
   (b) What does \( z \) represent?

3. Richard is \( m \) years old and Susan is \( n \) years old. The bar model represents their ages.
   (a) Express \( n \) in terms of \( m \).
   (b) What can you say about the relationship between their ages?

4. Tom’s age is four times the age of his son. If the age of his son is \( y \) years, express Tom’s age in terms of \( x \).

5. Mandy spends one-quarter of her weekly salary on a pair of sports shoes. If her weekly salary is \( w \)£, express the price of the pair of sports shoes in terms of \( w \).

6. The initial temperature of a cup of water is \( x \)°C. The temperature drops 23°C after an hour and reaches \( y \)°C. Write down a formula connecting \( x \) and \( y \).

LEVEL 2

7. The thickness of a dictionary is twice the thickness of another book. Let \( d \) cm and \( b \) cm be the thicknesses of the dictionary and the book respectively. Express \( d \) in terms of \( b \).

8. The height of a London Eye souvenir is one-thousandth of the actual height of the London Eye. Let \( h \) cm and \( H \) cm be the heights of the souvenir and the London Eye respectively. Express \( h \) in terms of \( H \).

9. The length of a folk song is \( x \) minutes. The length of a pop song is \( y \) minutes. The length of the pop song is half the length of the folk song.
   (a) Express \( y \) in terms of \( x \).
   (b) If the length of the folk song is 4 minutes, find the length of the pop song.

10. The area of Paul’s flat is \( 200 \) m² and the area of Queenie’s flat is \( q \) m². Paul’s flat is 15 m² more than Queenie’s flat.
   (a) Express \( q \) in terms of \( p \).
   (b) If Queenie’s flat is 90 m², find the area of Paul’s flat.

11. Mrs Atkins gives a money box with £9 in it to her son. She asks her son to put £2 into the money box each week.
   (a) Express the amount in the money box after \( n \) weeks in terms of \( n \).
   (b) Find the amount in the money box after 10 weeks.

12. In an exam, Emily’s science mark is 17 higher than her mathematics mark. Let \( x \) be her mark in mathematics.
   (a) Express Emily’s science mark in terms of \( x \).
   (b) Let \( T \) be the sum of the science and mathematics marks. Write a formula connecting \( x \) and \( T \).
   (c) If Emily gets 63 marks in mathematics, find the sum of the science and mathematics marks.
13. There are two consecutive even numbers. Let \( n \) be the smaller number.
   (a) Express the greater number in terms of \( n \).
   (b) Let \( P \) be the product of these two numbers. Write a formula connecting \( n \) and \( P \).
   (c) Find the product of the two numbers when \( n = 6 \).
   (d) What are the two original consecutive even numbers in (e)? Check whether their product is equal to the answer you gave to (e).

14. Frank has two part-time jobs. The weekly salary of the second job is three times the weekly salary of the first job. Let \( LW \) be the weekly salary of the first job. Let \( LW' \) be the weekly salary of Frank's part-time jobs. Write a formula connecting \( m \) and \( n \).
   (a) Express the weekly salary of the second job in terms of \( m \).
   (b) Let \( LT \) be the total weekly salary of Frank's part-time jobs. Write a formula connecting \( m \) and \( T \).
   (c) If the weekly salary of the first job is \( £30 \), find the total weekly salary of Frank's part-time jobs.

3.4 Like Terms and Unlike Terms

In an algebraic expression, the parts separated by plus or minus signs are called the terms of the expression. For example, there are three terms in the expression \( 2x - 3y + 8 \), and they are \( 2x, -3y \) and \( 8 \).

The number part, including the sign, of a term is called the coefficient of its variable part. In the example on the right, \( 2 \) is the coefficient of \( x \) and \( -3 \) is the coefficient of \( y \). The term \( 8 \) has no variable, so it is called a constant term.

Given the expression \( 4p - 7 + 3q - pq \), state

(a) the number of terms in the expression,
(b) the coefficient of \( p \),
(c) the coefficient of \( pq \),
(d) the constant term.

**Solution**

(a) The terms are \( 4p, -7, 3q \) and \( -pq \).
   ∴ there are four terms.

(b) Coefficient of \( p = 4 \)

(c) \( -pq = (-1)pq \)
   ∴ coefficient of \( pq = -1 \)

(d) Constant term = \(-7\)

15. Carbon and hydrogen form a compound. The number of hydrogen atoms in the compound is twice the number of carbon atoms.
   (a) If there are \( n \) carbon atoms, express, in terms of \( n \),
      (i) the number of hydrogen atoms,
      (ii) the total number of atoms in the compound.
   (b) If the compound has three carbon atoms, find the total number of atoms in the compound.

Note: Propene, whose molecular formula is \( C_3H_6 \), is an example of this type of compound.

16. Create a situation that forms a formula connecting variables \( M, x \) and \( y \) such that \( M = 52 \) when \( x = 4 \) and \( y = 7 \).

**Try It!**

Given the expression \( -9mn + n - 6 \), state

(a) the number of terms in the expression,
(b) the coefficient of \( n \),
(c) the coefficient of \( mn \),
(d) the constant term.

When two terms have identical variable parts, i.e. they consist of the same variable(s) having the same indices, they are called like terms.

For example, \( 3xy \) and \( -6xy \) are like terms as they have identical variable parts, \( xy \). All constant terms are like terms. When two terms are not like terms, they are called unlike terms.

**Like terms**

\[
\begin{array}{|c|c|}
\hline
\text{Like terms} & \text{Unlike terms} \\
\hline
3x, 7a & 4a, 8b \\
9x^2, -5x^2 & 2y, y^2 \\
-3y^4, -5y^4 & -pq^2, -6pq^2 \\
11, -2 & 15m, 15 \\
\hline
\end{array}
\]

You can simplify an algebraic expression by a process of simplification called **collecting like terms**.

**Class Activity 2**

**Objective:** To apply the process of collecting like terms using algebra discs.

As well as number discs, you have \( x \)-discs and \( y \)-discs.

Each disc has two sides. You can flip a disc to show either the front or the back. Just like \( 1 \) and \( -1 \), \( x \) and \( -x \) and \( y \) and \( -y \) form **zero pairs**. That is, \( 1 + (-1) = 0 \), \( x + (-x) = 0 \) and \( y + (-y) = 0 \). Examples of zero pairs are:

(a) \( 2x + (-2x) = 2x - 2x = 0 \)
(b) \( (-3y) + 3y = -3y + 3y = 0 \)

Some algebraic expressions can be represented by a set of algebra discs. For example,

(a) \( 2x + 3: \)

This set of algebra discs can also represent \( 3 + 2x \).
Chapter 3 Introduction to Algebra

1. Terms involving $x$ and $y$ can be simplified by collecting like terms as illustrated below.

\[
\begin{align*}
2x + 3y - 5x + y & \quad \text{Collect the like terms.} \\
2x + 3y - 5x + y & \quad \text{ } = 2x + 5y + 3y - 5x + y \\
& \quad \text{ } = -3x + 4y
\end{align*}
\]

2. Simplify these expressions. Use algebra discs to help you.

\[
\begin{align*}
(a) \quad 2x + 5x & \\
(b) \quad 2x - 5x & \\
(c) \quad -2x + 5x & \\
(d) \quad -2x - 5x & \\
(e) \quad 3y + 1 + 4y + 3 & \\
(f) \quad 3y + 1 - 4y - 3 & \\
(g) \quad -3y - 1 + 4y + 3 & \\
(h) \quad -3y - 1 - 4y - 3 & \\
(i) \quad 4x - 5y + x + 2y - 1 & \\
(j) \quad -3x + 2y - 4 - x + y + 1 &
\end{align*}
\]

Questions

1. Explain how you would simplify $ax + bx$, where $a$ and $b$ are given integers. You may substitute $a$ and $b$ with different integers to help you illustrate your explanation.

2. Can you write $3x + 4y$ as $7xy$? You may substitute $x$ and $y$ with numbers to help you illustrate your explanation.

Example 19

Simplify these expressions.

\[
\begin{align*}
(a) \quad 7x - 3x + x & \\
(b) \quad -5y + 4y - 6y &
\end{align*}
\]

Solution

\[
\begin{align*}
(a) \quad 7x - 3x + x & = (7 - 3 + 1)x \\
& = 4x + 1x \\
& = 5x \\
(b) \quad -5y + 4y - 6y & = (-5 + 4 - 6)y \\
& = (-1 - 6)y \\
& = -7y
\end{align*}
\]

Try It! 19

Simplify these expressions.

\[
\begin{align*}
(a) \quad 5x + 6t - 7t & \\
(b) \quad -4z - 3z + 5z &
\end{align*}
\]

Example 20

Simplify these expressions.

\[
\begin{align*}
(a) \quad 3a + 4b - 2a + 5b & \\
(b) \quad -3w + 8x - 1 + w - 5x - 2 &
\end{align*}
\]

Solution

\[
\begin{align*}
(a) \quad 3a + 4b - 2a + 5b & \quad \text{Arrange like terms together.} \\
& = 3a - 2a + 4b + 5b \\
& = (3 - 2)a + (4 + 5)b \\
& = a + 9b \\
& \quad \text{This cannot be further simplified.} \\
(b) \quad -3w + 8x - 1 + w - 5x - 2 & \quad \text{All terms are now unlike terms.} \\
& = -3w + w + 8x - 5x - 1 - 2 \\
& = (-3 + 1)w + (8 - 5)x - 3 \\
& = -2w + 3x - 3
\end{align*}
\]

Try It! 20

Simplify these expressions.

\[
\begin{align*}
(a) \quad 5c - 4d - 3c - d & \\
(b) \quad 2t - 7x + 3 + t - 2x - 1 &
\end{align*}
\]

Example 21

The total monthly salary of the employees at a small firm is \(5(5x + 3y + 4x + y)\).

\[
\begin{align*}
(a) \quad \text{Simplify the expression } 5x + 3y + 4x + y & \\
(b) \quad \text{Find their total monthly salary if } x = 3000 \text{ and } y = 3500.
\end{align*}
\]
Solution
(a) $5x + 3y + 4x + y = 5x + 4x + 3y + y = 9x + 4y$

(b) When $x = 3000$ and $y = 3500$,

\[
\text{total monthly salary} = \£(9(3000) + 4(3500)) = \£41,000
\]

The total value of four stacks of coins is $\£(10m + 4n + 7m + 5n)$.

(a) Simplify the expression $10m + 4n + 7m + 5n$.
(b) Find the total value if $m = 1$ and $n = 2$.

**Try It!**

**EXERCISE 3.4**

**Solution**

1. (a) $−7x − 3y − 2x + 3y$
   (b) $3p − 4q + 6p − q$
   (c) $7x + 4a − 5d + av$

2. (a) Simplify the expression $−4 − 2x + 5 + x$.
   (b) Find the value of the expression when $x = 2$.

3. Simplify these expressions.
   (a) $7a + 2a$
   (b) $5b − 8b$
   (c) $−4x + 6x$
   (d) $−2y − 3y$
   (e) $c + x + c$
   (f) $d + 2d − 9d$
   (g) $−3p + p + 4p$
   (h) $4y − 9y + 5y$
   (i) $−4m − 2m + 5n − m$

4. Simplify these expressions.
   (a) $3n + 10 − 4n − 11$
   (b) $−6 − 3a − 4x + 7$
   (c) $3m + 4a − 2n + 5m$

5. (a) Simplify the expression $7a − 2b + 5b − a − 3$.
   (b) Find the value of the expression when $a = −1$ and $b = 2$.

6. (a) Simplify the expression $3x − 2y − x + 1 − x$.
   (b) Find the value of the expression when $x = −2$ and $y = −6$.

7. The total number of atoms in the compound $C_2H_{11}$ is $n + 2a + 2$.
   (a) Simplify $n + 2a + 2$.
   (b) Find the total number of atoms if $n = 10$.

8. The height of a stack of 10 books is given by $(2x + 3y + 4x + y)$ cm.
   (a) Simplify the expression $2x + 3y + 4x + y$.
   (b) Find the height of the stack if $x = 2$ and $y = 3$.

9. (a) Express the perimeter of the rectangle in terms of $p$.
   (b) When $p = 12$, find the perimeter of the rectangle.

10. A woman works 2 hours each day from Monday to Friday. She works $(2t − 3y)$ hours on Saturdays. She does not work on Sundays.
   (a) Express her working hours in a week in terms of $t$ and $y$.
   (b) How many hours does she work each week when $t = 4$ and $y = 3$?

11. Write an algebraic expression that has three terms involving the variables $p$ and $q$.

12. Create a problem with an answer that can be simplified to $7x$.

**LEVEL 3**

**Objective:** To perform the addition and subtraction of linear expressions using algebra discs.

**Tasks**

1. To obtain the negative of an expression, you flip the discs that represent the expression.
   For example, simplify $−(3x − 2)$.

   \[−(3x − 2) = −3x + 2\]

2. The $−$ sign means flipping over all 8 discs to get 8 $+$ discs.

   \[−6 + 8\]

   The $-$ sign means flipping over all 8 discs to get 8 $+$ discs.

   \[−(−6) = 6\]

**REMARK**

The expressions $x^2 − 3$ and $6 − 5x$ are not linear expressions. This is because the term $x^2$ involves $x$ to the index 2 and the term $−5x$ involves two variables which are $x$ and $y$.

**Class Activity 3**

**RECALL**

$−(−6) = 6$.

The $−$ sign means flipping over all 8 discs to get 8 $+$ discs.

$−6 + 8$.

The $−$ sign means flipping over all 8 discs to get 8 $+$ discs.
2. To add two expressions, you remove the brackets and collect the like terms. For example, simplify $(2x - 3y) + (-3x + y)$.

\[
(2x - 3y) + (-3x + y) = 2x - 3y - 3x + y = -x - 2y
\]

3. Subtracting an expression is the same as adding the negative of that expression. For example, simplify $(2x - 3y) - (-3x + y)$.

\[
(2x - 3y) - (-3x + y) = 2x - 3y + 3x - y = 5x - 4y
\]

### Example 22

Simplify $(2a + 3b) + (5a - 4b)$.

\[
(2a + 3b) + (5a - 4b) = 2a + 3b + 5a - 4b = 7a - b
\]

### Try It! 22

Find the sum of $-2p + 3q - 4$ and $p + 5q - 3$.

\[
(-2p + 3q - 4) + (p + 5q - 3) = -2p + 3q + p + 5q - 4 - 3 = -p + 8q - 7
\]

### Example 23

Simplify $(4a + b) + (3a - 6b)$.

\[
(4a + b) + (3a - 6b) = 4a + b + 3a - 6b = 7a - 5b
\]

### Try It! 23

Find the sum of $5p - 4q + 7$ and $-3p - q + 2$.

\[
(5p - 4q + 7) + (-3p - q + 2) = 5p - 4q + 7 - 3p - q + 2 = 2p - 5q + 9
\]

### Example 24

Simplify $(4x - 5) - (2x - 3)$.

\[
(4x - 5) - (2x - 3) = 4x - 5 - 2x + 3 = 2x - 2
\]

### Try It! 24

Simplify $(7y - 2) - (4y - 9)$.

\[
(7y - 2) - (4y - 9) = 7y - 2 - 4y + 9 = 3y + 7
\]

### Example 25

There are two consecutive odd integers. If the smaller integer is $6n + 1$, find the sum of the two integers in terms of $n$.

\[
\text{Since the smaller integer is } 6n + 1, \text{ the larger integer is } (6n + 1) + 2 = 6n + 3
\]

\[
\text{Their sum } = (6n + 1) + (6n + 3) = 6n + 6n + 1 + 3 = 12n + 4
\]

### Try It! 25

There are two consecutive even integers. If the smaller integer is $2n$, find the sum of the two integers in terms of $n$. 

\[
\text{Their sum } = (2n + 1) + (2n + 2) = 2n + 2n + 1 + 2 = 4n + 3
\]
A car travels \((5d + 1)\) km due east, then \((3d – 2)\) km due west, and finally \((2d + 4)\) km due east. How far is the car to the east of its starting point?

**Analysis**

You may draw a diagram to illustrate the situation.

**Solution**

The required distance \(= (5d + 1) – (3d – 2) + (2d + 4)\)
\[= 5d + 1 – 3d + 2 + 2d + 4\]
\[= 5d – 3d + 2d + 1 + 2 + 4\]
\[= (4d + 7)\] km

A lift goes up \((7x + 3)\) metres, then goes down \((4x – 6)\) metres, and finally goes up \((2x – 1)\) metres. How high is the lift above its starting point?

---

**EXERCISE 3.5**

**LEVEL 1**

1. Simplify these expressions.
   (a) \(-2(2x + 1)\)  
   (b) \(-3x + 6\)  
   (c) \(-4y + 9\)  
   (d) \(-5x + 8y – 7\)

2. Simplify these expressions.
   (a) \(2a + 3 + (a – 4)\)  
   (b) \((-2b – 5) + (3b – 1)\)  
   (c) \((-4c + 2d) + (-3c + d)\)  
   (d) \((8m – 7n) + (-5m – 2n)\)

3. Simplify these expressions.
   (a) \(6p + 7 – (3p + q)\)  
   (b) \(-4 – (6x – 3)\)  
   (c) \((-2a – t) + (3a + t)\)  
   (d) \((-4v + 5y) – (3x – 6y)\)

4. Simplify these expressions.
   (a) \((2b – 3x + 6) + (8b – 5x – 2)\)  
   (b) \((-m – 8a + 1) + (-7m + 6a + 3)\)  
   (c) \((7x + 2y) + (4x – 6) – (-3 + 2y)\)  
   (d) \((2t – 3x) – (9 – 4z) + (2x + 3 – 5)\)

5. Add \(7x – 2y + 4z\) to \(-2x + 3y – 5z\).

6. Find the sum of \(5a – 3b, 7b – 3c\) and \(9c – a\).

7. Subtract \(a – 4b – 3c\) from \(a + 2b – 6c\).

8. Subtract \(t – 3v\) from the sum of \(7t – 2a – 3v\) and \(3t + 5u – 8v\).

**LEVEL 2**

9. There are three consecutive integers. If the smallest one is \(n\), find the sum of these three integers in terms of \(n\).  
   **Hint:** For consecutive numbers, the difference between one number and the next number is 1.

10. The masses of three boxes of chocolates are \((3p + 4q + 2)\) grams, \((4p + 6q + 5)\) grams and \((p + 7q + 9)\) grams.
    (a) Find their total mass in terms of \(p\) and \(q\).
    (b) If \(p = 10\) and \(q = 30\), find the total mass.

---

**CLASS ACTIVITY 4**

**Objective:** To rewrite expressions with brackets as expressions without brackets.

1. \((2\cdot 3x)\) can be considered as 2 groups of \(3x\).

2. \((-3\cdot 2)\) can be considered as \(-3\) groups of \(-2\).

---

**3.6 Expressions with Brackets**

There are some expressions that involve brackets such as \(3(2x + 1)\) and \(-4(5x – 4b)\). You can write them as a sum of terms by expanding the brackets.
Ungrouping all of the groups, you get $-6x$. That is

\[ 2(-3x) = 2 \text{ groups of } -3x = 2 \times (-3x) = -6x. \]

$-2(3x)$ can be considered as the negative of 2 groups of $3x$.

\[
\begin{align*}
\begin{array}{ccc}
\hline
\text{Group 1} & \text{Group 2} & \text{Result} \\
\hline
1 & 1 & 2 \\
1 & 1 & 2 \\
\end{array}
\end{align*}
\]

$-2(3x) = \text{negative of 2 groups of } 3x = 2 \times (-3x) = -6x$

$-2(-3x)$ can be considered as the negative of 2 groups of $-3x$.

\[
\begin{align*}
\begin{array}{ccc}
\hline
\text{Group 1} & \text{Group 2} & \text{Result} \\
\hline
1 & 1 & 2 \\
1 & 1 & 2 \\
\end{array}
\end{align*}
\]

$-2(-3x) = \text{negative of 2 groups of } -3x = 2 \times (-(-3x)) = 6x$

Expand the brackets in these expressions using algebra discs to help you.

(a) $3(2x)$

(b) $3(-2x)$

(c) $-4(5x)$

(d) $-4(-5x)$

2. The brackets in $2(3x + 2)$ and $-3(x - 3y)$ can be expanded in a similar way.

\[
\begin{align*}
\begin{array}{ccc}
\hline
\text{Group 1} & \text{Group 2} & \text{Result} \\
\hline
1 & 1 & 2 \\
1 & 1 & 2 \\
\end{array}
\end{align*}
\]

$2(3x + 2) = 2 \text{ groups of } (3x + 2) = 6x + 4$

\[
\begin{align*}
\begin{array}{ccc}
\hline
\text{Group 1} & \text{Group 2} & \text{Result} \\
\hline
1 & 1 & 2 \\
1 & 1 & 2 \\
\end{array}
\end{align*}
\]

$-3(x - 3y) = 3 \text{ groups of the negative of } (x - 3y) = 3 \times (-x + 9y) = -3x + 9y$

Class Activity 4 reveals these properties.

\[
\begin{align*}
a(bx) &= abx \\
a(x + y) &= ax + ay \\
a(x - y) &= ax - ay
\end{align*}
\]

The second two statements are known as the **distributive law** of multiplication over addition and over subtraction respectively.

**Example 27**

Expand these expressions.

(a) $4(-5x)$

(b) $-7(6y)$

**Solution**

(a) $4(-5x) = 4 \times (-5x) = -20x$

(b) $-7(6y) = -7 \times 6y = -42y$

**Try It! 27**

Expand these expressions.

(a) $2(8p)$

(b) $-6(5q)$

**Example 28**

Expand these expressions.

(a) $3(2a + 5)$

(b) $-9(-4 - 7y)$

**Solution**

(a) $3(2a + 5) = 3 \times 2a + 3 \times 5 = 6a + 15$

(b) $-9(-4 - 7y) = -9 \times (-4) - 9 \times (7y) = 36 + 63y$

**Try It! 28**

Expand these expressions.

(a) $4(3b + 8)$

(b) $-5(2 - n)$
In a Nutshell

Notation in Algebra
In algebra, letters are used to represent numbers.
The operation signs in algebra are the same as in arithmetic.

- Addition: \( a + b \)
- Subtraction: \( a - b \)
- Multiplication: \( ab \) or \( a \times b \)
- Division: \( \frac{a}{b} \) or \( a \div b \)

Algebraic Expressions
An algebraic expression involves numbers and letters connected by operations.

- Linear expressions: \( ax + b \) where \( a \) and \( b \) are numbers.
- Quadratic expression: \( ax^2 + bx + c \)

Substitution
When the variables in an algebraic expression are substituted by numbers, the value of the expression can be found.

- Example: \( x + 3 \) when \( x = 2 \)
  \[ x + 3 = 2 + 3 = 5 \]

Writing Algebraic Expressions and Formulae

1. Read the situation carefully.
2. Draw a bar model to represent the situation, if necessary.
3. Use variables to stand for some quantities.
4. Express other quantities in terms of the variables from the given information.
5. Set up a formula to connect the variables.

Formulae
A formula is an equality connecting two or more variables.

- Example: \( P = 2L + 2W \)

Substitution
The value of a variable in a formula can be found when the values of other variables are substituted into the formula.

- Example: when \( L = 6 \) and \( W = 4 \),
  \[ P = 2L + 2W = 2(6) + 2(4) = 20 \]

Like Terms and Unlike Terms
Terms are the parts separated by + or – signs in an expression.

- Like terms: terms with identical variable parts.
  e.g. \( 3x + 4y - 2 \) has three terms. They are \( 3x, 4y \), and \(-2\).
- Unlike terms: terms with different variable parts.
  e.g. \( 5a + 2b + 3c \) and \( 2x - 3y + 5z\) are pairs of unlike terms.

Addition and Subtraction of Algebraic Expressions
Addition and subtraction are carried out by removing brackets and collecting like terms.

- Example:
  \[ (2x - 5b) + (x + 4b) = 2x - 5b + x + 4b = 3x - b \]
  \[ (4x + 3y) - (2x - 5y) = 4x + 3y - 2x + 5y = 2x + 8y \]

Expressions with Brackets

1. \( a(b) = ab \)
   e.g. \( 4(6a) = 24a \)
2. \( a(a + y) = ax + ay \)
   e.g. \( 3(4x + 7) = 12x + 21 \)
3. \( a(x - y) = ax - ay \)
   e.g. \( -2(5 - 6y) = -10 + 12y \)

Chapter 3 Introduction to Algebra
REVISION EXERCISE 3

1. Simplify these expressions.
   (a) $2x \times 5$  
   (b) $2 \times x \times 5 \times y$  
   (c) $3 + 2x$  
   (d) $3 + 2x \times 3 - 4 \times y$

2. Simplify these expressions.
   (a) $p \times 7 \times p$  
   (b) $p \times 7 \times p \times p$  
   (c) $p \times p \times p + q$  
   (d) $3 + 2x \times 3 - 4 \times y$

3. Write these statements as algebraic expressions.
   (a) Add $c$ squared to $a$.
   (b) Subtract $2 \times h$ from $k$.
   (c) Multiply $p \times q$ and $p \times 3$.
   (d) Divide $x$ cubed by 7.

4. Find the values of these expressions when $n = 4$.
   (a) $2n + 5$  
   (b) $5n - 2$  
   (c) $7(1 - 3n)$

5. When $a = 3$ and $b = -2$, find the values of
   (a) $a^2$  
   (b) $b^3$  
   (c) $-5b^3$  
   (d) $a^2 - 5b^3$

6. Find the value of the expression $g^2 + 7k$ when $g = 5$ and $h = -4$.

7. Given the formula $E = \frac{1}{2}mv^2$, find the value of $E$ when $m = 3$ and $v = 2$.

8. Simplify these expressions.
   (a) $3 + 4a - 2 + 5n$  
   (b) $2a - 3b - 4a - b$  
   (c) $-3p + 5 + 2q - 6p$  
   (d) $3x - 7y + 8 - x - 2y$

9. Simplify these expressions.
   (a) $(1 + 5a) + (3 + 4a)$  
   (b) $(2x - 7d) + (-d - 5c)$  
   (c) $(-3m + n) - (2m + n)$  
   (d) $(5x - 6y) - (-7y + 2x)$

10. Expand these expressions.
    (a) $4(3x)$  
    (b) $-7(5y)$  
    (c) $5(3a + b)$  
    (d) $-6(4 + 7q)$

11. The price, $P$, of planting a square flowerbed of side $x$ metres is given by the formula $P = 8x + 20x^2$.
    Find the price of planting a square flowerbed of side 3 metres.

12. The temperature of an iron bar is $17^\circ C$. When it is heated, the temperature rises by $20^\circ C$ every minute.
   (a) Find the temperature of the iron bar after 3 minutes.
   (b) If the temperature of the iron bar after $t$ minutes is $T^\circ C$, express $T$ in terms of $t$.
   (c) Find the temperature of the iron bar after 9 minutes.

13. There are three consecutive odd numbers.
    The smallest number is $n$.
    (a) Express the other two numbers in terms of $n$.
    (b) Find the sum of the three numbers. Write a formula connecting $n$ and $S$.
    (c) Hence find the value of $S$ when the smallest number is 13.

14. The price for a bag of pasta is £2 and the price for a pack of sausages is £3. Mrs Stocks buys $m$ bags of pasta and $n$ packs of sausages.
    (a) Express the total price of the items in terms of $m$ and $n$.
    (b) Mrs Stocks pays with a £50 note at the counter and she gets £6 change. Write a formula connecting $m$, $n$ and $C$.
    (c) Find the change that Mrs Stocks receives if she buys four bags of pasta and two packs of sausages.

15. The mass of three apples is $3x$ grams. The mass of four apples and a basket is $(4x + 120)$ grams.
    (a) Express the total mass of these seven apples and the basket in terms of $x$.
    (b) Find the total mass of these seven apples and the basket when $x = 150$.

16. In the Leeds Triathlon, an athlete’s swimming time is $t$ minutes. Her cycling time is three times the swimming time. Her running time is 20 minutes more than the swimming time.
    (a) Express her cycling time and running time in terms of $t$.
    (b) Let $T$ minutes be the time taken by the athlete in the race. Write a formula connecting $t$ and $T$.
    (c) If the athlete’s swimming time is 25 minutes, find her time taken, in hours, to complete the race.

17. A rectangular lawn is $(2x + 3y)$ metres long and $(6y - x)$ metres wide.
    (a) Let $P$ metres be the perimeter of the lawn. Find a formula connecting $P$, $x$ and $y$.
    (b) The cost of making the fence around the lawn is £3 per metre. Find the cost of making the fence when $x = 5$ and $y = 2$.

18. A bottle contains 90 tablets of vitamin C. Each tablet provides 500 mg of vitamin C. David takes two tablets per day.
    (a) Express David’s intake of vitamin C from the tablets in terms of $x$.
    (b) Mr Stocks pays with a £50 note at the counter and gets £6 change. Write a formula connecting $m$, $n$ and $C$.
    (c) Find the number of tablets in the bottle after seven days.
    (d) Write a formula connecting $n$ and $y$.
    (e) Write the number of tablets in the bottle after 17 days.

19. The production cost of a shirt in a garment factory is £x. A retailer purchases 20 shirts and her purchase price of each shirt is £4 more than the production cost. The retailer marks the price of each shirt to be three times her purchase price. She sells 16 shirts at the marked price and the remaining shirts at half of the marked price.
    (a) Express, in terms of $x$,
       (i) the retailer’s purchase price of each shirt,
       (ii) the retailer’s marked price of each shirt.
    (b) Let $P$ be the retailer’s profit from selling the shirts.
       (i) Write a formula connecting $x$ and $P$.
       (ii) Find the profit of the retailer when $x = 9$.

Write in your Journal:
Write in your own words, using some examples, the meaning of the words ‘expression,’ ‘terms’ and ‘formulae’.