OXFORD
TEACHING GUIDES
HOW TO
Teach Mathematics
For Mastery
SECONDARY
SCHOOL EDITION
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OXFORD
About the author

Dr Helen Drury has taught mathematics for over a decade, including as a head of mathematics and senior leader, in Oxfordshire and London (UK). She is passionate about bringing research and best practice into the classroom to close the attainment gap and raise achievement for all. As Director of Mathematics for Ark, Helen founded the charitable, not-for-profit Mathematics Mastery programme to build up an evidence base to demonstrate that every student in mainstream education can succeed with mathematics, and that the high achievement of UK students can rival that of students anywhere in the world. In 2015, Helen published Mastering Mathematics: Teaching to transform achievement.

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Where children benefit from a high-quality learning environment in the early years, average achievement is higher and variation is reduced. Children who can count, identify written numbers, recognise shapes and have some sense of number before starting school have an important head start. High-performing systems make sure that their primary education overcomes these initial differences between learners. Teaching for mastery at primary is intended to reduce variance and raise achievement for all.

A well-established mastery approach in all feeder primary schools would narrow the gap between students’ attainment on entry and deepen all students’ understanding of the Key Stage 2 mathematics content. Of course, any activity in the primary phase will take some time to demonstrate impact.

This situation forces each secondary school to make an important decision – to wait to adopt teaching for mastery until the students joining them demonstrate higher and less varied understanding on entry, or to adopt teaching for mastery for the first few years of secondary school.

It can be argued that the present situation for secondary teachers in England and comparable countries has key aspects in common with the primary, rather than the secondary, schools in high-performing systems. Like the primary schools in these jurisdictions, secondary schools in England take students with a wide range of achievement. To choose to adopt a mastery approach given this intake is to choose to take on a significant challenge with the aspiration of achieving success for all students, regardless of background.

Differences in students’ attainment and understanding at the start of secondary school are due, for the most part, to different experiences in early years, different parental expectations, and different experiences of primary mathematics. If these students are to catch up, they need to have full access to mathematics learning from the start of secondary school, as well as being supported to learn any key concepts from primary school that are not yet sufficiently secure.

As soon as you offer opportunities to learn to some students but not others – you teach only ‘higher-attaining’ 12-year-olds to multiply and

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37 Ibid.
38 For example, you can find the full and most up-to-date Mathematics Mastery curriculum map at www.morticismastery.org.

Bar models can lay the foundations for students learning formal algebra. They help students derive, construct and simplify algebraic expressions and equations.

**GCSE exam question**

*Kieran, Jermaine and Chris play football.*

- **Kieran has scored 8 more goals than Chris.**
- **Jermaine has scored 5 more goals than Kieran.**
- **Altogether they have scored 72 goals.**

How many goals did they each score?\(^{39}\)

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**Figure 7.9**

Many of the topics on the secondary mathematics curriculum lend themselves to representation with bar models.

To demonstrate this, here’s a sample of one or two of the ways bar models are used early in *Mathematics Mastery*’s secondary teaching programme.

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**Try this in the classroom**

**Alicia and Bobby**

*Alicia had £6 more than Bobby. If Bobby had £10, how much did they have altogether?*

*Alicia had £6 more than Bobby. If they had £10 altogether, how much did each person have?*

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39 OCR Foundation Specimen Papers, Paper 1 Question 15.
Howard Gardner advanced the theory that different people are intelligent in different ways. Unfortunately, some schools interpreted Howard Gardner’s theory of multiple intelligences as a reason to label students as particular types of learners who were then taught in different ways. But students who are not yet confident visual thinkers or kinaesthetic learners arguably need to work with pictorial and concrete representations more than anyone.

Mixing up the order – multiple representations

Try this in the classroom

Always, sometimes or never true?

<table>
<thead>
<tr>
<th>Equation</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n + 4 = 7$</td>
<td>$u + 2 = u + 14$</td>
</tr>
<tr>
<td>$2n + 3 = 3 + 2n$</td>
<td>$2r - 4 = 4 - 2r$</td>
</tr>
<tr>
<td>$4y + 3 = 7y$</td>
<td>$m + 6 &lt; 20$</td>
</tr>
<tr>
<td>$x^2 = 5x$</td>
<td>$4t &gt; 8 + t$</td>
</tr>
</tbody>
</table>

Figure 7.16

Preparing to teach – do the mathematics!

In preparing to teach ‘Always, sometimes or never true?’, you might consider the following points:

- When talking about each equation or inequality, when do you use the term ‘variable’ and when ‘unknown’? Does your use of these terms pre-suppose the statement being ‘sometimes’ or ‘always’ true? What term would you expect students to use to refer to these letters?
- Ask students to explain and justify their answers using manipulatives.

Howard Gardner advanced the theory that different people are intelligent in different ways. Unfortunately, some schools interpreted Howard Gardner’s theory of multiple intelligences as a reason to label students as particular types of learner who were then taught in different ways. But students who are not yet confident visual thinkers or kinaesthetic learners arguably need to work with pictorial and concrete representations more than anyone.

Mixing up the order – multiple representations

Concrete manipulatives need not always be seen as the first and easiest step on a journey to understanding. Naturally, there will be learning sequences where it is appropriate to begin with concrete manipulatives, and progress through to the abstract. However, on some occasions students’ understanding will be deepened through beginning work with symbols, and asking them to represent the ideas using diagrams or objects.

If you try something and it doesn’t work or has unintended consequences, don’t hide it. Failure is a learning experience for teachers, just as it is for students. Dealing effectively with setbacks is critical if change is going to be an accepted part of your team’s culture.

Jumping in with both feet — trying to impose significant change on a team before its members are comfortable with new ideas and processes — may doom your adventures in mastery before they’ve even started.

Of the three dimensions of depth, multiple representations perhaps lends itself best to this kind of small-scale change. This may be because the change is (literally!) more visible than adopting strategies to improve language and communication, or mathematical thinking.

**Developing teaching for mastery**

**The school context:** The team at Cramlington Learning Village, Northumberland, UK, thought carefully about when and how to introduce a mastery approach. They identified mathematics as a key area on their School Improvement Plan. Although Key Stage 3 had always been a strong focus for the school, GCSE (i.e., 16+) outcomes often took priority. The decision to teach mathematics for mastery saw the start of a clear emphasis on developing pedagogical approaches in Year 7. This is expected to quickly have an impact on teaching and learning across all years, and improve outcomes in the long term.

Central to the school’s long-term emphasis on effective teaching is the timetabling of a weekly one-hour session every Wednesday after school. This is kept separate from the weekly department meeting for administrative matters. Teachers look together at key tasks for the following week’s Year 7 lessons, ‘doing the mathematics’ and discussing possible questions and scope for additional challenge and scaffolding.

The Year 7 teaching team includes newly qualified teachers, non-specialists from other departments and experienced mathematics teachers. The school and department leadership have viewed this as an opportunity. The head of mathematics finds that the mix of experience has led to a positive environment for professional development sessions, with some teachers using them as a platform to develop their existing practice, and others as a starting point for teaching mathematics.

**Their impact:** Within the first term of this work, the senior leadership team member noticed positive feedback from staff and students. In particular, several experienced members of the mathematics team have embraced the opportunity to reflect on their mindset and try new classroom approaches.

A learning walk through Year 7 lessons found students in all classes to be highly engaged with learning, and enjoying opportunities to use manipulatives, explain their reasoning and engage in problem solving. The mathematics team are finding that the mastery approach is supporting students who are not secondary-ready when joining the school, particularly boys, in accessing and engaging in mathematics. A student voice survey of over 240 respondents found that students have positive attitudes towards mathematics, and believe they have the opportunity to engage in problem solving.

**Why change? Where’s the evidence?**

The teaching approach described in this book is evidence-based by design. It begins with an ambition that every student will succeed in mathematics and a significant proportion will excel. It draws on worldwide effective practice, and on established research findings, to develop a coherent approach that will achieve this ambition.

Evidence of impact, then, can be seen as falling into three main types:

1. evidence of high achievement in the countries which inspired this approach
2. evidence for specific recommendations in the approach, such as use of concrete manipulatives, or student feedback
3. evidence of impact of the Mathematics Mastery programme in UK schools

An introduction to the evidence base for all three of these has been integrated throughout this book.

International comparisons have been drawn on, as relevant, in the preceding chapters. Case studies from schools implementing this programme have been included throughout the book.

In this section, I summarise key international high achievement. This section also includes early evidence of impact of the Mathematics Mastery programme. The intention here is to offer some indication that combining the specific, research-informed recommendations described in this book has the potential to transform student achievement.