How to use this book

Chapters

Chapters 1–3 each focus on one of the key concepts:
- Form, Relationships and Logic.

Chapters 4–15 each focus on one of the twelve related concepts for Mathematics:
- Representation, Simplification, Quantity, Measurement, Patterns, Space, Change, Equivalence, Generalization, Justification, Models, and Systems

Creating your own units

The suggested unit structure opposite shows just one way of grouping all the topics from different chapters, from both Standard and Extended, to create units. These units have been created following the official guidelines from the IB Building Quality Curriculum. Each unit is driven by a meaningful statement of inquiry and is set within a relevant global context.

- Each chapter focuses on one of the IB’s twelve related concepts for Mathematics, and each topic focuses on one of the three key concepts. Hence, these units, combining topics from different chapters, connect the key and related concepts to help students both understand and remember them.
- Stand-alone topics in each chapter teach mathematical skills and how to apply them, through inquiry into factual, conceptual, and debatable questions related to a global context. This extends students’ understanding and ability to apply mathematics in a range of situations.
- You can group topics as you choose, to create units driven by a contextualized statement of inquiry.

Using MYP Mathematics 4 & 5 Extended with an existing scheme of work

If your school has already established units, statements of inquiry and global contexts, you can easily integrate the concept-based topics in this book into your current scheme of work. The table of contents on page vii clearly shows the topics covered in each concept-based chapter. Your scheme’s units may assign a different concept to a given topic than we have. In this case, you can simply add the concept from this book to your unit plan. Most topics include a Review in context, which may differ from the global context chosen in your scheme of work. In this case, you may wish to write some of your own review questions for your global context, and use the questions in the book for practice in applying mathematics in different scenarios.

Suggested plan

The units here have been put together by the authors as just one possible way to progress through the content.

Units 1, 5, 6, 9 and 10 are made up of topics solely from the MYP Mathematics 4 & 5 Standard book.

UNIT 2

Topics: Scatter graphs and linear regression, drawing reasonable conclusions, data inferences
Global context: Identities and relationships
Key concept: Relationships

UNIT 3

Topics: Equivalence transformations, inequalities, non-linear inequalities
Global context: Identities and relationships
Key concept: Relationships

UNIT 4

Topics: Rational and irrational numbers, direct and indirect proportion, fractional exponents
Global context: Globalization and sustainability
Key concept: Form

UNIT 7

Topics: Finding patterns in sequences, making generalizations from a given pattern, arithmetic and geometric sequences
Global context: Scientific and technical innovation
Key concept: Form

UNIT 8

Topics: Simple probability, probability systems, conditional probability
Global context: Identities and relationships
Key concept: Logic

UNIT 11

Topics: Circle segments and sectors, volumes of 3D shapes, 3D orientation
Global context: Personal and cultural expression
Key concept: Relationships

UNIT 12

Topics: Using circle theorems, intersecting chords, problems involving triangles
Global context: Personal and cultural expression
Key concept: Logic

UNIT 13

Topics: Algebraic fractions, equivalent methods, rational functions
Global context: Algebraic fractions, equivalent methods, rational functions
Key concept: Form

UNIT 14

Topics: Evaluating logarithms, transforming logarithmic functions, laws of logarithms
Global context: Orientation in space and time
Key concept: Relationships

UNIT 15

Topics: The unit circle and trigonometric functions, sine and cosine rules, simple trigonometric identities
Global context: Orientation in space and time
Key concept: Relationships

Access your support website for more suggested plans for structuring units:
www.oxfordsecondary.com/myp-mathematics
About the authors

Marlene Torres-Skoumal has taught DP and MYP Mathematics for several decades. In addition to being a former Deputy Chief Examiner for IB HL Mathematics, she is an IB workshop leader, and has been a member of various curriculum review teams. Marlene has authored both DP and MYP books, including Higher Level Mathematics for Oxford University Press.

Clara Huizink has taught MYP and DP Mathematics at international schools in the Philippines, Austria and Belgium. She has also been through the IB experience herself as a student and she is a graduate of the IB program.

Rose Harrison is the Lead Educator for MYP Mathematics at the IB Organization. She is an MYP and DP workshop leader, a senior reviewer for Building Quality Curriculum in the MYP, and has held many positions of responsibility in her 20 years’ experience teaching Mathematics in international schools.

Aldan Sproat-Clements is Head of Mathematics at Wellington College in the UK, an IB World School. He has spent his career in British independent schools where he has promoted rigorous student-led approaches to the learning of Mathematics. He helped to develop the pilot MYP eAssessments for Mathematics.

Learning features

Each topic has three sections, exploring:

- factual
- conceptual
- debatable inquiry questions

Problem solving – where the method of solution is not immediately obvious, these are highlighted in the Practices.

Explorations are inquiry-based learning activities for students working individually, in pairs or in small groups to discover mathematical facts and concepts.

ATL highlights an opportunity to develop the ATL skill identified on the topic opening page.

Reflect and discuss – opportunities for small group or whole class reflection and discussion on their learning and the inquiry questions.

Worked examples show a clear solution and explain the method.

Practice questions written using IB command terms, to practice the skills taught and how to apply them to unfamiliar problems.

Objective boxes highlight an IB Assessment Objective and explain to students how to satisfy the objective in an Exploration or Practice.

Technology icon

Using technology allows students to discover new ideas through examining a wider range of examples, or to access complex ideas without having to do lots of painstaking work by hand.

This icon shows where students could use Graphical Display Calculators (GDC), Dynamic Geometry Software (DGS) or Computer Algebra Systems (CAS). Places where students should not use technology are indicated with a crossed-out icon.

The notation used throughout this book is largely that required in the DP IB programs.
Summary
The standard deviation is a measure of dispersion that gives an idea of how close the original data values are to the mean, and so how representative the mean is of the data. A small standard deviation shows that the data values are close to the mean. The units of standard deviation are the same as the units of the original data. 

\[ \sigma = \sqrt{\frac{\sum (x - \mu)^2}{n}} \]

The sample mean is 
\[ \bar{x} = \frac{\sum x}{n} \]

The normal distribution is a symmetric distribution, with most values close to the mean and tending off evenly in either direction. Its frequency graph is a bell-shaped curve.

Mixed practice
In questions 3 and 2 the data provided is for the population.

1. Find the mean and the standard deviation of each data set.
   a. 2, 3, 4, 5, 6, 6
   b. 21 kg, 22 kg, 24 kg, 25 kg, 27 kg, 29 kg

   \[ \mu = \frac{\sum x}{n} \]
   \[ \sigma = \sqrt{\frac{\sum (x - \mu)^2}{n}} \]

   Number of strawberries on each plant fed with special strawberry plant food:
   14 15 17 17 19 19 12 14 15 15

   a. Find the mean and standard deviation of the weight of food the hamster ate over the 9 days.
   b. Find the mean and standard deviation of the weight of food the hamster ate each day.
   c. Bernie assumes that his hamster’s food consumption is normally distributed.

Review in context
The ancient art of origami has roots in Japanese culture and involves folding flat sheets of paper to create models. Some models are designed to resemble animals, flowers or buildings, others just celebrate the beauty of geometric design.

Review in context – summative assessment questions within the global context for the topic.

The distance from Vevey to Saint-Gingolph is 6.5 km.

The distance from Vevey to Montreux is 6.5 km.

The distance from Vevey to Thonon-les-Bains is 14 km.

The distance from Vevey to Geneva is 18 km.

The distance from Vevey to Lausanne is 7.0 km.

The resulting creases are illustrated in this diagram.

Reflection on the statement of inquiry
How have you explored the statement of inquiry? Give specific examples.

Table of contents

Form
E1.1 What if we all had eight fingers? • Different bases

Relationships
E2.1 Multiple doorways • Composite functions
E2.2 Doing and undoing • Inverse functions

Logic
E3.1 How can we get there? • Vectors and vector spaces

Representation
E4.1 How to stand out from the crowd • Data inferences

Simplification
E5.1 Super powers • Fractional exponents

Quantity
E6.1 Ideal work for lumberjacks • Evaluating logs

Measurement
E7.1 Slices of pi • The unit circle and trigonometric functions

Patterns
E8.1 Making it all add up • Arithmetic and geometric sequences

Space
E9.1 Another dimension • 3D Orientation
E9.2 Mapping the world • Oblique triangles

Change
E10.1 Time for a change • Log functions
E10.2 Meet the transformers • Rational functions

Equivalence
E11.1 Unmistaken identities • Trigonometric identities

Generalization
E12.1 Go ahead and log in • Laws of logarithms

Justification
E13.1 Are we very similar? • Problems involving triangles

Models
E14.1 A world of difference • Nonlinear inequalities

Systems
E15.1 Branching out • Conditional probability

Answers
315

Index
350
E1.1 What if we all had eight fingers?

Global context: Scientific and technical innovation

Objectives
- Understanding the concept of a number system
- Counting in different bases
- Converting numbers from one base to another
- Using operations in different bases

Inquiry questions
- How have numbers been written in history?
- What is a number base?
- How can you write numbers in other bases?
- How are mathematical operations in other bases similar to and different from operations in base 10?
- Would you be better off counting in base 2?
- How does form influence function?

Numbers in different bases
- How have numbers been written in history?
- What is a number base?
- How can you write numbers in other bases?
- How are mathematical operations in other bases similar to and different from operations in base 10?
- Would you be better off counting in base 2?
- How does form influence function?

Humans have used many different ways to record numbers. How would you write down the number of green bugs in this diagram? You might have written a symbol 5, or the word “five”. You could have used a word in a different language, or maybe even a tally: . Each of these represents the number in a different way, but they all represent the same quantity.

Different cultures use different forms of notation to represent number. The ancient Egyptian hieroglyphic number system was an additive system. Each symbol has a different value and you find the total value of the number by adding the values of all the symbols together.

Egyptian numerals use these symbols:

<table>
<thead>
<tr>
<th>Stroke</th>
<th>Heelbone</th>
<th>Coiled rope</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lotus flower</th>
<th>Pointed finger</th>
<th>Tadpole</th>
<th>Scribe</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>10000</td>
<td>1000000</td>
<td></td>
</tr>
</tbody>
</table>

The number 11 is written Iι, and 36 is written IIIΙΧΧΧΧ.

Green shield bugs are sometimes called green stink bugs, as they produce a pungent odor if handled or disturbed.

You should already know how to:
- use the operations of addition, subtraction and long multiplication in base 10, without a calculator
- understand place value

1. Calculate these by hand:
   a. 10442 + 762
   b. 10887 – 7891
   c. 27 × 43
   d. 14078 × 71

2. Write down the value of the 5 in:
   a. 351
   b. 511002
   c. 15
   d. 1.5

Attempt 1:

- Use intercultural understanding to interpret communication

Communication

Global context: Scientific and technical innovation
Reflect and discuss 1

- Write each number in Egyptian numerals:
  - 5
  - 32
  - 126
  - 99
  - 100
  - 10240

- Write these numbers as ordinary (decimal) numbers:

  3642 and 4623

  3 thousands
  6 hundreds
  4 tens
  2 units

A digit in the furthest right column represents individual objects: units. Moving left, the second column represents collections of ten units: the tens column. The third column represents collections of ten tens: the hundreds column. The pattern continues: each column is worth 10 times more than the column to its right. The number 364 actually represents a collection of that many objects:

<table>
<thead>
<tr>
<th>hundreds</th>
<th>tens</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>6</td>
<td>4</td>
</tr>
</tbody>
</table>

Reflect and discuss 2

- Does the symbol 0 have different values in the numbers 205 and 2051? Does it represent something different?
- The Roman numerals do not have a symbol to represent zero, so why does the place value system need a symbol for zero?

Our number system, often called Arabic numerals, is a place value system. The position of a digit tells you its value.

The two numbers below have the same four digits, but digits do not always represent the same amount.

The binary number 10111₂ represents a set containing:

<table>
<thead>
<tr>
<th>sixteen</th>
<th>eight</th>
<th>four</th>
<th>two</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

The subscript 2 means that the number 10111₂ is written in base 2.

Tip

- Base 3 - ternary, or trinary
- Base 4 - quaternary
- Base 8 - octal
- Base 12 - duodecimal (widely used in computing)
The base of a number system tells you how many unique symbols the number system has. Base 10 has ten unique number symbols, 0 through 9. Base 2 has two unique symbols, 0 and 1.

Example 1

Find the value of 12011₂ in base 10.

<table>
<thead>
<tr>
<th>3⁴</th>
<th>3³</th>
<th>3²</th>
<th>3¹</th>
<th>3⁰</th>
</tr>
</thead>
<tbody>
<tr>
<td>81</td>
<td>27</td>
<td>9</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

Write the powers of 3 above the digits, starting with $3^0 = 1$ at the right-hand side.

$81 + 2 	imes 27 + 3 + 1 = 139$

Add up the parts that make up the number.

$12011₂ = 139₁₀$

Use subscripts to show the base.

Practice 1

1 Find the value of each binary number in base 10.
   a $10111₂$
   b $11001₁₂$
   c $1101₁₀₁₂$

2 Convert each number to base 10.
   a $2100₂$
   b $2210₁₂$
   c $412₂$
   d $33₂$
   e $64₆$
   f $77₉$

Problem solving

3 Write these numbers in ascending order:
   $101₀₁₁₂$, $1000₁₁₁₂$, $101₁₃$, $111₁₃$

4 The number $1101₁₀₁₀₀₀₁₀ = 856₁₀$.
   a Describe the relationship between $1101₁₀₁₀₀₀₁₀$ and $1101₁₀₁₀₀₂$.
   b Find the value of $1101₁₀₁₀₀₂$.

Exploration 1

1 Use this table to explore questions a to g.

<table>
<thead>
<tr>
<th>3⁷</th>
<th>3⁶</th>
<th>3⁵</th>
<th>3⁴</th>
<th>3³</th>
<th>3²</th>
<th>3¹</th>
<th>3⁰</th>
</tr>
</thead>
<tbody>
<tr>
<td>2187</td>
<td>729</td>
<td>243</td>
<td>81</td>
<td>27</td>
<td>9</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

a Explain how the value of $3^7$ tells you that the number $1038₁₀$ will have 7 digits in base 3.

b $1038₁₀ = 729₁₀ + 309₁₀$.
   Explain how this sum tells you that the first digit of $1038₁₀$ in base 3 will be 1.

---

The following algorithm produces the digits of a number $n$ in base $b$, starting with the units digit and then working to the left.

- Start with the number, $n$. Divide $n$ by $b$ and find the quotient $q$, and the remainder $r$.
- Write down the value of $r$.
- If $q > 0$, replace $n$ with the value of $q$; otherwise stop.
- Repeat from the beginning with the new value of $n$, and record new remainders to the left of any you have already written down.

4 Here the algorithm is used to convert $103₈₁₀$ into base 3.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$q$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>103₈₁₀</td>
<td>346</td>
<td>0</td>
</tr>
<tr>
<td>346</td>
<td>115</td>
<td>1</td>
</tr>
<tr>
<td>115</td>
<td>38</td>
<td>1</td>
</tr>
<tr>
<td>38</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Use the algorithm to convert:
   a $1000₁₁₂$ to base 2
   b $51₃₁₀$ to base 3
   c $67₃₁₀$ to base 4.

---

Music is the pleasure the human mind experiences from counting, without being aware that it is counting.

– Gottfried Leibniz
E1.1 What if we all had eight fingers?

Use the method you prefer to convert numbers to different bases.

Practice 2

1. Convert $999_{10}$ to:
   a. base 2
   b. base 3
   c. base 4
   d. base 5

   b. Hence find the value of $472_8$ in base 5.

3. By first converting to base 10, find the value of these numbers in the given base:
   a. $223_7$ in base 7
   b. $431_2$ in base 2
   c. $214_2$ in base 2
   d. $1011_2$ in base 6
   e. $110111_2$ in base 8
   f. $110213_2$ in base 9
   g. $8868_3$ in base 3
   h. $101101_2$ in base 4
   i. $2468_9$ in base 8

Problem solving

4. Four students write the same number in different bases:
   Alberto: $1331_q$
   Benito: $2061_q$
   Claudio: $3213_q$
   Donatello: $1000_q$
   a. Determine which of the four students used the largest base. Explain how you know.
   b. Use your answer to part a, and any other information you can gain from the students’ numbers, to list the numbers $a$, $b$, $c$ and $d$ in ascending order.
   c. Determine the minimum possible value for $b$.
   d. Find values of $a$, $b$, $c$ and $d$ such that $1331_q = 2061_q = 3213_q = 1000_q$.
   e. Use your answer to d to find the value of the number in base 10.

Exploration 2

A bottle factory packs 12 bottles to a box. There are 12 boxes in a crate.
A shipping container will hold 12 crates.

1. Determine the number of bottles in each crate.
2. Determine the number of bottles in each shipping container.
3. A customer orders 600 bottles. Find the number of crates and boxes to fulfil this order.
4. A customer orders 81 bottles. Determine the number of complete boxes and single bottles for this order.

The table on page 9 shows four different orders, with some information missing. Calculate appropriate values for the shaded cells.

<table>
<thead>
<tr>
<th>Customer name</th>
<th>Total ordered</th>
<th>Notes</th>
<th>Containers</th>
<th>Crates</th>
<th>Boxes</th>
<th>Singles</th>
<th>Total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mr Antinoro</td>
<td>a</td>
<td>n/a</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>b</td>
</tr>
<tr>
<td>Mr Drouhin</td>
<td>6000</td>
<td>10% discount on complete containers</td>
<td>c</td>
<td>d</td>
<td>e</td>
<td>f</td>
<td>g</td>
</tr>
<tr>
<td>Herr Müller</td>
<td>h</td>
<td>(%) discount on the whole order</td>
<td>4</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>68400</td>
</tr>
<tr>
<td>Mrs Symington</td>
<td>1500</td>
<td>n/a</td>
<td>j</td>
<td>k</td>
<td>l</td>
<td>m</td>
<td>n</td>
</tr>
</tbody>
</table>

5. An employee suggests that since all orders are made of a number of containers, crates, boxes and singles, the company does not need to repeat the headings every time, so an order of 3 crates, 7 boxes and 0 singles could be written as 370.

a. Write Mr Antinoro’s, Mr Drouhin’s and Herr Müller’s orders using this convention.

b. Explain how notating the orders in this way relates to writing numbers in non-decimal number bases.

c. Describe the problem in writing Mrs Symington’s order in this way.

Reflect and discuss 3

- Why do you think people most commonly use base 10 to count?
- When is the number 12 commonly used as a base? What makes the number 12 a convenient number to use?

Base 10 (decimal) uses ten different symbols to describe the whole numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Similarly, base 2 uses two symbols: 0 and 1. Base 12 (duodecimal) requires twelve symbols, but you cannot use ‘10’ or ‘11’ because these both involve two digits.

When you write numbers in bases greater than 10, letters are used for the extra symbols needed.
In base 12, the symbols are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A and B.

Sometimes lowercase letters a and b are used instead of uppercase A and B.
What if we all had eight fingers?

US President Abraham Lincoln’s “Gettysburg Address” began:

“Four score and seven years ago our fathers brought forth on this continent a new nation, conceived in liberty, and dedicated to the proposition that all men are created equal.”

A ‘score’ means a group or set of 20, so ‘four score and seven’ is 87.

In the French number system, 80 is ‘quatre-vingt’ or ‘four twenties’ and 90 is ‘quatre-vingt-dix’ or ‘four twenties and ten’.

**Objective C:** Communicating

iii. move between different forms of mathematical representation

Different number bases and symbols are different forms of representation. In this Exploration, show that you can move between different symbols and number bases.

**ATL**

Continued on next page
2. Find the value of these Mayan numerals:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Write these numbers in Mayan:

- 806
- 2005
- 10145
- 16125
- 43487
- 56267
- 100000
- 2 million

4. Find the value of this sum, showing all your steps:

- +

Express your answer in Mayan notation as well as in base 10.

5. Explain why the Mayan system is part additive and part place value.

6. Compare our base 10 number system with the Mayan system and write down any advantages or disadvantages that one has over the other.

C. Performing operations

- How are mathematical operations in other bases similar to and different from operations in base 10?

In a calculation like this:

\[
\begin{align*}
892 \\
+347
\end{align*}
\]

you add the units, then the tens, then the hundreds. When the addition gives you a number greater than or equal to 10 (the base) you write down the units digit and ‘carry’ the 10 to the next column.

---

“E1.1 What if we all had eight fingers?”

You can add numbers in other bases in the same way as decimal numbers, ‘carrying’ when a number is greater than or equal to the base number.

**Example 4**

Find the value of \(465_7 + 326_7\).

\[
\begin{align*}
465_7 + 326_7 &= 1124_7 \\
\end{align*}
\]

Units column: \(5 + 6 = 11_7\). In base 7 this is 1 seven and 4 units, or \(14_7\). Write 4 in the units column and carry a 1 into the sevens column.

Sevens column: \(6 + 2 + 1 = 8_7\). Rewrite \(8_7\) as \(12_7\). Write 2 in the sevens column and carry a 1 into the 7² column.

7² column: \(4 + 3 + 1 = 8_9 = 11_7\). Write 11 in the 7² column.

So, \(465_7 + 326_7 = 1124_7\).

**Reflect and discuss 4**

You could work out \(465_7 + 326_7\) by adding, as in Example 4, or by:

- converting \(465_7\) and \(326_7\) into base 10
- adding
- converting the result back to base 7.

Which method would you prefer? Explain.

For subtraction, you can ‘borrow’ the base number, in the same way that you ‘borrow’ a 10 in base 10.

**Example 5**

Calculate \(783_9 - 267_9\).

\[
\begin{align*}
783_9 - 267_9 &= 516_9 \\
7 > 3, \text{ so you cannot subtract } 7 \text{ from } 3. \text{ Rewrite } 783_9 \text{ as } 777_9, \text{ because } 3 \text{ units, } 8 \text{ nines and } 7 \text{ eighty-ones is the same as } 13 \text{ units, } 7 \text{ nines and } 7 \text{ eighty-ones.} \\
777_9 - 267_9 &= 516_9 \\
777_9 - 267_9 &= 516_9 \\
\end{align*}
\]

Find \(13_9 - 7_9\) by writing it in base 10:

\[13_9 - 7_9 = 12_{10} - 7_{10} = 5_{10} = 5_9\]
E1.1 What if we all had eight fingers?

Practice 4

1. Calculate:
   a) \(724 + 321\)
   b) \(77 + 261\)
   c) \(563 + 241 + 757\)

2. Calculate:
   a) \(453 - 231\)
   b) \(341 - 153\)
   c) \(1231 - 402\)

3. Calculate:
   a) \(1153 - 23 + 45\)
   b) \(2231 - 125 - 216\)
   c) \(8463 - 728 - 541 - 18\)

4. Calculate:
   a) \(92A12 + 443612\)
   b) \(1000012 - 12312\)

Problem solving

Vorbelar the alien does not count in base 10, but in order to make things easy for us to understand, she uses our base 10 symbols in the usual order. She calculates \(216 + 165\) and obtains the answer 403. Find the value of \(216 - 165\), giving your answer using Vorbelar’s base.

Reflect and discuss 5

When performing a calculation, is it important that the two numbers being added or subtracted are in the same base? Why or why not?

Exploration 4

There are many ways of setting out the calculation \(742_{10} \times 36_{10}\).

Here is one approach.

First multiply the top row by 6, one column at a time, starting with the 2 in the units column.

\[
\begin{array}{c}
742 \\
\times 36 \\
\hline
12 \\
\hline
252 \\
\hline
1648 \\
\hline
7728 \\
\hline
\end{array}
\]

6 × 2 = 12, so write 2 in the units column and 1 in the tens column.

Continued on next page
E1.1 What if we all had eight fingers?

Reflect and discuss 6

● Create a multiplication table in binary. Can you ‘learn your times tables’ in the new base?
● Try using it to perform some multiplications in base 2.
● Can you work out 1.011100110 × 110110100 in base 2?

You may find multiplication slow in base 2 because there are a lot of digits, and long multiplication involves a lot of steps. There are fewer symbols in numbers with small bases, and thus numbers have more digits. In numbers with larger bases, there are more symbols, and so numbers have fewer digits.

Hexadecimal is used in computers because each symbol in hexadecimal represents a string of 4 bits, or 4 binary digits. For example, 3F in hexadecimal converts to 00111111 in binary. That is why built-in passwords (in modems and routers for example) often use digits 0-9 and letters A-F.

Practice 5

1 Here is a multiplication table in base 5:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>11</td>
<td>13</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>11</td>
<td>14</td>
<td>22</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>13</td>
<td>22</td>
<td>31</td>
<td>40</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>100</td>
</tr>
</tbody>
</table>

Use it to complete these multiplications:

a 123₅ × 2₄₅
b 401₄ × 13₃₅
c 21₃₅ × 131₀₅

2 Copy and complete this multiplication table for base 6.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>12</td>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>12</td>
<td>40</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>20</td>
<td>40</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Hence complete these multiplications:

a 155₆ × 2₃₆
b 212₆ × 31₅₆

Problem solving

3 Find two numbers (both greater than 1) whose product is 121₅₆.

Because long multiplication relies on knowing multiplication table facts, multiplication in different bases is quite tricky. You can write out a multiplication table for the base you need, but in most cases it is easier to convert the numbers to base 10, perform the multiplication, and then convert the numbers back to the required base.

Division also relies on knowing multiplication table facts, so it is usually easiest to convert numbers to base 10 and then divide them.

A peek at binary numbers

● Would you be better off counting in base 2?
● How does form influence function?

Computers use base 2 (binary) for most of their operations. One reason for this is that the electrical signals that pass through the computer chips are either ‘on’ or ‘off’. As the computer’s memory exists in two states (on or off) it uses just two symbols for its counting system: 0 and 1.
Reflect and discuss 7

- Is counting in base 2 on your fingers a more efficient way of counting?
- How is counting in base 2 on your fingers more difficult than counting in base 10?
- When might it be useful to try to count in base 2 on your fingers?

Summary

Our number system, often called Arabic numerals, is a place value system. The position of a digit tells you its value.

Our number system is also known as decimal, or base ten, or denary, because the value of each place value column is 10 times the value of the column to its right.

You can make a place value system with any natural number base. In binary, or base two, each place value column is 2 times the value of the column to its right.

The subscript 2 means that the number 101111₂ is written in base 2.

The base of a number system tells you how many unique symbols the number system uses. Base 10 has ten unique number symbols: 0 through 9. Base 2 has two unique symbols: 0 and 1.

When you write numbers in bases greater than 10, letters are used for the extra symbols needed.

In base 12 the symbols are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A and B (or sometimes lowercase letters a and b).

You can add numbers in other bases in the same way as decimal numbers, ‘carrying’ when a number is greater than or equal to the base number.

For division and multiplication in a different base, it is usually most efficient to convert the numbers to base 10, do the calculations, then convert the answer back to the base in question.

Mixed practice

1 Find the value in base 10 of:
   a 110₁₂
   b 1101₁₀
   c 10001₁₀
   d 122₁₃
   e 1200₀
   f 452₃
   g 343₃₄
   h 18A₉₁₁
   i 4E₈₁₈

2 Find the value in the base specified of:
   a 100₁₀ in base 2
   b 100₀ in base 3
   c 100₁₀ in base 4
   d 255₁₀ in base 2
   e 1008₁₀ in base 3
   f 816₂₁₀ in base 5
   g 27₁₁₀ in base 12
   h 45₆₁₀ in base 12
   i 168₉₁₀ in base 12
   j 27₈₁₀ in base 12
   k 104₉₁₀ in base 16
   l 153₄₁₀ in base 16

3 You can measure angles in degrees, minutes and seconds. One degree = 60 minutes or 60', and one minute = 60 seconds or 60″. 45° 15’ 30″ = 45 degrees, 15 minutes and 30 seconds.

   Convert these angle measures:
   a 42.8° into degrees, minutes and seconds
   b 90.15° into degrees, minutes and seconds
   c 2.52° into degrees, minutes and seconds
   d 25°15’ into base 10 (including a decimal)
   e 62° 30’ 30″ into base 10 (including a decimal)
   f 58° 12’ 20″ into base 10 (including a decimal)

4 Calculate, giving your answers in the base used in the question:
   a 110₁₂ + 100₁₂
   b 110₁₂ + 101₁₂
   c 101₁₂ + 1011₁₂ + 110111₂

5 Calculate, giving your answers in the base used in the question:
   a 1101₁₁ – 10₁₂
   b 1110₁₁ – 110₁₂
   c 101₁₁ – 11₂
   d 22₁₀ – 10₂
   e 30₁₁ – 2₁₂
   f 44₁₂ – 5₁₀
   g 50₁₁₂ – 104₅
   h 7₈₁₂ – 8₉₀₁₂

6 If the current time was 10:45, use addition and subtraction to find the time that is:
   a 35 minutes from now
   b 2₂ hours from now
   c 3 hours and 40 minutes from now
   d 1 hour and 19 minutes ago
   e 6 hours and 50 minutes ago

7 Calculate, giving your answer in the base used in the question.

   a 110₁₁ × 10₂
   b 101₁₂ × 11₂
   c 110₁₂ × 10₂
   d 2₁₀ × 2₁₁
   e 1₂ × 2₃
   f 2₄ × 1₃₁

8 A set of weights contains weights in grams that are powers of 4. There are three of each size weight.

   - Determine which weights you would use to weigh out twice that amount. Ned weighs out twice as much as Ed and Ted combined. Combining weights that he has only three of each type of weight, determine the weights Ned should use.
   - Ed weighs out 1330 grams. Ed weighs out twice that amount. Ned weighs out twice as much as Ed and Ted combined. Combining weights that he has only three of each type of weight, determine the weights Ned should use.

9 A congressional committee decides to streamline the US monetary system. All coins will be phased out except for 1 cent, 5 cents and 25 cents. All existing bills (bank notes) will be phased out and replaced with bills valued $1, $5, $25, $125 and $625.

   a A mathematician suggests that the dollar should be revalued to be worth 125 cents, not 100 cents as it currently is.
   b Suggest reasons why it might not be a good idea.

   c Find 540₁₀ in base 5.
   d Find the smallest number of notes that you would need to give somebody $540 under the new system.

E1 Form

Number

E1.1 What if we all had eight fingers?
Review in context

In the 1960s and 1970s, NASA’s Pioneer and Voyager missions launched spacecraft to travel beyond the solar system. Each contained messages in case the craft was intercepted by extra-terrestrial life forms. Scientists tried to choose things that they hoped could be universally communicated, or which could be measured by aliens.

1. If an alien culture has a number system, there is a good chance that they will have some sort of place value system. They will probably have an idea of whole numbers, and maybe an awareness of prime numbers. One suggestion for a message to include might be to communicate the first prime numbers in binary. The first five numbers in binary are: 10, 11, 101, 111 and 1011.
   a. Write down the values of these numbers in denary (decimal).
   b. List the next ten prime numbers in denary.
   c. Hence list the first 15 prime numbers in binary.

Problem solving

2. For the messages, the scientists did not use 1s and 0s; these symbols are known only to humans. Instead, they used horizontal and vertical lines: — and |. Here are some sequences given in binary using the symbols — and |.

   Determine whether — or | represents 1. Justify your answer.

   Describe each sequence and predict, using — and |, the next three terms.

   a. | , — — , — — — , — — — — , — — — — —
   b. | , | , | — , — | , — — | , — — — | , — — — — |
   c. | , || , | — — | , ||—|| , |—|— — — |

3. The scientists also encoded some information about our solar system. They used information about quantities that can be counted, rather than measured, because measurements require units whereas counting does not. One quantity that can be counted is the number of ‘days’ in each planet’s year. Earth has (roughly) 365 Earth days per year because that is how many times it rotates around its own axis while orbiting the sun. The table gives some information about the planets in our solar system:

<table>
<thead>
<tr>
<th>Planet</th>
<th>Year length</th>
<th>Day length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>87.96 Earth days</td>
<td>1408 Earth hours</td>
</tr>
<tr>
<td>Venus</td>
<td>224.68 Earth days</td>
<td>5832 Earth hours</td>
</tr>
<tr>
<td>Earth</td>
<td>365.26 Earth days</td>
<td>24 Earth hours</td>
</tr>
<tr>
<td>Mars</td>
<td>686.98 Earth days</td>
<td>25 Earth hours</td>
</tr>
<tr>
<td>Jupiter</td>
<td>11.862 Earth years</td>
<td>10 Earth hours</td>
</tr>
<tr>
<td>Saturn</td>
<td>29.456 Earth years</td>
<td>11 Earth hours</td>
</tr>
<tr>
<td>Uranus</td>
<td>84.07 Earth years</td>
<td>17 Earth hours</td>
</tr>
<tr>
<td>Neptune</td>
<td>164.81 Earth years</td>
<td>16 Earth hours</td>
</tr>
</tbody>
</table>

   a. Calculate the number of Earth hours in a Mars year. Give your answer in denary.
   b. Hence find the number of Mars days in a Mars year. Give your answer in denary.
   c. Find (to the nearest whole number) the number of Mars days in a Mars year in binary.
   d. Find, in binary, the number of days in a year for each of the other outer planets (Jupiter to Neptune).
   e. Describe any problems you would encounter when trying to perform a similar calculation for Venus.