Global context:
In this unit, you will use linear systems to model relationships as you explore a relatively new idea, social entrepreneurship. As part of the global context of fairness and development, you will see how people are trying to make a difference in the lives of others through the companies they start and the products they sell. Along the way, these entrepreneurs will use tools, like linear systems, to make decisions that affect their ability to sustain their mission.

Statement of Inquiry:
Representing relationships with models can promote and support social entrepreneurship.

Objectives
In this unit, you will learn how to:
- Solve complex multi-step algebraic equations
- Represent and classify systems of linear equations
- Solve a system of linear equations using graphing, substitution and elimination
- Apply mathematical strategies to solve problems using a system of linear equations to help in decision-making.

Inquiry questions
- What is a linear system?
- What does it mean to “break even”?
- How do we represent relationships with models?
- Are models realistic?
- What is our responsibility to those in our community and other communities?
- How can I make a difference?
You should already know how to:

1. Find the degree of a polynomial:
   State the degree of each polynomial
   \(2x^2 + 3x - 5\) \(4x^3 - 5x^4 + x^2 - 8\)
   \(2x^2y + 6xy^2 - 7x^3y^2\)

2. Solve equations
   Solve the following equations:
   \(3(x - 2) + 5 = -4\) \(\frac{x}{2} - 3 = 1\) \(x^2 = 16\)

3. Draw linear functions
   Sketch the following lines:
   \(y = -2x + 7\) \(2x - 3y = 6\) \(4x + y - 2 = 0\)

4. Write the equation of a line in both standard form and gradient-intercept form.
   Write each equation in standard form and gradient-intercept form.
   \(4x - y = -12\) \(3x - 4 = -2y\) \(x = -5y + 9\)

5. Find \(x\)- and \(y\)-intercepts
   Find the \(x\)- and \(y\)-intercepts for each function in question 4.

6. Find the slope of a line using the formula.
   Determine the slope of the line through each pair of points:
   \((-1, 3)\) and \((2, 5)\) \((-2, -2)\) and \((3, -1)\)
   \((0, -7)\) and \((2, -1)\)

7. Find the slope of a line parallel and perpendicular to a given line.
   Find the slope of a line that is parallel to each of the lines in question 4.
   Find the slope of a line that is perpendicular to each of the lines in question 4.

8. Find the equation of a line.
   Find the equation of the line with the following characteristics:
   a. gradient of \(-2\) and \(y\)-intercept of 4
   b. gradient of \(\frac{1}{4}\) and a \(y\)-intercept of \(-8\)
   c. passes through the points \((-1, 2)\) and \((3, -2)\)
Introducing linear systems

What goes into the decisions that people make? Is cost always the most important factor or are there other elements that need to be considered? For instance, would you be willing to pay more for a product that was created using environmentally friendly materials? How important is it that the company you buy from uses a workforce that is well compensated and has good working conditions? Would you prefer to buy from a company that is using its profits to solve a social issue? Do you only support local companies?

In this unit you will analyse decisions by creating simple models that consider some of the important factors that are involved. These same types of models are used every day by individuals, companies and governments when they make decisions.

By the end of this unit, you may be ready to start your own company and dedicate your profits to a worthy cause of your choice!

Reflect and discuss 1

- When you shop for products, what factor(s) impact your final choice?
- Give an example of when you had a decision to make about which option was best where the result would impact someone else or a group of people. How did that impact your decision-making process?

Solving linear equations

Solving equations is a very important skill in mathematics. However, there are different kinds of equations, which may require different strategies to solve them.
In this unit, you will be solving linear equations. You may have noticed in the Investigation that each equation only contained one variable. When there is only one variable, a single equation is enough to find the value of that variable (or “solve the equation”). You have already learned strategies to solve different types of equations. You will apply those same strategies here to solve more complex equations.

Investigation – Types of equations

Perform the following investigation with a peer.

1 Fill in a table like the one below.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Degree</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3g + 2 = 12$</td>
<td></td>
<td>linear</td>
</tr>
<tr>
<td>$4m^2 - 8m + 3 = 0$</td>
<td></td>
<td>quadratic</td>
</tr>
<tr>
<td>$\frac{4x - 5}{7} = 9$</td>
<td></td>
<td>linear</td>
</tr>
<tr>
<td>$c^3 - 4c = 2$</td>
<td></td>
<td>cubic</td>
</tr>
<tr>
<td>$12 - 4w^3 + 2w^2 - w = 2$</td>
<td></td>
<td>cubic</td>
</tr>
<tr>
<td>$3y + 11 = -7y^2$</td>
<td></td>
<td>quadratic</td>
</tr>
<tr>
<td>$6z - 19 = 2(3z - 4)$</td>
<td></td>
<td>linear</td>
</tr>
<tr>
<td>$5p^3 - 2p + 4 = -2p^2 + p^3 - 8p$</td>
<td></td>
<td>cubic</td>
</tr>
<tr>
<td>$7h^2 - 9h = -12(3h + 2)$</td>
<td></td>
<td>quadratic</td>
</tr>
</tbody>
</table>

2 Based on your results, define the following terms:
   - Linear equation
   - Quadratic equation
   - Cubic equation

3 What do you think a “quartic equation” is? Give an example.

4 Does the choice of variable (x or m, etc.) impact the classification of the equation? Explain.
Example 1

Solve the following equations.

\[ a \quad \frac{x}{6} = 15 - \frac{2x}{3} \quad b \quad x^2 + 75 = 1000 \quad c \quad \frac{2(x + 7)}{5} = 2 + 6 \]

**Q** Solve the following equations.

**a** \[
\frac{x}{6} = 15 - \frac{2x}{3}
\]

\[
\frac{6}{6} \times \frac{x}{6} = \frac{6(15)}{6} - \frac{2 \times (2x)}{6}
\]

\[
x = 90 - 4x
\]

\[
x + 4x = 90 - 4x + 4x
\]

\[
\frac{5x}{5} = \frac{90}{5}
\]

\[
x = 18
\]

**b** \[
x^2 + 75 = 100
\]

\[
x^2 + 75x - 75 = 100 - 75
\]

\[
x^2 = 25
\]

\[
\sqrt{x^2} = \sqrt{25}
\]

\[
x = \pm 5
\]

**c** \[
\frac{2(x + 7)}{5} = 2x + 6
\]

\[
\frac{5 \times 2(x + 7)}{5} = 5(2x + 6)
\]

\[
2x - 2x + 14 = 10x - 2x + 30
\]

\[
14 - 30 = 8x + 30 - 30
\]

\[
\frac{-16}{8} = \frac{8x}{8}
\]

\[
x = -2
\]
Practice 1

1 Solve each equation.

\[3(x + 4) = -3x\]
\[\frac{5x}{2} = 4x + 3\]
\[x - 4 = \frac{x}{4} + 20\]
\[\frac{4(x - 5)}{5} - 3x = x - 36\]
\[\frac{2x + 3}{6} - \frac{x}{4} = 1\]
\[\frac{x - 5}{4} = 13 + \frac{3 - 4x}{9}\]
\[6x + 6 = \frac{4x + 6}{3}\]
\[\frac{2x^2 + 100}{5} = -407\]
\[\frac{4x - 9}{3} - \frac{5x + 12}{6} = 3x\]
\[\frac{2}{3}x - \frac{4}{3} = \frac{1}{2}x - \frac{3}{2}\]

2 Create a linear equation that requires multiple steps to solve where the answer is \(-4\).

3 Create a fractional linear equation that requires multiple steps to solve where the answer is 2.

4 Create a quadratic equation that requires multiple steps to solve where the answers are 7 and \(-7\).
Solving systems of linear equations

You know how to solve equations with one variable, but what if there is more than one unknown? What if there are two variables? A system of equations is a group of equations used to solve for each of the variables in it. With two variables, you will need at least two equations in order to find the value of both variables. If these two equations are both linear, you have what is called a system of linear equations or simply a linear system. You can solve this linear system using a variety of methods, the focus of this unit.

Solving linear systems by graphing

One of the methods that can be used to solve a linear system is graphing. Here, you will apply the skills you learned related to graphing lines on the coordinate plane, though now there will be two lines instead of just one.
Activity – Graphing linear systems

1. On the same coordinate grid, draw the following lines
   \[ y = 2x - 4 \] and \[ y = -3x + 1 \]

2. Why do these lines intersect? Justify your answer with mathematics.

3. Find the coordinates of the point where the two lines meet (the point of intersection).

4. Verify by substituting the coordinates into the original equations that the point of intersection is on both lines.

5. Explain why the point of intersection solves the linear system.

6. Repeat steps 1 through 5 using the following equations
   a. \[ y = -5x + 8 \] and \[ 2x + y = 2 \]
   b. \[ y = 3 - 2x \] and \[ 4y - 7 = -6x \]

Reflect and discuss 3

Answer the following questions in pairs:

- What are the different ways of graphing a line such as \[ 3x - 6y = 12 \]? Which method do you prefer?
- Describe any disadvantages you see to solving a system of linear equations by graphing.
- Why is it a good idea to verify your answer when solving a linear system? Explain.
- Is it possible for a linear system to have no solution? Explain.

Web link

If you do not have access to a graphic display calculator (GDC), you can use an online graphing tool such as Desmos to graph each of the lines. Visit www.desmos.com to use this graphing utility. Be sure to write your equations in gradient–intercept form.
Activity – Profit or loss?

A local artist in Columbia makes macramé cotton hammocks. The cost for the equipment needed to make the hammocks is $500 and the cost of materials for each hammock is $25. He decides to take out a microloan from a bank like Grameen Bank in the hopes of starting a business that can support his family. The selling price of a hammock is $70.

Let “y” be the amount in dollars and “x” be the number of hammocks sold.

1 Revenue is the amount of money a company takes in from sales of a product. Explain why the revenue can be given by the equation $y = 70x$.

2 Explain why the cost of making the hammocks can be given by $y = 25x + 500$.

3 Solve the linear system by graphing each line.

4 The point of intersection of a system that represents a company’s cost and revenue is called the break−even point. Why do you think this is?

5 How many hammocks does the artist have to produce before he begins to make a profit (in other words, his revenue is more than his costs)? Explain your answer.

6 The same hammock on the North American market would cost the customer $120? Do you think that is fair? Explain.
Practice 2

1 Rewrite each equation in gradient–intercept form:

a $2y = -4x + 8$

b $5x - 10y = 20$

c $0 = 2x - 3y + 9$

d $\frac{1}{2}y - 3x = 5$

e $\frac{2}{3}y = 2x + 6$

f $3y = \frac{1}{2}x - 12$

g $5x = 10y - 20$

h $x = 8y$

i $\frac{1}{2}x = 3y - 2$

j $\frac{2}{3}y - 8x = -6$

2 Find the $x$– and $y$–intercept of the following linear equations

a $3y + 4x = 20$

b $5x - 10y - 30 = 0$

c $0 = 2x - 4y + 12$

d $\frac{y - 3x}{2} = 6$

e $\frac{1}{4}y = 2x + 4$

f $4y = \frac{1}{2}x - 12$

3 Solve each linear system by graphing. Verify your answer.

$y = 2x - 6$ and $y = x + 3$

$y = -x + 2$ and $y = 5x - 4$

$y = -\frac{1}{2}x - 4$ and $y = \frac{1}{3}x + 1$

$y = -2x - 10$ and $y = \frac{3}{2}x - 3$

$y - 7 = 6x$ and $3y + 12x - 6 = 0$

$3x + 2y = 16$ and $2x + 3y = 14$

$5y - 35 = -5x$ and $0 = \frac{3}{2}x - 3 - y$

4 A video streaming company is putting together their pricing strategy and is considering a couple of new promotions. Two of them involve a loyalty programme where some of the proceeds will go to providing internet access to local students who do not have the financial resources to afford the internet. The current average rental cost is $6 per movie. With the loyalty program, you can either download as many movies as you want for a $35 monthly fee or you can pay $10 a month to join the loyalty programme and then movies are just $4 per rental.

Continued on next page
a Create a table of values for each of the three options that shows the cost of renting from 1 to 5 movies.

b Graph these options on the same set of axes.

c Based on a marketing survey it was determined that, on average, customers download 5 movie rentals each month. Which promotion is the better deal at this level of rentals? Explain.

d Write a general statement about when each option is more economically attractive to the customer depending on the number of movie rentals downloaded.

e How could such promotions increase a company’s ability to raise money for their cause? Explain.

Number of solutions to a linear system

The systems of linear equations that you have solved so far all produced one solution. Is it possible for there to be more than one answer? Could there ever be no solution? Graphing systems is an efficient way of investigating the answer to these questions.

Investigation – Classifying systems of equations I

Here are three linear systems:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$y = 2x + 1$</td>
<td>$2y + 8 = x$</td>
<td>$6x = 20 - 2y$</td>
</tr>
<tr>
<td></td>
<td>$y = -x + 7$</td>
<td>$4y - 2x = -16$</td>
<td>$y + 3x - 5 = 0$</td>
</tr>
</tbody>
</table>

1 Graph each pair of lines on a separate set of axes. With the permission of your teacher, you may use technology if you wish.

2 How many solutions does each linear system have? Explain your answers.

3 Research the names used to classify the three types of linear systems.
4 Is it possible to determine how the lines will intersect before actually trying to find the solution by graphing? Rewrite all the above equations in the form \( y = mx + b \) and fill in the table below.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear system in ( y = mx + b ) form</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope of the lines</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y )-intercept</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of solutions</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5 Write a brief summary of the results of your investigation into the three types of linear systems.

6 Verify your results for at least two more linear systems.

7 Justify why your results make sense.

---

**Practice 3**

Answer question #1 in pairs. If someone needs help, ask leading questions instead of giving answers.

**ATL2** 1 Without graphing, classify each linear system based on the number of solutions. Justify your answer using mathematics.

\[
\begin{align*}
    y & = 3x - 5, \ 2x - 6y = 8 \\
    4x & = 6y + 1, \ 9y = 6x - 5 \\
    y & = \frac{1}{3}x - \frac{5}{6}, \ 6y - 2x + 5 = 0 \\
    4x - 2y & = 14, \ 12x - 6y = 40
\end{align*}
\]

\[
\begin{align*}
    y & = 2 - \frac{3}{4}x, \ 3x + 4y = -2 \\
    12 - 2x & = 4y, \ 5x + 10y - 30 = 0 \\
    x & = -1, \ y = 3 \\
    -1x + 2y & = 5 - y, \ -x + 3y = 5
\end{align*}
\]

Continued on next page
2 Given the following system:
\[ y = 2kx + 6 \]
\[ 4x + y = 2 \]

a Find the value of k that makes the linear system have no solution.

b Find the value of k that makes the linear system have infinite solutions.

3 The YMCA is a non-profit, worldwide organization operating facilities in 119 countries. YMCAs differ from country to country and offer vastly different programs in response to local community needs. Local YMCAs raise money so that they can engage in a wide variety of charitable activities, including giving classes and mentorship to young people who want to become social entrepreneurs.

A local YMCA has a membership offer of a $90 one-time initiation fee and $20 per month for a youth. Instead, a day pass (which costs $5 each day) can be bought to use the facilities.

a Set up a table of values to represent both options.

b Graph the system and check your answer.

c If you think you will go 6 times a month, how long will it take for the membership to pay off and be the better option?

d If you ended up only going once a week (4 times a month), would getting a membership be a good idea? Explain.

e Can you think of a reason why you would only use the day pass and not sign up for a membership?
Sometimes the YMCAs run promotions and offer a significantly lower initiation fee. What if the initiation fee was dropped to $1 and you plan on going once a week. Would you sign up for the membership then? Justify your answer by solving the system of equations.

How would the problem have to be modified in order for there to be no solution?

Given the system of equations: \(3x + 2y = 2\) and \(-6x + 4y = -1\).

Classify this system. Justify your answer.

Verify your classification by graphing the system.

Change one term in the equation \(-6x + 4y = -1\) so that the system has an infinite number of solutions. Explain why this works.

Show that your answer is correct by graphing the system with your new equation.

Create a linear system where there is no solution but this is not immediately obvious.

Create a linear system where there are infinitely many solutions but this is not immediately obvious.

While graphing gives you a good visual representation of the linear system, answers that are decimals or fractions may be difficult to determine. Sometimes it is easier and more efficient to solve the linear system algebraically. There are two methods commonly used – substitution and elimination.

Solving linear systems by substitution

Activity – The substitution method

In pairs, complete the following activity:

1. Given the linear system

\[
\begin{align*}
3x - 2y &= 8 \\
y &= -2x + 3
\end{align*}
\]

a. Substitute the expression for \(y\) from equation (2) into the “\(y\)” in equation (1).

b. Solve this linear equation for “\(x\)”.

Continued on next page
c Substitute the value for “x” in equation 2 to find the value of “y”.

d Write your answer as an ordered pair, (x, y).

2 Given the linear system:

\[4x + y = -2, \ -2x - 2y = 4\]

a Which variable would be easiest to isolate in either equation? Explain.

b Isolate one of the variables in one of the two equations.

c Substitute the expression for this variable in the other equation, as seen previously.

d Solve the linear equation.

e Find the value of the missing variable.

3 Summarize how to solve a linear system using the method of substitution.

**Reflect and discuss 4**

In pairs, answer the following:

- What made equation 2 easy to substitute into equation 1?
- What would you have to do if equation 2 were written in standard form \((Ax + By + C = 0)\)?
- What disadvantages do you see to the substitution method? Explain.

**Example 2**

Solve the following linear system using the substitution method.

\[4x - 3y = -11, \ -5x + 2y = 12\]

\[-5x + 2y = 12\]

\[\begin{align*}
+5x & \quad +5x \\
2y & = 5x + 12 \\
\end{align*}\]

\[\frac{2y}{2} = \frac{5x + 12}{2}\]

\[y = \frac{5}{2}x + 6\]

Isolate one of the variables in one of the equations.

Continued on next page
\[
\begin{align*}
4x - 3 \left( \frac{5}{2} x + 6 \right) &= -11 \\
4x - \frac{15}{2} x - 18 &= -11 \\
2(4x - \frac{15}{2} x - 18) &= -11 \\
\end{align*}
\]

Substitute the expression into the other equation to solve for the other variable.

Use the distributive property to expand the brackets.

Multiply each side of the equation by 2 to eliminate all of the denominators.

Solve for \(x\).

Substitute the value of \(x\) in the equation for \(y\) to find the value of the other variable.

Verify your answer in each of the original equations.

Write the solution as an ordered pair

\[
\begin{align*}
8x - 15x - 36 &= -22 \\
-7x - 36 &= -22 \\
\text{+36} &\text{+36} \\
-7x &= 14 \\
x &= -2 \\
\end{align*}
\]

\[
\begin{align*}
y &= \frac{5}{2}(-2) + 6 \\
y &= 1 \\
4(-2) - 3(1) &= -11 \\
-11 &= -11 \\
-5(-2) + 2(1) &= 12 \\
12 &= 12 \\
\end{align*}
\]

The solution is \((-2, 1)\)
Activity – The Paradigm Project

According to the World Health Organization, approximately 4 million women and children die every year from lower respiratory diseases related to indoor cooking smoke. Globally, it is the number one cause of death in children under 5 years of age. The Paradigm Project is a social entrepreneurial organization that is trying to address this serious issue by supplying clean-burning cook stoves to people in developing countries.

A family living in a poor rural area in Guatemala is making $5 per day. They use 30% of their income to pay for cooking fuel each day. If they buy a clean burning stove, they would only use 15% of their income on cooking fuel. How many days will it take for the stove to pay for itself with the money they saved on cooking fuel?

1. Complete a table like the following:

<table>
<thead>
<tr>
<th>Number of days</th>
<th>Total cost of fuel (no stove)</th>
<th>Total cost of fuel (including stove)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>$35</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Graph each relationship and solve the linear system.

3. Determine the equation of each line, hence create a system of linear equations.

4. Verify your answer by solving the system by substitution.

5. What are the other benefits of the stove top, apart from the fuel efficiency savings?

6. Once the stove is paid off, what else do you think the money saved on fuel would be spent on?

7. How does the stove improve the overall quality of life for the family?
Practice 4

1. Solve the following linear systems using substitution. Verify each answer.
   
   \[
   \begin{align*}
   4x + y &= 1, \quad 2x + 3y &= -7 \\
   5x - y &= -1, \quad -10x + 3y &= 4 \\
   5x - 2y &= 6, \quad x = 3y - 4 \\
   x - 7y &= 0, \quad 3x - 4y &= 17 \\
   2x - 5y &= 6, \quad -3x + 4y &= -6 \\
   x &= -5, \quad 4x + 2y = -18
   \end{align*}
   \]

2. Solve the following systems by graphing and by substitution. Verify each answer.
   
   \[
   \begin{align*}
   y &= 6x - 5, \quad y = 3 - 2x \\
   5x - 3y &= -2, \quad -3x + 7y = 9 \\
   y &= -2, \quad -5x + 6y = -2 \\
   4x - y &= -8, \quad 2x + 3y = 10 \\
   3x + 7y - 1 &= 0, \quad 2y = 8 + 3x \\
   6x + 4y &= 14, \quad 5x - 2y = 9
   \end{align*}
   \]

3. You are starting a business that sells cell phone covers in the hopes of donating money to a local charity. Your start-up costs for the business are $1300 and each cell phone cover costs $30 to manufacture. You sell the phone covers for $60.
   
   a. Explain why the equations \( y = 60x \) and \( y = 30x + 1300 \) represent the revenue and cost function.
   
   b. Solve the system by substitution in order to find how many covers you must sell to break even.
   
   b. Find how much revenue is made at the break-even point.
   
   c. When will you start to make a profit? Explain.

4. Which method do you prefer, graphing or substitution, or does it depend on the question? Explain.

Solving linear systems by elimination

As you have seen, solving a linear system by substitution can be very efficient, especially when it is fairly straightforward to isolate one of the variables. The method of elimination can also be used to solve any linear system though it does not rely on isolating a variable.
Activity – Equivalent equations

1. Given the equation \(4x - 2y = 8\),
   a. Multiply the equation by 2.
   b. Multiply the equation by \(-3\).
   c. Divide the equation by \(-2\).
   d. Explain why the equations you created in a) through c) are equivalent to the original.

2. Given the equation \(-4x + 5y = -7\),
   a. Multiply the equation by an integer so that the co-efficient of “\(x\)” is 12.
   b. Multiply the equation by an integer so that the co-efficient of “\(y\)” is \(-25\).
   c. Multiply the equation by an integer so that the co-efficient of “\(x\)” is 4.
   d. By what number could you multiply the equation so that the new equation is NOT equivalent to the original? Explain.

The method of elimination uses the concept of equivalence in order to solve a linear system. Creating equivalent equations allows you to manipulate them more easily.

Activity – Elimination method

Perform the following Activity in pairs.

Your school is putting on an event to support the upcoming fundraising walk. At the event, you can get the \(\frac{2}{1}\) combo (2 slices of pizza and a drink) for \$5.20. You can also buy the \(\frac{6}{2}\) combo (6 slices and 2 drinks) for \$14.40.

How much does one slice of pizza cost? How about one drink?

In pairs, answer the following questions:

1. Starting with the more expensive combo, what happens if you subtract the first combo from it? How many slices of pizza and how many drinks are left and what is their cost? Explain your answer.

2. Repeat the same process, subtracting the 2/1 combo from your result in step 1.
3 What does this tell you? What is the price of a slice of pizza? What is the price of a drink?

4 Represent the original combos with a system of equations. Use the variable “p” to represent the number of slices of pizza and “d” to represent the number of drinks.

5 Multiply the combo 1 equation by 2 and then subtract this equivalent equation from the combo 2 equation.

6 How does this relate to what you did in steps 1 and 2? Why do you think this method is called elimination?

7 Your original system of equations was 

\[ 2p + d = 5.20 \]
\[ 6p + 2d = 14.40 \]

8 Multiply the first equation by \(-3\) and then add it to the second equation.

9 What are the similarities and differences between step 5 and step 8? Explain.

10 Find the value of “d”. Verify your solution in both original equations.

Given the linear system

\[ 2x - 5y = 12 \]
\[ 2x + 20y = -18 \]

11 How could you combine the two equations to eliminate the variable “x”? Explain.

12 How could you combine the two equations to eliminate the variable “y”? Explain.

13 Solve the system by elimination. Verify your answer.

14 Summarize the process of solving a linear system by elimination.

**Reflect and discuss 5**

- You can either add or subtract equations when you use the method of elimination. How does your choice of operation affect the value by which you multiply the equation(s)? Explain.

- Which one will result in making fewer mistakes, adding or subtracting equations? Why?

- Once you have the value of the first variable, you need to find the value of the other one. What options do you have to find the value of the second variable?
Example 3

Solve the following linear system using the elimination method.

\[
\begin{align*}
2x - 3y &= 1 \\
3x + 5y &= \frac{3}{2}
\end{align*}
\]

(A) \((2x - 3y = 1) \times 3\)

\[
\begin{align*}
(3x + 5y &= \frac{3}{2})x - 2
\end{align*}
\]

\[
\begin{align*}
6x - 9y &= 3 \\
-6x + 10y &= -3 \\
y &= 0
\end{align*}
\]

\[
\begin{align*}
2x - 3(0) &= 1 \\
2x &= 1 \\
x &= \frac{1}{2}
\end{align*}
\]

\[
\begin{align*}
2\left(\frac{1}{2}\right) - 3(0) &= 1 \\
1 - 0 &= 1
\end{align*}
\]

\[
\begin{align*}
3\left(\frac{1}{2}\right) + 5(0) &= \frac{3}{2} \\
\frac{3}{2} &= \frac{3}{2}
\end{align*}
\]

The solution is \(\left(\frac{1}{2}, 0\right)\).
Activity – Benefit concerts

A benefit concert is a musical performance in which musicians use their celebrity status to help raise money and awareness for a particular humanitarian crisis, either local or abroad. Some large and famous concerts have been Live Aid (1985), Conspiracy of Hope Tour (1986), The Concert for New York City (2001), The SARS Benefit Concert (2003), Live 8 (2005) and Live Earth (2007), One Love Manchester (2017) and Hand in Hand: A Benefit for Hurricane Relief (2017).

a Research these big concerts to see what they were raising money and/or awareness for.

Your school has an ongoing connection with an orphanage in a less developed country. You would like to organize a concert to raise money to help build a school for the children in the orphanage. You have managed to get some great local bands to play and they have agreed to do it for 20% of each ticket sold. The rental of the auditorium and crew is $4000. You are going to sell tickets to the event for $20.

b Write down an equation to represent the revenue from the concert in terms of the number of tickets sold. Justify your equation.

c Write down an equation to represent the cost of putting on the concert in terms of the number of tickets sold. Justify your equation.

d Solve the revenue–cost system by graphing and find the break–even point.

e Solve the system by elimination.

f The auditorium has a capacity of 1000 people and your goal is to raise a total of $10,000. Is this going to be possible? If so, how many tickets will you need to sell? Show your working.
Practice 5

1 Find the point of intersection for the following linear systems using the method of elimination.

\[
\begin{align*}
2x - 5y &= 9 \\
3x + 2y &= 4
\end{align*}
\]

\[
\begin{align*}
5x - 3y &= 5 \\
3x + 2y &= 4
\end{align*}
\]

\[
\begin{align*}
x + 5y &= -2, -3x - 6 &= 15y
\end{align*}
\]

\[
\begin{align*}
6x + 9y &= 12 \\
-4x - 6y &= -8
\end{align*}
\]

\[
\begin{align*}
3x - y - 11 &= 0 \\
x + 2y &= 6
\end{align*}
\]

\[
\begin{align*}
x - 3y &= 5 \\
3x + 2y &= 4
\end{align*}
\]

\[
\begin{align*}
8x + 5y &= 2 \\
5x + 2y &= 8
\end{align*}
\]

\[
\begin{align*}
4x - 8y &= -3, -10x + 20y &= 12
\end{align*}
\]

\[
\begin{align*}
3x - y &= 11 \\
x + 2y &= 6
\end{align*}
\]

\[
\begin{align*}
x - \frac{y}{3} &= 1 \\
x - \frac{y}{4} &= 2
\end{align*}
\]
Problem solving with linear systems

When decisions need to be made based on a variety of factors, they can often be modelled using linear relationships. Knowing how to solve a linear system can help you to make an informed choice.

2 a Verify three of the linear systems in question 1 by graphing.

b How did you decide which linear system to verify using the method of graphing? Explain.

3 a Verify three of the systems in question 1 using the method of substitution.

b How did you decide which linear system to verify using the method of substitution? Explain.

Reflect and discuss

Discuss the following with a peer:

● Explain when you would use the graphing method to solve a linear system.

● Explain when you would use substitution to solve a linear system.

● Explain when you would use elimination to solve a linear system.

● Make a flow chart of the different strategies you would use depending on the information given. Be sure to include which graphing method to use when graphing a linear system (intercept and gradient, find \(x\)- and \(y\)-intercepts, find two points).
Activity – Creating a system of equations

Translating information into a system of equations is an important first step in solving a problem. With a partner, fill in a table like the following.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>The sum of two numbers is 1211 and their difference is 283.</td>
<td>Let $S$ represent the smaller number. Let $L$ represent the larger number.</td>
<td>$S + L = 1211$ [L - S = 283]</td>
<td></td>
</tr>
<tr>
<td>Four times the mass of a baseball is 16g less than the mass of a basketball. The sum of their masses is 756g.</td>
<td>Let $b$ represent ____ Let $B$ represent ____</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two hamburgers and a drink cost $5.00. Three hamburgers and two drinks cost $7.90.</td>
<td>Let ____ represent ____ Let ____ represent ____</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A rectangle with a perimeter of 28m has a length which is 2m less than triple its width.</td>
<td>Let ____ represent ____ Let ____ represent ____</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The Tritons made a total of 42 shots in a basketball game. They scored a total of 96 points through a combination of 2-point and 3-point shots.</td>
<td>Let ____ represent ____ Let ____ represent ____</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Seven times the smaller of two numbers plus nine times the larger number is 178. When ten times the larger number is increased by eleven times the smaller number, the result is 230.</td>
<td>Let ____ represent ____ Let ____ represent ____</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Organizing the information in problems is sometimes made easier with the use of a table.

Example 4

Q You find $3.25 in 5–cent and 10–cent coins in your piggy bank. If you counted 46 coins in total, how many of each coin do you have?

A Let $x$ represent the number of 5 cent coins. Let $y$ represent the number of 10 cent coins.

<table>
<thead>
<tr>
<th>Number</th>
<th>Value (cents)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5–cent coin</td>
<td>$x$</td>
</tr>
<tr>
<td>10–cent coin</td>
<td>$y$</td>
</tr>
<tr>
<td>TOTAL</td>
<td>46</td>
</tr>
<tr>
<td></td>
<td>325</td>
</tr>
</tbody>
</table>

$x + y = 46$
$5x + 10y = 325$

$x + y = 46$
$y = 46 - x$

$5x + 10y = 325$
$5x + 10(46 - x) = 325$
$5x + 10(46) - 10x = 325$
$-5x + 460 = 325$
$-5x = -135$
$x = 27$

$x + y = 46$
$27 + y = 46$
$y = 19$

$27 + 19 = 46$
$5(27) + 10(19) = 325$

You have 19 ten cent coins and 27 five sent coins in your piggy bank.
Example 5

You invested $12,000 into two types of bonds. One bond yields 8% interest each year and has minimal risk. The other bond yields 12% interest each year but more risk. If you earned $1160 in a year, how much did you invest in each bond?

Let $x$ be the amount invested at 8%
Let $y$ be the amount invested at 12%

Note: risk is not factored into the question at all.

<table>
<thead>
<tr>
<th>Amount invested</th>
<th>Interest</th>
<th>Amount earned</th>
</tr>
</thead>
<tbody>
<tr>
<td>8% $x$</td>
<td>0.08</td>
<td>0.08($x$)</td>
</tr>
<tr>
<td>12% $y$</td>
<td>0.12</td>
<td>0.12$y$</td>
</tr>
<tr>
<td>Total $12000$</td>
<td>$1160$</td>
<td></td>
</tr>
</tbody>
</table>

$x + y = 12000$
$0.08x + 0.12y = 1160$

$0.08x + 0.08y = 960$
$0.08x + 0.12y = 1160$

$0x - 0.04y = -200$
$y = 5000$

$x + y = 12000$
$x + 5000 = 12000$
$x = 7000$

$7000 + 5000 = 12000$

$0.08(7000) + 0.12(5000) = 1160$
$560 + 600 = 1160$

You invested $5000 in the higher risk bond (12%) and $7000 in the lower risk bond (8%).
Can you think of a real life situation that could be modelled using a linear system in which there is no solution? Infinite solutions?

Reflect and discuss 7

Task – Go big?

A coffee farmer in Costa Rica currently has a farm that produces 6000 pounds of coffee per year. It costs him 80 cents per pound to grow and harvest the coffee with $1000 of fixed costs in land management per year. He can sell his green, unroasted beans for a guaranteed $1.50 per pound.

a Create a linear system model for this scenario.

b Solve the system using two of the three methods (graphing, substitution, elimination) in order to calculate the breakeven point.

The farmer sees the coffee being sold to consumers for $8 per pound and is considering getting into the rest of the supply side of the business. A social entrepreneur said she would help him start up a co–op where they will roast, package and export the coffee direct to the customer. The costs associated with the entire process are $5 per pound and the fixed costs for all of the extra processing and exporting would be substantially more at $8000 per year.

c Create a linear system model for this new scenario.

d Solve the system using the method you did not use before in order to calculate the breakeven point.
Practice 6

Answer the first 3 questions in pairs. When someone encounters difficulty, offer leading questions instead of answers.

1 Ann has $300 made up of $5 and $10 bills. If there are 39 bills in all, how many $5 bills does she have?

2 The parking machine contained $3.05 made up of dimes and quarters. There were 20 coins in all. How many dimes were there?

3 Saira invested $1000, part at 8% per annum and the remainder at 9% per annum. After one year, her total interest from these investments was $84. How much did she invest at each rate?

4 Plan international offer gifts of hope options where you select a gift to support a community in a developing country. Most of the gifts are matched to provide more funding for communities in need. It costs $500 to equip a schoolroom and $295 to send a girl to school. How many classrooms did you supply equipment for and how many girls did you send to school if there were 20 gifts that cost $6515.

Verify your answer using one of the other two methods.

Determine the profit function for both scenarios. At 6000 pounds per year, what would his income be in both scenarios?

Based on your analysis, what would you recommend that he do? Justify your answer.

Why is there such a large difference between these incomes?

What are the advantages and disadvantages of starting the co–op?

Continued on next page
5 A professional soccer player is negotiating her contract. Using the advice of her manager, she asked for $800 000 for the year, plus an additional $1500 for every game she starts. The team offered $6000 for every game played and $600 000 for the year. How many games would she need to start to earn more money on the team’s offer?

6 Supplying a sheep to a family costs $50 and a bee hive costs $35. If there were 24 gifts totalling $990, how many of each were bought? What would be the benefit of supplying a sheep or bee hive to a family?

7 Jen invested $8000, part at 9% and the rest at 10%. The interest after one year was $740. How much did she invest at each rate?

8 Two social events have been organized by the local scout group to raise funds for a well in Tanzania. During the first night, 25 children and 20 adults attended and the revenue for that evening was $150.00. On the second night there were 30 children and 22 adults, with revenues of $170.00. How much would each adult have paid to attend this social?

9 Many companies today are moving toward more ethical supply chains. Costco Wholesale Corporation has a global supplier Code of Conduct which prohibits human rights abuses in their supply chain, such as human trafficking, physical abuse of workers and unsafe work environments. To ensure the code of conduct is being enforced, they may audit the facilities of suppliers, especially those in countries that are more at risk of such violations. A basic Costco membership in the United States is $60 per year, while the premium membership is $120 per year. If you get 2% back on Costco purchases with the premium membership, how much money would you have to spend at Costco in a year before that is the better membership for you to buy? Set up a linear system to show the membership options.

10 A contract employed 5 adults and 3 teenagers for one day and paid them a total of $224 per hour. The following day he employed 3 adults and 5 teenagers for $160 per hour. What was the hourly rate paid to each adult and teenager?

11 Over one billion people around the world live where electrical grid service is unreliable and at least 1 billion more live completely out of range of the grid. A social entrepreneur manufactures solar lanterns for which the material cost is Rs 350 per lantern and fixed costs are Rs 800 000 per month.

a If each lantern sells for Rs 475 (costs are kept low to ensure the people who really need them can buy them), find how many lanterns must be made and sold each month for the company to break even.

b Convert this amount into your local currency.
12 Find the equation of the line, in standard form, that passes through \((-7, 3)\) and the point of intersection of \(4x - y = 3\) and \(2x + y = 9\).

13 A rectangle has vertices at \(A(2, 2), B(2, 6), C(4, 6),\) and \(D(4, 2)\). Develop an **algebraic** solution to determine the coordinates of the point of intersection of the diagonals.

14 Thailand is one of the world’s top rice producers. For a typical rice farm, the fixed costs are $16,000THB per growing season and the variable costs are $12,000THB per hectare. Rice is sold for $19,500THB per hectare.

**a** Set up a linear model to find how many hectares you would need to break even.

**b** If the average size of a farmer-owned rice farm is 6 hectares, what profit are they making?

**c** Convert this amount to your local currency. Do you think that is enough money to live on for the whole growing season? Explain.

**d** Suppose the price of rice is trending downward. The selling price of rice is set by the global market and the local small farmer has no control over fluctuations in price, which have ranged between increases of 10% to decreases of 10% monthly, with overall trends of a decrease in price over the past 5 years of approx 15%. If the selling price drops by 15%, what will the new break even point be? How will this affect local farmers?

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**Formative assessment – Sustainable Denim**

More companies are starting corporate social responsibility (CSR) reporting, sometimes referred to as the **triple bottom line** (triple for “people, planet and profit”). It is when companies take the initiative to ensure their supply chains are ethically and environmentally responsible and that they have positive interactions with the communities in which they operate.

Unfortunately profit is still the driving force of many companies and in the search to cut costs, employees, especially those in developing countries, may work in dangerous and unhealthy conditions. Pollution in manufacturing and shipping can be extensive and the Earth’s natural resources can be used in an unsustainable way.

> Continued on next page
Case Study:

Two premium denim jean factories in Asia have the following costs per month. The first one is a socially responsible factory and the second is a traditional factory that is still common in Asia.

<table>
<thead>
<tr>
<th>Cost</th>
<th>Socially responsible factory</th>
<th>Traditional factory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Materials per pair of jeans</td>
<td>$12*</td>
<td>$7</td>
</tr>
<tr>
<td>Wash/dye process per pair of jeans</td>
<td>$2**</td>
<td>$0.25</td>
</tr>
<tr>
<td>Labour cost per pair of jeans</td>
<td>$1.50***</td>
<td>$0.50</td>
</tr>
<tr>
<td>Rental of building, utilities, admin costs, programmes etc.</td>
<td>$10,000****</td>
<td>$5800</td>
</tr>
</tbody>
</table>

* All raw materials that go into making the denim jeans meet the highest ethical and environmental standards.

** The factory uses organic cotton and a water recycling system to reduce environmental impact and not contaminate local community water supplies. It also uses non–toxic organic vegetable dye to ensure no harm to the workers, as opposed to harmful chemical dyes used in traditional jean manufacturing.

*** Workers earn a living wage, which is 3 times the average of workers in the industry, to better support themselves and their families and even budget to save for the future.

**** Fixed costs are higher due to better ventilation, light, space and general facilities in the factory. English language courses and training courses in all components of jean production require more time and every worker is rotated to ensure they are skilled in every component of jean making.

Build a Linear Systems Model:

1. Derive a cost function for both factories.
2. If the jeans are sold to the wholesaler for $30 a pair, determine a revenue function.
3. Graph both factory cost functions and the revenue function on one set of axes.
4. What is the break–even point for both factories? Comment on the difference between the two.
5. Verify your answer by solving the system algebraically. Show your working.
6. What selling price would the socially responsible factory have to sell a pair of jeans for if they wanted the same break–even point as the traditional factory? What would you round that to?
7. How much more is that per pair as a percentage?
Reflect & Discuss

8 Explain the degree of accuracy of your solution.

9 Describe whether or not your solution makes sense in the context of the problem.

10 Given the steps still needed to get these jeans to the end customer, including shipping, marketing and product placement in stores, the final selling price of the jeans could be up to 5 times the amount they are sold from factory door. What would the approximate final price be of both pairs of jeans?

11 Do you think companies should put sustainable supply chains in place, even though it decreases the profit? Explain.

Discuss in groups of 3:

12 Whose responsibility is it to ensure sustainable growth? Companies? Governments? The Consumer?

13 Each person takes the role of one of these key players and lists how they could help generate change. Who do you think is in the best position to generate change?

14 How could a factory with ethical practices such as the one described affect current and future generations?

15 If you were to write a headline that captured the most important aspect of this task and your discussion, what would that headline be? Share headlines with the class.

Activity – Urban planning (extension)

A 10 acre plot of forest has been designated to build a housing development due to the urban sprawl of a nearby city. One proposal comes from a new company that tries to keep its planned communities CO2 neutral. In other words, they want to develop the housing estate so that enough trees are left on the land to convert the CO2 of the people living in the estate. The problem they must solve is determining the number of houses they can build.

Factors to consider:
An average human exhales around 2.3 pounds of CO2 in a day.
The average tree can convert 48 pounds of CO2 per year.
There are approximately 700 trees per acre in the forested region currently.
The houses will have 3000 square foot plots.

Continued on next page
Process:
1. What units will you use? Explain.
2. Given that “x” will be the number of houses and “y” will be CO2 production per year, what does the slope of each line represent?
3. Assuming an average of 4 people living in each house, determine the linear equation to represent the people living in the housing development. Make sure you show all of your working to calculate the slope.
4. Is the slope positive or negative? Explain
5. What does the y–intercept represent?
6. Determine the linear equation to represent the trees living in the housing development. Make sure you show all of your working to calculate the slope.
7. Will the slope of this line be positive or negative? Explain.
8. What does the y–intercept represent in this scenario?
9. Graph both linear equations on the same set of axes, making sure to clearly show the point of intersection. This can be done using technology (graphic calculator or graphing software). If using technology, make sure you take screen shots of your solutions and copy them into your report to be handed in.
10. What does the point of intersection represent?
11. Verify your solution using an algebraic method. Show all working.
12. Given the number of houses that could be built and still remain ecologically viable, calculate the percentage of the original plot of land that would be used for housing and the percentage that will be left forested.
13. When housing developments are being built, are these percentages maintained? How are they different? Why do you think that is?

Changing the model
14. What would happen if you used 5 people living in each house as the average? Which line would change? How would it change? Determine the new linear equation and compare to the originals. How would that affect the point of intersection?
15. What would happen if the number of trees was only 500 per acre? Which line would change? How would it change? Determine the new linear equation and compare to the originals. How would that affect the point of intersection?
Unit summary

A linear equation is an equation of degree one.
A quadratic equation is an equation of degree two.
A cubic equation is an equation of degree three.
An equation generally has the same number of solutions as its degree.
A system of equations is a group of equations used to solve for each of the variables in it. With two variables, you will need at least two equations in order to find the value of both variables.
If the two equations are both linear, you have what is called a system of linear equations or simply a linear system.
Linear systems can be solved by graphing each of the lines and finding the point of intersection.
Linear systems can also be solved algebraically using substitution or elimination.
To solve a linear system by substitution, rewrite one of the equations to isolate one of the variables. Then, substitute this expression into the other equation. Solve this new equation.
To solve a linear system by elimination, multiply one or both equations so that the coefficients of “x” or “y” are exact opposites and then add the equations. This will eliminate the variable and then you can solve for the other one.
Always verify your answers by substituting into the original equations.
Linear systems can have no solution, one solution or infinitely many solutions.
Graphically, a linear system has no solution when the lines are parallel to each other. There are infinitely many solutions when you are given the same line twice.
Algebraically, there are infinitely many solutions when all of the variables are eliminated and you end up with a true equation, like 0 = 0 or –4 = –4.
Algebraically, there is no solution when all of the variables are eliminated and you end up with an equation that is not true, like 0 = 2 or –4 = 7.
Unit review

1. Solve for x and verify your answer.
   
   \[ 2x + 7 = 11 \]
   \[ 3(5 + 2x) = -21 \]
   \[ 5(x + 4) = 3(x - 3) + 5 \]
   \[ \frac{x + 3}{4} = \frac{x - 1}{2} \]
   \[ \frac{1}{2} y - 3 = \frac{2}{3} y + 4 \]
   \[ \frac{3}{4}(y + 2) = 2y - 11 \]

2. Sketch the following graphs and write down the coordinates of the point of intersection.
   
   \[ y = -3x + 10 \text{ and } y = 2x - 5 \]
   \[ y = 2x - 6 \text{ and } y = \frac{3}{4}x - 6 \]

3. Find the EXACT intersection point of the following two linear systems using technology.
   
   a. \[ y = x - 8 \text{ and } y = -\frac{1}{3}x - 4 \]
   b. \[ x = 5y - 75 \text{ and } x + \frac{1}{2}y - 7 = 0 \]

4. Solve the following linear systems algebraically:
   
   \[ y = 6x + 7 \text{ and } 3y + 12x - 6 = 0 \]
   \[ 3a + 2b = 16 \text{ and } 2a + 3b = 14 \]
   \[ 14x + 21 = -21y \text{ and } 2x + 3y = 12 \]
   \[ 2y - 4 = x \text{ and } 3y - 6x = -3 \]
   \[ 6x = 2y - 8 \text{ and } 5y - 5x + 10 = 0 \]

5. Classify each of the following linear systems and justify your answer.
   
   a. \[ 4x - 3y = 15 \text{ and } 8x - 9y = 15 \]
   b. \[ 4x - 3y = 5 \text{ and } 8x - 6y = 10 \]
   c. \[ 2x = 3y - 6 \text{ and } 4x - 6y = 24 \]
6 When the sum of four times a number and ten is divided by five, the result is negative 2. Find the number.

7 Two consecutive integers are both smaller than 20. If three times the smaller integers is equal to eight less than double the larger integer, find both integers.

8 A four-sided figure with a perimeter of 120 cm has a width which is 30 cm less than twice its length. What kind of quadrilateral is this? Justify your answer.

9 The difference between two numbers is then divided by 2 and the result is 3. Find these two numbers if triple the smaller number is 4 less than double the larger number.

10 You have three dollars’ worth of coins in your piggy bank consisting of nickels, dimes and quarters. If there are double the number of dimes than nickels and one third the number of quarters than nickels, how many coins are in your piggy bank in total?

11 You could use a music streaming service and pay $9.99 a month or pay a range of $0.99 to $1.29 for each song you download.
   a Graph this scenario using technology.
   b How many songs a month would you have to download to make this service a viable option?
   c Can you think of why you would choose the pay per song option instead, even if you downloaded 20 songs a month?

12 Some teachers are taking a group of Year 3 students to a talk on sustainable supply chains and hearing some social entrepreneurs speak. 42 tickets cost $582 in total. If each student’s ticket cost $12 and each teacher’s ticket cost $25, use an algebraic method to find the number of teachers and students going to the talk.
13 A micro-lender invested $20000 in a combination of two ventures, one of low risk yielding 3% per annum and one with higher risk yielding 8% per annum. If the interest after one year was $850, calculate the amount invested in each venture algebraically.

14 In some communities in the United States, local food trucks have started giving out meals to the homeless. While they still have delicious meals for sale, those that cannot afford it can get a meal for free. The cost to make each meal is $2.50 and it is sold for $8. The food truck costs $3,000 per month to operate. They must make a profit of at least $2000 to cover the costs of the free meals. What is the least number of meals they need to make and sell in a month (assume 30 days) to cover the cost of the free meals?

15 In some areas of the world, you may be living on land which you could drill your own gas. A farmer pays $5000 for natural gas each year. If you drill yourself, it costs $40,000 to start up your own gas line and you could drill enough to produce $7500 or gas each year and can sell the rest back into the grid. How many years would it take to pay off the drill costs. Would this be a good investment? What other factors would you have to consider?
You are boxing up care packages to send to developing countries. The first one contains bed nets to stop mosquito bites and the spread of malaria ($10 each) and solar power kits ($60 each) to provide light in areas with restricted or no electricity. The second containing medicine kits for mum and baby ($20 each) and baby blankets and supplies ($15 each). There are 140 items in the first package and double the number of baby blankets to medicine kits in the second package. If the package with the medicine kits and blankets cost $3000 and the package with nets and solar kits cost $3400, calculate how many of each of the 4 items there are in the packages.
Summative Task

Running your own business for a day

Part One: Your school will be hosting a fundraiser, in which students will set up booths to sell products that they have made. Research and find a charity/organization that you would like to raise awareness for and donate all profits that you make. You will create a one page flyer summarizing the charity that you will be able to show customers and hand out during the event.

Part Two: The school's event will be set up as a market, where students can set up booths to sell a product that they have made.

a Decide what you would like to make (e.g. baked goods, simple toy or jewellery etc.).

b Go online to approximate the cost to start up your business (including buying all the utensils needed to make your goods). Make sure you itemize all of the costs to be included in your report.

c Go online to approximate the cost to make each individual item to be sold (materials used to make the product). Make sure you show all of your working to calculate the cost per item and include it in your report.

d Decide how much you would like to charge for your product. It must be a realistic selling price so that you will be able to sell the product but also raise money for your charity.

Part three: Create a linear model

a Determine the equation that represents the cost to make the product (what you pay to create the product). What do the slope and $y$–intercept represent in this equation?

b Determine the equation that represents revenue (the money that you make by selling the product). What do the slope and $y$–intercept represent in this equation?

c Graph the linear system that you have created. Remember to label the axes as well as the point of intersection.

d The point of intersection represents the break–even point – where costs equal revenue, hence profit is zero. How many of your product do you have to sell to break even? (solve graphically)
e  Verify your point of intersection by solving the system algebraically.

f  Write a general statement relating the quantity of your product sold to when you are losing money and when you are making a profit.

g  How much money will you lose or make if you make and sell
   i  10 units of your product
   ii  1000 units of your product

h  What is happening to the original line (slopes and y–intercept) and the point of intersection (number of product needed to sell in order to break even) when you alter a particular part of either equation:
   i  change of start–up costs
   ii  change of variable costs
   iii  change of selling price

i  Show each of the above three scenarios in a sketch on three separate sets of axes – drawing in the original cost and revenue lines in one colour and the new lines (both the increase & decrease) in different colours. Label the original point of intersection and the new break even points. (This is conceptual so you do not need to calculate the actual break even points – just show how the lines have shifted and how the point of intersection has increased or decreased)

In your report include:

- Flyer containing the summary of information about the charity/organization
- List, explanation and calculations on how all costs were derived and selling price determined
- Linear model showing cost and revenue functions and break–even point graphically
- Verification of break–even point using algebra
- 3 sketches showing how changes in costs and selling price alter the graphs and break even points
Reflect and discuss

- Explain the degree of accuracy of your solution.
- Describe whether or not the answer makes sense in the context of the problem.
- Given you will only have one day to sell your product, how many units of your product would you anticipate selling? Using the point of intersection that you calculated would you make money or lose money using your estimated selling quantity?
- What could you do to ensure you make money? Can you even make such assumptions?
- What other factors need to be taken into consideration when determining how many of your product you will make for the school fundraiser?
- Given the projected break-even point, is there something about your product/model that you would change? If so, what would it be?