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The study of very large and very small numbers can lead you on a journey through some interesting discoveries and developments, as you will see in this unit. However, there are other contexts where the ability to work with these types of numbers is just as fundamental.

Globalization and sustainability

Population and consumption

Large and small quantities play an important part in the study of populations on Earth, the consumption of resources and an analysis of sustainability.

In 1960, the global population was just over 3 billion people. That number has since grown to almost 8 billion. In order to feed our growing population, there are over 1.5 billion cows, more than 1 billion pigs and goats, and 150 million square kilometers of agricultural land.

All inhabitants of the planet need water and food of their own. With just 4 million cubic kilometers of fresh water available, is there enough and, if so, how long will it last? Numbers can help us to understand and plan for the future.

Is there sufficient land for the growing human population, our crops and other animals? One method for making land more arable involves microbes. Microbes are microscopic organisms that can help soil become more fertile and produce more crops.
The internet itself is made up of tiny particles known as electrons. Electricity is the flow of electrons, each with a mass so small that it would take 30 zeros after the decimal place before writing the first non-zero number!

Scientific and technical innovation

The Digital Age

The Digital Age began in the 1970s when the personal computer was introduced. In 1980, early versions of modern laptops or desktops had only 16 000 bytes of memory. In less than 40 years, that number has soared to over 16 billion bytes.

Every second, one search engine manages over 40 000 internet searches, which means over 3.5 billion search queries in total every day. In addition to this, 500 million tweets and 3 billion snaps are sent every day.

What did people do with this time before the internet?
In this unit, you will explore some amazing human discoveries and developments as you expand your understanding of the global context of orientation in space and time. You will see that being able to manipulate quantities, simplify them and represent them in a variety of ways is one tool that can be used to uncover everything from far-off galaxies to the tiniest of environments, and all that lies in between.

### Statement of Inquiry:
Representing and simplifying quantities in different forms can help explore remarkable discoveries and developments.

### Related concepts: Quantity, Representation, Simplification

### Objects
- Identifying and representing rational numbers
- Evaluating negative and zero exponents
- Simplifying expressions with exponents
- Representing numbers in scientific notation
- Performing operations with numbers in scientific notation

### Inquiry questions
- What is a quantity?
- What are the laws of exponents?
- How are quantities represented in different forms?
- How does simplification lead to equivalent forms?
- What does it take to make the next great discovery?
- Are great discoveries planned or accidental?
You should already know how to:

1. Solve simple equations
   Solve the following equations. Write your answers as integers or simplified fractions.
   a. \(14x = 28\)  
   b. \(72x = 34\)  
   c. \(189x = 354\)

2. Evaluate positive exponents
   Evaluate each of the following.
   a. \(3^2\)  
   b. \(2^4\)  
   c. \(5^3\)

3. Multiply fractions
   Multiply each of the following.
   a. \(\frac{1}{4} \times \frac{1}{4}\)  
   b. \(\frac{2}{3} \times \frac{3}{4}\)  
   c. \(\frac{4}{15} \times \frac{5}{12}\)

4. Apply the distributive property
   Simplify each of the following expressions.
   a. \(3(2x - 5)\)  
   b. \(4(5m + 9)\)  
   c. \(-9(-3t - 4)\)

5. Solve problems involving rates
   a. If a car travels at 30 km/h for 2 hours, how far does it travel?
   b. If an Olympic athlete runs an average speed of 40 km/h, how far can he run in 10 seconds?

6. Find the area and circumference of a circle
   Find the area and circumference of a circle with the following measurements.
   a. a radius of 10 cm  
   b. a diameter of 10 cm
Interestingly, answering all of these questions requires the same basic skill: being able to work with numbers. From the very small to the very large, numbers are necessary to make all kinds of discoveries and developments. Whether you want to calculate how far away a new planet may be or describe the dimensions of a new strain of bacteria, you will need to know how to manipulate and perform operations with numbers represented in a variety of ways. Your ability to do this, and to describe your results accurately, may lead you to the world’s next great discovery!

**Rational and irrational numbers**

You are familiar with integers and natural numbers, which are both types of real numbers. However, real numbers can also be classified in other ways. A number can be either *rational* or *irrational*. 
Did you know...?

Our understanding of numbers has developed throughout history. We started with whole numbers since we needed to count. Fractions were developed as we discovered the need to break whole things into smaller pieces. The discovery of irrational numbers is often attributed to the Ancient Greek philosopher Hippasus, who was a Pythagorean mathematician. He used an isosceles right triangle to prove that the measure of the hypotenuse was an irrational number. It is rumoured that his discovery was so shocking that he was sentenced to death for it.

How can a number be rational or irrational?

It is said that Hippasus used a right triangle like the one shown here in his discovery of irrational numbers. In this isosceles right triangle, the sides next to the right angle measure 1 unit each, while the hypotenuse measures $\sqrt{2}$ units.

What kind of number is 1? How about $\sqrt{2}$? What makes a number rational or irrational? The activity below will help you see the difference between these two types of number.

Activity 1 – What does it mean to be rational?

Some examples of rational and irrational numbers are given in the table below.

<table>
<thead>
<tr>
<th>Rational numbers</th>
<th>Irrational numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.43256931…</td>
</tr>
<tr>
<td>1.4</td>
<td>−1.559824…</td>
</tr>
<tr>
<td>−3.141414…</td>
<td>23.057419…</td>
</tr>
<tr>
<td>25.659</td>
<td>56.1416278…</td>
</tr>
<tr>
<td>4.5555555…</td>
<td>−423.745782309…</td>
</tr>
<tr>
<td>105.37</td>
<td>72.2290004681…</td>
</tr>
<tr>
<td>−11.258258258…</td>
<td>2.71828182845…</td>
</tr>
</tbody>
</table>

1 Based on these examples, how would you define a rational number? How would you define an irrational number? Compare your definition with a peer.
A rational number can be defined as “any number that can be written as the ratio of two integers, as long as the denominator is not zero”. How do the examples you have seen so far fit that definition?

Is 7 a rational or irrational number? Justify your answer.

Jeremiah says: “An irrational number is a number that cannot be written as a fraction.” Is he correct? Explain.
Representing rational numbers

If a rational number can be written as a ratio of two integers where the denominator is not zero, how do you go about converting a decimal to a fraction?

Investigation 1 – Rational numbers in different forms

Take a look at the following decimal numbers:
0.5   2.7   37.4   0.3   9.12   2.4567

1 Classify each number as either “terminating/finite” or “infinite”. Justify your answer.

2 Express each number as a fraction in the form \( \frac{p}{q} \), where \( p \) and \( q \) are both integers and \( q \neq 0 \).

3 Write down a general rule for how to represent a finite decimal number as a fraction.

Take a look at the following fractions:
\[
\frac{1}{9}, \frac{5}{99}, \frac{34}{99}, \frac{52}{99}, \frac{8}{9}, \frac{68}{99}
\]

4 Represent each fraction in decimal form. Classify each number in as many ways as possible.

5 What patterns do you notice in your answers?

6 How would you write each of the following numbers as fraction? Use a calculator to verify that each answer is correct.
   a 2.4…   b 0.26   c 23.125125125…

7 Summarize your rules for how to represent finite decimal numbers and infinite, periodic decimal numbers as fractions.

8 Verify each of your rules for two other numbers.

9 Justify why each of your rules works.
So far, you have seen how to represent finite decimals and certain kinds of periodic decimals in fraction form. What happens if the infinite decimal number has a part that repeats and a part that does not?

Explain why it is impossible to write a non-repeating, infinite decimal as a fraction.

You perform a calculation on your calculator and the answer fills the screen. How do you know whether that number is a finite decimal, an infinite periodic decimal or an infinite non-periodic decimal?

Based on the patterns you found in step 5 in Investigation 1, how should \( \frac{9}{9} \) be represented as a decimal? How does that conflict with the value of \( \frac{9}{9} \)?

Example 1

Represent each of the following as a fraction with integer numerator and denominator.

a 0.0353535...

b 2.13789789789...

Example 1

<table>
<thead>
<tr>
<th>Q</th>
<th>Represent each of the following as a fraction with integer numerator and denominator.</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.0353535...</td>
</tr>
<tr>
<td>b</td>
<td>2.13789789789...</td>
</tr>
</tbody>
</table>

Example 1

<table>
<thead>
<tr>
<th>A</th>
<th>If ( x = 0.0353535... )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Multiply the number by two different powers of 10 so that the same decimal part is repeating in both numbers.</td>
</tr>
<tr>
<td></td>
<td>1000 ( x = 35.35353535... )</td>
</tr>
<tr>
<td></td>
<td>( -10x = 0.35353535... )</td>
</tr>
<tr>
<td></td>
<td>990 ( x = 35 )</td>
</tr>
<tr>
<td></td>
<td>( x = \frac{35}{990} )</td>
</tr>
<tr>
<td></td>
<td>( x = \frac{7}{198} )</td>
</tr>
<tr>
<td></td>
<td>7 ÷ 198 = 0.0353535...</td>
</tr>
<tr>
<td></td>
<td>0.0353535... = ( \frac{7}{198} )</td>
</tr>
</tbody>
</table>

Example 1

<table>
<thead>
<tr>
<th>A</th>
<th>If ( x = 2.13789789789... )</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>Multiply the number by two different powers of 10 so that the same decimal part is repeating in both numbers.</td>
</tr>
<tr>
<td></td>
<td>100 000 ( x = 213789.789789789... )</td>
</tr>
<tr>
<td></td>
<td>( -100x = 213.789789789... )</td>
</tr>
<tr>
<td></td>
<td>99 900 ( x = 213576 )</td>
</tr>
<tr>
<td></td>
<td>( x = \frac{213576}{99900} )</td>
</tr>
<tr>
<td></td>
<td>( x = \frac{17798}{8325} )</td>
</tr>
<tr>
<td></td>
<td>17 798 ÷ 8 325 = 2.13789789789...</td>
</tr>
<tr>
<td></td>
<td>2.13789789789... = ( \frac{17798}{8325} )</td>
</tr>
</tbody>
</table>
Reflect and discuss 3

- Show how you can use the procedure in Example 1 to convert 0.7777… to a fraction.
- Show how you can use the procedure in Example 1 to convert 1.242424… to a fraction.
- Create a two- or three-line rhyme that will help you remember how to convert decimals to fractions. Share your rhyme with a few peers and give positive feedback to each other.
- Write down what is going well so far in this unit in terms of your learning.

Practice 1

1 Classify each of the following numbers as either rational or irrational. Justify your answer.
   a −4    b \( \frac{1}{2} \)    c 14.23    d \( \sqrt{12} \)    e 104.25    f 2.089999…
   g \( \sqrt{60} \)    h \( \sqrt{81} \)    i −19.34862147…    j 0

2 Represent each of the following rational numbers as a fraction in simplified form.
   a 0.38    b 0.2\( \overline{1} \)    c 1.6    d 2.\( \overline{4} \)    e 12.874
   f 0.1333…    g 5    h −4.1392    i 5.6\( \overline{72} \)    j 9.03555…

3 a Represent 0.3 and 0.6 as fractions.
   b Show that 0.\( \overline{9} \) = 1 by adding together 0.3 and 0.6 and then adding together their equivalent fractions which you found in step a.
   c Repeat steps a and b with 0.2\( \overline{5} \) and 0.74.
   d Find your own example of two rational numbers that can be used to demonstrate that 0.999… = 1.

Exponents

An exponent is written in the form \( a^x \), where “a” is the base and “x” is the exponent or power. You have already seen how this can be used to simplify longer expressions. It would be difficult to write 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 over and over again: \( 2^{10} \) is much simpler. \( 2^{10} \) is a power of 2, with a base of 2 and an exponent of 10.

base \( 2^{10} \) exponent
Mathematical puzzles have been developed over the centuries, dating as far back as Ancient Egypt. More contemporary versions include Rubik’s Cube and Sudoku. The Tower of Hanoi is a puzzle invented by Édouard Lucas in 1883 that is based on a legend.

According to the legend, there is a Hindu temple that contains three large posts and 64 different gold disks. All the disks are on one post, ordered in size with the largest disk at the bottom and the smallest on top. The temple priests must move the disks one at a time from one post to another, never placing a larger disk on top of a smaller one. The puzzle is completed (and the world ends!) when all of the disks are again stacked on another post in order from largest (on the bottom) to smallest (on top). How many moves would it take to complete the puzzle?

a  Simulate the puzzle using three disks to start with. You can draw the posts and disks or you can use coins to represent the disks.

b  What is the minimum number of moves you need to transfer three disks from the first post to the last, following the rule of never placing a larger disk on top of a smaller one?

You can play an online version of the game by going to the mathisfun.com website and searching for Tower of Hanoi. The game will keep track of the number of moves and you can gradually increase the number of disks as you solve the puzzle.

Continued on next page
Investigation 2 – Positive and negative bases

1 Find the value of each of the following.

\[
\begin{align*}
2^2 & \quad 2^3 & \quad 2^4 & \quad 2^5 & \quad 2^6 \\
3^2 & \quad 3^3 & \quad 3^4 & \quad 3^5 & \quad 3^6 \\
4^2 & \quad 4^3 & \quad 4^4 & \quad 4^5 & \quad 4^6 \\
5^2 & \quad 5^3 & \quad 5^4 & \quad 5^5 & \quad 5^6 \\
(-1)^2 & \quad (-1)^3 & \quad (-1)^4 & \quad (-1)^5 \\
(-2)^2 & \quad (-2)^3 & \quad (-2)^4 & \quad (-2)^5 \\
(-3)^2 & \quad (-3)^3 & \quad (-3)^4 & \quad (-3)^5 \\
\end{align*}
\]

2 Generalize your results and write down a rule related to the result of raising a positive and negative base to an exponent.

3 Verify your rule for two more examples of your own choosing.

4 Justify why your rule works.

Like bases, exponents can also be integers. But what would an exponent of 0 or \(-1\) mean?

c Gradually increase the number of disks as you solve the puzzle and record your results in a table like the one below.

<table>
<thead>
<tr>
<th>Number of disks</th>
<th>Minimum number of moves</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

d Based on the pattern in your table, what is the minimum number of moves required for the priests to move all 64 of the disks to the last pole? Write your answer using an exponent.

e Use exponents to write a rule for the minimum number of moves required to solve the puzzle with \(n\) disks.

f If there were no disks, show that your rule correctly predicts the minimum number of moves necessary to solve the puzzle.

In all examples you have met before, the base has been a positive integer. What happens if the base is negative? How does that affect the value of the quantity?
Zero and negative powers

The typical definition of an exponent is “the number of times you multiply a base number by itself”. However, exponents can be integers, decimals and even fractions. How do these unfamiliar powers fit into this very familiar definition?

**Investigation 3 – Zero and negative exponents**

1. What do you think $6^0$ is equal to? Explain your thinking.

2. What do you think $4^{-1}$ is equal to? Explain your thinking.

3. Copy and complete the table by finding the value of each of the following powers. Write each answer as either a whole number or a fraction. **No decimals allowed**! If you are unsure of the value of a power, simply follow the pattern you can see in the powers above it.

   - $2^5 =$ ______
   - $2^4 =$ ______  $3^4 =$ ______
   - $2^3 =$ ______  $3^3 =$ ______  $5^3 =$ ______
   - $2^2 =$ ______  $3^2 =$ ______  $5^2 =$ ______
   - $2^1 =$ ______  $3^1 =$ ______  $5^1 =$ ______
   - $2^0 =$ ______  $3^0 =$ ______  $5^0 =$ ______
   - $2^{-1} =$ ______  $3^{-1} =$ ______  $5^{-1} =$ ______
   - $2^{-2} =$ ______  $3^{-2} =$ ______  $5^{-2} =$ ______
   - $2^{-3} =$ ______  $3^{-3} =$ ______  $5^{-3} =$ ______
   - $2^{-4} =$ ______  $3^{-4} =$ ______  $5^{-4} =$ ______
   - $2^{-5} =$ ______

4. Repeat the same process, starting with $\left( \frac{1}{2} \right)^4$ and ending with $\left( \frac{1}{2} \right)^{-3}$.

5. Based on your results, what conclusion can you draw about an exponent of zero?

6. Based on your results, what conclusion can you draw about negative exponents?

7. Write your conclusions as general rules, using variables instead of specific numeric examples.

8. Verify your rules for two more examples of your own, using a pattern similar to the ones above.

9. Justify why each of your rules works.
Reflect and discuss 4

- Explain how the results of Investigation 3 challenge the typical definition of an exponent.
- What do you think is the value of $\left(\frac{2}{3}\right)^{-2}$? Explain your answer.
- Create a memory aid to help you to remember your rules for zero and negative exponents.

Example 2

Find the value of each of the following:

a $4^{-3}$

\[
\left(\frac{1}{4}\right)^3 \text{ or } \frac{1^3}{4^3}
\]

\[
\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \text{ or } \frac{1 \times 1 \times 1}{4 \times 4 \times 4}
\]

\[
\frac{1}{64}
\]

b $(-3)^{-2}$

\[
\left(-\frac{1}{3}\right)^2
\]

\[
-\frac{1}{3} \times -\frac{1}{3} = \frac{1}{9}
\]

c $-5^{-2}$

\[
-\left(\frac{1}{5}\right)^2
\]

\[
-\left(\frac{1}{5} \times \frac{1}{5}\right) = -\frac{1}{25}
\]

A negative exponent indicates that you take the reciprocal of the base.

Two fractions that multiply to 1 are called reciprocals. For example, $\frac{2}{3}$ and $\frac{3}{2}$ are reciprocals since $\frac{2}{3} \times \frac{3}{2} = 1$.

A negative exponent indicates that you take the reciprocal of the base.

As you found before, a negative base raised to an even exponent produces a positive answer.

A negative exponent indicates that you take the reciprocal of the base. Note that the original base is 5, not $-5$. 
Practice 2

1 Find the value of each of the following. Write your answer as either a whole number or a fraction.

\[
a. 6^{-2} \\
b. (-3)^0 \\
c. 10^{-3} \\
d. 1^{-5} \\
e. (-8)^2 \\
f. (-9)^{-1} \\
g. 7^3 \\
h. 4^{-3} \\
i. 5^0 \\
j. (-2)^{-6} \\
k. \left(\frac{-1}{3}\right)^3 \\
l. \left(\frac{3}{4}\right)^{-2} \\
m. \left(-\frac{2}{11}\right)^0 \\
n. \left(\frac{4}{7}\right)^{-1} \\
o. \left(\frac{2}{5}\right)^{-3} \\
p. -12^{-2} \\
q. (-12)^{-2} \\
r. 0^2 \\
s. -\left(\frac{2}{3}\right)^{-2} \\
t. \left(\frac{1}{2}\right)^5
\]

2 Rewrite each of the following in the form \(a^b\), where \(a\) and \(b\) are integers (no fractions).

\[
a. 4 \times 4 \times 4 \times 4 \times 4 \\
b. \frac{1}{5 \times 5 \times 5 \times 5 \times 5} \\
c. \frac{1}{10 \times 10 \times 10 \times 10} \\
d. -7 (-7) (-7) (-7) (-7)
\]

3 Order the quantities in each set of four from lowest to highest. Show your working.

\[
a. 5^0, 4^{-2}, 3^1, \left(\frac{1}{3}\right)^{-2} \\
b. 2^{-2}, 3^{-1}, \left(\frac{1}{4}\right)^{-1}, \left(\frac{5}{8}\right)^0 \\
c. 7^{-1}, 2^{-2}, 1^{-3}, \left(\frac{2}{5}\right)^2 \\
d. 1^0, 3^{-1}, 0^1, \left(\frac{1}{2}\right)^2
\]

4 Ricardo says: “\(5^0\) equals zero because you have zero 5s, which means you don’t have anything. You have zero.” Explain any faults in his thinking.

5 Talei says: “Negative exponents make fractions. If the negative exponent is already in the denominator of a fraction, then it makes an integer.” Is Talei correct? Explain.

6 Anna says: “\(8\frac{1}{2}\) must be 4 since you have half of 8.” Explain why this thinking is faulty.

7 Thomas states: “A positive exponent is the number of times you multiply the base. A negative exponent is the number of times you divide by the base.” Do you agree? Explain your answer.
The development of the current metric system of units began in France in the 18th century. Basic units for measurements like angles, lengths, mass and capacity were created, often derived from the properties of natural objects such as water. For example, 1 liter of water has a mass of 1 kg. Multiples or divisions of these units could be created by using prefixes, such as those used in the units millimeter and kilogram. Some of the prefixes are given in the table below. Copy and complete the table.

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Exponential form</th>
<th>Expanded form</th>
</tr>
</thead>
<tbody>
<tr>
<td>gigi</td>
<td>$10^9$</td>
<td></td>
</tr>
<tr>
<td>mega</td>
<td>$10^6$</td>
<td>1 000 000</td>
</tr>
<tr>
<td>kilo</td>
<td>$10^3$</td>
<td></td>
</tr>
<tr>
<td>deci</td>
<td>$10^{-1}$</td>
<td></td>
</tr>
<tr>
<td>centi</td>
<td></td>
<td>$\frac{1}{100}$</td>
</tr>
<tr>
<td>milli</td>
<td></td>
<td>$\frac{1}{1000}$</td>
</tr>
<tr>
<td>micro</td>
<td>$10^{-6}$</td>
<td></td>
</tr>
<tr>
<td>nano</td>
<td></td>
<td>$\frac{1}{100000000}$</td>
</tr>
<tr>
<td>pico</td>
<td>$10^{-12}$</td>
<td></td>
</tr>
<tr>
<td>femto</td>
<td></td>
<td>$\frac{1}{100000000000000}$</td>
</tr>
<tr>
<td>atto</td>
<td>$10^{-18}$</td>
<td></td>
</tr>
</tbody>
</table>

Multiplying powers

You already know that $2^2$ means $2 \times 2$ and $2^4$ means $2 \times 2 \times 2 \times 2$. Is it possible to multiply $2^2$ and $2^4$ and, if so, how is this similar or different to the multiplication you do already? In the following investigations you will look for patterns and determine general rules about how to find the product of two or more powers.
Investigation 4 – Product rule

1 Copy and complete this table.

<table>
<thead>
<tr>
<th>Question</th>
<th>Expanded form</th>
<th>Simplified form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^2 \times 2^4$</td>
<td>$2 \times 2 \times 2 \times 2 \times 2 \times 2$</td>
<td>$2^6$</td>
</tr>
<tr>
<td>$3^5 \times 3^3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2^4 \times 2^5$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y^9 \times y^2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w \times w^6$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h^2 \times h^3 \times h^3$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2 Describe any patterns that you see among your answers in each row.

3 Describe a rule that you could use to multiply $4^{12} \times 4^{20}$.

4 Generalize your rule to multiplying powers with any base. Be sure to use variables rather than specific numeric examples.

5 You know that $2^6$ means $2 \times 2 \times 2 \times 2 \times 2 \times 2$. Rearrange this in as many different ways as possible by inserting brackets. For example, $2 \times (2 \times 2 \times 2 \times 2 \times 2)$ or $(2 \times 2) \times (2 \times 2) \times (2 \times 2)$.

6 Compare your representations with those of a peer. How many different representations can you find?

7 Knowing that all your representations should all equal $2^6$, show that your representations follow the rule that you found in step 4.

8 Verify your rule from step 4 for two more examples of your own, with a base other than 2.

9 Justify why your rule works.

Reflect and discuss 5

- Can you use the rule you discovered in investigation 4 with an expression like $2^3 \times 3^4$? Explain your answer, describing any limitations on the bases involved in the multiplication.

- Can you use your rule with an expression like $4^5 \times 4^{-2}$? Explain.

- What limitations, if any, are there on the exponents? Explain.
Activity 3 – Reinforcing zero and negative exponents

1 Define the term “multiplicative inverse” – research this if necessary. What happens when you multiply a number by its multiplicative inverse? Demonstrate with an example.

2 Write down the multiplicative inverse of $7^3$.

3 Rewrite the multiplicative inverse from step 2 in the form $a^b$, where $a$ and $b$ are both integers.

4 Multiply $7^3$ and its multiplicative inverse from step 3 using the product rule you have just established.

5 Repeat the same procedure for $5^6$.

6 Repeat the same procedure for $\left(\frac{1}{2}\right)^4$.

7 How do your results reinforce what you have already learned about zero and negative powers? Explain.

Dividing exponents

While it is always possible to expand and multiply powers, finding a general rule allows you to accomplish the same operation much more efficiently. Is there a similar rule for dividing powers?

Investigation 5 – Quotient rule

1 Copy and complete this table.

<table>
<thead>
<tr>
<th>Question</th>
<th>Expanded form</th>
<th>Simplified form</th>
<th>Exponential form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^5 \div 2^3$</td>
<td>$2 \times 2 \times 2 \times 2 \div 2 \times 2 \times 2$</td>
<td>$2 \times 2$</td>
<td>$2^2$</td>
</tr>
<tr>
<td>$2^7 \div 2^4$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$5^6 \div 5^2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$4^6 \div 4$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t^5 \div t^4$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m^8 \div m^4$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Continued on next page
2 Describe any patterns that you see among your answers in each row.

3 Generalize a rule that you could use to divide powers. Be sure to use variables rather than specific numeric examples.

4 Verify your rule for two other examples.

5 Justify why your rule works.

Reflect and discuss 6

- Show how the quotient rule that you discovered in Investigation 5 can be used to derive the zero exponent rule.
- Show how the quotient rule can be used to derive the rule for negative exponents.
- Write down what is going well so far in this unit in terms of your learning.
- Identify the strengths that you have as a student. How have they helped you succeed in this unit?

Example 3

Q Simplify the following using the laws of exponents. Write your answer with positive exponents only.

\[ \text{a} \quad 9^5 \times 9^{-8} \quad \text{b} \quad \frac{m^3}{m^{-6}} \quad \text{c} \quad \frac{4n^2 p^{-4}}{10n^{-1} p^3} \]

A

\[ \text{a} \quad 9^5 \times 9^{-8} \]
\[ 9^{5+(-8)} = 9^{-3} \]
\[ \frac{1}{9^3} \]

\[ \text{b} \quad \frac{m^3}{m^{-6}} \]
\[ m^{3-(-6)} \]
\[ m^9 \]

The laws of exponents that you need here are the product rule and the quotient rule, both of which you have discovered in this unit.

When multiplying expressions with the same base, simply add the exponents.

Rewrite your answer with positive exponents only.

When dividing expressions with the same base, simply subtract the exponents.

Continued on next page
c \[ \frac{4n^2 p^{-4}}{10n^{-1} p^3} \] 
\[ 2n^{2(-1)} p^{-4-3} \] 
\[ \frac{2n^3 p^{-7}}{5} \] 
\[ 2n^3 \] 
\[ \frac{5 p^7}{5} \]

Practice 3

1 Simplify the following. Write your answers with positive exponents only.

\[ a \quad 5^6 \times 5^3 \]
\[ b \quad 6^2 \times 6^{-6} \]
\[ c \quad 8^{-6} \times 8^2 \]
\[ d \quad \frac{x^9}{x^4} \]
\[ e \quad \frac{y^3}{y^{-2}} \]

\[ f \quad \frac{s^8}{s^3} \]
\[ g \quad \frac{10^{-3}}{10^4} \]
\[ h \quad \frac{12^{-4}}{12^8} \]
\[ i \quad \frac{9^{-4}}{9^6} \]
\[ j \quad \frac{2^{-7}}{2^{-5}} \]

\[ k \quad \frac{x^{-4}}{x^6} \]
\[ l \quad \frac{a^3}{a^{-2}} \]
\[ m \quad \frac{w^{-11q}}{w^{-6q}} \]
\[ n \quad \frac{m^{-7t}}{m^{-5t}} \]
\[ o \quad \frac{y^{-3t}}{y^{8t}} \]

\[ p \quad \frac{a^{-5} \times a^3}{a^4} \]
\[ q \quad 5x^{-2} \times 6x^{-5} \]
\[ r \quad 2ab^2c^4 \times 3a^2c \]
\[ s \quad (-2pq^3)(7p^3q) \]
\[ t \quad (-6y^{-3}z^{-3})(\frac{1}{2}y^{-1}z) \]

\[ u \quad \frac{42q^5 r^4}{6pq^4 r} \]
\[ v \quad \frac{12d^2 ef^3}{36d^2 f^2} \]
\[ w \quad \frac{3p^{-4} q^{-1}}{6p^2 q^{-4}} \]
\[ x \quad \frac{45a^{-4} b^{-2}}{9ab} \]

2 With the development of the metric system it became easier to compare, multiply and divide quantities because of the use of prefixes.

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Exponential form</th>
</tr>
</thead>
<tbody>
<tr>
<td>giga</td>
<td>10^9</td>
</tr>
<tr>
<td>mega</td>
<td>10^6</td>
</tr>
<tr>
<td>kilo</td>
<td>10^3</td>
</tr>
<tr>
<td>deci</td>
<td>10^{-1}</td>
</tr>
<tr>
<td>centi</td>
<td>10^{-2}</td>
</tr>
<tr>
<td>milli</td>
<td>10^{-3}</td>
</tr>
<tr>
<td>micro</td>
<td>10^{-6}</td>
</tr>
<tr>
<td>nano</td>
<td>10^{-9}</td>
</tr>
<tr>
<td>pico</td>
<td>10^{-12}</td>
</tr>
<tr>
<td>femto</td>
<td>10^{-15}</td>
</tr>
<tr>
<td>atto</td>
<td>10^{-18}</td>
</tr>
</tbody>
</table>

a How many picograms are in a kilogram?
b How many nanobytes are in a gigabyte?
c How many femtometers are in a millimeter?
d What fraction of a megameter is an attometer?
e What fraction of a decigram is a microgram?
3 Which quantity is bigger? Justify each answer.
   a 2³ or 3²   b 3⁴ or 4³   c 5⁻² or 10⁻¹   d 4⁻³ or 8⁻²   e 6⁻² or 2⁻⁵   f 2⁻⁴ or 4⁻²

4 Evaluate the following. Write your answers as integers or fractions.
   a 3⁻² × 2⁻³ ÷ 6⁻²   b 7⁰ × 4⁻² × 2⁵   c 3⁻³ ÷ 9⁻² ÷ 6²   d \( \frac{4² \times 6⁻²}{9⁻¹} \)

5 In physics, dimensional analysis is often used to verify that the units in a formula produce the correct units for the answer. This process was first published by François Daviet de Foncenex in 1761, and was later developed by other scientists, for example James Clerk Maxwell.

   a In 1784, Charles-Augustin de Coulomb discovered that the force of attraction between two electric charges, \( q_1 \) and \( q_2 \), can be calculated using the formula
      \[ F = \frac{kq_1q_2}{r^2} \]
      where:
      each electric charge is measured in coulombs (C),
      \( r \) (the distance between the charges) is measured in meters,
      the constant \( k \) has the units \( \text{Nm}^2 \text{C}^{-2} \).
      Substitute these units into the formula and show that the formula produces the correct units of force, newtons (N).

   b The electric potential \( V \) created by a charge \( q \) (measured in coulombs) at a distance \( r \) (in metres) from the charge was discovered to be given by the formula
      \[ V = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{r} \]
      where \( \varepsilon_0 \) is measured in \( \text{C}^2\text{N}^{-1}\text{m}^{-2} \). What should the units of electric potential be?

---

Formative assessment

“Those who can, teach.”

In this task, you will work in small groups to develop an investigation and lesson to teach your peers one of the remaining laws of exponents. The list of possible topics is given on the next page.
### Topic Example

<table>
<thead>
<tr>
<th>Topic</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power of a product</td>
<td>((x , y)^m)</td>
</tr>
<tr>
<td>Power of a quotient</td>
<td>(\left(\frac{x}{y}\right)^m)</td>
</tr>
<tr>
<td>Power of a power</td>
<td>((x^a)^b)</td>
</tr>
<tr>
<td>Roots as exponents</td>
<td>([\sqrt[3]{x} = x^{\frac{1}{3}}])</td>
</tr>
<tr>
<td></td>
<td>([\sqrt[3]{x} = x^{\frac{1}{3}}])</td>
</tr>
</tbody>
</table>

You must hand in a detailed lesson plan which includes:

- lesson objectives/goals
- an investigation
- examples that you are going to use
- practice problems (but no word problems or problems in a global context are needed).

**Objectives/goals:**

- What are the desired outcomes of the lesson?
- What are the main concepts that you would like the students to understand by the end of the lesson?

**Investigation:**

- Create an activity that will allow your peers to discover the rule for themselves and to write it in their own words. Make sure the investigation includes at least five problems so that the pattern is clear.

**Examples:**

- You must guide the class through at least two examples (or as many examples as necessary to teach each type of question within your topic).
- Start with a basic one-step (single variable) question and move into multi-step questions with multiple variables.

**Handout for the lesson (scaffolded notes):**

- Prepare a handout to help students follow your lesson and record their learning. It should contain headings, questions and examples in the order that they will appear in your lesson. Leave blanks for students to fill in the answers to your worked examples and the key concepts that they discover during the lesson.

**Practice:**

- Create a worksheet to help reinforce the concepts that you have just taught. You and your group members should walk around while the class is working and help with any questions/difficulties that your peers may have.
- Provide an answer key and submit this at least one day before you teach your lesson so that your teacher can check it.
Activity/Game:
- In order to keep the class’s attention and check their understanding, plan a game or activity for the end of the lesson. You must submit the details and questions for this activity at least one day prior to your lesson for your teacher to check.

Every group member is expected to understand the material and should be prepared to answer any and all questions from the rest of the class.

Example 4

Q
Simplify the following:

\[ a \left( \frac{3i^3 j^2}{i^2 jk^4} \right)^3 \]

\[ b \left( \frac{4t^{-1}u^6}{18t^{-2}u^{-4}} \right)^{-2} \]

A

\[ a \left( \frac{3i^3 j^2}{i^2 jk^4} \right)^3 \]

\[ \frac{3^3 i^9 j^6}{i^6 j^3 k^{12}} \]

\[ \frac{27i^3 j^3}{k^{12}} \]

\[ b \left( \frac{4t^{-1}u^6}{18t^{-2}u^{-4}} \right)^{-2} \]

\[ \left( \frac{2tu^{10}}{9} \right)^{-2} \]

\[ \left( \frac{9}{2tu^{10}} \right)^2 \]

\[ \frac{81}{4t^2 u^{20}} \]

Use the power of a power rule to simplify the outer exponent. Remember that the “3” has an exponent of 1.

Use the quotient rule to simplify again by subtracting exponents.

NOTE: Alternatively, you could use the quotient rule first inside the parentheses and then use the power of a power rule.

Simplify using the quotient rule.

A negative exponent indicates that you need to work with the reciprocal.

Use the power of a power rule to simplify.

There is no order to the laws of exponents. You may use them in the order that you wish, as long as you apply them correctly.

The student-created activities could be presented to the class using the ShowMe app.

Instead of a whole class activity, all of the student work can be used as a jigsaw activity where each student in a group learns one rule and teaches it to his/her peers.
Practice 4

1 Evaluate each of the following. Write your answer as either an integer or a simplified fraction.

\[a \left(\frac{5}{9}\right)^{-2}\]
\[-9^2\]
\[(16)^2\]
\[\left(\frac{3}{4}\right)^2\]
\[\left(\frac{9}{10}\right)^{-1}\]
\[\left(\frac{9}{49}\right)^{\frac{1}{2}}\]
\[\left(\frac{1}{8}\right)^{-\frac{1}{3}}\]
\[\left(\frac{3}{5}\right)^{-2}\]
\[\left(-\frac{1}{64}\right)^3\]
\[(81)^{\frac{1}{2}}\]

2 Simplify the following. Write your answers with positive exponents only.

\[\frac{8^{-2}\times 8^4}{(8^2)^{-3}}\]
\[(9^{-1}\times 9^{-6})^{-3}\]
\[(a^4)^2\times a^{-3}\]
\[a^{-3}\times a^0\times a^4\]
\[\frac{x^{-2}\times y^{10}\times z^{-8}}{(x^2\times y^2\times z^3)^4}\]
\[2t^3\times (3t)^2\]
\[\frac{3x^2}{6x^4}\]
\[(-9y)^2\times (2y)^3\]
\[(-3x^2)^2\times (-y^5)\]
\[\left(\frac{2ab^4}{c^3}\right)^{-1}\]
\[\frac{1}{2a^2}\times 3a^3\]
\[\frac{(x^3y^2)(3xy^5)^2}{9x^4y^9}\]
\[\frac{(7ax^2z^4)(3xy^2)^3}{a^2zy}\]
\[(4g^3h)^2\]
\[7\sqrt{78125a^{-7}b^{-49}c^{21}}\]
\[\left(4a^2b^3\right)^{-2}\times 3(6b^3)^{\frac{1}{3}}\]
\[\left(\frac{3\sqrt[3]{8c^9d^{12}e^{15}}}{4}\right)^2\]
\[\left(-3c^6d^5\right)^2\times 4(c^2d^3)^{\frac{1}{3}}\]
\[\frac{(6j^{-2}k^3)^{-2}}{2(j^{-3}k^{-1})^{3}}\times \left(\frac{3j^{-1}}{4k^3}\right)^{-3}\]
\[\left(\frac{1}{4f^3g^{-1}}\right)^{-3}\]
\[2\left(\frac{1}{4fg^2}\right)^3\]

3 Create a bookmark with the laws of exponents on it. Make sure that all the laws fit on a small, rectangular piece of paper. Include the name of each rule and the rule itself. Enhance your bookmark through the use of colors, pictures, arrows, etc.
Scientific notation

Did you know...?

In 216 BC, Archimedes wrote his book *The Sand Reckoner*, in which he established an upper limit for the number of grains of sand in the universe. It was the first time that humans had considered such large numbers so Archimedes had to invent a way to name them! However, using the Greek number system made it difficult to write such a large quantity. It wasn't until 1637 that humans could represent large quantities more efficiently when René Descartes developed the system of exponents that we use today. The term “scientific notation” was first used in the 1900s, most likely by computer scientists.

The 17th century was a time in which very large and very small numbers were at the forefront of science. With the development of the telescope in 1610 by Galileo Galilei and with Antonie van Leeuwenhoek’s improvements to the microscope roughly 50 years later, humans could now see things that were either a great distance away or were incredibly small. Being able to describe such diverse numbers with ease required the development of a new system: this is now called scientific notation (or standard form).

Writing really large and really small quantities

A water molecule measures approximately 275 picometers, where one picometer can be written as 0.000 000 000 001 m. A single drop of water contains roughly 1.7 quintillion of these water molecules. One quintillion is 1 followed by 18 zeros! Are you really supposed to write all of those zeros or is there a more efficient way to represent these quantities?
Investigation 6 – Developing scientific notation I

In 1800, a group of 25 astronomers calling themselves the Celestial Police were searching for a planet whose existence was predicted by the astronomer Johann Titius. Around the same time, another astronomer, Giuseppe Piazzi, discovered the first asteroid in a location now known as the asteroid belt. The asteroid belt is a ring of rocks and debris left over from the formation of the universe, located between Mars and Jupiter. Its average distance from the Sun is approximately 415 000 000 km.

1. 415 000 000 can be represented in a variety of ways. Copy the table and fill in the missing values.

<table>
<thead>
<tr>
<th>415 000 000 using products</th>
<th>415 000 000 using powers of 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>41 500 000 × ___</td>
<td>41 500 000 × 10^1</td>
</tr>
<tr>
<td>4 150 000 × ___</td>
<td>4150 × 10^5</td>
</tr>
<tr>
<td>___ × 1000</td>
<td>415 × 10^6</td>
</tr>
<tr>
<td>___ × 10 000</td>
<td></td>
</tr>
<tr>
<td>41.5 × ____</td>
<td></td>
</tr>
<tr>
<td>4.15 × ____</td>
<td></td>
</tr>
</tbody>
</table>

2. Piazzi named that first asteroid Ceres and it is now known to be the largest asteroid in the asteroid belt. It measures approximately 950 000 meters in diameter. Use the same procedure as in step 1 to represent this quantity in a variety of ways.

<table>
<thead>
<tr>
<th>950 000 using products</th>
<th>950 000 using powers of 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>95 000 × ___</td>
<td>95 000 × 10^7</td>
</tr>
<tr>
<td>9500 × ___</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>9.5 × 10^7</td>
</tr>
</tbody>
</table>

3. Look at the powers of 10 representation in the last row of each table. These are both examples of scientific notation or standard form. Describe the components of a quantity represented in scientific notation.

4. Write down a general rule for writing a large number in scientific notation.

5. The Celestial Police discovered two large asteroids in the asteroid belt: Juno and Vesta. Vesta has a mass of 259 quintillion grams. At its closest, Juno is 297 000 000 000 meters from the Sun. Verify your rule by writing each of these quantities in scientific notation.

6. Justify why your rule works.
A quantity represented in scientific notation has a decimal component, called the **coefficient**, multiplied by a power of 10.

\[ 2.75 \times 10^{-4} \]

**Example 5**

**Q** Represent the following quantities using scientific notation.

- **a** 2139
- **b** 4 980 000
- **c** 310 billion

**A**

- **a** 2139
  
  \[ 2.139 \times 1000 = 2.139 \times 10^3 \]

- **b** 4 980 000
  
  \[ 4.980000 \times 1000000 = 4.98 \times 10^6 \]

- **c** 310 billion
  
  \[ 310 \times 10^9 = 3.10 \times 10^{11} \]

**Reflect and discuss 7**

- Describe two advantages of representing quantities using scientific notation.
- Which quantity is greater, \( 7.32 \times 10^{12} \) or \( 4.2 \times 10^{14} \)? Explain how you know.
Investigation 7 – Developing scientific notation II

1. Using the same procedure as in the previous investigation, represent 0.000006 using products of 10 and powers of 10.

<table>
<thead>
<tr>
<th>0.000006 using products</th>
<th>0.000006 using powers of 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00006 × 0.1</td>
<td>0.00006 × 10⁻¹</td>
</tr>
<tr>
<td>0.0006 × ___</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>6.0 × ___</td>
<td></td>
</tr>
</tbody>
</table>

2. Write 0.000000304 in scientific notation.

3. Write down a general rule for writing a small number in scientific notation.

4. Van Leeuwenhoek made other discoveries as well. He was the first to see sperm cells, which have a length of 0.0047 cm, and red blood cells, whose minimum thickness is 0.8 micrometers. Verify your rule by writing each of these quantities in meters using scientific notation.

5. Justify why your rule works.

Reflect and discuss 8

- Compare and contrast scientific notation for very large and very small numbers.
- Which quantity is greater, 3.8 × 10⁻⁴ or 9.2 × 10⁻⁷. Explain how you know.
- When a number is represented in scientific notation, how can you tell if it is less than or greater than 1? Explain.
Practice 5

1 Represent these numbers in scientific notation.
   a \(23\,500\)  \(b\) \(365\,800\)  \(c\) \(210\,000\,000\)  \(d\) \(3\,650\,000\)  \(e\) \(569\,000\)  \(f\) \(7\,800\,000\,000\)

2 Represent these numbers as numbers in expanded form (e.g. write \(1.034 \times 10^2\) as 103.4).
   a \(1.45 \times 10^6\)  \(b\) \(2.807 \times 10^{-3}\)  \(c\) \(9.8 \times 10^3\)  \(d\) \(3.7 \times 10^9\)  \(e\) \(5.06 \times 10^{-5}\)  \(f\) \(2 \times 10^{-8}\)

3 Order the following numbers on a number line. Explain your reasoning.
   0.0025  \(1.42 \times 10^4\)  \(9.83 \times 10^{-4}\)  \(7.8 \times 10^3\)  \(302 \times 10^{-6}\)  \(14\)  \(2.876 \times 10^2\)

4 Important discoveries in physics are listed in the table below. Represent each quantity in scientific notation.

<table>
<thead>
<tr>
<th>Year</th>
<th>Discovery</th>
<th>Quantity represented as an ordinary number</th>
<th>Quantity represented in scientific notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1676</td>
<td>Speed of light in air</td>
<td>299 792 km/s</td>
<td>km/s</td>
</tr>
<tr>
<td>1798</td>
<td>Acceleration due to gravity</td>
<td>980.665 cm/s²</td>
<td>cm/s²</td>
</tr>
<tr>
<td>1835</td>
<td>Earth’s magnetic field (average)</td>
<td>45 microteslas</td>
<td>teslas</td>
</tr>
<tr>
<td>1850</td>
<td>Speed of light in water</td>
<td>225 000 000 m/s</td>
<td>m/s</td>
</tr>
<tr>
<td>1998</td>
<td>Average diameter of an atom</td>
<td>1 nanometer</td>
<td>meters</td>
</tr>
</tbody>
</table>

5 The word “googol” was introduced by the mathematician Edward Kasner, who asked his 9-year-old nephew what he should call the number 1 followed by 100 zeros. It is said that the company name “Google” was an accidental misspelling of the word “googol”, since Google’s founders planned to make incredibly large amounts of information available to people.

a Write 1 googol in scientific notation.

b Research how the quantity of information available on the internet compares with 1 googol.

c Research “googolplex” and write it in scientific notation.

d How do you think you would add 2 googols and 5 googols? How would you represent the operation and sum in scientific notation?

6 You have learned that a quantity written as a number in expanded form can be represented in scientific notation using a number between 1 and 10 multiplied by the appropriate power of 10. Write a six-word memory aid personal to you that will help you to remember the conversion process (e.g. “Big move left, small move right”).
The development of scientific notation allows you to represent large and small numbers efficiently. How does it affect your ability to perform mathematical operations with these numbers?

**Addition/subtraction with scientific notation**

Our decimal system allows for numbers to be added and subtracted with ease. Do these operations become more complicated with the use of scientific notation? In this section, you will develop rules for adding and subtracting quantities represented in scientific notation.

### Activity 4 – Addition and subtraction

1. Copy this table and complete the first two columns.

<table>
<thead>
<tr>
<th>Question and answer</th>
<th>Question in scientific notation</th>
<th>Using the distributive property</th>
<th>Answer in scientific notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>127 + 345 = _______</td>
<td>($\ldots \times 10^3$) + ($\ldots \times 10^3$)</td>
<td></td>
<td>$\ldots \times 10^3$</td>
</tr>
<tr>
<td>5212 – 3158 = ______</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.044 – 0.031 = ______</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.00025 + 0.00071 = ______</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>33 208 + 19 117 = ______</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Use the *distributive property* to rewrite the questions written in scientific notation. Write this new representation in the 3rd column.

3. Use the order of operations to evaluate the expressions in step 2. Write your answers in scientific notation in the last column.

### Reflect and discuss

- What allows you to use the distributive property in Activity 4?
- What does this tell you about the conditions necessary to add or subtract numbers represented in scientific notation?
- What will you do if you get an answer of $14.5 \times 10^6$? How will you express this in correct scientific notation?
4 You are asked to add the following numbers without changing them from scientific notation.

\[(4.11 \times 10^2) + (5.08 \times 10^3)\]

What will you need to do before being able to use the distributive property?

5 Subtract the following numbers \((3.72 \times 10^5) - (2.56 \times 10^3)\) by first making sure they have the same power of 10. Express your answer in correct scientific notation.

6 Create an acronym (e.g. BEDMAS) for the process of adding or subtracting quantities represented in scientific notation. Share with a few peers, explaining what each of the letters in your acronym means.

Reflect and discuss 10

- How is adding and subtracting quantities represented in scientific notation similar to adding and subtracting fractions?
- How do you decide which power of 10 to change so that quantities written in scientific notation can be added or subtracted?

Multiplication/division with scientific notation

In 1676, the Danish astronomer Ole Rømer discovered that light travels at a specific speed that can actually be calculated. His results were based on observations of the eclipses of several moons of Jupiter by this large planet. The speed of light is approximately \(3 \times 10^8\) m/s. If light travels for 1 million seconds, how far will it have traveled? Can scientific notation make multiplying these large numbers easier?
Activity 5 – How far can light travel?

How far will light travel in 1 million seconds at a speed of $3.0 \times 10^8$ m/s?

1. Provide a reason or justification for each of the following steps.

<table>
<thead>
<tr>
<th>Step</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(3 \times 10^8) \times (1 \times 10^6)$</td>
<td></td>
</tr>
<tr>
<td>$3 \times 1 \times 10^8 \times 10^6$</td>
<td></td>
</tr>
<tr>
<td>$(3 \times 1) \times (10^8 \times 10^6)$</td>
<td></td>
</tr>
<tr>
<td>$3 \times 10^{14}$</td>
<td></td>
</tr>
</tbody>
</table>

Light will travel $3 \times 10^{14}$ meters in 1 million seconds.

2. Perform the same procedure to calculate how far light will travel in 63 million seconds (approximately 2 years). Be sure to represent your answer in correct scientific notation.

3. How far can light travel in 350,000 microseconds (the blink of a human eye)? Show your working.

4. The light from the Sun takes approximately 480 seconds to reach Earth. How far does it have to travel? Use division of quantities in scientific notation to calculate this value and to represent your answer.

5. Generalize the procedure for multiplying and dividing quantities written in scientific notation.

Reflect and discuss 11

- Does scientific notation make multiplying and dividing numbers easier or more difficult? Explain.
- At what point would you represent quantities in scientific notation rather than ordinary numbers when finding their product or quotient? Explain your answer with an example.
- Write down what went well in this unit in terms of your learning.
- What strengths in mathematics did you develop or enhance in this unit?
Practice 6

1 \[ a = 1.3 \times 10^6, \quad b = 4.9 \times 10^{-2}, \quad c = 8.32 \times 10^4, \quad d = 7.6 \times 10^{-3}, \quad e = 5.32 \times 10^5 \]

Evaluate each of the following. Represent your answers in scientific notation.

\[
\begin{align*}
\text{a} & \quad 5a \\
\text{b} & \quad 7b \\
\text{c} & \quad b + d \\
\text{d} & \quad 2e - 4c \\
\text{e} & \quad ae \\
\text{f} & \quad bd \\
\text{g} & \quad \frac{c}{a} \\
\text{h} & \quad \frac{e}{c} \\
\text{i} & \quad bc - de \\
\text{j} & \quad \frac{ac}{be} \\
\text{k} & \quad d^2 \\
\text{l} & \quad 12a - 5c + 2e \\
\text{m} & \quad \frac{a + b}{e + d} \\
\text{n} & \quad e^2 - c^2 \\
\text{o} & \quad \frac{1}{d} \\
\text{p} & \quad 2abd \\
\text{q} & \quad (ce)^2
\end{align*}
\]

2 In the Tower of Hanoi puzzle (see pages 12–13), you found that it would take the priests \(2^{64} - 1\) moves to transfer the 64 disks to a different post, maintaining their order.

- a Using a calculator, write down the number of moves using scientific notation, correct to 3 significant figures.
- b Calculate the number of seconds in one year. Represent your answer in scientific notation.
- c Legend has it that the world will end when the monks move the final disk to the new post. Suppose each move takes 1 second. Find the number of years it will take the monks to complete the puzzle if there are 64 disks. Show your working using scientific notation.

3 Considered one of the world’s greatest discoveries, penicillin was actually found by accident. In 1928, Alexander Fleming returned to his laboratory to find that a sample of bacteria had been left out and had become contaminated by mold. However, where the mold had grown, the bacteria had been destroyed. A single bacterium has a diameter of \(8 \times 10^{-7}\) meters and a penicillium mold spore has a diameter of \(3.5 \times 10^{-6}\) meters.

- a How many times larger is the diameter of a penicillium mold spore than that of a bacterium?
- b Assuming a roughly circular shape, find the area of one bacterium.
- c If there are 5 million bacteria, find the area they occupy.
- d If there are 1 million bacteria and 1 million mold spores, find the total area they occupy.
- e Find the number of mold spores that would equal the area of 1 million bacteria.
4 The development of the rocket began over 2000 years ago with experiments and models that used steam to propel objects into the air. It was Robert Goddard who, in 1919, published a paper on how rockets could reach extreme altitudes and paved the way for the development of modern rockets. His designs and experiments formed the basis of the space programs that eventually took humans into outer space and the Moon.

a The average distance from the Earth to the Moon is $3.84 \times 10^5$ km. A rocket can travel $3.60 \times 10^4$ km on one tank of fuel. Use standard form to represent each quantity and to find the number of fuel tanks the rocket would have to take to ensure it could make the return journey.

b At its closest, Mars is 50 million km from Earth. If a rocket can travel at a speed of $5.8 \times 10^4$ km/h, how long will it take to reach Mars when at its closest distance?

c How many times further from Earth is Mars (at its closest) than the moon?

5 Satellite technology was developed in the 1960s as a means of communication, but also as a way to spy on other people. Satellites are now used for so much more, including collecting weather data, broadcasting television signals and even helping you navigate to a new destination. A satellite travels around the Earth in a circular orbit 500 kilometres above the Earth’s surface. The radius of the Earth is 6375 kilometres. Calculate the maximum distance traveled by the satellite in one orbit of the Earth. Use the value of $\pi$ as 3.14, or the pi button on your calculator. Write your answer in standard form with the coefficient rounded to two decimal places.
The idea that all matter is composed of atoms is an incredibly important development. Before atomic theory, people had a variety of beliefs, such as the idea that all objects were made of some combination of basic elements: earth, air, fire and water. Atomic theory helps to explain the different phases of matter (solid, liquid, gas) and it allows you to predict how materials will react with each other. However, even the theory of the atom and its structure has developed over time, owing to important discoveries of the particles that make up an atom.

The current model of the atom includes three types of particle: the nucleus contains neutrons and protons and is surrounded by electrons that orbit around it.

Neutrons and protons have roughly equivalent masses that can be written as $16.6 \times 10^{-28}$ kg. Using the same power of 10, electrons have a mass of $0.00911 \times 10^{-28}$ kg.

**Formative assessment**

The idea that all matter is composed of atoms is an incredibly important development. Before atomic theory, people had a variety of beliefs, such as the idea that all objects were made of some combination of basic elements: earth, air, fire and water. Atomic theory helps to explain the different phases of matter (solid, liquid, gas) and it allows you to predict how materials will react with each other. However, even the theory of the atom and its structure has developed over time, owing to important discoveries of the particles that make up an atom.

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**Atomic particles**

a  Represent each of these masses using correct scientific notation.

b  Which particle has the smallest mass? Explain how you know.
Current elements

For each of the following questions, perform all of your calculations using scientific notation and show your working. Express your answers in correct scientific notation.

c How many times greater is the mass of a proton than the mass of an electron?

d Oxygen is an element that was discovered in 1772 by Carl Wilhem Scheele. One atom of oxygen has 8 electrons, 8 protons and 8 neutrons. Calculate the mass, in kg, of one atom of oxygen (the atomic mass). Show your working.

e The element carbon-14, which is used in dating very old objects, was discovered in 1940 by Martin Kamen and Sam Ruben. Its atomic mass is $2.324728 \times 10^{-26}$ kg. If one atom of carbon-14 has 6 electrons and 6 protons, find the number of neutrons in one atom of the element. Show your working.

f Because individual atoms are so small, they are often grouped together in a larger amount, called a mole. The number of atoms in a mole of a substance is $6.02 \times 10^{23}$, which is referred to as Avogadro’s number. If one mole of water has a mass of 18 g, find the number of atoms in 1000 grams (1 liter) of water.

g The distance from Earth to Mars is 54.6 million kilometres. A helium molecule has a length of approximately 280 picometers. Find how many helium molecules could fit between these two planets.

A new discovery

h Imagine you have discovered an element unlike any other on the planet. Prepare an info-graphic that includes the following information, showing all necessary calculations using standard form:

- the name of the element and its symbol
- the number of protons, neutrons and electrons
- the properties of the element that make it so unique and valuable
- the mass of one atom of the element in grams
- the number of atoms in 25 grams of the substance.

You can refer to the metric prefixes table you produced in Practice 2 question 4 (see page 16).
Unit summary

A rational number can be defined as “any number that can be written as the ratio of two integers, as long as the denominator is not zero”. Irrational numbers cannot be written as fractions.

To convert a decimal number to a fraction:

If the decimal number is finite, read the number using its place value and write down the corresponding fraction. For example, 0.324 is “three hundred and twenty four thousandths” and can be written as \( \frac{324}{1000} \) or \( \frac{81}{250} \).

If the decimal number is periodic, multiply it by one or more powers of 10 until you have two numbers in which the decimal parts are exactly the same. Subtract them and solve the equation, as shown below:

If \( x = 0.0353535… \),

then \( 1000x = 35.353535… \)

\[-10x = 0.35353535…\]

\[990x = 35\]

\[x = \frac{35}{990} \text{ or } \frac{7}{198}\]

Expressions containing quantities raised to an exponent can be simplified using the following laws of exponents.

Product rules:

Product rule with same base: \( a^n \times a^m = a^{n+m} \)

Product rule with same exponent: \( a^n \times b^n = (ab)^n \)

Quotient rules:

Quotient rule with same base: \( \frac{a^n}{a^m} = a^{n-m} \)

Quotient rule with same exponent: \( \frac{a^n}{b^n} = \left( \frac{a}{b} \right)^n \)

Power of a power rule: \( (a^n)^m = a^{nm} \)

Zero power rule: \( a^0 = 1 \)

Negative power rule: \( \frac{1}{a^m} = a^{-m} \)

Fractional exponents: \( \sqrt[a]{x} = x^{\frac{1}{a}} \)
Quantities represented in *scientific notation* (also known as *standard form*) have a *coefficient* between 1 and 10 multiplied by a power of 10. For example:

\[
1.67 \times 10^5
\]

**coefficient**  **power of 10**

To multiply or divide quantities represented in scientific notation, multiply/divide the coefficients and multiply/divide the powers of 10 using the laws of exponents. For example:

\[
(2.4 \times 10^7) \times (1.5 \times 10^{-3}) = (2.4 \times 1.5) \times (10^7 \times 10^{-3})
\]

\[
= 3.6 \times 10^4
\]

To add or subtract quantities represented in scientific notation, rewrite the quantities so that they have the same power of 10 and then add or subtract the coefficients. For example:

\[
(2.4 \times 10^8) - (1.5 \times 10^7) = (24 \times 10^7) - (1.5 \times 10^7)
\]

\[
= 10^7 \times (24 - 1.5)
\]

\[
= 22.5 \times 10^7 \text{ or } 2.25 \times 10^8
\]
Unit review

Key to Unit review question levels:

<table>
<thead>
<tr>
<th>Level 1–2</th>
<th>Level 3–4</th>
<th>Level 5–6</th>
<th>Level 7–8</th>
</tr>
</thead>
</table>

1. Classify each of the following numbers as either rational or irrational. **Justify** your answer.
   
   a. \( \frac{2}{3} \)  
   b. \( \pi \)  
   c. 7.68  
   d. \( \sqrt{17} \)  
   e. 33.914  
   f. 8.725555…  
   g. \( \sqrt{49} \)  
   h. 18

2. Represent each of the following rational numbers as a fraction in simplified form.
   
   a. 0.222…  
   b. 11.\( \overline{6} \)  
   c. 3.1  
   d. 2.\( \overline{4} \)  
   e. 2.4111…  
   f. −0.862  
   g. 3.04\( \overline{3} \)  
   h. 7.40111…

3. Our current number system, the Hindu–Arabic numeral system, was developed because humans required an efficient way to represent the quantities they were working with. Several mathematicians from India are credited with the development of the place-value system and the introduction of the number zero in the 5th and 6th centuries. Represent each of the following place values as a power of 10.
Evaluate each of the following. Write your answer as either an integer or a simplified fraction.

\[ \begin{align*}
    a & \quad 5^{-3} \\
    b & \quad -3^{-3} \\
    c & \quad (-27)^{\frac{1}{3}} \\
    d & \quad \left(\frac{3}{4}\right)^2 \\
    e & \quad \left(\frac{2}{5}\right)^{-1} \\
    f & \quad \left(\frac{4}{25}\right)^{\frac{1}{2}} \\
    g & \quad \left(\frac{1}{64}\right)^{-\frac{1}{3}} \\
    h & \quad \left(\frac{6}{7}\right)^{-2}
\end{align*} \]

Simplify each of the following. Express your answers with positive exponents only.

\[ \begin{align*}
    a & \quad a^3b^2 \times a^4b^{-5} \\
    b & \quad (28x^2y^{-3})^0 \\
    c & \quad -(14m^2n^3)(-2m^3n^2) \\
    d & \quad \frac{9a^2}{(-3a)^2} \\
    e & \quad (6x^5y^{-2})(3x^2y^3) \\
    f & \quad \frac{-14x^6y^7}{7x^{-3}y^9} \\
    g & \quad (6x^4y^3)(-4x^{-8}y^{-2}) \\
    h & \quad (15x^4c)(7x^{-6}c) \\
    i & \quad \frac{20n^4m^{-3}}{8n^8m^{-5}}
\end{align*} \]

Find the following products and quotients. Write your answers using scientific notation.

\[ \begin{align*}
    a & \quad (3 \times 10^5) \times (4 \times 10^8) \\
    b & \quad (2.4 \times 10^9) \div (1.2 \times 10^6) \\
    c & \quad (2.5 \times 10^{-4}) \times (3.1 \times 10^{-3}) \\
    d & \quad \frac{9.2 \times 10^3}{4.2 \times 10^6} \\
    e & \quad (8.1 \times 10^{-2}) \div (6.8 \times 10^{-7}) \\
    f & \quad (6.2 \times 10^{11}) \times (4.9 \times 10^{-13})
\end{align*} \]

Our understanding of our own place in the universe has developed over time, often influenced by personal beliefs rather than scientific evidence. While Nicolaus Copernicus is credited with formulating the current model of our solar system, a Greek astronomer, Aristarchus, promoted the idea of the Sun being at the center of our universe about 1800 years earlier.

Johannes Kepler was the first to propose the laws of planetary motion that are still in use today.
The distance between the Earth and Mars is constantly changing as the planets rotate about the Sun. The smallest distance between them is 54.6 million km and the greatest distance between them is 401 million km. The fastest spaceship to leave Earth was NASA’s New Horizons with a recorded speed of 58 000 km/h. Use this information to answer the following questions.

**a Calculate** the minimum number of days the spaceship would take to travel from Earth to Mars when the planets are at their closest to each other.

**b Calculate** the minimum number of days the spaceship would take to travel from Earth to Mars when the planets are at their furthest apart.

**8** Find the following sums and differences. Write your answers using standard form.

**a** $(6 \times 10^8) - (2 \times 10^7)$  
**b** $(5.5 \times 10^{-2}) + (3.1 \times 10^{-4})$

**c** $(7.3 \times 10^{-4}) - (8.6 \times 10^{-3})$  
**d** $(6.27 \times 10^3) + (5 \times 10^4)$

**e** $(9.1 \times 10^{11}) + (4.4 \times 10^{13})$  
**f** $(5 \times 10^9) - (2.7 \times 10^7)$

**9** The Human Genome Project began in 1990 with the goal of discovering and recording the complete sequence of DNA base pairs in human genetic material. It was a 13-year project that was almost completely successful, mapping over 99% of the human genome.

The table below gives the number of base pairs mapped during each three-year period from 1990 to 1999.

<table>
<thead>
<tr>
<th>Years</th>
<th>Number of base pairs mapped in each period</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990–1993</td>
<td>$2.0 \times 10^6$</td>
</tr>
<tr>
<td>1993–1996</td>
<td>$1.4 \times 10^8$</td>
</tr>
<tr>
<td>1996–1999</td>
<td>$4.7 \times 10^9$</td>
</tr>
</tbody>
</table>

**a** Find the total number of base pairs that were mapped between 1990 and 1999.

**b** If there are $3.2 \times 10^{10}$ base pairs in the human genome, find the number of base pairs that were mapped in the final four years of the project.
Simplify each of the following. Express your answers with positive exponents only.

\[a \left( \sqrt[49]{p^{26}q^{10}r^{-12}} \right)\]

\[b \left( \frac{2u^3v^{-2}}{(uv^5)^2} \right)^{-6}\]

\[c \left( \frac{(a^5b^{-6})^{-2}}{(a^2b^7)^{3}} \right)\]

\[d \left( \frac{x^2y^6}{a^{-2}b} \right)^2 \left( \frac{x^3y^0}{ab} \right)^{-3}\]

\[e \left( 16a^4b^{-2}c^6 \right)^{\frac{1}{2}} \left( -27a^{-6}b^9c^3 \right)^{\frac{1}{3}}\]

\[f \left( \frac{36m^3n^0}{121m^{-4}n^{-5}} \right)^{\frac{1}{2}}\]

\[g \left( 25g^{-5}h^{-1}j^7 \right)^{\frac{1}{2}} \left( -8g^4h^{-2}j^{-1} \right)^{\frac{1}{3}}\]

Neuroscience is the study of the nervous system, with neuroscientists focusing on the brain and its structure. It is a discipline that has developed since 500 BC, with major advances being made since the middle of the 20th century. While the brain has a high composition of water and fat, it also contains a large number of neurons that transmit and receive signals.

The cerebellum contains roughly 69 billion neurons. The cerebral cortex, made up of the frontal, parietal, occipital and temporal lobes, contains roughly 16 billion neurons. The remaining structures contain 690 million neurons.

\[a\] How many neurons are there in the brain in total? Express your calculations and answer in scientific notation.

\[b\] It has often been said that the number of neurons in the brain is the same as the number of stars in the universe. If there are \(7.0 \times 10^{22}\) stars in the universe, is this statement true? If not, which quantity is greater and by what factor?

\[c\] Each neuron has a length of approximately \(1 \times 10^{-4}\) meters. If they were placed end to end, what would be the length of all of the neurons in the brain?
The Richter Scale was devised by Charles Richter in 1940 to compare the intensities of earthquakes. The intensity of an earthquake is determined by the amount of ground motion measured on a seismometer. Each increase of 1 unit in magnitude on the Richter scale corresponds to an increase of 10 times the intensity measured on a seismometer.

a Using this ratio, how many times more intense was the 1556 earthquake in China with a magnitude of 8 compared with the 2010 earthquake in Haiti with a magnitude of 7?

By expressing the intensity \( I \) as an exponential function of the magnitude \( M \), you can compare the intensities of earthquakes that do not differ by a whole integer.

\[
I = 10^M
\]

b The world’s most powerful earthquake was in Chile in 1960 and registered magnitude 9.5 on the Richter Scale. The deadliest recorded tsunami was caused by an earthquake which registered magnitude 9.1 on the Richter scale off the coast of Indonesia in 2004. Using the formula given above, how many times more intense was the earthquake in Chile than the one in Indonesia?

c The two most costly earthquakes both occurred in Japan: the earthquake of 2011 had a magnitude of 9.1; the earthquake of 1995 had a magnitude of 6.9. Using the formula given above, how many times more intense was the earthquake in 2011 compared with the one in 1995?
Microchip technology

How is it possible to surf the internet? How can a smartphone control so many devices? How does a pacemaker help control a heart’s contractions? At the core of all of these is a single device called a transistor. The transistor was invented in 1945 in Bell Labs and the inventors had little idea how much it would revolutionize our way of life. In this task, you will analyze the growth of transistor technology and the development of the microprocessor chip.

You will present your work for each part in a single report. Show your working in each section. Perform all your calculations and write all your answers using scientific notation.

Part 1 – Moore’s law

Gordon Moore, one of the founders of Intel, helped build a company that produces processors for computer manufacturers. Processors, or microprocessors, are small chips inside devices such as smartphones and computers that receive input and produce output using transistors. In what has been named Moore’s law, Moore predicted that the number of transistors that would fit on a chip would double every two years.

If the very first chip had four transistors, use Moore’s law to calculate the number of transistors on a chip every two years over the next 10 years. Copy and complete this table, writing your answers as powers of 2.

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of transistors on chip</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>$4 \times 2 = 2^2$</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>
b If there were four transistors on a chip in 1965, predict the number of transistors on a chip in the year 2015. Write your answer both as a number in expanded form and in scientific notation.

c During a speech in 2014, one of Intel’s vice presidents said that, by 2026, the company would make a processor with as many transistors as there are neurons in a human brain. If there are $1.0 \times 10^{11}$ neurons in the human brain, would Moore’s law agree with the vice president’s statement?

Part 2 – Chip technology

Transistors can perform two functions. They can amplify current so that an input current is greatly increased as it passes through the transistor. Because of this, transistors were originally used to develop hearing aids. They can also act as switches, being either “on” or “off”. This allows the transistor to store two different numbers, either a 0 (off) or a 1 (on). Originally, vacuum tubes were used as switches, but these were large and required a lot of power.

a Smartphones have chips in them that can contain 3.3 billion transistors. If each transistor weighs $5.1 \times 10^{-23}$ grams, find the total mass of the transistors in a smartphone.

b If each chip has a length of 35 nanometers (nm), how many would you need to circle the Earth, which has a radius of 6371 km?

c Supercomputers have been developed that are much larger and can perform many more calculations than ordinary desktop or laptop computers. One such supercomputer, the Titan, has $4.485 \times 10^{10}$ transistors in its central processing unit (CPU) and another $1.3268 \times 10^{11}$ transistors in its graphics processing unit (GPU). Find the total number of transistors in the Titan. Show your working and give your answers in standard form.

d Intel estimates that about 12 quintillion transistors are shipped around the globe each year. If that represents 10 000 times the number of ants on the planet, find the number of ants on Earth.
Part 3 – Design your own

What if you could design your own processor? How small would you make it? How many transistors could you fit on it?

The size of transistors has decreased dramatically since they were first invented. Assume you will use transistors that are approximately rectangular and measure 35 nm by 14 nm.

a Select a chip size that sounds impressive (e.g. a fingernail). Find its area. (You may choose to research the area or calculate it after taking measurements.)

b Find the number of transistors that you will be able to fit on your chosen area.

c If transistors costs $0.000000003 USD each, find the cost of the transistors on your chip.

d Create a headline to announce your technology to the world.

e Write a newspaper article about your invention and create a snazzy name for your chip. Your article must include the following:

- **Headline** – usually only a few words. It’s purpose is to attract the interest of the reader by giving a hint as to what the article is about in a concise way.

- **By-line** – the author of the article.

- **Introduction** – sets the scene and summarizes the main points of the article: *who, what, when, where*.

- **Body** – provides more detail about the event, in particular it answers the questions *how* and *why*.

- **Quotes** – what a person (such as an eye-witness or an expert) has said about the invention. These will be in speech marks.

- **Photograph and caption** – include a drawing or photograph of your invention as well as a caption that describes what is in the photo.

- **Answers to these questions** – What does it take to make the next great discovery? Are great discoveries planned or accidental?