# Mastering Mathematics

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Introduction

What are children capable of achieving in mathematics? Though far from perfect, data from national assessments do give us some sense of how children in England are doing now. However, such data tell us almost nothing about what they might be able to do, given optimal conditions. We have become used to seeing roughly a fifth of children fall below national expectations by the end of primary school, with twice as many behind by age sixteen. We’re accustomed to seeing two-fifths of children ‘exceeding expectations’ by age ten. But are these national expectations high enough? Is this distribution of achievement inevitable? Could more children achieve more in mathematics?

In 2012, the OECD’s PISA study tested more than 510,000 students in 65 countries and economies, with a focus on mathematics.1 Shanghai and Singapore topped the table in mathematics, with students in Shanghai scoring the equivalent of nearly three years of schooling above most OECD countries. Several other Asian countries were also in the top performing group, as well three of our European neighbours – Liechtenstein, Switzerland and the Netherlands. These findings have made national news many times – some seem to find it surprising or even depressing that we are lagging behind. To others, these stories of international success are inspiring, and show just how much children are capable of. They demonstrate that our children’s current achievement in mathematics is not the best they can do; they can achieve much more.

The mastery approach is driven by a commitment to transform achievement in mathematics. Whilst every small improvement in understanding for every child merits celebration and can be transformative for that individual, the mastery approach is not just about slightly increasing the proportions who meet and exceed existing national expectations. It is driven by a determination to dramatically shift national expectations themselves, and to ensure that every single child meets them, and that many excel.

This handbook has been written for teachers and school leaders who share this commitment to transforming mathematics education. It shares some of the key principles of the mastery approach to teaching mathematics. It draws on the wide range of research that underpins the mastery approach, and uses examples of the approach in action to illustrate how teaching can be transformed and achievement raised. Teachers who teach for mastery expect every single child to succeed. This handbook introduces the frameworks and classroom approaches they use to ensure that these high expectations are met. To support its use as a professional development resource, discussion points are included at relevant points. These may provide a prompt for individual reflection, or could be used to initiate discussion in staff training.

The first chapter explores the idea of a commitment to success in mathematics for every child, and introduces the rationale for a focus on mastery. In subsequent chapters, the handbook addresses a mastery curriculum (Chapter 2), deep understanding (Chapter 3), purposeful learning (Chapter 4) and whole-school commitment and leadership (Chapter 5).

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About the author

Dr Helen Drury has more than a decade of experience as a mathematics teacher, head of mathematics and senior leader in Oxfordshire and London. She is passionate about bringing research and best practice into the classroom to close the achievement gap and raise achievement for all. As Director of Mathematics for ARK Schools, Helen founded the charitable, not-for-profit, Mathematics Mastery partnership to build up an evidence base to demonstrate that every child really can succeed with mathematics, and that the high achievement of our children can rival that of children anywhere in the world. Since 2012, the collaboration has extended beyond ARK’s network of schools to a large national partnership of primary and secondary member schools. These schools combine research-suggested teaching practices, professional judgement and classroom experience to collaboratively transform mathematics education.

Initially inspired by the high performance of countries such as Singapore, the underpinnings of the mastery approach provided by this handbook are theoretical and practical, local and international. It draws on national and international evidence and research findings, and is further developed through ongoing action research in schools in the UK. Throughout this handbook, examples of classroom practice are given to illustrate why or how to apply key principles of the approach. The vast majority of these examples are from the inspirational practice of schools in the Mathematics Mastery partnership, both within and beyond the ARK network of schools.

Though international successes have provided much of the inspiration, the mastery approach we explore here is grounded in UK classrooms.

In the foreword to Ofsted’s Made to Measure report, Sir Michael Wilshaw stated, “We must all play our part to ensure that all of our pupils receive the best possible mathematics education.” This handbook introduces some of the efforts teachers and school leaders have been making to do just that, and is part of an ongoing commitment to transformation.

Motivation is the most important factor in determining whether you succeed in the long run. What I mean by motivation is not only the desire to achieve, but also the love of learning, the love of challenge, and the ability to thrive on obstacles. These are the greatest gifts we can give our students.

Carol Dweck

Chapter 1

Every Child

What’s really possible?

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Mastery of mathematics

Mastery is the ultimate aim of learning

The need for the term ‘mastery’ stems from the wide spectrum of meanings of the verb ‘to learn’. If a child tells you, “today we learnt to round decimals with one decimal place to the nearest whole number”, they could mean any number of things. They might mean that after months working with decimal numbers, they fully understand decimal place value and relative size and can give the approximate size of numbers stated as decimals. What is perhaps more likely is that they have been given a rule for rounding decimals with one decimal place, and have correctly rounded a series of these numbers. Some teachers get frustrated that a child who, having ‘learnt’ to round decimals in one lesson, appears to have entirely ‘unlearnt’ this skill by the end-of-term test, or in class the following week or even day. This happens when the focus is on ‘learning’ mathematics in unconnected chunks, rather than on ‘mastering’ the subject over time.

In mathematics, you know you’ve mastered something when you can apply it to a totally new problem in an unfamiliar situation.

Whilst an hour’s lesson might be sufficient for someone to say that they have learnt something, mastery is a much longer term investment. Let’s take the example of mastering ‘counting from one to ten’. A child explores the concept of counting by experiencing it in a wide variety of contexts. They have the skill clarified when an adult or another child tells them “no, eight comes after seven”, or “you missed four”. They practise counting stairs, people, biscuits and much more. Almost every lesson will offer further opportunities to explore the idea, moments of clarification, and plenty of practice. As the child moves to mastery of ‘counting from one to ten’, there will be many chances to apply this skill. It is through this application – once sufficient exploration and practice have taken place and there is no longer any need for clarification (as no errors are made) – that the child comes to master the skill. As a Year 4 teacher, setting out with the intention that a class of eight- to nine-year-olds will ultimately master adding and subtracting fractions with the same denominator, the first thing that is clear is that this is not going to happen in sixty minutes. There may well be a lesson which begins with many children flummoxed by the question ‘add two-sixths to three-sixths’ and ends with them all confidently and correctly answering it – but this in itself is not necessarily a sign of successful mathematics teaching. Rather the curriculum throughout Year 4 (arguably throughout the school years) must be planned so that the necessary concepts and skills have already been explored, clarified and practised (the skill of adding positive integers, for example, and the concept of fractions) and appropriate relevant opportunities to apply the skill of adding fractions are built in to the subsequent months.
A mathematical concept or skill has been mastered when, through exploration, clarification, practice and application over time, a person can represent it in multiple ways, has the mathematical language to be able to communicate related ideas, and can think mathematically with the concept so that they can independently apply it to a totally new problem in an unfamiliar situation.

The ‘mastery approach’ that this handbook introduces is a teaching methodology that was born out of a passion for achieving mastery for every child. That mastery of primary mathematics is achievable by every child is therefore a key underpinning to the mastery approach.

This chapter looks at the rationale behind the mastery approach. The approach is motivated by a commitment to transforming achievement for all. This means placing a high value on mathematics education, adopting and promoting a growth mindset, and focusing on every child achieving a high expected standard and a high proportion excelling.