Number and algebra 1

Answers

Skills check

1 a \( y = 3x^2 (x - 1) \)
\( y = 3(-0.1)^2 (-0.1 - 1) \)
\( y = -0.033 \)
b \( y = \frac{(x-1)^2}{x} \)
\( y = \frac{(-0.1-1)^2}{-0.1} \)
\( y = -12.1 \)
c \( y = (1-x)(2x+1) \)
\( y = (1-(-0.1))(2\times-0.1+1) \)
\( y = 0.88 \)

2 a \( 3x - 7 = 14 \)
\( 3x = 14 + 7 \)
\( x = \frac{21}{3} \)
\( x = 7 \)
b \( 2(x-6) = 4 \)
\( x - 6 = \frac{4}{2} \)
\( x = 2 + 6 \)
\( x = 8 \)
c \( \frac{1}{2}(1-x) = 0 \)
\( 1-x = 0 \)
\( x = 1 \)
d \( x \cdot x = 16 \)
\( x = 4 \) or \( x = -4 \)

2 a \( 4x + 2 = 0 \)
\( 4x = -2 \)
\( x = \frac{-2}{4} \) (or \( x = -0.5 \))
b It is not an integer.

2 a \( x \cdot x = 4 \)
\( x = 2 \) or \( x = -2 \)
b Both are integers.

3 a \( \frac{a-b}{a+b} = \frac{-2-4}{-2+4} = \frac{-6}{2} = -3 \)
b i It is an integer.
ii It is not an integer.

Exercise 1B

1 a \( 4x + 2 = 0 \)
\( 4x = -2 \)
\( x = \frac{-2}{4} \)

Exercise 1C

1 Look for the decimal expansion of each of the fractions
\( \frac{2}{3} = 0.66666... \) Therefore the decimal expansion of this fraction recurs.
\( \frac{-5}{4} = -1.25 \) Therefore the decimal expansion of this fraction is finite.
\[
\frac{2}{9} = 0.22222\ldots \text{ Therefore the decimal expansion of this fraction recurs.}
\]

\[
\frac{4}{7} = 0.5714285714\ldots \text{ Therefore the decimal expansion of this fraction recurs.}
\]

\[
\frac{-11}{5} = -2.2. \text{ Therefore the decimal expansion of this fraction is finite.}
\]

\[
\frac{2}{a} \quad a = 0.5
\]

\[
\frac{a}{2} = 0.5555\ldots
\]

\[
10a = 5.5555\ldots
\]

\[
a = 0.5
\]

\[
\frac{b}{10} = 1.8
\]

\[
b = 1.8888
\]

\[
10b = 18.8888\ldots
\]

\[
b = 1.8
\]

\[
\frac{5}{9} + \frac{17}{9} = \frac{22}{9}
\]

\[
3 \ a \quad \text{It could be 0.8; 0.5; 2.1; 3.08; etc}
\]

\[
b \quad \text{It could be} \quad 0.12; 0.5; 1.24; 1.02; \text{etc}
\]

\[
c \quad \text{It could be} \quad 3.4578; 0.0002; 1.0023
\]

**Exercise 1D**

1. either work out the arithmetic mean of these numbers as shown in the book or look for their decimal expansion.

   The numbers are 2 and \( \frac{9}{4} \)
   Therefore 2 and 2.25
   Numbers in between may be for example 2.1; 2.2; 2.23

2. \( \sqrt{2(y-x)} \) when \( y = 3 \) and \( x = -\frac{1}{8} \)
   \[
   \sqrt{2 \left(3 - \left(-\frac{1}{8}\right)\right)} = \frac{5}{2} \text{ (or 2.5)}
   \]
   \[
b = \frac{5}{2} \text{ is a rational number}
   \]

3. a The numbers are \( \frac{9}{5} \) and \( \frac{11}{6} \)
   Therefore 1.8 and 1.83
   Numbers in between may be for example 1.81; 1.82; 1.83.

   b i The numbers are \( \frac{28}{13} \) and \( -2 \)
   Therefore \( -2.15384\ldots \) and \( -2 \)
   Numbers in between may be for example \( -2.14; -2.12; -2.1 \)
   ii infinite

   Exercise 1E

1. a It is a right angled triangle.
   \( h^2 = 2^2 + 1.5^2 \)
   \( h^2 = 6.25 \)
   \( h = 2.5 \text{ cm} \)

   b \( h \) is rational.

2. a \( r = \frac{10}{2} = 5 \text{ cm} \)
   \( A = \pi \times 5^2 \)
   \( A = 25\pi \text{ cm}^2 \)

   b \( A \) is irrational.

**Exercise 1F**

1. a i \( 0.5 \leq \frac{x}{2} \leq 1.5 \)
   multiply by 2
   \( 2 \times 0.5 < 2 \times \frac{x}{2} \leq 2 \times 1.5 \)
   \( 1 < x \leq 3 \)

   ii make \( x \) the subject of the inequality
   \( 3 - x \geq 1 \)
   \( 3 \geq 1 + x \)
   \( 2 \geq x \)

   b i \( 1 \)
   ii \( 3 \)
   iii \( 2 \)

   c i \( q = 1.5 \) is solution as \( 1 < 1.5 \leq 3 \)
   \( t = \sqrt{5} \) is solution as \( 1 < \sqrt{5} \leq 3 \)

   ii \( q = 1.5 \) is solution as \( 2 \geq 1.5 \).
   \( t = \sqrt{5} \) is not solution as the inequality \( 2 \geq \sqrt{5} \) is not true.

2. a i \( 2x + 1 > -1 \)
   \( x > \frac{-2}{2} \)
   \( x > -1 \)

   ii \( 4 \leq x + 1 \leq 8 \)
   \( 4 - 1 \leq x + 1 - 1 \leq 8 - 1 \)
   \( 3 \leq x \leq 7 \)

   iii \( 2 - x > -1 \)
   \( 3 > x \)

   b i \( -1 \)
   ii \( 3 \)
   iii \( 3 \)

   c substitute each of these numbers in the inequalities

<table>
<thead>
<tr>
<th>( p )</th>
<th>( 2x + 1 &gt; -1 )</th>
<th>( 4 \leq x + 1 \leq 8 )</th>
<th>( 2 - x &gt; -1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{2}{3} )</td>
<td>( \sqrt{\checkmark} )</td>
<td>( \sqrt{\checkmark} )</td>
<td>( \checkmark )</td>
</tr>
<tr>
<td>( \sqrt{10} )</td>
<td>( \sqrt{\checkmark} )</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
</tr>
<tr>
<td>( 2\pi )</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
</tr>
</tbody>
</table>
Exercise 1G
1 i 358.4 = 358 to the nearest unit
   ii 24.5 = 25 to the nearest unit
   iii 108.9 = 109 to the nearest unit
   iv 10016.01 = 10016 to the nearest unit
2 i 246.25 = 250 correct to the nearest 10
   ii 109 = 110 correct to the nearest 10
   iii 1015.03 = 1020 correct to the nearest 10
   iv 269 = 270 correct to the nearest 10
3 i 140 = 100 correct to the nearest 100.
   ii 150 = 200 correct to the nearest 100.
   iii 1240 = 1200 correct to the nearest 100.
   iv 3062 = 3100 correct to the nearest 100.
4 i 105 607 = 106 000 correct to the nearest 1000.
   ii 1500 = 2000 correct to the nearest 1000.
   iii 9640 = 10 000 correct to the nearest 1000.
   iv 952 = 1000 correct to the nearest 1000.

Any $x$ where $150 \leq x < 250$

Any $x$ where $2500 \leq x < 3500$

Any $x$ where $5.5 \leq x < 6.5$

Exercise 1H
1 i $45.67 = 45.7$ correct to 1 d.p.
   ii $301.065 = 301.1$ correct to 1 d.p.
   iii $2.401 = 2.4$ correct to 1 d.p.
   iv $0.09 = 0.1$ correct to 1 d.p.
2 i $0.0047 = 0.00$ correct to 2 d.p.
   ii $201.305 = 201.31$ correct to 2 d.p.
3 i $10.0485 = 10.049$ correct to the nearest thousandth.
   ii $3.9002 = 3.900$ correct to the nearest thousandth.
   iii $201.7805 = 201.781$ correct to the nearest thousandth.
   iv $0.00841 = 0.008$ correct to the nearest thousandth.
4 $\frac{\sqrt{1.8}}{3.04 \times 0.012^2} = 3064.786153$.
   i 3064.8 (1 d.p.)
   ii 3064.79 (2 d.p.)
   iii 3064.786 (3 d.p.)
   iv 3100 correct to the nearest 100
   v 3000 correct to the nearest 1000
5 $\left(\frac{p + q}{p + q}\right)^3 = 15.6025$
   i 15.60 (2 d.p.)
   ii 15.603 (3 d.p.)
   iii 16 correct to the nearest unit
   iv 20 correct to the nearest 10
6 Any $x$ where $2.365 \leq x < 2.375$
7 Any $x$ where $4.05 \leq x < 4.15$

Exercise 1I
1 i 106 has 3 significant figures as all non-zero digits are significant and zeros between non-zero digits are significant.
   ii 200 has 1 significant figure as trailing zeros in a whole number are not significant.
   iii 0.02 has 1 significant figure as all non-zero digits are significant and zeros to the left of the first non-zero digit are not significant.
   iv 1290 has 3 significant figures as trailing zeros in a whole number are not significant.
   v 1209 has 4 significant figures as all non-zero digits are significant and zeros between non-zero digits are significant.
2 i $280 = 300$ (1 s.f.)
   ii $0.072 = 0.07$ (1 s.f.)
   iii $390.8 = 400$ (1 s.f.)
   iv $0.001 32 = 0.001$ (1 s.f.)
3 i $355 = 360$ (2 s.f.)
   ii $0.0801 = 0.080$ (2 s.f.)
   iii $1.075 = 1.1$ (2 s.f.)
   iv $1560.03 = 1600$ (2 s.f.)
4 i $2971 = 2970$ (3 s.f.)
   ii $0.3259 = 0.326$ (3 s.f.)
   iii $10410 = 10400$ (3 s.f.)
   iv $0.5006 = 0.501$ (3 s.f.)
5 $\sqrt[3]{8.7 + 2 \times 1.6} = 425.881 192 9$
   a 400 correct to 1 significant figures
   b 426 correct to 3 significant figures
   c 425.9 correct to 1 decimal place
   d 425.88 correct to the nearest hundredth
6 $\pi = 3.141592654$
   a 3 correct to the nearest unit
   b 3.14 correct to 2 d.p.
   c 3.1 correct to 2 s.f.
   d 3.142 correct to 3 d.p.
7  
\( a \) \( 238 = 200 \) (1 s.f.)  
\( b \) \( 4609 = 4610 \) (3 s.f.)  
\( c \) \( 2.700 \approx 2.70 \) (3 s.f.)

8  
\( a \) \( \frac{\sqrt{3.375}}{1.5^2 + 1.8} = 0.370 \)  
\( b \) \( 0.37 \)  
\( i \) \( 0.370 \)  
\( ii \) \( 0.3704 \)  
\( iii \) \( 0.3704 \)

Exercise 1J

1  
\( a \) \( A = \pi r^2 \) \( 10.5 = \pi r^2 \) \( r = \frac{\sqrt{10.5}}{\pi} \) \( r = 1.828 \) cm (4 s.f.)  
\( b \) \( C = 2\pi \) \( C = 2\pi \times \frac{\sqrt{10.5}}{\pi} \) \( C = 11 \) cm (2 s.f.)

2  
\( a \) \( \frac{\sqrt{2} + \sqrt{10}}{2} = 2.288 \) (4 s.f.)  
\( b \) substitute the values of \( p \) and \( q \) in the formula. \( (p + q)^2 = (\sqrt{2} + \sqrt{10})^2 = 20.9 \) (3 s.f.)  
\( c \) \( \sqrt{2} \times \sqrt{10} = 4.5 \) cm² (2 s.f.)

Exercise 1K

1  
\( a \) \( 298 \times 10.75 = 300 \times 10 = 3000 \)  
\( b \) \( 3.8^2 = 3.8 \times 3.8 = 4 \times 4 = 16 \)  
\( c \) \( 147 \approx 150 = 15 \)  
\( d \) \( 11.02 \approx 10 \)  
\( e \) \( \sqrt{103} = \sqrt{100} = 10 \)  
\( f \) \( 210 \times 18 = 200 \times 20 = 4000 \) pipes.  
\( g \) population density = \( \frac{\text{total population}}{\text{land area}} \)  
\( h \) population density = \( \frac{127,076,183}{377,835} \)  
\( i \) population density = \( \frac{120,000,000}{40,0000} \)  
\( j \) population density = \( 300 \) people per km²

2  
Number of reams = \( \frac{9000}{500} \)  
Number of reams = \( \frac{10000}{500} \)  
Number of reams = \( 20 \)

3  
Average speed = \( \frac{\text{distance travelled}}{\text{time taken}} \)  
Average speed = \( \frac{33}{1.8} \)  
Average speed = \( \frac{30}{2} \)  
Average speed = \( 15 \) km h⁻¹

4  
Number of visitors per year = \( 53,000 \times 365 \)  
Number of visitors per year = \( 50,000 \times 400 \)  
Number of visitors per year = \( 20,000,000 \)

6  
\( 7 \) estimate the area of the square using reasonable numbers.  
Area of square = \( 100.1 \times 100.1 \)  
Area of square = \( 100 \times 100 \)  
Area of square = \( 10,000 \) m²  
Therefore Peter's calculation is not correct. \( 10,000 \) is far bigger than \( 1020.01 \)

Exercise 1L

1  
\( a \) substitute the values of \( a \) and of \( b \) in the given formula. \( 3a + b^3 = 3 \times 5.2 + 4.7^3 = 119.423 \)  
\( b \) Percentage error = \( \frac{v_4 - v_E}{v_E} \times 100\% \)  
Percentage error = \( \frac{140 - 119.423}{119.423} \times 100\% \)  
Percentage error = \( 17.2\% \) (3 s.f.)

2  
\( a \) Actual final grade = \( \frac{8.3 + 6.8 + 9.4}{3} \)  
Actual final grade = \( 8.17 \) (3 s.f.)  
\( b \) The three grades rounded are 8, 7 and 9.  
Approximate final grade = \( \frac{8 + 7 + 9}{3} \)  
Approximate final grade = \( 8 \)

3  
\( a \) Exact area = \( 5.34 \times 3.48 \)  
Exact area = \( 18.5832 \) m²  
\( b \) Length = \( 5.3 \) m  
Width = \( 3.5 \) m  
\( c \) Approximate area = \( 18.55 \) m²  
Percentage error = \( \frac{18.55 - 18.5832}{18.5832} \times 100\% \)  
Percentage error = \( 0.179\% \) (3 s.f.)

4  
\( a \) \( A = \pi r^2 \) \( 89 = \pi r^2 \) \( r = \frac{\sqrt{89}}{\pi} \) cm \( r = 5.323 \) m (3 d.p.)  
\( b \) \( C = 2\pi \) \( C = 2\pi \times \frac{\sqrt{89}}{\pi} \)  
\( C = 33.4 \) m (3 s.f.)
c Approximate value for perimeter = 30 m
Accepted value for perimeter = 33.4 m
Percentage error = \( \frac{30 \text{ m} - 33.4 \text{ m}}{33.4 \text{ m}} \times 100\% \)
Percentage error = 10% (2 s.f.)

Exercise 1M
1 \( 2.5 \times 10^{-3}; 10^9 \)
2 a number is written in standard form if it is written as
\( a \times 10^k \) where \( 1 \leq a < 10 \) and \( k \) is an integer.

a \( 135 \ 600 = 1.356 \times 10^5 \) or \( 1.36 \times 10^5 \) (3 s.f.)
b \( 0.00245 = 2.45 \times 10^{-3} \)
c \( 16 \ 000 \ 000 \ 000 = 1.6 \times 10^{10} \)
d \( 0.000 \ 108 = 1.08 \times 10^{-4} \)
e \( 0.23 \times 10^3 = 2.3 \times 10^2 \)

Exercise 1N
1 a \( x \times y = 6.3 \times 10^6 \times 2.8 \times 10^{10} = 1.764 \times 10^{17} \)
or \( 1.76 \times 10^{17} \) (3 s.f.)
b \( \frac{x}{y} = \frac{6.3 \times 10^6}{2.8 \times 10^{10}} = 2.25 \times 10^{-4} \)
c \( \sqrt{\frac{x}{y}} = \sqrt{\frac{6.3 \times 10^6}{2.8 \times 10^{10}}} = 1.5 \times 10^{-2} \)
2 a the arithmetic mean between \( a \) and \( b \) is simply \( \frac{a + b}{2} \).
Arithmetic mean = \( \frac{2.5 \times 10^6 + 3.48 \times 10^6}{2} \)
Arithmetic mean = 2990000
Arithmetic mean = 2.99 \times 10^6

Exercise 10
1 a \( \text{km h}^{-2} \) or \( \text{km/h}^2 \)
b \( \text{kg m}^{-3} \) or \( \text{kg/m}^3 \)
c \( \text{m s}^{-1} \) or \( \text{m/s} \)

Exercise 1P
1 a \( 2.36 \text{ m}^2 = 2.36 \times 10^4 \text{ cm}^2 = 23600 \text{ cm}^2 \)
b \( 1.5 \text{ dm}^2 = 1.5 \times 10^{-4} \text{ dm}^2 = 0.00015 \text{ dm}^2 \)
c \( 5400 \text{ mm}^2 = 5400 \times 10^{-2} \text{ cm}^2 = 54 \text{ cm}^2 \)
d \( 0.06 \text{ m}^2 = 0.06 \times 10^2 \text{ mm}^2 = 60000 \text{ mm}^2 \)
e \( 0.8 \text{ km}^2 = 0.8 \times 10^4 \text{ hm}^2 = 80 \text{ hm}^2 \)
f \( 35000 \text{ m}^2 = 35000 \times 10^{-6} \text{ km}^2 = 0.035 \text{ km}^2 \)
2 a \( 5 \text{ m}^3 = 5 \times 10^6 \text{ cm}^3 = 5000000 \text{ m}^3 \)
b \( 0.1 \text{ dam}^3 = 0.1 \times 10^3 \text{ m}^3 = 1 \times 10^2 \text{ m}^3 \)
c \( 350000 \text{ mm}^3 = 350000 \times 10^{-6} \text{ dm}^3 = 3.5 \times 10^2 \text{ dm}^3 \)
d \( 255 \text{ m}^3 = 255 \times 10^9 \text{ mm}^3 = 2.55 \times 10^{11} \text{ mm}^3 \)
e \( 12000 \text{ m}^3 = 12000 \times 10^{-3} \text{ dam}^3 = 1.2 \times 10^4 \text{ dam}^3 \)
0.7802 \text{ hm}^3 = 0.7802 \times 10^3 \text{ dam}^3
\begin{align*}
&= 7.802 \times 10^2 \text{ dam}^3 \\
&= 7.80 \times 10^2 \text{ dam}^3 \text{(3 s.f.)}
\end{align*}

the area of a square with side length \( l \) is \( l^2 \).

Area = \( l \times l \)

Area = 13 \text{ cm}^2

Area = 169 \text{ cm}^2

Area = 169 \times 10^{-4} \text{ m}^2 = 0.0169 \text{ m}^2

the volume of a cube with side length (or edge) \( l \) is \( l^3 \).

Volume = \( l^3 \)

Volume = 0.85 \text{ m}^3

Volume = 0.614125 \text{ m}^3

Volume = 0.614125 \times 10^6 \text{ cm}^3

Volume = 614125 \text{ cm}^3 \text{ or } 614000 \text{ cm}^3 \text{ (3 s.f.)}

convert all the measurements to the same unit.

0.081 \text{ dam}^2 = 8.1 \text{ m}^2

8000000 \text{ mm}^2 = 8 \text{ m}^2

82 \text{ dm}^2 = 0.82 \text{ m}^2

7560 \text{ cm}^2 = 0.756 \text{ m}^2

Therefore the list from smallest is

7560 \text{ cm}^2; 0.8 \text{ m}^2; 82 \text{ dm}^2 8000000 \text{ mm}^2; 0.081 \text{ dam}^2

convert all the measurements to the same unit.

11.2 \text{ m}^3

1200 \text{ dm}^3 = 1.2 \text{ m}^3

0.01 \text{ dam}^3 = 10 \text{ m}^3

11020000000 \text{ mm}^3 = 11.02 \text{ m}^3

10900000 \text{ cm}^3 = 10.9 \text{ m}^3

Therefore the list from smallest is

1200 \text{ dm}^3; 0.01 \text{ dam}^3; 10 \text{ m}^3; 11020000000 \text{ mm}^3; 11.2 \text{ m}^3

Exercise 1Q

1 a change all to seconds

1 \text{ d} = 24 \text{ h} = 24 \times 60 \text{ min}

= 24 \times 60 \times 60 \text{ s} = 86400 \text{ s}

2 \text{ h} = 2 \times 60 \text{ min} = 2 \times 60 \times 60 \text{ s} = 7200 \text{ s}

23 \text{ min} = 23 \times 60 \text{ s} = 1380 \text{ s}

Therefore

1 \text{ d} 2 \text{ h} 23 \text{ min} = 86400 \text{ s} + 7200 \text{ s} + 1380 \text{ s}

= 94980 \text{ s}

b 94980 \text{ s} = 95000 \text{ (nearest 100)}

2 a change all to seconds

2 \text{ d} = 48 \text{ h} = 48 \times 60 \text{ min} = 48 \times 60 \times 60 \text{ s}

= 172800 \text{ s}

5 \text{ min} = 5 \times 60 \text{ s} = 300 \text{ s}

Therefore

2 \text{ d} 5 \text{ min} = 172800 \text{ s} + 300 \text{ s} = 173100 \text{ s}

b 173100 \text{ s} = 1.731 \times 10^5 \text{ s or } 1.73 \times 10^5 \text{ s (3 s.f.)}

3 a \( 5.1 = 5 \times 10^1 \text{ ml} = 5000 \text{ ml} \)

b \( 0.56 \text{ ml} = 0.56 \times 10^{-3} \text{ hl} = 0.0000056 \text{ hl} \)

c \( 4500 \text{ dal} = 4500 \times 10^1 \text{ cl} = 4500000 \text{ cl} \)

4 a \( 1 \text{ l} = 1 \text{ dm}^3 \)

b \( 500 \text{ l} = 500 \text{ dm}^3 = 500 \times 10^1 \text{ cm}^3 = 5 \times 10^6 \text{ cm}^3 \)

b \( 145.8 \text{ dl} = 14.58 \text{ l} = 1.458 \times 10^1 \text{ dm}^3 \)

or \( 1.46 \times 10^1 \text{ dm}^3 \) (3 s.f.)

c \( 8 \text{ hl} = 800 \text{ l} = 800 \text{ dm}^3 = 800 \times 1000 \text{ cm}^3 = 8 \times 10^3 \text{ cm}^3 \)

4 a Area = \( l \times l \)

Area = 13 \text{ cm}^2

Area = 169 \text{ cm}^2

4 b the volume of a cube with side length (or edge) \( l \) is \( l^3 \).

Volume = \( l^3 \)

Volume = 0.85 \text{ m}^3

Volume = 0.614125 \text{ m}^3

Volume = 0.614125 \times 10^6 \text{ cm}^3

Volume = 614125 \text{ cm}^3 \text{ or } 614000 \text{ cm}^3 \text{ (3 s.f.)}

5 a 12.5 \text{ m}^3 = 12.5 \text{ l} = 13 \text{ correct to the nearest unit.}

b 0.368 \text{ m}^3 = 0.368 \times 10^3 \text{ dm}^3 = 368 \text{ dm}^3

= 368 \text{ l} = 3.68 \text{ hl}

= 4 \text{ hl} correct to the nearest unit.

c 809 \text{ cm}^3 = 809 \times 10^{-3} \text{ dm}^3 = 0.809 \text{ dm}^3

= 0.809 \text{ l} = 80.9 \text{ cl}

= 81 \text{ cl} correct to the nearest unit.

6 a Average speed = \frac{\text{distance travelled}}{\text{time taken}}

40 \text{ m min}^{-1} = \frac{3000 \text{ m}}{\text{time taken}}

\text{time taken} = \frac{3000 \text{ m}}{40 \text{ m min}^{-1}}

\text{time taken} = 75 \text{ min}

b 75 \text{ min} = 75 \times 60 \text{ s} = 4500 \text{ s}

7 a Volume = \( 1.5^3 \)

Volume = 3.375 \text{ m}^3

b 3.375 \text{ m}^3 = 3.375 \times 10^3 \text{ dm}^3 = 3375 \text{ dm}^3

c 3375 \text{ dm}^3 = 3375 \text{ l} \text{ and } 3375 \text{ l} < 4000 \text{ l}

\text{therefore } 4000 \text{ l} \text{ of water cannot be poured in this container. Only } 3375 \text{ l}

\text{can be poured.}

8 a \( \frac{4}{5} \) of 220 \text{ cm}^3 = 176 \text{ cm}^3

176 \text{ cm}^3 = 176 \times 10^{-3} \text{ dm}^3 = 0.176 \text{ l}

b \( \frac{15}{0.176} = 8.52 \text{ tea cups therefore Mercedes can serve up to 8 tea cups.}

9 a Average speed = \frac{\text{distance travelled}}{\text{time taken}}

800 \text{ km h}^{-1} = \frac{6900 \text{ km}}{\text{time taken}}

\text{time taken} = \frac{6900 \text{ km}}{800 \text{ km h}^{-1}}

\text{time taken} = 8.625 \text{ h} \text{ or } 8.63 \text{ h (3 s.f.)}

b Average speed = \frac{\text{distance travelled}}{\text{time taken}}

\text{Average speed} = \frac{1393 \text{ km}}{2 \text{ h}}

\text{Average speed} = 696.5 \text{ km h}^{-1} \text{ or } 697 \text{ km h}^{-1}
c  Time travelling = 8.625 h + 2 h + 1.5 h
     = 12.125 h
Arrival time = 10 + 12.125 = 22.125 h or 10:13 PM

**Exercise 1R**

1 a  \( t_c = t_F - 273.15 \)
   \( t_c = 280 - 273.15 = 6.85 \)
   6.85 °C = 6.9 °C correct to one tenth of degree

b  \( 80 = \frac{9}{5} t_c + 32 \)
   \( t_c = \frac{(80 - 32) \times 5}{9} \)
   \( t_c = 80 \times \frac{5}{9} = 26.6 \)
   26.6 °C = 26.7 °C correct to one tenth of degree

Exercise 1R

1 a  \( t_c = t_F - 273.15 \)
   \( t_c = 280 - 273.15 = 6.85 \)
   6.85 °C = 6.9 °C correct to one tenth of degree

b  \( 80 = \frac{9}{5} t_c + 32 \)
   \( t_c = \frac{(80 - 32) \times 5}{9} \)
   \( t_c = 80 \times \frac{5}{9} = 26.6 \)
   26.6 °C = 26.7 °C correct to one tenth of degree

Exercise 1R

1 a  \( t_c = t_F - 273.15 \)
   \( t_c = 280 - 273.15 = 6.85 \)
   6.85 °C = 6.9 °C correct to one tenth of degree

b  \( 80 = \frac{9}{5} t_c + 32 \)
   \( t_c = \frac{(80 - 32) \times 5}{9} \)
   \( t_c = 80 \times \frac{5}{9} = 26.6 \)
   26.6 °C = 26.7 °C correct to one tenth of degree

**Review exercise**

**Paper 1 style questions**

1  \( \begin{array}{ccccc}
5 & \pi & 2 & -3 & 0.8 \\
N & \checkmark & & & \\
Z & \checkmark & \checkmark & & \\
Q & \checkmark & \checkmark & \checkmark & \checkmark \\
R & \checkmark & \checkmark & \checkmark & \checkmark \\
\end{array} \)

2 a  \( \sqrt{2} \)
   \( \sqrt{2} = 1.4142 \)
   \( 14.1 \times 10^{-1} = 1.41 \times 10^{0} \)

b  \( \sqrt{2} = 1.4142 \)
   \( 0.00139 \times 10^{2} = 1.39 \times 10^{-3} \)
   \( 1414 \times 10^{-2} = 1.414 \times 10^{1} \)
   \( 0.00139 \times 10^{3}; 14.1 \times 10^{-1}; \)
   \( \sqrt{2}; 1414 \times 10^{-2}; 1.4 \times 10^{2} \)

3 a  2690 kg = 2.69 \times 10^{3} kg

b  i  2.7 \times 10^{3} kg = 2700 kg
   ii  \[ \text{percentage error formula} \]
   \[ \text{Percentage error} = \frac{|v_A - v_E|}{v_E} \times 100\% \]
   \[ \text{Percentage error} = \frac{2700 - 2690}{2690} \times 100\% \]
   \[ \text{Percentage error} = 0.372\% \ (3 \text{ s.f.}) \]

4 a  299 792 458 m s\(^{-1}\) = 300 000 000 m s\(^{-1}\)
   b i  m s\(^{-1}\) means metres per second therefore the answer from a gives you the distance traveled in 1 second.
   \( 1 \text{ s } \rightarrow 300 000 000 \text{ m} \)
   \( 300 000 000 \text{ m} = 300 000 000 \times 10^{-3} \text{ km} \)
   \( = 300 000 \text{ km} \)
   \( 3600 \text{ s } \rightarrow 300 000 \text{ km} \times 3600 \)
   \( = 1080 000 000 \text{ km} \)
   Therefore the average velocity is 1.08 \times 10^{9} \text{ km h}^{-1}

5 a  52200 \times 90 = 580 g
   580 g = 580 \times 10^{-3} \text{ kg} = 0.580 \text{ kg}
   b 0.580 kg = 0.6 \text{ kg} (1 \text{ s.f.})
   c Accepted value = 0.6 kg
   Estimated value = 0.4 kg
   \[ \text{Percentage error} = \frac{v_A - v_E}{v_E} \times 100\% \]
   \[ \text{Percentage error} = \frac{0.4 - 0.6}{0.6} \times 100\% \]
   \[ \text{Percentage error} = 33.3\% \ (3 \text{ s.f.}) \]

6 a  1560 \text{ cm}^{3} = 1560 \times 10^{-3} \text{ dm}^{3} = 1.56 \text{ dm}^{3}
   b 1.56 \text{ dm}^{3} = 1.56 l
   \( \frac{3}{4} \text{ of } 1.56 \text{ l} = 1.17 l \)
   c i  \( \frac{25}{37} \) = 21.4 jars
   Therefore Sean pours 21 jars.
   ii  21 \times 1.17 = 24.571
   25 - 24.57 = 0.431
7 a  \( x = \frac{30y^2}{\sqrt{y^4 + 1}} \) when \( y = 1.25 \)
\[
x = \frac{30(1.25)^2}{\sqrt{1.25^4 + 1}}
\]
\( x = 31.25 \)
b  \( 31.25 = 31.3 \) (3 s.f.)
c  \( 31.3 = 3.13 \times 10^1 \)

8 a  \( A = x^2 \)
b i  \( x = 2.56 \) km \( = 2.56 \times 10^6 \) m\(^2 = 2560000 \) m\(^2 \)
\[
x = 2 \times 2560000
\]
\( x = \sqrt{2} \times 2560000 
\]
\( x = 1600 \) m

9 a  \( t_F = \frac{9}{5} \times t_K - 459.67 \)
\[
t_K = \frac{9}{5} \times 100 = 459.67
\]
\( t_K = \frac{5}{9} (100 + 459.67) \)
\( t_F = 310.927... = 311 \) correct to the nearest unit

10 a  \( 4x + 5 > x + 6 \)
\( x > 1 \)

b  
\[
\begin{array}{c|c|c|c|c|c}
  & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
\hline
\hline
1 & & & & & \checkmark & & \\
0.75 & & & & & & & \checkmark \\
0.75 & & & & & & & \checkmark \\
\hline
\end{array}
\]
Therefore \( \sqrt{3}; 2.06; \frac{101}{100} \) ...

11 a  Area = 210 mm \( \times 297 \) mm
\[
\text{Area} = 62370 \text{ mm}^2
\]
b  \( 62370 \text{ mm}^2 = 62370 \times 10^{-6} \text{ m}^2 = 0.062370 \text{ m}^2 \)
c  \( 1 \text{ m}^2 = \frac{75 \text{ g}}{0.062370 \text{ m}^2} \)
\[
0.062370 \text{ m}^2 \text{ per page weight} \rightarrow 0.062370 \times 75
\]
\( = 4.67775 \text{ g} = 4.68 \text{ g} \) (3 s.f.)
d  \( 4.68 \times 500 = 2340 \text{ g} \)
\( 2340 \text{ g} = 2340 \times 10^{-3} \text{ kg} = 2.34 \text{ kg} \)

Review exercise
Paper 2 style questions
1 a  Perimeter of the field = 2 \( \times 2500 + 2 \times 1260 \)
\[
\text{Perimeter of the field} = 7520 \text{ m}
\]
\( 7520 \text{ m} = 7520 \times 10^{-3} \text{ km} = 7.52 \text{ km} \)

b  Cost of fencing the field = 7.52 \times 327.64
Cost of fencing the field = 2463.85 \text{ (2 d.p.)}
\[
V_A = 7.6 \times 327.64 = 2490.064
\]
c  Percentage error = \( \frac{v_A - v_E}{v_E} \times 100\% \)
\[
\text{Percentage error} = \frac{2490.064 - 2463.85}{2463.85} \times 100\%
\]
Percentage error = 1.06\% (3 s.f.)
d  Area of the field = 2500 \times 1260
Area of the field = 315000 \text{ m}^2
Area of the field = 315000 \times 10^{-6} \text{ km}^2
\[
= 3.15 \text{ km}^2
\]

2 a  Radius of semicircles = \( \frac{400}{2} = 200 \text{ m} \)
Length of circumference = \( 2\pi \)
Length of circumference = \( 2\pi \times 200 = 400\pi \)
Perimeter = \( 2 \times 800 + 400\pi \)
Perimeter = 2856.637... m
\[
= 2857 \text{ m correct to the nearest metre.}
\]
b  Number of laps that Elger runs = \( \frac{\text{total distance run by Elger}}{\text{perimeter of running track}} \)
Number of laps that Elger runs = \( \frac{14200}{2856.637...} \)
Number of laps that Elger runs = 4.97
Therefore Elger runs 4 complete laps around the track.
c  convert the distance to km
\[
2856.637... \text{ m} = 2856.637... \times 10^{-3} \text{ km}
\]
\[
= 2.856637... \text{ km}
\]
average speed = \( \frac{\text{distance travelled}}{\text{time taken}} \)
\[
19 \text{ km h}^{-1} = \frac{2.856637... \text{ km}}{19 \text{ km h}^{-1}}
\]
time taken = \( \frac{2.856637... \text{ km}}{19 \text{ km h}^{-1}} \)
time taken = 0.150 h (3 s.f.)
d  average speed = \( 19 \text{ km h}^{-1} = \frac{19 \text{ km}}{1 \text{ h}} = \frac{19000 \text{ m}}{60 \text{ min}} \)
\[
= \left( \frac{19000}{60} \right) \text{ m min}^{-1}
\]
\[
\left( \frac{19000}{60} \right) \text{ m min}^{-1} = 14200 \text{ m time taken}
\]
time taken = \( \frac{14200 \text{ m}}{14200 \text{ m time taken}} \)
time taken = 44.842 min (5 s.f.)
e  Percentage error = \( \frac{v_A - v_E}{v_E} \times 100\% \)
\[
\text{Percentage error} = \frac{44 - 44.842}{44.842} \times 100\%
\]
Percentage error = 1.88\% (3 s.f.)
3  a  Diameter = 2.5 cm
   Radius = \( \frac{2.5}{2} = 1.25 \) cm
   Volume of one chocolate = \( \frac{4}{3} \pi r^3 \)
   Volume of one chocolate = \( \frac{4}{3} \pi (1.25)^3 \)
   Volume of one chocolate = \( 8.18123 \ldots \) cm\(^3\)
   = \( 8.18 \) cm\(^3\) (2 d.p.)

3  b  first convert the measurements to cm.
   Radius of cylindrical box = 12.5 mm
   = 1.25 cm
   Volume of cylindrical box = \( \pi r^2 h \)
   Volume of cylindrical box = \( \pi (1.25)^2 15 \)
   Volume of cylindrical box = \( 73.63107 \ldots \) cm\(^3\)
   = \( 73.63 \) cm\(^3\) (2 d.p.)

3  c  Number of chocolates in the box = \( \frac{15}{2.5} = 6 \) chocolates

3  d  Volume occupied by the chocolates
   = \( 8.18123 \ldots \times 6 = 49.087 \ldots \) cm\(^3\)
   Volume not occupied by the chocolates
   = volume of box – volume occupied by chocolates
   Volume not occupied by the chocolates
   = \( 73.63107 \ldots - 49.087 \ldots = 24.5 \) cm\(^3\) (3 s.f.)

3  e  24.5 cm\(^3\) = \( 24.5 \times 10^3 \) mm\(^3\) = \( 24500 \) mm\(^3\)

3  f  \( 2.45 \times 10^4 \) mm\(^3\)