Answers
Skills check
1 a \(1 - \frac{3}{7} = \frac{7}{7} - \frac{3}{7} = \frac{4}{7}\)
b \(\frac{2}{3} + \frac{5}{7} = \frac{14+25}{35} = \frac{39}{35} = \frac{14}{35}\)
c \(\frac{1}{5} \times \frac{2}{3} = \frac{2}{15}\)
d \(1 - \left(\frac{1}{3} \times \frac{5}{9}\right) = 1 - \frac{5}{27} = \frac{27-5}{27} = \frac{22}{27}\)
e \(\frac{3}{7} \times \frac{3}{20} \times \frac{20}{27} = \frac{3}{7}\)
2 a \(1 - 0.375 = 0.625\)
b \(0.65 + 0.05 = 0.7\)
c \(0.7 \times 0.6 = 0.42\)
d \(0.25 \times 0.64 = 0.16\)
e \(50\%\) of \(30 = 0.5 \times 30 = 15\)
f \(22\%\) of \(0.22 = 0.22 \times 0.22 = 0.0484\)
g \(12\%\) of \(10\%\) of \(0.8 = 0.12 \times 0.1 \times 0.8 = 0.0096\)

Exercise 3A
1 a \(P(2, 4, 6, 8) = \frac{4}{8} = \frac{1}{2}\)
b \(P(3, 6) = \frac{2}{8} = \frac{1}{4}\)
c \(P(4, 8) = \frac{2}{8} = \frac{1}{4}\)
d \(P(1, 2, 3, 5, 6, 7) = \frac{6}{8} = \frac{3}{4}\) or 
\(1 - P(4, 8) = 1 - \frac{1}{4} = \frac{3}{4}\)
e \(P(1, 2, 3) = \frac{3}{8}\)
2 \(P(\text{defective car}) = \frac{\text{number defective}}{\text{number of cars}} = \frac{30}{150} = \frac{1}{5}\)
3 a i \(0.21\)
ii \(0.19 + 0.14 = 0.33\)
b Proportion of 15 year old students = 0.21
Therefore \(0.21 \times 1200 = 252\) students who are 15.
4 a \(\frac{27}{100} = 0.27\)
b No – the frequencies for different numbers are very different
c \(\frac{15}{100} \times 3000 = 450\)
5 a \(\frac{\text{number of c's}}{\text{number of letters}} = \frac{2}{11}\)
b \(\frac{\text{number of p's}}{\text{number of letters}} = \frac{0}{11} = 0\)
c \(\frac{\text{number of vowels}}{\text{number of letters}} = \frac{5}{11}\)
6 P(red) + P(yellow) + P(green) + P(blue) = 1
Let P(yellow) = \(x\) so P(green) = \(2x\)
\(0.4 + 2x + 0.3 = 1\)
\(3x = 0.3\)
x = 0.1
Therefore P(green) = 0.2
7 a \(\frac{\text{number of even numbers}}{\text{number of possible outcomes}} = \frac{20}{40} = \frac{1}{2}\)
b \(\left\{1, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 31\right\} = \frac{13}{40}\)

Exercise 3B
1 \(n(\text{blond and brown}) = 4\)
\(n(\text{blond and not brown}) = 10 - 4 = 6\)
\(n(\text{brown and not blond}) = 14 - 4 = 10\)
\(n(\text{neither blond or brown}) = 35 - (6 + 4 + 10) = 15\)
\(P(\text{blond hair or blue eyes}) = \frac{6+4+10}{35} = \frac{20}{35} = \frac{4}{7}\)
2 \(n(\text{French and Malay}) = x\)
\(n(\text{F and not M}) = 15 - x\)
\(n(\text{M and not F}) = 13 - x\)
\(n(\text{neither F or M}) = 5\)
Therefore \(x + (15 - x) + (13 - x) + 5 = 25\)
\(33 - x = 25\)
x = 8
\(P(F \text{ and M}) = \frac{8}{25}\)
3 \(n(\text{Aerobics and Gymnastics}) = x\)
\(n(\text{A and not G}) = 13 - x\)
\(n(\text{G and not A}) = 17 - x\)
\(n(\text{neither A or G}) = 1\)
Therefore \(x + (13 - x) + (17 - x) + 1 = 25\)
31 – x = 25
x = 6

a  P(A and G) = \frac{6}{25}
b  P(G and not A) = \frac{11}{25}

4  n(Golf and Piano) = 7
n(G and not P) = 18 – 7 = 11
n(P and not G) = 16 – 7 = 9
n(neither G or P) = 32 – (7 + 11 + 9) = 5

a  P(G and not P) = \frac{11}{32}
b  P(P and not G) = \frac{9}{32}

5  a  A = \{\text{integers that are multiples of 3}\} = \{3, 6, 9, 12, 15\}
B = \{\text{integers that are factors of 30}\} = \{1, 2, 3, 5, 6, 10, 15\}

b  i  P(both a multiple of 3 and a factor of 30) = \frac{3}{15} = \frac{1}{5}
ii  P(\text{Neither a multiple of 3 or a factor of 30}) = \frac{6}{15} = \frac{2}{5}

6  n(A \& B, not C) = 5\% - 2\% = 3\%
n(A \& C, not B) = 4\% - 2\% = 2\%
n(B \& C, not A) = 3\% - 2\% = 1\%
n(A, not B or C) = 40\% - (2\% + 3\% + 2\%) = 33\%
n(B, not A or C) = 30\% - (2\% + 3\% + 1\%) = 24\%
n(C, not A or B) = 10\% - (2\% + 2\% + 1\%) = 5\%

Exercise 3C
1  a  \text{number that are divisible by 5} \over \text{number of possible outcomes} = \frac{34 + 68}{500} = \frac{102}{250}

b  \text{number that are even} \over \text{number of possible outcomes} = \frac{6 + 21 + 65 + 63 + 68 + 42}{500} = \frac{265}{500} = \frac{53}{100}

c  \text{number that are divisible by 5 or even} \over \text{number of possible outcomes} = \frac{6 + 21 + 65 + 63 + 68 + 42 + 34}{500} = \frac{299}{500}
or P(\text{sum divisible by 5} \cup \text{sum even}) = P(\text{sum divisible by 5}) + P(\text{sum even}) - P(\text{sum divisible by 5} \cap \text{sum even})
= \frac{102}{500} + \frac{265}{500} - \frac{53}{100}

2  a  P(\text{prime}) = \frac{4}{10} = \frac{2}{5} [\text{primes are 2, 3, 5, 7}]
b  P(\text{prime or multiple of 3}) = \frac{4}{10} + \frac{3}{10} - \frac{1}{10} = \frac{6}{10} = \frac{3}{5}
c  P(\text{multiple of 3 or 4}) = \frac{3}{10} + \frac{2}{10} + \frac{5}{10} = \frac{1}{2}

3  P(\text{camera owner or female}) = P(\text{camera owner}) + P(\text{female}) - P(\text{female camera owner})
= \frac{40}{80} + \frac{50}{80} - \frac{22}{80} = \frac{68}{80} = \frac{17}{20}

4  a  8 \text{ different letters in MATHEMATICS}\{M, A, T, H, E, I, C, S\} \over 26 = \frac{4}{13}
b  9 \text{ different letters in TRIGONOMETRY}\{T, R, I, G, O, N, M, E, Y\} \over 26 = \frac{9}{26}
c  \{M, T, E, I\} \over 26 = \frac{4}{26} = \frac{2}{13}
d  \{M, A, T, H, E, I, C, S, R, G, O, N, Y\} \over 26 = \frac{13}{26} = \frac{1}{2}

5  a  P(\text{work of fiction, non-fiction, or both}) = 0.40 + 0.30 - 0.20 = 0.5
b  P(\text{no book}) = 1 - 0.5 = 0.5
6  Let \( P(\text{local and national}) = x \)
\( P(\text{national and not local}) = \frac{1}{4} - x \)
\( P(\text{local and not national}) = \frac{3}{5} - x \)
\( \frac{2}{3} = \left( \frac{1}{4} - x \right) + \left( \frac{3}{5} - x \right) + x \)
\( x = \frac{11}{60} \)

7  a  \( P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) \)
\( = \frac{1}{4} + \frac{1}{8} - \frac{1}{8} = \frac{4}{8} = \frac{1}{2} \)

b  \( P(X) \cup P(Y)' = 1 - P(X \cup Y) = 1 - \frac{1}{2} = \frac{3}{4} \)

8  a  \( P(A \cup B) = P(A) + P(B) - P(A \cap B) \)
\( = 0.2 + 0.5 - 0.1 = 0.6 \)

b  \( P(A \cup B)' = 1 - P(A \cup B) = 1 - 0.6 = 0.4 \)

c  \( P(A' \cup B) = 1 - P(A \cap B) \)
\( = 1 - [P(A) - P(A \cap B)] \)
\( = 1 - [0.2 - 0.1] = 0.9 \)

Exercise 3D

1  a  A and B = N  b  A and C = Y
c  A and D = N  d  A and E = Y
e  B and E = N  f  C and D = N
g  B and C = N

2  Let \( P(N \cap M) = x \). If \( P(N \cap M) = 0 \) then \( N \) & \( M \) are mutually exclusive.
Now \( P(N \cup M) = P(N) + P(M) - P(N \cup M) \), so
\( \frac{3}{10} = \frac{1}{5} + \frac{1}{10} - x \)
\( x = \frac{1}{2} \)
Therefore yes
\( \frac{30}{89} + \frac{27}{89} = \frac{57}{89} \)

4  a  \( \frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12} \)

b  \( \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{20 + 15 + 12}{60} = \frac{47}{60} \)

c  \( 1 - \frac{47}{60} = \frac{13}{60} \)

Exercise 3E

1  \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}
a  \{HHH, HHT, HTH, THH\} \( \frac{1}{2} \)
b  \{HHH, HHT, THH\} \( \frac{3}{8} \)
c  \{HTH, THT\} \( \frac{1}{4} \)

2  a  \( \frac{6}{16} = \frac{3}{8} \)

b  \( \frac{6}{16} = \frac{3}{8} \)

c  \( \frac{4}{16} = \frac{1}{4} \)

3  a  \( \frac{9}{16} \)

d  \( \frac{9}{16} \)

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Worked solutions: Chapter 3

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<td>k When rolling the dice twice, there are 36 possible outcomes</td>
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| a | $\{(1, 3), (2, 4), (3, 1), (4, 2), (5, 5), (5, 6), (6, 5), (6, 6)\}; \frac{8}{36} = \frac{2}{9}$ |
| b | $\{(1, 1), (2, 2), (3, 3), (4, 4)\}; \frac{4}{36} = \frac{1}{9}$ |
| c | $\{(1, 2), (1, 4), (2, 1), (2, 3), (3, 2), (3, 4), (4, 1), (4, 3)\}; \frac{8}{36} = \frac{2}{9}$ |
Exercise 3F

1 \[ \frac{1}{5} \times \frac{1}{5} = \frac{1}{25} \]

2 \[ P(K) \times P(10) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{169} \]

3 \[ \left( \frac{4}{5} \right)^3 = \frac{64}{125} \]

4 \[ P(C) \times P(H) = 0.75 \times 0.85 = 0.6375 = 0.638 \]

5 a \[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]
   Let \( P(B) = x \).
   \[ 0.4 = 0.2 + x - 0 \]
   \[ x = 0.2 \]
   \[ P(B) = 0.2 \]
   \[ P(B \cup C) = P(B) + P(C) - P(B \cap C) \]
   Let \( P(B \cap C) = y \).
   \[ 0.34 = 0.2 + 0.3 - y \]
   \[ y = 0.16 \]
   \[ P(B \cap C) = 0.16 \]

b \[ P(B) \times P(C) = 0.2 \times 0.3 = 0.06 \]
   \[ P(B \cap C) = 0.16 \]
   \[ P(B \cap C) \neq P(B) \times P(C) \]
   Not independent

6 \[ P(H) \times P(\overline{H}) = \frac{1}{2} \times \frac{5}{6} = \frac{5}{12} \]

7 \[ \left( \frac{1}{9} \right)^3 = \frac{1}{59049} \]

8 \[ P(H) = \frac{1}{4}, \text{ therefore for 4 hearts } \left( \frac{1}{4} \right)^4 = \frac{1}{256} \]

9 a \[ P(E) = 1 - P(E') = 1 - 0.6 = 0.4 \]

b \[ P(E) \times P(F) = 0.4 \times 0.6 = 0.24 = P(E \cap F) \]

c \[ P(E \cap F) \neq 0 \]

d \[ P(E \cup F') = P(E) + P(F') - P(E \cap F') \]
   We know that since \( E \) & \( F \) are independent, \( P(E \cap F') = P(E) \times P(F') = 0.4 \times 0.4 \)
   \[ P(E \cup F') = 0.4 + 0.4 - (0.4 \times 0.4) = 0.64 \]

10 \[ P(R_1, \text{ and } B_2 \text{ and } R_2) = \frac{4}{12} \times \frac{8}{12} \times \frac{4}{12} = \frac{2}{27} \]

11 \( \{2, 2, 2\}; \left( \frac{1}{3} \right)^3 = \frac{1}{27} \)

12 a \[ P(A \cap B) = P(A) \times P(B) = 0.9 \times 0.3 = 0.27 \]
   since \( A \) & \( B \) are independent

b \[ P(A \cap B') = 0.9 \times 0.7 = 0.63 \]
   (since \( P(B') = 1 - P(B) = 0.7 \))

c \[ P(A \cup B') = 1 - P(A \cap B) \]
   \[ = 1 - [P(A) + P(B) - P(A \cap B)] \]
   \[ = 1 - (0.9 + 0.3 - 0.27) \]
   \[ = 0.07 \]

Exercise 3G

1 Let \( n(A \cap D) = x \)

\[ 15 - x + x + 20 - x + 4 = 27 \]
\[ 39 - x = 27 \]
\[ x = 12 \]

a \[ P(\text{Drama not Art}) = \frac{8}{27} \]

b \[ P(\text{Takes at least one of the two subjects}) \]
   \[ = 1 - P(\text{takes none}) = 1 - \frac{4}{27} = \frac{23}{27} \]

c \[ P(\text{Takes both subjects, given that he takes Art}) \]
   \[ = \frac{12}{27} = \frac{12}{15} = \frac{4}{5} \]
2 a \( P(A \cup B) = 1 - P(A' \cap B') = 1 - 0.35 = 0.65 \)
\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]
\[ 0.65 = 0.25 + 0.6 - P(A \cap B) \]
\[ P(A \cap B) = 0.85 - 0.65 = 0.2 \]

b \( P(A | B) = \frac{P(A \cap B)}{P(B)} \)
\[ = \frac{0.2}{0.6} \]
\[ = \frac{1}{3} \]

c \( P(B' | A') = \frac{P(B' \cap A')}{P(A')} \)
\[ = \frac{0.35}{0.75} \]
\[ = \frac{7}{15} \]

3 \[ P(R | S) = \frac{P(R \cap S)}{P(S)} \]
\[ = \frac{0.39}{0.48} \]
\[ = 0.8125 = \frac{13}{16} \]

4 a \[ P(E | M') = \frac{P(E \cap M')}{P(M')} \]
\[ = \frac{0.25}{0.75} \]
\[ = \frac{1}{3} \]

b \[ P(<15 | >5) = \frac{P(<15 \cap >5)}{P(>5)} \]
\[ = \frac{2}{3} \]
\[ = \frac{2}{3} \]

c \[ P(<5 | <15) = \frac{P(<5 \cap <15)}{P(<15)} \]
\[ = \frac{3}{5} \]
\[ = \frac{3}{5} \]

d \[ P(\text{between 10 and 20} | \text{between 5 and 25}) \]
\[ = \frac{P(\text{between 10 and 20 and between 5 and 25})}{P(\text{between 5 and 25})} \]
\[ = \frac{2}{8} \]
\[ = \frac{1}{4} \]

5 \[ P(L | D) = \frac{P(D \cap L)}{P(D)} = \frac{0.61}{0.95} = \frac{61}{95} \]

6 \[ P(S | T) = \frac{P(T \cap S)}{P(T)} = \frac{0.1}{0.6} = \frac{1}{6} \]

7 a \( P(U \text{ and } V) = 0 \text{ by definition} \)
b \( P(U | V) = 0 \text{ by definition} \)
c \( P(U \text{ or } V) = P(U) + P(V) = 0.26 + 0.37 = 0.63 \)

8 \[ P(\text{Pass both}) = \frac{0.35}{0.52} = 0.673. \text{ Therefore } 67.3\% \]

9 \[ P(B_1 \text{ and } W_2) = 0.34; P(B_1) = 0.47. \]
\[ P(W_2 | B_1) = \frac{P(B_1 \text{ and } W_2)}{P(B_1)} = \frac{0.34}{0.47} = \frac{34}{47} \]

10 a \[ P(\text{male and left handed}) = \frac{5}{10} = \frac{1}{2} \]

<table>
<thead>
<tr>
<th>Left handed</th>
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</thead>
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<tr>
<td>Male</td>
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<td></td>
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<tr>
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<td>2</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>13</td>
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<tr>
<td>7</td>
<td>43</td>
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b \[ P(\text{right handed}) = \frac{43}{50} \]

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<tr>
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<td>11</td>
<td>16</td>
</tr>
<tr>
<td>10</td>
<td>43</td>
<td>50</td>
</tr>
</tbody>
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c \[ P(\text{right handed, given that the player selected is female}) = \frac{11}{13} \]

11 \[ P(J | K) = \frac{P(J \cap K)}{P(K)} \]
\( J \& K \text{ are independent, so } P(J \cap K) = P(J) \times P(K) \)
\[ \therefore P(J | K) = \frac{P(J \times K)}{P(K)} = P(J) \]
\[ \text{so } P(J) = P(J | K) = 0.3 \]

12 Let \( T \) be the event the neighbor has 2 boys and \( S \) be the event that the neighbor has a son
The possible options are \{BB, BG, GB, GG\}
Event \( S \), the neighbor has a son is the set \( S = \{BB, BG, GB\} \)
Event \( T \), that the neighbor has two boys is the set \( T = \{BB\} \)
We require \[ P(T | S) = \frac{P(T \cap S)}{P(S)} = \frac{P(BB)}{P(BB, BG, GB)} \]
\[ = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3} \]

Exercise 3H

1 a \[ \begin{align*}
\text{Left handed} & \quad \text{Right handed} \\
\text{Male} & \quad 2 \quad 32 \quad 37 \\
\text{Female} & \quad 2 \quad 11 \quad 13 \\
\text{Total} & \quad 7 \quad 43 \quad 50 \\
\end{align*} \]

b \[ P(\text{right handed}) = \frac{43}{50} \]

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c \[ P(\text{right handed, given that the player selected is female}) = \frac{11}{13} \]

11 \[ P(J | K) = \frac{P(J \cap K)}{P(K)} \]

J & K are independent, so \( P(J \cap K) = P(J) \times P(K) \)
\[ \therefore P(J | K) = \frac{P(J \times K)}{P(K)} = P(J) \]

so \( P(J) = P(J | K) = 0.3 \)

12 Let \( T \) be the event the neighbor has 2 boys and \( S \) be the event that the neighbor has a son
The possible options are \{BB, BG, GB, GG\}
Event \( S \), the neighbor has a son is the set \( S = \{BB, BG, GB\} \)
Event \( T \), that the neighbor has two boys is the set \( T = \{BB\} \)
We require \[ P(T | S) = \frac{P(T \cap S)}{P(S)} = \frac{P(BB)}{P(BB, BG, GB)} \]
\[ = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3} \]

Exercise 3H

1 a
\[ P(C \text{ and I}) \text{ or } P(I \text{ and C}) = \left( \frac{2}{3} \times \frac{1}{3} \right) + \left( \frac{2}{3} \times \frac{2}{3} \right) = \frac{2}{9} + \frac{2}{9} = \frac{4}{9} \]

\[ 1 - P(\text{none correct}) = 1 - \left( \frac{1}{3} \times \frac{1}{5} \right) = 1 - \left( \frac{1}{15} \right) = \frac{8}{9} \]

P\text{ neither will score in the next game } = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}

\[
\begin{array}{l}
\text{Laura} & \frac{1}{2} & S \\
\text{Michelle} & \frac{1}{2} & S'
\end{array}
\]

\[
\begin{array}{l}
\text{Boy} & \frac{1}{2} & \text{Walk} \\
\text{Girl} & \frac{1}{2} & \text{Lift}
\end{array}
\]

\[
\begin{array}{l}
\text{Coach} & \frac{1}{2} & \text{Walk} \\
\text{Lift} & \frac{1}{2} & \text{Coach}
\end{array}
\]

\[ a \quad \frac{1}{2} \times \frac{2}{5} = \frac{1}{5} \]

\[ b \quad \left( \frac{1}{2} \times \frac{2}{5} \right) + \left( \frac{1}{2} \times \frac{17}{30} \right) = \frac{1}{5} + \frac{17}{60} = \frac{29}{60} \]

P\text{ (Head) } = \frac{2}{3} \quad \text{We require HHT or HTH or THH. Each has probability } \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{4}{27}. \quad \text{Therefore}

P \text{ (HHT or HTH or THH) } = 3 \times \frac{4}{27} = \frac{4}{9}

\[ a \quad P(\text{Prime}) = 0.4. \]

\[ b \quad P(\text{exactly one Prime}) = P(\text{Prime and not prime}) + P(\text{not prime and prime}) = (0.4 \times 0.6) + (0.6 \times 0.4) = 0.48 \]

\[ b \quad P(\text{at least one prime}) = 1 - P(\text{no primes}) = 1 - (0.6 \times 0.6) = 1 - 0.36 = 0.64 \]

\[ a \quad \begin{array}{c}
\text{R} \\
0.6
\end{array}
\]

\[ \frac{0.4}{W} \]

\[ b \quad P(\text{rainy} ) = P(\text{W and R} ) \text{ or } P(\text{W' and R} ) = (0.6 \times 0.4) + (0.4 \times 0.2) = 0.24 + 0.08 = 0.32 \]

\[ c \quad P(\text{two successive days not being rainy}) = P(\text{not rainy} ) \times P(\text{not rainy}) \]

P\text{ (not rainy) } = 1 - 0.32 = 0.68

0.68 \times 0.68 = 0.4624

Exercise 3I

1 \[ a \quad P(\text{picture card}) = \frac{12}{52} \]

We require \[ \frac{12}{52} \times \frac{11}{51} \times \frac{10}{50} = \frac{11}{1105} \]

\[ b \quad \text{We require } PFP \text{ or } FPP \text{ or } PPP. \text{ Each of these has equal probability of } \frac{12}{52} \times \frac{11}{51} \times \frac{10}{50} = \frac{44}{1105} \]

\[ P(PPF \text{ or } FPP \text{ or } PPP) = 3 \times \frac{44}{1105} = \frac{132}{1105} \]

\[ a \quad P(\text{two faulty}) = \frac{5}{12} \times \frac{4}{11} = \frac{5}{33} \]

\[ b \quad P(\text{exactly one faulty}) = \left( \frac{5}{12} \times \frac{7}{11} \right) + \left( \frac{7}{12} \times \frac{5}{11} \right) = \frac{35}{132} + \frac{35}{132} = \frac{35}{66} \]

\[ c \quad P(F_2 \mid \text{exactly one faulty pen}) = \frac{P(F_2 \text{ and exactly one faulty pen})}{P(\text{exactly one faulty pen})} = \frac{7}{35} \times \frac{5}{66} = \frac{1}{2} \]

\[ a \quad \frac{3}{9} \times \frac{2}{8} = \frac{1}{12} \]

\[ b \quad P(\text{RR or GG or YY}) = \left( \frac{4}{9} \times \frac{3}{8} \right) + \left( \frac{3}{9} \times \frac{2}{8} \right) + \left( \frac{2}{9} \times \frac{1}{8} \right) = \frac{5}{18} \]

\[ c \quad \text{We require } P(YY \text{ or YG or GG or GY}) = \left( \frac{2}{9} \times \frac{1}{8} \right) + \left( \frac{2}{9} \times \frac{3}{8} \right) + \left( \frac{3}{9} \times \frac{2}{8} \right) + \left( \frac{3}{9} \times \frac{1}{8} \right) = \frac{5}{18} \]

\[ d \quad \text{We require } 1 - P(\text{RR or RG or GR or GG}) = 1 - \left[ \left( \frac{4}{9} \times \frac{3}{8} \right) + \left( \frac{4}{9} \times \frac{3}{8} \right) + \left( \frac{3}{9} \times \frac{2}{8} \right) + \left( \frac{3}{9} \times \frac{2}{8} \right) \right] = 1 - \frac{7}{12} = \frac{5}{12} \]

\[ 4 \]

\[ P(\text{one of each color}) = P(\text{RBOP in any order}) \]

\[ P(\text{RBOP}) = \frac{5}{14} \times \frac{4}{13} \times \frac{3}{12} \times \frac{3}{11} = \frac{5}{1001} \]

We can arrange RBOP in 24 ways.

Therefore required probability = 24 \times \frac{5}{1001} = \frac{120}{1001} \]

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5  \( a \) \[ \frac{4}{10} = \frac{2}{5} \]
\[ b \] \[ \left( \frac{4}{10} \times \frac{6}{9} \right) + \left( \frac{6}{10} \times \frac{4}{9} \right) = \frac{8}{15} \]

6

\[ a \] P(at least one of the students answers the question correctly) = 1 – P(both incorrect)
\[ = 1 - \left( \frac{2}{7} \times \frac{4}{9} \right) = \frac{55}{63} \]
\[ b \] P(Billy correct given that the answer is correct) = \[ \frac{5}{7} \]
\[ c \] P(Natasha correct given that the answer is correct) = \[ \frac{5}{9} \]
\[ d \] P(two correct answers given that there were one) = \[ \frac{5}{7} \times \frac{5}{9} = \frac{25}{63} = \frac{5}{11} \]

Review exercise

1 There are 90 numbers from 10 to 99 inclusive.
\[ a \] \{10, 15, 20, …, 90, 95\} or every 5th number is divisible by 5 so \[ \frac{18}{90} = \frac{1}{5} \]
\[ b \] \{3, 6, 9, 12, …, 96, 99\} or every 3rd number is divisible by 3 so \[ \frac{1}{3} \]
\[ c \] \{51, 52, 53, …98, 99\} \[ \frac{49}{90} \]
\[ d \] \{16, 25, 36, 49, 64, 81\} \[ \frac{6}{90} = \frac{1}{15} \]

2 Let \( n(C \cap D) = x \)
\[ 18 - x + x + 20 - x + 3 = 30 \]
\[ 41 - x = 30 \]
\[ x = 11 \]
\[ P(\text{Cat and Dog}) = \frac{11}{30} \]

3 Let \( P(C \cap D) = x \)
\[ a \] \[ 0.7 - x + x + 0.2 - x + 0.25 = 1 \]
\[ 1.15 - x = 1 \]
\[ x = 0.15 \]
\[ \therefore P(C \cap D') = P(C) - P(C \cap D) \]
\[ = 0.7 - 0.15 = 0.55 \]
\[ b \] Not independent since \( P(C \cap D) = 0.15 \) and \( P(C) \times P(D) = 0.7 \times 0.2 = 0.14 \)

4 \[ a \] We require \( P(A \cap B) \)
\[ P(A \cap B) = \frac{P(A \cap B)}{P(B)} \]
\[ 0.1 = \frac{P(A \cap B)}{0.2} \]
\[ P(A \cap B) = 0.1 \times 0.2 = 0.02 \]
\[ b \] We require \( P(A \cup B) \)
\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]
\[ P(A \cup B) = 0.6 + 0.2 - 0.02 \]
\[ P(A \cup B) = 0.78 \]
\[ c \] We require
\[ P(A \cup B) - P(A \cap B) \]
\[ = 0.78 - 0.02 \]
\[ = 0.76 \]
\[ d \] We require \( P(B \mid A) \)
\[ P(B \mid A) = \frac{P(B \cap A)}{P(A)} \]
\[ = \frac{0.02}{0.6} = \frac{1}{30} = 0.0333 \]

5 \[ a \] 6\( x \)
\[ b \]
\[ c \] \[ 6x + 3 + 2x + 20 + 15 + 7 + x + 10 = 100 \]
\[ 9x + 55 = 100 \]
\[ 9x = 45 \]
\[ x = 5 \]
Review exercise

1. a  \( P(C \cap D) = P(C|D) \times P(D) \)
   \[ = 0.6 \times 0.5 \]
   \[ = 0.3 \]

   b  No since \( P(C \text{ and } D) \neq 0 \)

   c  No since \( P(C) \times P(D) = 0.4 \times 0.5 = 0.2 \neq P(C \text{ and } D) \)

   d  \( P(C \cup D) = P(C) + P(D) - P(C \cap D) \)
   \[ = 0.4 + 0.5 - 0.3 \]
   \[ = 0.6 \]

   e  \( P(D|C) = \frac{P(D \cap C)}{P(C)} \)
   \[ = \frac{0.3}{0.4} = 0.75 \]

2. 
   \[ \begin{array}{c}
   \text{Jack} \\
   \frac{3}{5} \\
   \text{Jill} \\
   \frac{2}{5} \\
   \end{array} \]

   a  \( P(\text{Properly}) = P(\text{Jack and Properly}) + P(\text{Jill and Properly}) \)
   \[ = \left(\frac{3}{5} \times 0.35\right) + \left(\frac{2}{5} \times 0.55\right) \]
   \[ = 0.21 + 0.22 = 0.43 \]

   b  \( P(\text{Jill} | \text{Not Properly}) = \frac{P(\text{Jill and not proper})}{P(\text{Not Properly})} \)
   \[ = \frac{\frac{2}{5} \times 0.45}{0.57} = 0.316 \]

3. 
   \[ \begin{array}{c}
   \text{Bicycle} \\
   0.3 \\
   \text{Bus} \\
   0.6 \\
   \end{array} \]

   a  

   b  i  \( P(\text{Travels by bicycle on Monday and Tuesday}) = 0.3 \times 0.3 = 0.09 \)

   ii  \( P(\text{Travels by bicycle on Monday and by bus on Tuesday}) = 0.3 \times 0.6 = 0.18 \)

4. a  \( \frac{6}{16} = \frac{3}{8} \)

   b  \( \frac{10}{15} = \frac{2}{3} \)

   c  \( \frac{5}{15} \times \frac{4}{14} = \frac{2}{21} \)

5. 

   \( n(\text{Female and not eating carrots}) = 23 \)

   \( n(\text{Female and eating carrots}) = 42 - 23 = 19 \)

   \( n(\text{not female and eating carrots}) = x \)

   Now \( 70 - (19 + x) = 34 \)

   \[ x = 17. \]

   a  \( P(\text{a rabbit is male and not eating carrots}) = \frac{11}{70} \)

   b  \( P(\text{a rabbit is female | that it is eating carrots}) = \frac{19}{70} = \frac{19}{36} \)

   c  No; \( P(F) \times P(C) = \frac{42}{70} \times \frac{36}{70} = \frac{78}{2450} \neq P(F \text{ and } C) \)

iii  \( P(\text{Travels by the same method of travel on Monday and Tuesday}) = (0.3 \times 0.3) + (0.6 \times 0.6) + (0.1 \times 0.1) = 0.46 \)

   c  \( P(\text{not by bicycle on 3 days}) = 0.7 \times 0.7 \times 0.7 = 0.343 \)

   d  \( P(\text{twice by car & once by bus}) = P(\text{car} \cap \text{car} \cap \text{bus}) + P(\text{car} \cap \text{bus} \cap \text{car}) + P(\text{bus} \cap \text{car} \cap \text{car}) \)

   Now \( P(\text{car} \cap \text{car} \cap \text{bus}) = (0.1 \times 0.1 \times 0.6) \)

   So \( P(\text{twice by car & once by bus}) = 3 \times (0.1 \times 0.1 \times 0.6) = 0.018 \)

   P(\text{twice by bicycle & once by car}) = \( P(\text{bicycle} \cap \text{bicycle} \cap \text{car}) + P(\text{bicycle} \cap \text{car} \cap \text{bicycle}) + P(\text{car} \cap \text{bicylce} \cap \text{bicycle}) \)

   Now \( P(\text{bicycle} \cap \text{bicycle} \cap \text{car}) = (0.3 \times 0.3 \times 0.1) \)

   So \( P(\text{twice by bicycle & once by car}) = 3 \times (0.3 \times 0.3 \times 0.1) = 0.027 \)

   Thus, \( P(\text{twice by car & once by bus or twice by bicycle & once by car}) = 0.018 + 0.027 = 0.045 \)