<table>
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<th>NELSON MATHEMATICS FOR CAMBRIDGE INTERNATIONAL A LEVEL</th>
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**Syllabus overview**

**Unit P2: Pure Mathematics 2 (Paper 2)**

Knowledge of the content of unit P1 is assumed and candidates may be required to demonstrate such knowledge in answering questions.

### 1. Algebra

- understand the meaning of \(|x|\), and use relations such as \(a = b \Leftrightarrow a^2 = b^2\) and \(|x - a| < b \Leftrightarrow a - b < x < a + b\) in the course of solving equations and inequalities;
- divide a polynomial, of degree not exceeding 4, by a linear or quadratic polynomial, and identify the quotient and remainder (which may be zero);
- use the factor theorem and the remainder theorem, e.g. to find factors, solve polynomial equations or evaluate unknown coefficients.

### 2. Logarithmic and exponential functions

- understand the relationship between logarithms and indices, and use the laws of logarithms (excluding change of base);
- understand the definition and properties of \(e^x\) and \(\ln x\), including their relationship as inverse functions and their graphs;
- use logarithms to solve equations of the form \(ax = b\), and similar inequalities;
- use logarithms to transform a given relationship to linear form, and hence determine unknown constants by considering the gradient and/or intercept.

### 3. Trigonometry

- understand the relationship of the secant, cosecant and cotangent functions to cosine, sine and tangent, and use properties and graphs of all six trigonometric functions for angles of any magnitude;
- use trigonometrical identities for the simplification and exact evaluation of expressions and in the course of solving equations, and select an identity or identities appropriate to the context, showing familiarity in particular with the use of:
  - \(\sec^2 \theta = 1 + \tan^2 \theta\) and \(\csc^2 \theta = 1 + \cot^2 \theta\),
  - the expansions of \(\sin(A \pm B)\), \(\cos(A \pm B)\) and \(\tan(A \pm B)\),
  - the formulae for \(\sin 2A\), \(\cos 2A\) and \(\tan 2A\),
  - the expressions of \(\sin \theta + b \cos \theta\) and \(a \sin(\theta + \alpha)\).
### 4. Differentiation

- use the derivatives of $e^x$, in $x$, sin $x$, cos $x$, tan $x$, together with constant multiples, sums, differences and composites;
- differentiate products and quotients;
- find and use the first derivative of a function which is defined parametrically or implicitly.

Integration

- extend the idea of ‘reverse differentiation’ to include the integration of $e^{ax+b}$, $\frac{1}{ax+b}$, $\sin(ax + b)$, $\cos(ax + b)$ and $\sec^2(ax + b)$ (knowledge of the general method of integration by substitution is not required);
- use trigonometrical relationships (such as double-angle formulae) to facilitate the integration of functions such as $\cos^2 x$;
- use the trapezium rule to estimate the value of a definite integral, and use sketch graphs in simple cases to determine whether the trapezium rule gives an over-estimate or an under-estimate.

### 6. Numerical solution of equations

- locate approximately a root of an equation, by means of graphical considerations and/or searching for a sign change;
- understand the idea of, and use the notation for, a sequence of approximations which converges to a root of an equation;
- understand how a given simple iterative formula of the form $x_n = F(x_n)$ relates to the equation being solved, and use a given iteration, or an iteration based on a given rearrangement of an equation, to determine a root to a prescribed degree of accuracy (knowledge of the condition for convergence is not included, but candidates should understand that an iteration may fail to converge).

### Unit P3: Pure Mathematics (Paper 3)

Knowledge of the content of unit P1 is assumed and candidates may be required to demonstrate such knowledge in answering questions.

### 1. Algebra

- understand the meaning of $|x|$, and use relations such as $|a| = |b| \iff a^2 = b^2$ and $|x| < a < b \iff a - b < x < a + b$ in the course of solving equations and inequalities;
- divide a polynomial, of degree not exceeding 4, by a linear or quadratic polynomial, and identify the quotient and remainder (which may be zero);
- use the factor theorem and the remainder theorem, e.g. to find factors, solve polynomial equations or evaluate unknown coefficients.
- recall an appropriate form for expressing rational functions in partial fractions, and carry out the decomposition, in cases where the denominator is no more complicated than:
  - $(ax + b)(cx + d)(ex + f)$,
  - $(ax + b)(cx + d)^2$,
  - $(ax + b)(x^2 + c^2)$,
  and where the degree of the numerator does not exceed that of the denominator;
- use the expansion of $(1 + x)^n$, where $n$ is a rational number and $|x| < 1$ (finding a general term is not included, but adapting the standard series to expand e.g. $(2 - \frac{1}{2}x)^{-1}$ is included).
2. Logarithmic and exponential functions

- understand the relationship between logarithms and indices, and use the laws of logarithms (excluding change of base);  
- understand the definition and properties of \( e^x \) and \( \ln x \), including their relationship as inverse functions and their graphs;  
- use logarithms to solve equations of the form \( a^x = b \), and similar inequalities;  
- use logarithms to transform a given relationship to linear form, and hence determine unknown constants by considering the gradient and/or intercept.  

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3. Trigonometry

- understand the relationship of the secant, cosecant and cotangent functions to cosine, sine and tangent, and use properties and graphs of all six trigonometric functions for angles of any magnitude;  
- use trigonometrical identities for the simplification and exact evaluation of expressions and in the course of solving equations, and select an identity or identities appropriate to the context, showing familiarity in particular with the use of:  
  - \( \sec^2 \theta = 1 + \tan^2 \theta \) and \( \csc^2 \theta = 1 + \cot^2 \theta \),  
  - the expansions of \( \sin(A \pm B) \), \( \cos(A \pm B) \) and \( \tan(A \pm B) \),  
  - the formulae for \( \sin 2A \), \( \cos 2A \) and \( \tan 2A \),  
  - the expressions of \( a \sin \theta + b \cos \theta \) in the forms \( R \sin(\theta \pm \alpha) \) and \( R \cos(\theta \pm \alpha) \).  

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4. Differentiation

- use the derivatives of \( e^x \), \( \ln x \), \( \sin x \), \( \cos x \), \( \tan x \), together with constant multiples, sums, differences and composites;  
- differentiate products and quotients;  
- find and use the first derivative of a function which is defined parametrically or implicitly.  

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5. Integration

- extend the idea of ‘reverse differentiation’ to include the integration of \( e^{ax+b} \), \( \frac{1}{ax+b} \), \( \sin(ax + b) \), \( \cos(ax + b) \) and \( \sec^2 (ax + b) \)  
- use trigonometrical relationships (such as double-angle formulae) to facilitate the integration of functions such as \( \cos^2 x \);  
- integrate rational functions by means of decomposition into partial fractions (restricted to the types of partial fractions specified in paragraph 1 above);  
- recognise an integrand of the form \( \frac{Kf'(n)}{f(n)} \), and integrate, for example, \( \frac{x}{x^2+1} \) or \( \tan x \);  
- recognise when an integrand can usefully be regarded as a product, and use integration by parts to integrate, for example, \( x \sin 2x \), \( x^2 e^x \) or \( \ln x \);  
- use a given substitution to simplify and evaluate either a definite or an indefinite integral;  
- use the trapezium rule to estimate the value of a definite integral, and use sketch graphs in simple cases to determine whether the trapezium rule gives an over-estimate or an under-estimate.  

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### 6. Numerical solution of equations

- **locate approximately a root of an equation, by means of graphical considerations and/or searching for a sign change;**
- **understand the idea of, and use the notation for, a sequence of approximations which converges to a root of an equation;**
- **understand how a given simple iterative formula of the form \( x_{n+1} = F(x_n) \) relates to the equation being solved, and use a given iteration, or an iteration based on a given rearrangement of an equation, to determine a root to a prescribed degree of accuracy (knowledge of the condition for convergence is not included, but candidates should understand that an iteration may fail to converge).**

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### 7. Vectors

- **understand the significance of all the symbols used when the equation of a straight line is expressed in the form \( \mathbf{r} = \mathbf{a} + t\mathbf{b} \);**
- **determine whether two lines are parallel, intersect or are skew;**
- **find the angle between two lines, and the point of intersection of two lines when it exists;**
- **understand the significance of all the symbols used when the equation of a plane is expressed in either of the forms \( ax + by + cz = d \) or \( (\mathbf{r} - \mathbf{a}) \cdot \mathbf{n} = 0 \);**
- **use equations of lines and planes to solve problems concerning distances, angles and intersections, and in particular:**
  - find the equation of a line or a plane, given sufficient information,
  - determine whether a line lies in a plane, is parallel to a plane, or intersects a plane, and find the point of intersection of a line and a plane when it exists,
  - find the line of intersection of two non-parallel planes,
  - find the perpendicular distance from a point to a plane, and from a point to a line,
  - find the angle between two planes, and the angle between a line and a plane.

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### 8. Differential equations

- **formulate a simple statement involving a rate of change as a differential equation, including the introduction if necessary of a constant of proportionality;**
- **find by integration a general form of solution for a first order differential equation in which the variables are separable;**
- **use an initial condition to find a particular solution;**
- **interpret the solution of a differential equation in the context of a problem being modelled by the equation.**

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### 9. Complex numbers

- **understand the idea of a complex number, recall the meaning of the terms real part, imaginary part, modulus, argument, conjugate, and use the fact that two complex numbers are equal if and only if both real and imaginary parts are equal;**
- **carry out operations of addition, subtraction, multiplication and division of two complex numbers expressed in cartesian form \( x + iy \);**
- **use the result that, for a polynomial equation with real coefficients, any non-real roots occur in conjugate pairs;**
- **represent complex numbers geometrically by means of an Argand diagram;**

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- carry out operations of multiplication and division of two complex numbers expressed in polar form \( r (\cos \theta + i \sin \theta) = re^{i\theta} \);
- find the two square roots of a complex number;
- understand in simple terms the geometrical effects of conjugating a complex number and of adding, subtracting, multiplying and dividing two complex numbers;
- illustrate simple equations and inequalities involving complex numbers by means of loci in an Argand diagram, e.g. \(|z - a| < k\), \(|z - a| = |z - b|\), \(\text{arg}(z - a) = \alpha\).